Financial Intermediation and Capital Reallocation

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Abstract

To understand the link between financial intermediation activities and the real economy, we build a general equilibrium model in which agency frictions in the financial sector affect the efficiency of capital reallocation across firms and generate aggregate economic fluctuations. We develop a recursive policy iteration approach to fully characterize the nonlinear equilibrium dynamics and the off-steady-state crisis behavior. In our model, adverse shocks to agency frictions exacerbate capital misallocation and manifest themselves as variations in total factor productivity at the aggregate level. Our model endogenously generates countercyclical volatility in aggregate time-series and countercyclical dispersion of marginal product of capital and asset returns in the cross-section.

Keywords: Financial Intermediation, Capital Misallocation, Volatility, Crisis, Limited enforcement

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I Introduction

We study the mechanism by which financial intermediation affects macroeconomic fluctuations and asset prices. We present a general equilibrium model to link intermediation activities in the financial sector to capital reallocation across non-financial firms in the real sector. We show that shocks originated from the financial sector can account for a significant fraction of macroeconomic fluctuations.

Two main features distinguish our approach from those of the previous literature. The first is the emphasis on capital reallocation across firms with heterogeneous productivity. The second is the recursive policy function iteration approach, which allows us to obtain global solutions of a general equilibrium model with occasionally binding incentive compatibility constraints.

We focus on a heterogeneous firm setup for two reasons. In the aggregate, the U.S. corporate sector is rarely constrained: it typically has more cash flow than what is needed to finance investment. As is shown in Chari (2014), a typical feature of models with agency frictions is that firms do not pay dividends when financially constrained. However, the net dividend payment of the U.S. corporate sector as a whole is almost always positive, and significantly so most of the time. To understand why some firms are constrained in downturns while others are not, we use a model with heterogeneous firms.

From a quantitative point of view, models with capital reallocation allow financial frictions to play a significant role in generating large economic fluctuations. In representative firm models, financial frictions affect the efficiency of intertemporal investment. Previous researchers (for example, Kocherlakota (2000)) argued that this mechanism alone is unlikely to cause large economic fluctuations because investment is only a small fraction of the total capital stock of the economy.[1] In contrast, recent study on capital misallocation, for example, Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), found that large efficiency gains can be achieved by improving capital misallocation, on the order of $30\% - 50\%$.

We develop a recursive policy function iteration approach to fully account for the dynamics of the occasionally binding constraints in our model. A prominent feature of major financial crisis is elevated volatility at the aggregate level and sudden increases in the cross-sectional dispersions in prices and quantities. The majority of models with financial frictions have been solved using local approximation methods, which typically cannot capture the time variation of volatility implied by the model. The recursive policy function iteration method allows us to characterize the variation of the tightness of the incentive compatibility constraints across

[1] In standard real business cycle (RBC) models, annual investment is about ten percent of capital stock, and capital contributes to roughly one-third of the total output. In line with this calculation, the maximum effect of investment on output is about 3.3%.

time and across firms; this is the key feature of our model.

To formalize the link between financial intermediation and capital reallocation, we develop a model of financial intermediation in which firms are subject to idiosyncratic productivity shocks and credit transactions must be intermediated. Because of the heterogeneity in productivity, reallocating capital across firms improves efficiency in production, but requires high productivity firms to borrow from the rest of the economy. In addition, because of the limited enforcement of lending contracts, the accumulation of intermediaries' debt or declines in their net worth increase their incentive to default and limit their borrowing capacity. These features of our model have two implications. In the time series, adverse shocks to intermediary net worth weaken their borrowing capacity and slow down the formation of new capital. In the cross-section, intermediaries who finance for high productivity firms are more likely to be affected, because they need to borrow more from the rest of the economy and have a higher incentive to default. The later mechanism amplifies negative primitive shocks by lowering the efficiency of the reallocation of the existing capital stock.

We consider two versions of our model in the calibration: one with total factor productivity (TFP) shocks and the other with financial shocks. We calibrate the volatility of the primitive shocks to match the volatility of output in the U.S. data and evaluate the quantitative importance of financial frictions in both specifications. In our model with TFP shocks, the amplification effect from agency frictions accounts for about 10% of the total volatility of output and is fairly temporary. The magnitude of amplification is modest because of the well-known difficulty for real business cycle (RBC) models to generate large volatilities in asset prices: Because productivity shocks are not associated with significant variations in asset prices and intermediary net worth, they induce only a limited amount of amplification from financial frictions.

Motivated by the lack of volatility in asset prices in the model with TFP shocks and the finding in the asset pricing literature that a large fraction of asset price variations can be attributed to discount rate shocks, we model, in our second version of calibration, financial shocks as exogenous variations in bank managers' discount rate. This model generates two features distinct from the one with TFP shocks: persistence and asymmetry. A temporary shock to banks' net worth lowers their borrowing capacity and reduces the efficiency of capital reallocation in the subsequent period. Elevated capital misallocation depresses output and triggers another round of drop in banks' net worth. This effect propagates over time and has a long-lasting impact on future economic growth. In addition, negative shocks tighten banks' financing constraints and make the economy more vulnerable to future shocks, whereas positive shocks relax these constraints and have a smaller impact on capital misallocation. In the extreme case, a sequence of negative shocks depletes the banking sector's net worth, lowers the borrowing capacity of all banks to suboptimal levels, and sends the economy into a financial crisis marked by heightened macroeconomic volatility, large and persistent drops in output and asset prices, and sharp increases in interest rate spreads.

In our benchmark calibration, the standard deviation of the banker's discount rate is about 2.3% at the annual level. This is much smaller than the variation in discount rates typically found in the asset pricing literature (see, for example, Campbell and Shiller (1988), and more recently, Lettau and Ludvigson (forthcoming).) Nevertheless, the model matches well the macroeconomic moments in the United States and produces a volatility of aggregate output of 3.6% from the capital reallocation channel. More importantly, it endogenously generates a countercyclical volatility in the time series of aggregate output and consumption, a countercyclical dispersion in the cross section of firm output and stock returns, and a countercyclical efficiency of capital reallocation and capital utilization like in the data.

Related literature Our paper belongs to the literature on macroeconomic models with a financial intermediary sector.[2] The papers most related to our's are Gertler and Kiyotaki (2010), He and Krishnamurthy (2014),Brunnermeier and Sannikov (2014), and Rampini and Viswanathan (2014). The nature of agency frictions in our model is the same as that in Gertler and Kiyotaki (2010). Different from those papers, we allow heterogeneity in firms' productivity and evaluate the quantitative importance of the capital reallocation channel.

Our paper builds on the literature that emphasizes the importance of the cyclical properties of capital reallocation and capital mis-allocation. Eisfeldt and Rampini (2006) provide empirical evidence that the amount of capital reallocation is procyclical and the benefit of capital reallocation is countercyclical. They also present a model in which the cost of capital reallocation is correlated with TFP shocks to rationalize these facts. Eisfeldt and Rampini (2008), Kurlat (2013), Fuchs, Green, and Papanikolaou (2013), and Li and Whited (2014) analyze models of capital reallocation with adverse selection. Shourideh and Zetlin-Jones (2012) developed a model with financial frictions and heterogenous firms to study the impact of financial shocks. Kehrig (2015) documents the cyclical nature of productivity distribution over the business cycle.

Our paper is also related to the literature that links total factor productivity at the aggregate level to capital mis-allocation at the firm level, for example, Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015). Buera et al. (2011) develop a quantitative

[2]There is a vast literature on macro models with credit market frictions, but we do not attempt to summarize it here. A partial list includes Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Kiyotaki and Moore (2005), Bernanke et al. (1999), Krishnamurthy (2003), Kiyotaki and Moore (2008), Mendoza (2010), Gertler and Karadi (2011), Jermann and Quadrini (2012), He and Krishnamurthy (2012), He and Krishnamurthy (2013), Li (2013), and Bianchi and Bigio (2014). Quadrini (2011) and Brunnermeier et al. (2012) provide comprehensive reviews of this literature.

model to explain the relationship between aggregate TFP and financial constraints. Gopinath et al. (2015) develop a model with financial frictions to account for the decline in total factor productivity in south Europe. None of the above papers focus on the effect of financial frictions and capital reallocation on aggregate volatility dynamics as we do.

The idea that shocks may directly originate from the financial sector and affect economic activities is related to the setup of Jermann and Quadrini (2012). Different from that of Jermann and Quadrini (2012), our paper focuses on financial intermediation and capital reallocation and their connections with the macroeconomy.

Our paper also relates to the literature in economics and finance that emphasizes the importance of countercyclical volatility in understanding the macroeconomy and asset markets. Many authors have documented a strong countercyclical relationship between real activity and uncertainty as is proxied by stock market volatility and/or dispersion in firm-level earnings and productivity (see, for example, Bloom (2009), Bloom et al. (2012), Bachmann et al. (2013), and Jurado et al. (2015), among others). A large literature in asset pricing emphasizes the importance of countercyclical volatility in understanding stock market returns (see, for example, Bansal and Yaron (2004), Bansal et al. (2012), and Campbell et al. (2013)). Our model generates countercyclical volatility as an endogenous equilibrium outcome even though the primitive shocks are homoscedastic.

Our computational approach is related to the recent development in using global methods to solve macro models with financial frictions. Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2014), and Maggiori (2013) use continuous-time methods to obtain global solutions. Models in those papers all have a single state variable, and equilibrium conditions can be reduced to ordinary differential equations, whereas our model involves multiple state variables to quantitatively capture a rich set of macroeconomic moments. Mendoza and Smith (2006) study small open economies with margin requirements and use value function iteration to solve their model. We use a policy function iteration approach that greatly improves the numerical efficiency in our general equilibrium setup because it does not involve multiple recursive operators and it uses first-order conditions to reduce optimization problems to solving nonlinear equations. Our method potentially can be applied to many other models in this literature that are often solved using local approximation methods.

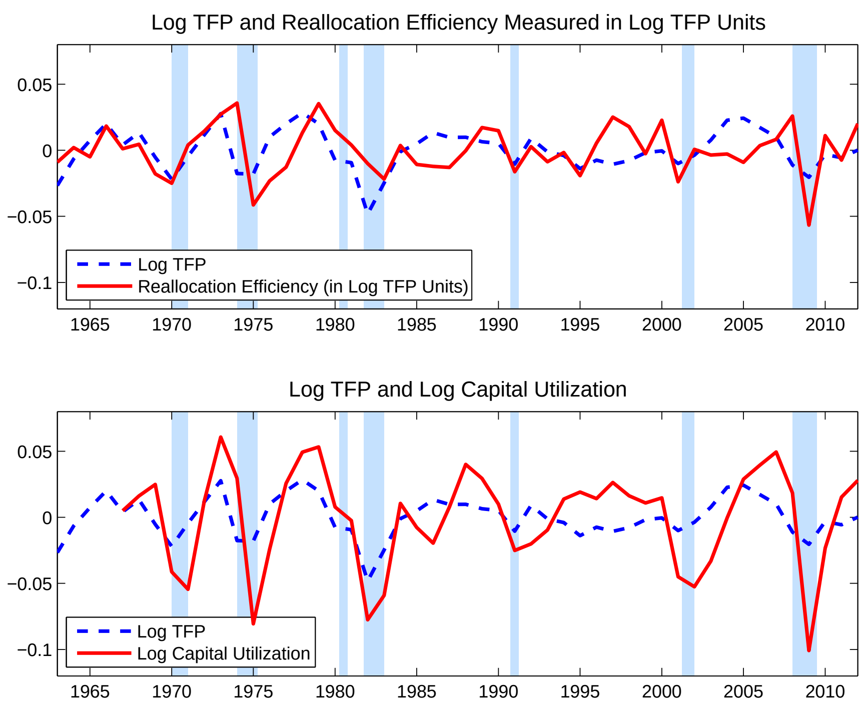
The rest of the paper is organized as follows. We provide a summary of some stylized facts that motivate the development of our model in Section II. We describe the model setup in Section III. In Section IV, we discuss the construction of the Markov equilibrium of our model and the recursive policy function iteration approach. In Section V, we analyze a deterministic version of our model to illustrate qualitatively the link between financial intermediation and capital reallocation. We calibrate our model and evaluate its quantitative implications on macroeconomic quantities and asset prices in Section VI. Section VII concludes.

II Stylized Facts

Below, we present several stylized facts that motivate our interest in studying the link between financial intermediation and capital reallocation. Here, we provide a brief description of this evidence in this section, and we provide the details of the data construction in Appendix A.

1. Measured total factor productivity (TFP) is highly correlated with a measure of the efficiency of capital reallocation and the rate of capital utilization.[3]

Figure 1: **capital reallocation and capital utilization**



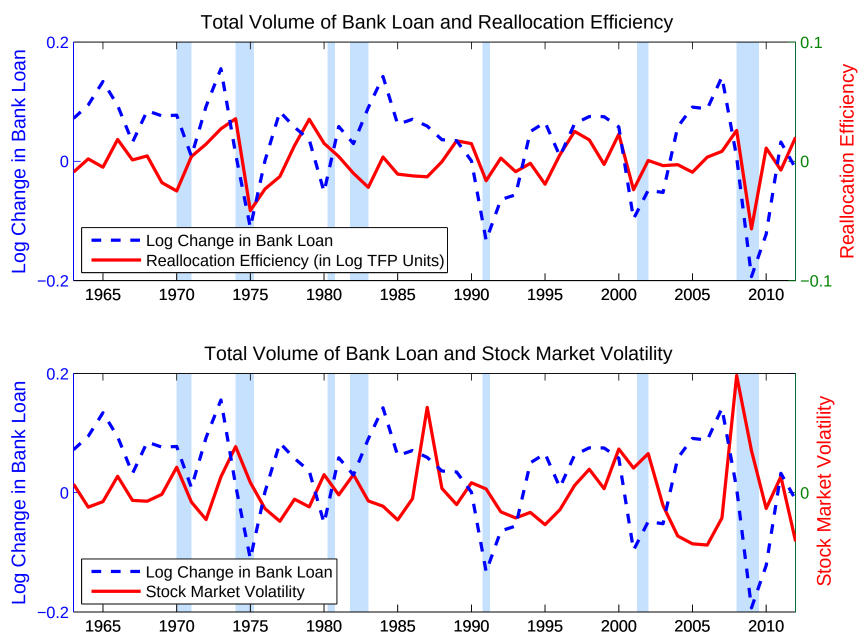
In the top panel of Figure 1, we plot the time series of log TFP (dashed line), and the measured efficiency of capital reallocation (solid line) in the United States, where all series are HP filtered. The shaded areas indicate NBER classified recessions. We follow a procedure similar to that of Hsieh and Klenow (2009) and measure capital misallocation by the variance of the cross-sectional distribution of log marginal product of capital within narrowly defined industries (classified by the four-digit standard

[3]Capital underutilization can be interpreted as a special form of misallocation.

industry classification code) and translate this measure into log TFP units. The measured efficiency of capital reallocation closely tracks the time series of log TFP, with a correlation of 0.33, indicating that the efficiency of capital reallocation may account for a significant fraction of variations in measured TFP. In the bottom panel, we plot the time series of log TFP (dashed line) and capital utilization rates (solid line), where capital utilization is measured using the capacity utilization rate published by Federal Reserve Bank of St Louis. Clearly, capital utilization also exhibits pronounced procyclicality, with a correlation of 0.62 with log TFP. Economic downturns are typically associated with sharp declines in capital utilization.

2. The total volume of bank loans is procyclical, and is positively correlated with the efficiency of capital reallocation and negatively correlated with measures of volatility.

Figure 2: **total volume of bank loans**



This fact motivates our theory of financial intermediation and its connection with capital reallocation. We calculate the total volume of bank loans of the non-financial corporate sector in the United States from the Flow of Funds Table, and plot the time series of changes in the total volume of bank loans (dashed line) and the measured efficiency of capital reallocation (solid line) in the top panel of Figure 2. We also plot the changes in the total volume of bank loans and the stock market volatility (solid line) in the bottom panel of the same figure, where stock market volatility is calculated by aggregating realized variance of monthly returns. The shaded areas in both panels indicate NBER-classified recessions. The total volume of bank loans is strongly procyclical. In addition, the total volume of bank loan is positively correlated with the efficiency of capital reallocation, with a correlation of 0.43, and negatively correlated with stock market volatility, with a correlation of -0.33.[4]

The rest of the stylized facts are well known. We therefore do not provide detailed discussion here, but refer readers to the relevant literature.

3. The amount of capital reallocation is procyclical, and the cross-sectional dispersion of marginal product of capital is countercyclical (see, for example, Eisfeldt and Rampini (2006)).

   The fourth, the fifth, and the sixth fact are about the cyclical properties of the volatility of macroeconomic quantities and asset returns and are well known in the macroeconomics literature and the asset pricing literature (see, for example, Bloom (2009), Bansal et al. (2012) and Campbell et al. (2001)).

4. The volatility of macroeconomic quantities, including consumption, investment, and aggregate output, is countercyclical.

5. The volatility of aggregate stock market return is also countercyclical. Equity premium and interest rate spreads are countercyclical.

6. The volatility of idiosyncratic returns on the stock market is countercyclical.

In the following sections, we set up and analyze a general equilibrium model with financial intermediation and capital reallocation to provide a theoretical and quantitative framework to interpret the above facts.

III Model Setup

In this section, we describe a general equilibrium model with heterogeneous firms and with agency frictions in the financial intermediation sector.

A Non-financial Firms

There are three types of non-financial firms in our model: intermediate goods producers, final goods producers, and capital goods producers. Because non-financial firms do not make

[4] This pattern is robust to other measures of aggregate volatility as well.

intertemporal decisions in our model, we suppress the dependence of prices and quantities on state variables in this subsection.

Final goods producers The specification of the production technology of intermediate goods and final goods follows the standard setup in the capital misallocation literature (see, for example, Hsieh and Klenow (2009)). Final goods are produced by a representative firm using a continuum of intermediate inputs indexed by $j \in [0,1]$. We normalize the price of final goods to one and write the profit maximization problem of the final goods producer as:

$$\max_{\{y_j\}} \left\{ Y - \int_{[0,1]} p_j y_j dj \right\} \Bigg|_{Y=\left[\int_{[0,1]} y_j^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}}, \tag{1}$$

where p_j and y_j are the price and quantity of input j produced on island j, respectively. Y stands for the total output of final goods. The parameter η is the elasticity of substitution across input varieties.

Intermediate goods producers There is a continuum of competitive intermediate goods producers, $j \in [0,1]$, who each produces a different variety on a separate island. We use j as the index for both the intermediate input and the island on which it is produced. The profit maximization problem for the producer on island j is given by

$$\begin{aligned} &\max \{p_j y_j - MPK_j \cdot k_j - MPL \cdot l_j\}, \\ &\textit{subject to}: \qquad y(j) = \bar{A} a_j k_j^{\alpha} l_j^{1-\alpha}. \end{aligned} \tag{2}$$

Here, the production of variety j requires two factors, capital k_j and labor l_j. $\bar{A}$ is the aggregate productivity common across all firms. a_j is island j-specific idiosyncratic productivity shock that we assume to be i.i.d. over time. MPK_j is the rental price of capital on island j, and MPL is the *economy-wide* wage rate. Because our focus is on capital reallocation across islands with different idiosyncratic productivity shocks, we allow the rental price of capital to be island specific, but assume frictionless labor market across the whole economy.

We assume, for simplicity, that there are only two possible realizations of idiosyncratic productivity shocks, a_H and a_L. $Prob\,(a = a_H) = 1 - Prob\,(a = a_L) = \pi$, and we adopt a convenient normalization,

$$\pi a_H^{1-\eta} + (1-\pi)\, a_L^{1-\eta} = 1. \tag{3}$$

As will become clear later, the above condition implies that the average idiosyncratic productivity is one and total output is given by the standard Cobb-Douglas production function,

$\bar{A}K^{\alpha}N^{1-\alpha}$ in the absence of misallocation.

Our setup adopts the Dixit-Stiglitz aggregate production function so that we can use the elasticity of substitution parameter η to quantify the effect of capital misallocation like in Hsieh and Klenow (2009). Instead of using monopolistic competition like in Hsieh and Klenow (2009), we assume that the intermediate goods producers are perfectly competitive. This assumption allows us to focus on financial frictions as the only source of inefficiency in our dynamic setup.[5]

Capital goods producers To allow for variable capital utilization, we assume that current period capital, K can be used for two purposes, production of intermediate goods (K_U) and storage (K_S). Let D_K denote the profit of capital goods producers, which is paid back to the household as dividend, and Q denotes the market price of capital. The capital goods producers maximize profit by operating the following storage technology:

$$D_K = \max_{K_S} \left\{ g\left(\frac{K_S}{K}\right) K - QK_S \right\}, \tag{4}$$

where K_S is the total amount of current period capital used in the storage technology and $g\left(\frac{K_S}{K}\right)$ is an increasing and concave production technology.

We assume that capital depreciates at rate δ if used for production. Therefore, the law of motion of next-period capital is $K' = g\left(\frac{K_S}{K}\right) K + (1-\delta) K_U + I$, where I is the total amount of new investment in the current period. We define $u = \frac{K_u}{K}$ as the capital utilization rate. Using the resource constraint, $K_U + K_S = K$, the law of motion of capital can be written as:

$$K' = [g(1-u) + (1-\delta)u] K + I. \tag{5}$$

Aggregation Let K_H denote the total amount capital used on high productivity islands, and K_L denotes that used on low productivity islands. Define $\phi = \frac{K_H}{K_L}$. It is straightforward to show that the efficient level of ϕ is $\hat{\phi} = \left(\frac{a_H}{a_L}\right)^{\eta-1}$. Because all islands start in a period with the same amount of capital, $\phi = 1$ corresponds to the case of no capital reallocation. In general, we can interpret $\phi \in \left(1, \hat{\phi}\right)$ as a measure of capital reallocation.

By using the normalization condition (3), we can write a_H and a_L as functions of $\hat{\phi}$: $a_H = \left(\frac{\hat{\phi}}{\pi\hat{\phi}+1-\pi}\right)^{\frac{1}{\eta-1}}$, $a_L = \left(\frac{1}{\pi\hat{\phi}+1-\pi}\right)^{\frac{1}{\eta-1}}$. Together, ϕ and u determine the efficiency of capital utilization. If we assume $\bar{A} = AK^{1-\alpha}$, where A is the exogenous productivity, and

[5] Hsieh and Klenow (2009) assume monopolistic competition, which does not distort the allocation of capital across firms in their static model, but leads to inefficiency in capital accumulation in our dynamic setup.

K is the economy-wide total capital stock, then we can express aggregate output and the marginal product of capital as functions of (u, ϕ).[6]

Proposition 1 *(Aggregation of the Product Market)*

The total output of the economy is $Y = Au^{\alpha} f(\phi) K$, *where the function* $f : \left[1, \hat{\phi}\right] \to [0, 1]$ *is defined as:*

$$f(\phi) = \frac{\left(\pi \hat{\phi}^{1-\xi} \phi^{\xi} + 1 - \pi\right)^{\frac{\alpha}{\xi}}}{(\pi\phi + 1 - \pi)^{\alpha} \left(\pi\hat{\phi} + 1 - \pi\right)^{\frac{\alpha}{\xi} - \alpha}} \tag{6}$$

The marginal product of capital on low productivity islands, MPK_L, *and the marginal product of capital on high productivity island,* MPK_H, *can be written as*

$$MPK_L(A, \phi, u) = \alpha A u^{\alpha-1} f(\phi) \frac{\pi\phi + 1 - \pi}{\pi\hat{\phi}^{1-\xi}\phi^{\xi} + 1 - \pi}, \tag{7}$$

$$MPK_H(A, \phi, u) = MPK_L(A, \phi, u) \left(\frac{\hat{\phi}}{\phi}\right)^{1-\xi}, \tag{8}$$

where the parameter $\xi \in (0, 1)$ *is defined as* $\xi = \frac{\alpha\eta - \alpha}{\alpha\theta - \alpha + 1}$.

Proof. *See Appendix B.* ■

It is straightforward to show that f is strictly increasing with $f\left(\hat{\phi}\right) = 1$. In general, $f(\phi) \leq 1$ and misallocation happens when strict inequality holds. Also, the first-order condition for capital-goods-producing firm implies: $Q = g'(1-u)$. We use this optimal condition to define the price of capital Q as a function of u:

$$Q(u) = g'(1-u). \tag{9}$$

B Household

There is a representative household with log preferences, and it is endowed with one unit of labor in every period that it supplies inelastically to firms. The representative household owns the ultimate claims of all assets in the economy. To make the intermediation problem non-trivial, and to prevent the model from collapsing into a single representative agent setup, like in Gertler and Kiyotaki (2010), we have assumed incomplete market between the household and the intermediary. That is, the only financial contract allowed between the household

[6] This specification follows Frankel (1962) and Romer (1986) and is a parsimonious way to inject endogenous long-run growth. From a technical point of view (see Section V), we can explore homogeneity and reduce the number of state variables in the construction of the Markov equilibrium.

and the financial intermediary is a risk-free deposit account. The household does not have access to markets that trade aggregate state-contingent payoffs, but instead must delegate its investment decisions in capital markets to financial intermediaries. The household starts the current period with a total amount of disposable wealth W, and decides the allocation of wealth between consumption and investment in the risk-free account with banks. The household's utility maximization problem in a recursive equilibrium can be written as

$$V(\mathbf{Z}, W) = \max_{C, B_f} \ln C + \beta E\left[V(\mathbf{Z}', W')\right],$$

$$\textit{subject to}: \qquad C + B_f = W$$

$$W' = B_f R_f(\mathbf{Z}) + \int D_B(j)(\mathbf{Z}')\, dj + D_K(\mathbf{Z}') + MPL(\mathbf{Z}'). \tag{10}$$

In the above maximization problem, we assume that there exists a vector of Markov state variables, $\mathbf{Z}$, the law of motion of which will be specified later, that completely summarize the history of the economy.[7] Taking the equilibrium interest rate $R_f(\mathbf{Z})$, and the dividend payment from the capital goods producers, $D_K(\mathbf{Z}')$, from the banks, $\{D_B(j)(\mathbf{Z}')\}_{j\in[0,1]}$ as given, the household makes its optimal consumption and saving decisions given its initial amount of disposable wealth, W.[8] Since labor supply is inelastic, the total amount of labor income is $MPL(\mathbf{Z}')$.

C Financial Intermediaries

There is one financial intermediary on each island.[9] Financial intermediaries or bankers are the only agents in the economy who have access to the capital markets.

Consider a bank who enters into a period with initial net worth N. It chooses the total amount of borrowing from the household, B_f, amount of borrowing from peer banks, B_I, and the total amount of capital stock for the next period K'. Because there is no capital adjustment cost, the price of capital is one and banks' budget constraint is:

$$K' = N + B_f + B_I.^{10} \tag{11}$$

[7] In other words, we will focus on Markov equilibria with state variable, $\mathbf{Z}$. We do not explicitly specify $\mathbf{Z}$ here. We construct the Markov equilibrium with the state variable $\mathbf{Z}$ in Section IV.

[8] Note that final goods producers and intermediate goods producers do not earn any profit because they operate constant return to scale technologies and the market is perfectly competitive.

[9] Because financial intermediaries on each island face competitive capital markets, one should interpret our model as having a continuum of identical financial intermediaries on each island.

[10] With a slight abuse of notation, we use B_f as both the amount of household savings and the amount of borrowing from the bank. We do so to save notation, because market clearing requires that the demand and supply of bank loans must equal.

In our model, the total amount of capital for the next period, K', is determined at the end of the current period, but before the realization of shocks of the next period. That is, we assume one period time to plan like in standard real business cycle models. However, different from the standard representative firm setup, capital can be reallocated across firms after idiosyncratic productivity shocks are realized; we turn to this next.

The market for capital reallocation opens after the realization of aggregate productivity shock, A', and idiosyncratic productivity shocks, a'. Let $Q(\mathbf{Z}')$ denote the price of capital on the capital reallocation market in state $\mathbf{Z}'$, and let $Q_j(\mathbf{Z}')$ denote the price of capital on an island with idiosyncratic productivity shock a_j for $j = H, L$, in aggregate state $\mathbf{Z}'$. We use $RA_j(\mathbf{Z}')$ to denote the total amount of capital purchased on the reallocation market by intermediary j in state $\mathbf{Z}'$. The total net worth of intermediary j at the end of the next period after the repayment of household loan and interbank borrowing is:

$$N_j' = Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')] - Q(\mathbf{Z}')RA_j(\mathbf{Z}') - R_f(\mathbf{Z})B_f - R_I(\mathbf{Z})B_I. \tag{12}$$

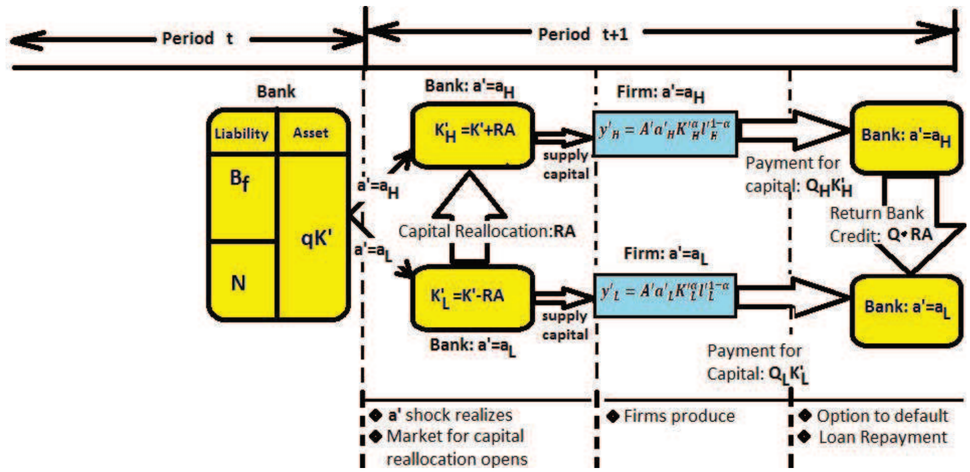
Here, we allow $Q(\mathbf{Z}')$, $Q_H(\mathbf{Z}')$, and $Q_L(\mathbf{Z}')$ to be potentially different because financial constraints may prevent the marginal product of capital from being equalized to the price of capital on the reallocation market when they are binding. The interpretation of (12) is that at the end of the next period, the total value of capital on island j, including the capital purchased in the current period, K', and the capital obtained on the reallocation market, $RA_j(\mathbf{Z}')$, is $Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$. The intermediary also needs to pay back the cost of capital obtained on the reallocation market, $Q(\mathbf{Z}')RA_j(\mathbf{Z}')$, and one-period risk-free loans borrowed from the household and other banks, B_f, and B_I.

Note that capital on the reallocation market only can be purchased by issuing a within-period interbank loan. This is because the purchase of capital on the reallocation market happens before production and the receipt of payment from local firms, $Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$.

Figure 3 illustrates the timing of events in period t and in period $t+1$. At the end of period t, the household has total disposable income, W, and the total net worth of the intermediary sector is N. The household wealth is allocated between consumption in the current period, C, and a risk-free deposit with the banks, B_f. From the bank's perspective, its total asset holdings, including total net worth, N, consumer loans, B_f, and interbank loans, B_I are used to purchase capital at price q. At the end of period t, a typical bank purchased K' amount of capital for period $t+1$ production before the realization of the productivity shocks in $t+1$.

Period $t+1$ is divided into four subperiods. In the first subperiod, the aggregate productivity shock, A', and the idiosyncratic productivity shock, a', are realized and the capital reallocation market opens. Banks on the high (idiosyncratic) productivity islands have an

Figure 3: **timing of events**



incentive to purchase more capital on the reallocation market, and banks on the low productivity islands have an incentive to sell. Note that transactions on the capital reallocation market must be performed by issuing interbank credit, because, at this point, production has not begun and banks have not yet received payment from the firms. Production happens in the second subperiod, and firms pay back the cost of capital to local banks at the end of the second subperiod.

In the third subperiod, banks payback their interbank loans and household deposit. Importantly, after banks receive payment from local firms, but before they pay back loans to creditors, banks have an opportunity to default. Following default, bankers can abscond with a fraction of their assets, and set up a new bank to operate on some other island. We assume that the amount of assets bankers can abscond with following default is

$$\theta Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right] - \omega\left[Q\left(\mathbf{Z}\right)RA_j\left(\mathbf{Z}'\right) + R_I\left(\mathbf{Z}\right)B_I\right]. \tag{13}$$

The total amount of capital on the island is $[K' + RA_j(\mathbf{Z}')]$, where $RA_j(\mathbf{Z}')$ is purchases on the capital reallocation market under within-period interbank loans. Following default, bankers take away all of the capital on the island, but they can only sell a fraction, θ, of it in the market. Therefore, following default, the total receipt of bankers on island j is $\theta Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')]$. Similar to Gertler and Kiyotaki (2010), we assume that bankers have better technology to enforce contracts than do households. This is captured by the parameter $\omega \in [0, \theta]$. The interpretation is that in the event of default, a fraction, ω, of interbank borrowing can be recovered. The case $\omega = 0$ means bankers are no better than households in enforcing contracts, and $\omega = 1$ corresponds to the case of a frictionless interbank market. The possibility of default implies that the contracting between borrowing and lending banks must respect the following limited enforcement constraint:

$$N_j' \geq \theta Q_j(\mathbf{Z}')[K' + RA_j(\mathbf{Z}')] - \omega [Q(\mathbf{Z}) RA_j(\mathbf{Z}') + R_I(\mathbf{Z}) B_I], \quad \forall\, \mathbf{Z}' \text{ and } \forall\, j, \tag{14}$$

where N' is given by (12). Inequality (14) is the incentive compatibility constraint for banks. It implies that in anticipating the possibility of default, lending banks ensure that the borrowing banks do not have an incentive to default on loans in all possible states of the world.

In the fourth, and last, subperiod, bankers clear their interbank transactions and consumers receive dividend payments from banks and firms, a risk-free return from bank deposits, and make their consumption and saving decisions. At this point, banks' net worth is allowed to move freely across islands.

Like in Gertler and Kiyotaki (2010), the assumption that banks' net worth moves freely at the end of every period is made for tractability. It implies that the expected return on all islands are equalized, and therefore the ratio of banks' net worth to capital must be equalized across all islands. As a result, the decision problems for banks on all islands are identical at the end of the last subperiod. This allows us to use the optimal decision problem of the representative bank to construct the equilibrium. Without this assumption, banks' net worth depends on the history of the realization of idiosyncratic productivity shocks and the distribution of banks' net worth across islands becomes a state variable in the construction of Markov equilibria. In our setup, the heterogeneity in the realization of idiosyncratic productivity shocks at the beginning of a period motivates the need for capital reallocation. At the same time, the possibility of moving banks' net worth across islands at the end of a period avoids the need to keep track of the distribution of banks' net worth across islands.

We note that no arbitrage on the capital markets within an island implies that

$$Q_j(\mathbf{Z}') = MPK_j(\mathbf{Z}') + 1 - \delta, \quad for \quad j = H, L. \tag{15}$$

The interpretation is that one unit of capital on island j produces an additional current period output $MPK_j(\mathbf{Z}')$ in the current period and depreciates at rate δ after production. In a frictionless market the above condition and the fact $Q_j(\mathbf{Z}') = Q(\mathbf{Z}')$ for all j guarantees that the marginal product of capital must be equalized across all islands. In our model, misallocation may happen in equilibrium due to limited enforcement of financial contracts.

We assume that the representative household is divided into bankers and workers, and there is perfect consumption insurance between bankers and workers within the household. Under this assumption, banks evaluate future cash flows using the "stochastic discount factor" implied by the marginal utility of the household.[11] Let $C(\mathbf{Z})$ denote the consumption policy that is consistent with household optimality in (10).[12] Under the assumption of log utility, the stochastic discount factor takes a simple form:

$$M' = \beta \left(\frac{C(\mathbf{Z}')}{C(\mathbf{Z})} \right)^{-1}. \tag{16}$$

As is standard in the dynamic agency literature, for example, DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007), we assume that bank managers are less patient than households and use Λ to denote the ratio of bankers' discount rate relative to that of the households. Equivalently, with probability Λ, where $\Lambda \in (0,1)$, bankers survive to the next period. With probability $1-\Lambda$, bankers' net worth is liquidated and paid back to the household as a dividend. This assumption is a parsimonious way to capture the idea that the managers of banks have a shorter investment horizon than the representative household and is a necessary condition for agency frictions to persist in the long-run.

Because banks' objective function is linear and the constraints (11), (12), and (14) are homogenous, the value function of banks, taking equilibrium prices as given, must be linear in banks' net worth, N. In addition, since banks' net worth can be freely moved across islands at the end of every period, the marginal value of bank networth must be equalized across all islands at the end of every period. This feature of the model greatly simplifies our analysis, because it implies that banks on different islands are just scaled versions of one another after banks' net worth is redistributed. $\mu(\mathbf{Z})N$ denotes the value function of banks. A typical bank maximizes

$$\mu(\mathbf{Z})N = \max_{B_f, B_I, K' \{RA_j(\mathbf{Z}')\}_{\mathbf{Z}',j}} E\left[M' \left\{ (1-\Lambda(\mathbf{Z}'))N' + \Lambda(\mathbf{Z}')\mu(\mathbf{Z}')N' \right\} \middle| \mathbf{Z}\right]$$

by choosing total capital stock for the next period, K', total borrowing from households, B_f, total borrowing from peer banks, B_I, and a state-contingent plan for capital reallocation, $RA_j(\mathbf{Z}')$ for all possible realizations of $\mathbf{Z}'$ and j, subject to constraints (11), (12), and (14).

D Market Clearing

Because market-clearing conditions have to hold in every period, we suppress the dependence of all quantities on time and state variables in this section to save notation. We list the

[11] See Gertler and Kiyotaki (2010) for details.

[12] The policy functions of the dynamic programming problem (10) have two state variables $(\mathbf{Z}, W)$. In our construction of the Markov equilibrium, the equilibrium level of wealth, W, is a function of the aggregate state variable, $\mathbf{Z}$. Therefore, we represent the equilibrium C as a function of $\mathbf{Z}$ without loss of generality.

resource constraints and market-clearing conditions below:

First, the total amount of capital utilized on island j is $K_j = K + RA_j, \quad for\ j = H, L$. The resource constraint requires that the amount of capital used for production on all islands must sum to uK, which is the total amount of utilized capital in the economy:

$$\pi \left(K + RA_H\right) + \left(1 - \pi\right) \left(K + RA_L\right) = uK. \tag{17}$$

Second, the total amount of interbank borrowing in the economy, B_I must be zero. This is because banks are ex ante identical before the realization of idiosyncratic productivity shocks and interbank borrowing is determined before the realization of these shocks. The possibility of interbank bank borrowing on the intertemporal bank loan market does not affect equilibrium allocations, but determines the interbank borrowing rate, which can be measured empirically and used to discipline our quantitative exercise.

Third, the total net worth of the banking sector equals the sum of banks' net worth across all islands:

$$N = \pi N_H + \left(1 - \pi\right) N_L. \tag{18}$$

Fourth, labor market clearing requires $\pi l_H + (1 - \pi)\, l_L = 1$, because we assume inelastic labor supply and normalized total labor endowment to one.

Fifth, and finally, market clearing for final goods requires that total consumption and investment sum to total output: $C + I = Y$, where Y is the total output of final goods defined in (1).

Note that market clearing implies that the sum of the household's disposable wealth, W, and the total net worth of the banking sector, N, must be equal to the total financial wealth of the economy. We do not list this condition here because it is redundant, given all other market-clearing conditions due to Walras' law.

IV Recursive Formulation

A Markov equilibrium consists of (1) a set of equilibrium prices and quantities as functions of the state variable $\mathbf{Z}$ and (2) the law of motion of the state variable $\mathbf{Z}$, such that households maximize utility, non-financial firms and financial intermediaries maximize their profit, and all markets clear. We construct the Markov equilibrium as follows. First, we assume, but do not explicitly specify, the existence of a vector of Markov state variables $\mathbf{Z}$ and derive a set of equilibrium conditions from optimality and market-clearing conditions. Second, we explicitly identify the state variables $\mathbf{Z}$ and use equilibrium conditions to construct the law of motion of $\mathbf{Z}$, as well as the equilibrium functionals (equilibrium prices and allocations as functions of $\mathbf{Z}$). Third, and finally, we verify that given the construction of the state variable $\mathbf{Z}$, our proposed pricing functions and quantities constitute a Markov equilibrium. Because our construction of the Markov equilibrium is a recursive procedure, it naturally leads to an iterative procedure to numerically solve the model; we describe this in detail in Appendix D.

Thanks to the assumption $\bar{A}_t = A_t K_t^{1-\alpha}$, equilibrium quantities are homogenous of degree one in K and equilibrium prices do not depend on K. It is therefore convenient to work with normalized quantities. We define

$$c = \frac{C}{K},\ i = \frac{I}{K},\ n = \frac{N}{K},\ b_f = \frac{B_f}{K}. \tag{19}$$

Using the above notation, equation (5) can be written as:

$$\frac{K'}{K} = g\left(1-u\right) + \left(1-\delta\right)u + i. \tag{20}$$

Clearly, K must be one of the state variables in the construction of the Markov equilibrium. We denote $\mathbf{Z} = (\mathbf{z}, K)$, where $\mathbf{z}$ is a vector of state variables to be specified later. The homogeneity property implies that normalized equilibrium quantities do not depend on K and only depend on $\mathbf{z}$.

A Equilibrium Conditions

In this section, we use banks' optimality conditions to derive a set of equations that the equilibrium prices quantities have to satisfy. We first simplify the limited enforcement constraints. Combining (13) and (14), the limited enforcement constraint can be written as:

$$(1-\theta)\, Q_j\left(\mathbf{z}'\right) K' - \left[(1-\omega)\, Q\left(\mathbf{z}'\right) - (1-\theta)\, Q_j\left(\mathbf{z}'\right)\right] RA_j\left(\mathbf{z}'\right) \geq R_f\left(\mathbf{z}\right) B_f\left(\mathbf{z}\right), \tag{21}$$

for $j = H, L$. We observe that the market-clearing condition (17) and the definition of ϕ and u jointly imply

$$\frac{RA_H}{K} = \frac{u\phi}{\pi\phi + 1 - \pi} - 1, \quad \frac{RA_L}{K} = \frac{u}{\pi\phi + 1 - \pi} - 1. \tag{22}$$

Note also that the no-arbitrage condition (15) and equations (7) and (8) imply that $Q_H\left(\mathbf{z}\right)$ and $Q_L\left(\mathbf{z}\right)$ depend on state variables only through (A, ϕ, u). We denote $Q_j\left(A, \phi, u\right) = MPK_j\left(A, \phi, u\right) + 1 - \delta$, for $j = H, L$ (with a slight abuse of notation). Dividing both sides of (21) by K' and using equation (22), we can show that $Q_H\left(A, \phi, u\right)$ and $Q_L\left(A, \phi, u\right)$ must satisfy

$$(1-\theta) Q_H(A', \phi', u') - [(1-\omega) Q(u') - (1-\theta) Q_H(A', \phi', u')] \left(\frac{u'\phi'}{\pi\phi' + 1 - \pi} - 1 \right) \geq 0 \tag{23}$$

$$(1-\theta) Q_L(A', \phi', u') - [(1-\omega) Q(u') - (1-\theta) Q_L(A', \phi', u')] \left(\frac{u'}{\pi\phi' + 1 - \pi} - 1 \right) \geq 0 \tag{24}$$

where s' is defined as

$$s' = \frac{R_f b_f}{g(1-u) + (1-\delta) u + i}. \tag{25}$$

Let ζ_H and ζ_L denote the Lagrangian multipliers on the limited enforcement constraint, (14) for $j = H, L$, respectively. The first-order conditions with respect to $RA(\mathbf{Z}')$ can be used to derive a relationship between Lagrangian multipliers and the prices of capital on high and low productivity islands. We use this relationship to define:

$$\zeta_H(A', \phi', u') = \frac{\pi [Q_H(A', \phi', u') - Q(u')]}{(1-\omega) Q(u') - (1-\theta) Q_H(A', \phi', u')} \geq 0, \tag{26}$$

$$\zeta_L(A', \phi', u') = \frac{(1-\pi) [Q_L(A', \phi', u') - Q(u')]}{(1-\omega) Q(u') - (1-\theta) Q_L(A', \phi', u')} \geq 0. \tag{27}$$

If both of the limited enforcement constraints (23) and (24) hold with equality, then they jointly determine ϕ' and u' as functions of (A', s'). If neither (23) nor (24) is binding, then $\zeta_H(A', \phi', u') = \zeta_H(A', \phi', u') = 0$, implying $Q_H(A', \phi', u') = Q_L(A', \phi', u') = Q(u')$. Again, ϕ' and u' can be determined as functions of (A', s'). In general, equations (23), (24), (26), (27) and the complementary slackness condition determine ϕ' and u' as functions of (A', s'), which we will denote as $\phi(A', s')$ and $u(A', s')$. The following proposition builds on this observation and characterizes the nature of the binding constraints.

Proposition 2 *(Characterization of Binding Constraints)*
There exist functions $\hat{s}(A)$ and $\bar{s}(A)$ such that

1. *If $s' \leq \hat{s}(A')$, then none of the limited commitment constraints bind, and $\phi(A', s')$ and $u(A', s')$ are determined by (26) and (27), with equality for both.*

2. *If $\hat{s}(A') < s' \leq \bar{s}(A')$, then the limited commitment constraint for banks on high productivity islands binds, and $\phi(A', s')$ and $u(A', s')$ are determined by (25) with equality and (27).*

3. *If $\bar{s}(A') < s'$, then the limited commitment constraint for all banks bind, and $\phi(A', s')$ and $u(A', s')$ are determined by (25) and (24) with equality.*

Proof. *See Appendix C.* ■

The result of the above proposition is intuitive. s' is the total amount of liability that banks need to pay back to households (normalized by capital stock). When s' is below $\hat{s}(A')$, the debt level is low enough and the limited enforcement constraints never bind. As the debt level increases, when $\hat{s}(A') < s' \leq \bar{s}(A')$, the limited enforcement constraint bind only if the island receives a high productivity shock. Capital reallocation efficiency requires that banks on high-productivity islands borrow more than those on low-productivity islands. Therefore, the limited enforcement constraint is more likely to bind for banks on high-productivity islands. In the region in which $s' > \bar{s}(A')$, the banking sector accumulated too much debt and the limited enforcement constants bind for all realizations of idiosyncratic productivity shocks.

The above proposition has two important implications. First, in the cross-section, the limited enforcement constraint is more likely to bind for intermediaries on high-productivity islands. This is the mechanism for misallocation in our model: when banks are constrained, more productive projects cannot be financed and measured TFP drops. Second, in the time series, the limited enforcement constraint is more likely to bind when banks' net worth is low and/or when aggregate productivity drops. This is the amplification mechanism in our model. Adverse shocks to TFP and banks' net worth are amplified because they tighten the limited enforcement constraints and exacerbate capital misallocation.

Given our definition of the Lagrangian multipliers in (26) and (27), we can use other first-order conditions to characterize the equilibrium policy functions. Here, we use the property that equilibrium prices only depend on $\mathbf{z}$, but not on K, to simplify notation. First, the first-order condition for households' optimal investment decision leads to the usual intertemporal Euler equation,

$$E\left[M\left(z, z'\right)\right] R_f\left(z\right) = 1, \tag{28}$$

where $M\left(z, z'\right)$ denotes the stochastic discount factor of households:

$$M\left(z, z'\right) = \frac{\beta\left[Au^{\alpha}\left(z\right) f\left(\phi\left(z\right)\right) - i\left(z\right)\right]}{c\left(z'\right)\left[g\left(1 - u\left(z\right)\right) + \left(1 - \delta\right) u\left(z\right) + i\left(z\right)\right]}. \tag{29}$$

Second, banks' optimal choice for intertemporal investment implies

$$\mu\left(z\right) = E\left[\widetilde{M}\left(z, z'\right)\left\{1 + \left(1 - \omega\right)\left(\zeta_H\left(A', \phi\left(z'\right), u\left(z'\right)\right) + \zeta_H\left(A', \phi\left(z'\right), u\left(z'\right)\right)\right)\right\} Q\left(u'\right)\right], \tag{30}$$

where $\widetilde{M}\left(z, z'\right)$ is defined as

$$\widetilde{M}\left(z, z'\right) = M\left(z, z'\right)\left\{1 - \Lambda' + \Lambda' \mu\left(z'\right)\right\}. \tag{31}$$

Third, banks' optimal choice for interbank loan implies

$$\frac{R_I(z)}{R_f(z)} = \frac{E_t\left[\widetilde{M}(z,z')\left\{1+\zeta_H(A',\phi(z'),u(z'))+\zeta_H(A',\phi(z'),u(z'))\right\}\right]}{E_t\left[\widetilde{M}(z,z')\left\{1+(1-\omega)\left(\zeta_H(A',\phi(z'),u(z'))+\zeta_H(A',\phi(z'),u(z'))\right)\right\}\right]}. \quad (32)$$

Fourth, the envelope condition on banks' optimization problem is

$$\mu(z) = E\left[\widetilde{M}(z,z')\left\{1+\zeta_H(A',\phi(z'),u(z'))+\zeta_H(A',\phi(z'),u(z'))\right\}\right]R_f(z). \quad (33)$$

Fifth, and finally, we note that the resource constraint requires

$$c(z)+i(z) = Au^{\alpha}(z)\,f(\phi(z)). \quad (34)$$

Note that the four unknown equilibrium functionals, $c(z)$, $i(z)$, $\mu(z)$, and $R_f(z)$, can be determined by the four functional equations (28), (30), (33), and (34). Given the equilibrium functionals, $c(z)$, $i(z)$, $\mu(z)$, and $R_f(z)$, the interbank interest rate, $R_I(z)$, can be determined by equation (32).

B Construction of the Markov Equilibrium

Subject to some technical details, the four functional equations (28), (30), (33), and (34) can be used to determine the four equilibrium functionals, $\{c(z), i(z), \mu(z), R_f(z)\}$ once the law of motion of the state variables are specified. Proposition 2 suggests that it is convenient to include $s' = \frac{R_f b_f}{h(1-u)+(1-\delta)u+i}$ as one of the state variables. Motivated by this observation, we define $x = (\Lambda, A)$ as the vector of exogenous shocks. We conjecture, and then verify, that a Markov equilibrium can be constructed with $z = (x, s)$ as the state variables. In the rest of this section, we detail the construction of the Markov equilibrium of our model as the fixed point of an appropriate recursive operator.

Because x is an exogenous Markov process, we only need to specify the law of motion of the endogenous state variable, s. By using the law of motion of banks' net worth on high- and low-productivity islands, equation (12), and the definition of banks' total net worth, equation (18), we can derive the following expression for the law of motion of banks' normalized net worth, $n = \frac{N}{K}$:

$$n' = \Lambda'\left\{\alpha A'(u')^{\alpha} f(\phi') + (1-u')\,MPK(u') + (1-\delta) - s'\right\}, \quad (35)$$

where the notation $MPK(u)$ is defined by $MPK(u) = Q(u) - (1-\delta)$, with $Q(u)$ given in equation (9).

Divide both sides of the bank budget constraint (11) by K to obtain

$$h(1-u)+(1-\delta)u+i=n+b_f. \tag{36}$$

By the definition of s, we have

$$s'=\frac{R_f b_f}{h(1-u)+(1-\delta)u+i}=\frac{h(1-u)+(1-\delta)u+i-n}{h(1-u)+(1-\delta)u+i}R_f.$$

Now we can replace n in the above equation using (35) to obtain the law of motion of s:

$$s'=R_f(z)\left\{1-\frac{\Lambda\left\{\alpha\left(1-\frac{1}{\eta}\right)Au(z)f(\phi(z))+(1-u(z))MPK(u(z))+(1-\delta)-s\right\}}{h(1-u(z))+(1-\delta)u(z)+i(z)}\right\}. \tag{37}$$

Our construction of the Markov equilibrium is formally summarized by the following proposition:

Proposition 3 *(Markov Equilibrium)*

Suppose there exists a set of equilibrium functionals, $\{c(z),i(z),\mu(z),R_f(z)\}_z$ *such that with the law of motion of* s *given by (37),* $\{c(z),i(z),\mu(z),R_f(z)\}_z$ *satisfies the funcitonal equations (28), (30), (33), and (34), then* $\{c(z),i(z),\mu(z),R_f(z)\}_z$ *constitutes a Markov equilibrium.*

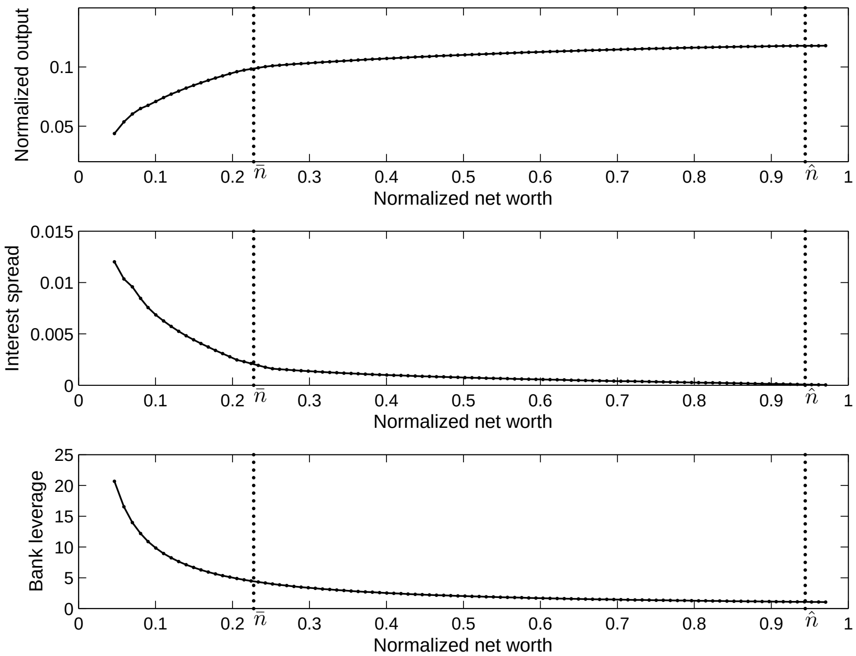
Proof. *See Appendix C.* ■

The recursive method used in our construction of the Markov equilibrium naturally leads to a numerically efficient way to compute the solution of the model; we detail this in Section D of the appendix. Although (x,s) is an efficient choice of the state variable from the numerical perspective, any one-to-one function of (x,s) can be used as state variables as well. From an economics point of view, it is more intuitive to use banks' net worth as a state variable. Equation (35) defines the mapping between (x,s) and (x,n). In the rest of the paper, we discuss the implications of our model using (x,n) as the state variable.

V Deterministic Dyanmics

In this section, we use the policy functions of the deterministic version of our model to illustrate the mechanism through which banks' net worth affects capital misallocation and economic fluctuations. In the absence of stochastic shocks, all equilibrium prices and normalized quantities are functions of the normalized net worth n. We use $n(s)$ to denote n as a function of s.

Figure 4: **macro moments and bank net worth**



Macroeconomic quantities and prices In Figure 4, we plot aggregate output (top panel), interest rate spread (middle panel), and banking sector leverage (bottom panel) as functions of banks' net worth, n. In the figure, $\hat{n} = n(\hat{s})$ is the level of banks' net worth above which the limited enforcement constraints do not bind for any bank and capital misallocation does not occur. Further increases in banks' net worth above $\hat{n}$ do not affect output because productivity is constant and capital reallocation stays at its first-best level. As n decreases toward $\bar{n}$, only the limited commitment constraint for high-productivity islands, (23) binds. Here, as n declines, capital misallocation deteriorates. As n drops below $\bar{n} = n(\bar{s})$, the limited enforcement constraint for both islands bind, and output drops sharply.

As is shown in the second panel of Figure 4, as banks' net worth drops, the spread between interbank interest rate and the household deposit rate, $R_I - R_f$, widens. As banks on high-productivity islands become more constrained, the expected return on capital rises, even though the household deposit rate is relatively low, and banks cannot take advantage of the return spread because their borrowing is restricted by the binding limited commitment constraint. At the same time, banks are less constrained in the interbank market because peer banks have better contract enforcement technologies. As a result, banks race to the interbank lending market and drive up the interest rate, R_I. This effect is particularly pronounced in the crisis region in which $n < \bar{n}$; all banks are constrained, and the interest rate spread rises sharply.

Similar to that of Gertler and Kiyotaki (2010), our model features countercyclical leverage. As bank networth declines, banks need to borrow more from the household to finance investment. Consequently, leverage increases. This is the key mechanism in our model that generates countercyclical volatility in aggregate quantities. As we show in Section VI, adverse shocks lower banks' net worth and raise banking sector leverage. In that event, exogenous shocks are more likely to be amplified, and the volatility in all aggregate quantities increase.

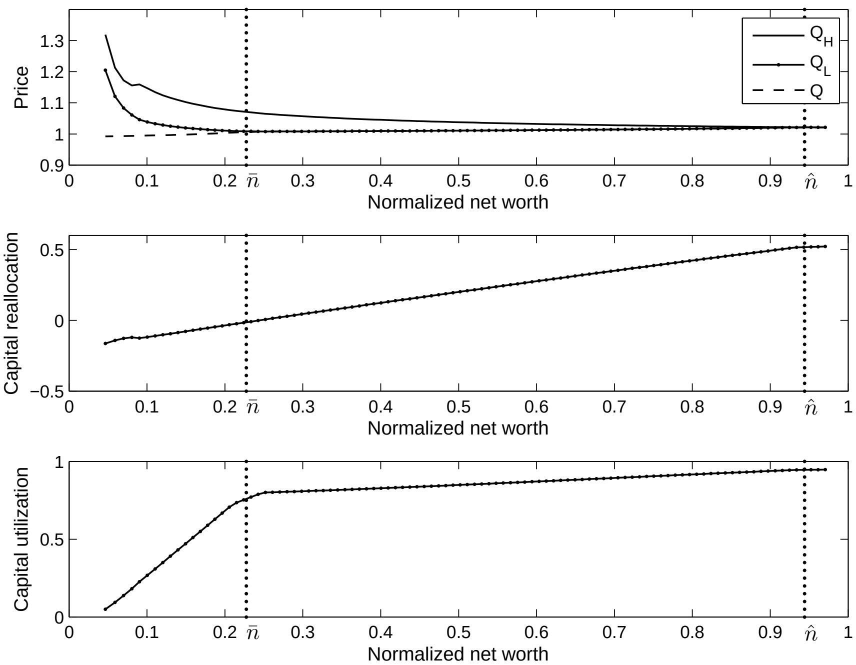
Capital reallocation As is shown in Eisfeldt and Rampini (2006), the amount of capital reallocation is procyclical and the benefit to capital reallocation is countercyclical. Our model is consist with this fact. We plot the dispersion in the marginal product of capital (top panel), the total amount of capital reallocation (second panel), and the percentage of capacity utilization (third panel) as functions of banks' net worth in Figure 5. As shown in the top panel of the figure, in the region $n \geq \hat{n}$, capital reallocation is fully efficient, and the marginal product of capital equalize across all islands. As banks' net worth drops below $\hat{n}$, the marginal product of capital on high- and low-productivity islands diverges, but the allocation of capital between low-productivity islands and the storage technology is fully efficient. As banks' net worth decreases further, $n < \bar{n}$, low-productivity islands become constrained as well, and capital "fly to safety", that is, they are invested in the risk-free storage technology despite its low marginal product. The benefit of capital reallocation increases as banks' net worth declines.

The divergence of the marginal product of capital is echoed by reductions in the total amount of capital reallocation (second panel) and decreases in the capital utilization rate (third panel). Again, drops in capital reallocation and capital utilization are much more pronounced in the crisis region in which the limited enforcement constraints bind for all banks.

VI Quantitative Results

In this section, we consider two specifications of our model and quantitatively evaluate the impact of financial frictions: a specification with TFP shocks only and a specification with shocks to agency frictions only. In the model with TFP shocks only, productivity shocks are the only source of primitive shocks and are amplified by financial frictions. Consistent with previous literature (for example, Kocherlakota (2000) and Chen and Song (2013)), we find that financial frictions do amplify TFP shocks, but the effect is quantitatively small. Amplification accounts for about 11% of the macroeconomic fluctuations in the model with

Figure 5: **capital reallocation and bank net worth**



TFP shocks. In addition, the economy almost rarely runs to the crisis region in which the limited enforcement constraint binds for all banks, because TFP shocks do not generate large enough variations in banks' net worth.

In a second specification of the model, we introduce stochastic shocks to bankers' discount rate. We show that relatively small shocks generate large fluctuations in capital misallocation and can account for most of the macroeconomic fluctuations in the U.S. economy. We show that this model endogenously generates countercyclical volatility at the aggregate level and countercyclical dispersion in the cross-section. In addition, this version of the model captures several salient features of the recent financial crisis, such as spikes in macroeconomic volatility, sharp drops in capital reallocation and capital utilization, and sudden increases in interest rate spreads. Our purpose here is not to argue that financial shocks are the only source of economic fluctuations, but merely to analyze their impact on the efficiency of capital reallocation and the macroeconomy.

To facilitate a comparison, we choose the same parameters, except for the volatility of exogenous shocks, for both specifications of our model. This approach guarantees that both the model with productivity shocks and that with financial shocks have the same deterministic steady state. We then calibrate the volatility of the exogenous shocks in both models to match the volatility of aggregate output in the data and evaluate the model's implications on the dynamics of macroeconomic quantities and asset prices.

A Calibration

We calibrate our model at the quarterly frequency. The calibrated parameter values are listed in Table 1. We set the standard preference and technology parameters to be consistent with the previous literature. We set the quarterly discount rate as $\beta = 0.99$, and the quarterly depreciation rate as $\delta = 2\%$. We set capital share as $\alpha = 0.33$ like in the standard RBC models, and the elasticity of substitution across varieties as $\eta = 4$,which is consistent with the value used in Hsieh and Klenow (2009).

The second group of technology parameters are specific to our model, and we calibrate them to jointly match relevant moments in the data. We set the capital storage technology in the CES form:

$$g(x) = a_0 + \frac{b_0}{\nu} x^{\nu}.$$

We set a_0 and b_0 to match the time-series average of capital utilization rate of 80% in the data and an average capital depreciation rate of 10% like in RBC models. We set the elasticity parameter as $\nu = 0.98$ so that the volatility of capital utilization rate in our model with financial shocks matches that in the data, 5% per year.[13]

This procedure leaves six remaining parameters, which we chose jointly to match six moments in the data. We set the mean productivity level as $E[\ln A] = 0.1805$ to match a mean growth rate of the U.S. economy of 1.8% per year. We set $\frac{a_H}{a_L} = 4.28$ and $\pi = 0.19$ so that the capital reallocation-to-investment ratio is 22%, as is reported in Eisfeldt and Rampini (2006) and Chari (2014), and high- and low-productivity islands each account for 50% of total output in the deterministic steady state. The banker's discount rate, Λ, determines the banking sector's total net worth. The fraction of asset that bankers can abscond with, θ, determines the borrowing capacity of the banking sector for a given level of net worth and therefore the total investment in the economy. The recovery rate of interbank loans upon default, ω, is a parameter that governs the amount frictions on the interbanking lending market. We calibrate $E[\Lambda]$, θ, and ω to jointly match a banking sector leverage ratio of 4 as in Gertler and Kiyotaki (2010), an investment-to-output ratio of 25% like in the United States post-war data, and the historical average of the TED spread (the spread between T-bills and the LIBOR) of 0.69% per year. The calibrated parameters and targeted moments are listed in Table 1.

We calibrate both $\ln A_t$ and Λ_t to be i.i.d. over time and examine the propagation

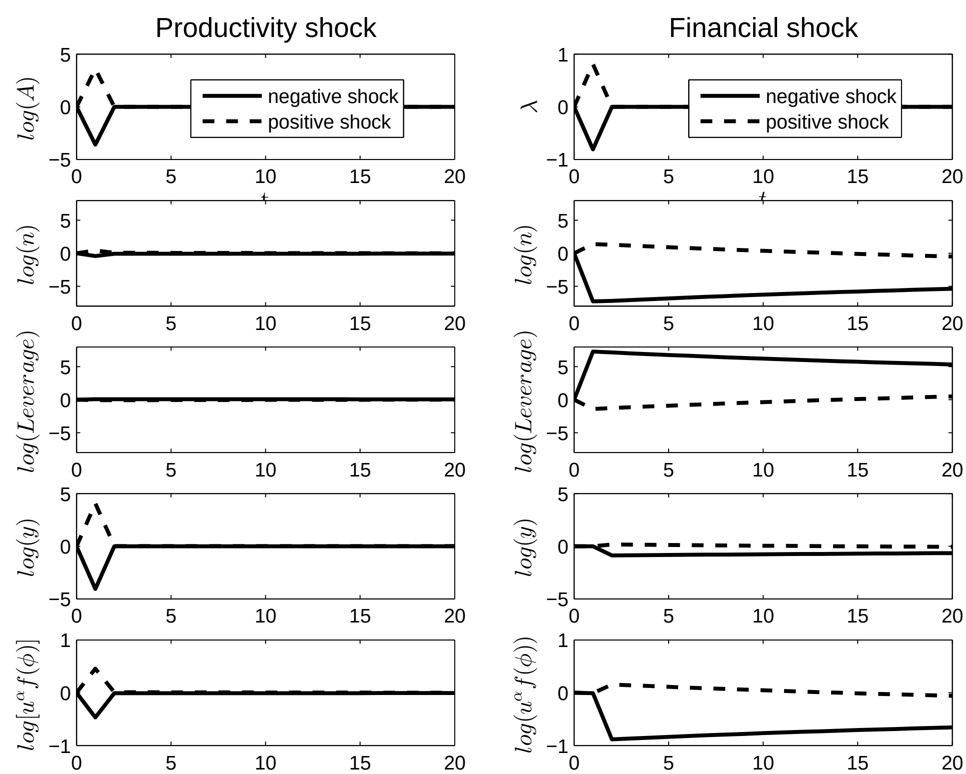
[13] Elasticity ν is the only technology parameter pinned down by a second moment in the data. We choose ν so that the volatility of capital utilization matches our preferred model with financial shocks.

of shocks endogenously generated by the model. In the model with TFP shocks only, we assume $\ln A_t$ follows a normal distribution and calibrate the volatility of $\ln A_t$ to match a volatility of aggregate output of 3% like the U.S. post-war data. Note that $\Lambda \in (0, 1)$. For parsimony, we set $\Lambda_t = \frac{\exp\{\lambda_t\}}{\exp\{\lambda_t\}+\exp\{-\lambda_t\}}$ and assume λ_t follows a normal distribution. This specification allows us choose one parameter, the standard deviation of λ_t, to match the volatility of aggregate output.

B Impulse responses

To understand the different implications of TFP shocks and discount rate shocks on financial frictions, we use the policy function iteration method introduced in Section IV to numerically solve the model and plot the impulse functions for one-standard-deviation shocks to $\ln A$ (left column) and those for one-standard-deviation shocks to λ (right column) in Figure 6. The horizontal axis represents the number of quarters after the initial shock, and the vertical axis is log deviations from the steady state. In both panels, we use dashed lines for positive shocks and solid lines for negative shocks.

Figure 6: **impulse responses to productivity and financial shocks**



We make two observations. First, shocks to λ have much more persistent effects than shocks to productivity, even though both shocks occur for one period and immediately return to steady state afterward. Like in standard RBC models, the price of capital is inelastic with respect to productivity shocks. Because capital is the only asset banks own, the response of banks' net worth to productivity shocks is very limited: a one-standard-deviation (5%) increase in productivity shock increases net worth by about 0.2% on impact (second panel in the left column). As a result, the tightness of the limited commitment constraints is almost unaffected. Because productivity returns back to steady state immediately, and because the tightness of the limited commitment constraint hardly reacts to these shocks, their effect on the growth rate of macroeconomic quantities virtually disappears after one period.

The model with λ shocks is very different in this respect. As we show in the second panel in the right column of Figure 6, a negative shock in banker's discount rate raises the current period's dividend payment and lowers banks' net worth immediately. Because banks' borrowing capacity reduces, so does the efficiency of capital reallocation (bottom panel). First, net worth is a persistent state variable and takes time to recover. Second, because of the lower efficiency of capital reallocation, capital rental rate, and therefore bank income, drops, making it more difficult for banks to rebuild their net worth. These two effects reinforce each other to generate long-lasting effects. In fact, as we show in the figure, with a 1% reduction in λ, total output drops by about 1%, and the effect is so persistent that the system is still far from its steady state even after twenty quarters.

Second, the variation in the efficiency of capital reallocation is quantitatively small in the model with productivity shocks, but accounts for most of the fluctuations in aggregate output in the model with financial shocks. Because the effect of productivity shocks on banks' net worth is small and transitory, the immediate impact of productivity shocks must be responsible for most of the fluctuations of output. In the model with productivity shocks, following impact, output, $y = Au^{\alpha} f(\phi)$ drops by about 5%, and most of the change comes from the direct effect of productivity, $\ln A$ (top panel), not from the efficiency of capital reallocation, $u^{\alpha} f(\phi)$ (bottom panel). In contrast, the changes in output in the model with financial shocks (right column) are completely due to the efficiency of capital reallocation, $u^{\alpha} f(\phi)$. Following a one-standard-deviation financial shock, although the initial reaction of output is fairly modest (less than 1%), it propagates over time and lasts for many periods. Note that both models are calibrated to match a standard deviation of output growth rate of 3% per year. The model with financial shocks requires much smaller shocks and a much smaller initial impact of these shocks, because financial shocks provides a powerful propagation mechanism due to the feedback effect between the real economy and the banking sector's net worth.

Third, the effect of productivity shocks is largely symmetric: positive and negative shocks in productivity result in changes in quaternities and prices of similar magnitude. Qualitatively, as we have seen in the policy functions in the deterministic case, negative shocks to net worth have a larger impact on capital misallocation than do positive ones, especially in the "crisis" region. Quantitatively, however, productivity shocks induce very modest changes in banks' net worth and the asymmetry and countercyclical volatility generate by the model is insignificant.

In contrast, the asymmetry in the impulse responses of quantity and prices with respect to shocks to agency frictions is apparent in Figure 6. A positive shock to λ relaxes the limited enforcement constraint, lowers leverage (third panel), and reduces the effect of future shocks. A negative shock to λ tightens the limited enforcement constraint and makes the system more sensitive to additional disturbances. As a result, negative shocks are amplified and positive shocks are dampened, leading to endogenous countercyclical volatility in our model.

C Simulation

To understand the quantitative implications of the model, we simulate the model for 800 quarters and discard the first 400 quarters; we aggregate the quarterly quantities in the remaining part of the simulation into annual quantities; and we compute moments for annualized quantities.

Unconditional moments We report the moments of macroeconomic quantities (top panel), those of interest rates (second panel), and the cyclicality of variables of interest (bottom panel) in Table 2. Both versions of our model are consistent with the basic features of the data in terms of the relatively low volatility of consumption growth, the high volatility of investment, and the comovement between consumption and investment.[14] Both feature a low and smooth risk-free interest rate. The model with TFP shocks generates very little variations in interest rates and in interest rates spread. The model with financial shocks produces a low but significant volatility in interest spread, as in the data.

Focusing on the bottom panel of Table 2, the data features have pronounced countercyclical volatility in aggregate output, stock market return, and interest rate spread. In Table 2, $Corr[\Delta lnY, Vol(\Delta lnY')]$ measures the correlation between output growth and the volatility of growth rates in the following year, where $Vol(\Delta lnY')$ is computed as the realized standard deviation of the growth rates of quarterly industrial production in the data and in the model.[15] $Corr[\Delta lnY_t, Vol(R'_M)]$ stands for the correlation between output growth and

[14]Relative to the data, our model produces too much volatility in investment and too little volatility in consumption. This counterfactual implication of the model can be corrected by adding an investment adjustment cost.

[15]We interpret all production as industrial production in the model.

the realized volatility of stock market return in the next year, where the volatility of stock market return is computed as the realized standard deviation of monthly market return. We also report the correlation between output growth and the spread between interbank lending rate and household savings rate, $Corr\left[\Delta lnY, R_I - R_f\right)]$, where the interest rate spread in the data is measured by the difference between T-bill rates and LIBOR. As we explained in the previous section, positive shocks lower banking sector leverage and reduce the effect of future shocks, whereas negative shocks raise leverage and enhance the amplification mechanism. This asymmetry translates into significant countercyclical volatility in our model with financial shocks. On the other hand, volatility is almost constant in the model with TFP shocks, because these shocks have little effect on the banking sector's net worth.

To quantify the impact of capital reallocation, we compute the fraction of measured TFP shocks attributable to variations in the efficiency of capital reallocation. In our model, the measured log TFP equals $\ln \bar{A}_t + \ln u_t^{\alpha} f\left(\phi_t\right)$, where the component $\ln u_t^{\alpha} f\left(\phi_t\right)$ is the efficiency of capital reallocation. We compute the following variance ratio as a measure of the amplification effect of financial frictions

$$\Delta = \frac{Var\left[\ln\left(u_{t+1}^{\alpha} f\left(\phi_{t+1}\right)\right) - \ln\left(u_t^{\alpha} f\left(\phi_t\right)\right)\right]}{Var\left[\ln \bar{A}_{t+1} - \ln \bar{A}_t + \ln\left(u_{t+1}^{\alpha} f\left(\phi_{t+1}\right)\right) - \ln\left(u_t^{\alpha} f\left(\phi_t\right)\right)\right]}.$$

In the model with TFP shock, $\Delta = 0.11$; that is, financial frictions account for 11% of the total variation in TFP. In the model with financial shocks only, $\Delta = 0.98$, and the efficiency of capital reallocation accounts for virtually all of the macroeconomic fluctuations.

Capital reallocation and financial intermediation We document the statistics related to capital reallocation in Table 3.[16] Both models are calibrated to match a deterministic steady-state capital reallocation-to-investment ratio, $\frac{RA}{I}$ of a 22%. However, because the model with financial shocks displays a significant nonlinearity, the stochastic steady state is quite different from the deterministic steady state and the average capital reallocation-to-investment ratio is 50% in the stochastic steady state. Although this ratio is considerably higher than its data counterpart for publicly traded firms, it is broadly consistent with empirical evidences that also include private firms.[17] In both versions of the model, the average cross-section dispersion of the marginal product of capital, $E\left[Var\left(MPK\right)\right] = 0.16$, accounting for only a small fraction of the same moment in the data.[18] Despite the low average level

[16]We provide the construction details for the relevant series in Appendix A.

[17]Shourideh and Zetlin-Jones (2012) document that external financing accounts for a much larger fraction of investment for private firms, about $70\% - 95\%$.

[18]This aspect of our model is consistent with the previous literature that finds financial constraints contributing to a relatively small fraction of measured misallocation in the data, for example, Midrigan and Xu

of misallocation, the model with financial shocks exhibits a significantly more time variation in the dispersion of the marginal product of capital (4%) than the model with TFP shocks (0.2%).

In the model with financial shocks only, the quantity of capital reallocation is procyclical. The correlation of capital reallocation-to-investment ratio with measured TFP is 0.31, which is close to its empirical counterpart. Like in the data, the benefit of capital reallocation, measured by the dispersion of the marginal product of capital, has a negative correlation with measured TFP of a similar magnitude, −0.32. Our result is therefore consistent with the empirical evidence in Eisfeldt and Rampini (2006). As we have explained in our discussion of the impulse response functions, an adverse shock to banks' net worth lowers bank's borrowing capacity and limits capital reallocation across firms. As a result, the dispersion in the marginal product of capital rises. This mechanism of our model is confirmed by the positive correlation between changes in total amount of bank loan and capital reallocation, 0.52 and the negative correlation between changes in bank loan and the variance of marginal product of capital of −0.50, both of which are consistent with the pattern in the data.

The model with TFP shocks also produces a procyclical capital reallocation. A positive productivity shock implies that the production technology become more efficient. As a result, capital moves from the storage technology to the productive sectors. However, a positive productivity shock is associated with a relative small increase in banks' net worth, but a surge in total investment. As a result, most of the new investment flows into the unconstrained low-productivity firms, and the cross-sectional dispersion of the marginal product of capital increases. The model with TFP shock therefore generates a counterfactually procyclical measured benefit of capital reallocation.

Crisis dynamics To further understand the implications of our model on volatility dynamics and economic recessions, in Table 4, we separately report the moments of macroeconomic quantities and interest rate spreads in the data and those in our model with financial shocks for recession periods and for non-recession periods. For simplicity, we use a "rule-of-thumb" classification and define recession as years with two consecutive quarters of declines in real GDP both in the data and in the model. Our definition yields results similar to the NBER definition of recession; this results in about 20% of the sample being classified as a recession, both in the data and the model simulation.

As shown in Table 4, in the data and in our model, the volatility of output, consumption, and investment is significantly more volatile in recession periods due to a stronger amplification mechanism from the banking sector. Our model matches the countercyclical pattern

(2014).

of the volatility of aggregate output quite well. Because of the absence of adjustment costs, most of the volatility in output is absorbed by investment, not by consumption.[19] In addition, like in the data, in recessions, the level of capital utilization rate drops, its volatility increases, and the spread between interbank lending rate and household deposit rate widens. All the above features are the endogenous outcomes of the financial frictions in the model.

VII Conclusion

We have presented a general equilibrium model with financial intermediary and capital reallocation. Our model emphasizes the role of financial intermediary in reallocating capital across firms with heterogeneous productivity. Shocks to financial frictions alone may account for a large fraction in the fluctuations of measured TFP and aggregate output. Our calibrated model is consistent with the salient features of business cycle variations in macroeconomic quantities and asset prices. In particular, our model successfully generates countercyclical volatility in aggregate consumption and output, and countercyclical dispersion in the cross-section.

An important next step is to infer or impute shocks to financial frictions from the data and investigate whether our model can account for the realized variations in macroeconomic quantities and asset prices once these shocks are fed into the model. We leave this for future research.

[19] Although we do not entertain an extension with adjustment cost for parsimony, the fit of our model can be improved by allowing such an extension.

Table 1

Calibrated Parameters and Targeted Moments

Parameters chosen from previous literature			Reference	
β	discount rate	0.99		
α	capital share	0.33	Kydland and Prescott (1982)	–
$\frac{\eta}{\eta-1}$	markup	1.33	Hsieh and Klenow (2009)	–
δ	depreciation	0.02	Kydland and Prescott (1982)	–
Model-specific parameters			**Targeted moments**	
$\frac{a_H}{a_L}$	relative productivity	4.28	mean capital reallocation	22%
π	prob. of a_H	0.19	output ratio $\left(\frac{\pi y_H}{(1-\pi)y_L}\right)$	1
$E[A]$	aggregate productivty	0.12	mean aggregate growth	2%
$E[\Lambda]$	banker discount rate	0.95	leverage of banking sector	4
θ	banker outside option	0.286	investment-output ratio	25%
ω	interbank friction	0.033	mean TED spread	0.64%
a_0	storage technology	-0.08	average capital utilization	80%
b_0	storage technology	0.99	average depreciation	10%
ν	storage technology	0.99	volatility of capital utilization	5%

Table 1 lists the parameter values we use in our model and the macroeconomic moments used to calibrate these parameter values.

Table 2
Unconditional moments

Moments	Data	TFP Shocks	Financial Shocks
Macroeconomic moments			
$E\left[\Delta \ln Y\right]$	2.00%	2.00%	2.00%
$Std\left[\Delta \ln Y\right]$	3.05%	3.05%	3.05%
$Std\left[\Delta \ln C\right]$	2.04%	0.67%	0.92%
$Std\left[\Delta \ln I\right]$	6.53%	11.01%	14.82%
$Corr\left[\Delta \ln C, \Delta \ln I\right]$	39.70%	59.00%	15.53%
Interest rates			
$E\left[\ln R_f\right]$	0.86%	0.25%	0.21%
$Std\left[\ln R_f\right]$	2.29%	0.01%	1.00%
$E\left[R_I\right] - E\left[R_f\right]$	0.69%	0.64%	0.70%
$Std\left[\ln R_I - \ln R_f\right]$	0.39%	0.01%	0.25%
Cyclical properties			
$Corr\left[\Delta \ln Y, Vol\left(\Delta \ln Y'\right)\right]$	-0.19	0.00	-0.15
$Corr\left[\Delta \ln Y, Vol\left(R_M'\right)\right]$	-0.37	0.00	-0.04
$Corr\left[\Delta \ln Y, \ln R_I - \ln R_f\right]$	-0.14	-0.01	-0.69

Table 2 documents moments of macroeconomic quantities and interest rates in the U.S. (1930-2009), those generated by our model with TFP shocks, and those generated by our model with financial shocks. $Corr\left[\Delta \ln Y, Var\left(\Delta \ln Y\right)\right]$ stands for the correlation of output growth and the variance of future output growth. The latter is calculated as the realized variance of quarterly output growth for the next two years. $Var\left[\Delta uf\left(\phi\right)\right]/Var\left[\Delta TFP\right]$ stands for the fraction of variance in output that can be accounted for by changes in the efficiency of capital reallocation.

Table 3

Capital Reallocation and Financial Intermediation

Moments	Data	TFP Shocks	Financial Shocks
Capital reallocation and capital utilization			
$E\left[RA/I\right]$	0.22	0.22	0.50
$E\left[Var\left(\ln MPK\right)\right]$	0.95	0.16	0.16
$Std\left[Var\left(\ln MPK\right)\right]$	0.24	0.002	0.04
$E\left[u\right]$	80%	80%	78%
$Vol\left[u\right]$	5%	0.66%	5%
Cyclical properties			
$Corr\left[\Delta \ln TFP, RA/I\right]$	0.25	0.25	0.31
$Corr\left[\Delta \ln TFP, Var\left(\ln MPK\right)\right]$	-0.24	0.55	-0.32
$Corr\left[\Delta \ln TFP, \ln u\right]$	0.56	0.62	0.60
$Corr\left[\Delta \ln BL, RA/I\right]$	0.18	0.60	0.52
$Corr\left[\Delta \ln BL, Var\left(\ln MPK\right)\right]$	-0.37	0.83	-0.50

Table 3 documents the moments of capital reallocation and capital utilization in the data, and those generated by our models. Our construction of capital reallocation series follows Eisfeldt and Rampini (2006). Details of the calculation of the cross-section dispersion in log marginal product of capital ($\ln MPK$) can be found in Appendix A of the paper. The capacity utilization rate (u) is published by Federal Reserve Bank of St. Louis.

Table 4
Crisis Dynamics

Moments	Non-Recession Years		Recession Years	
	Data	Model	Data	Model
$Vol\left[\Delta \ln Y\right]$	2.29%	2.83%	3.15%	3.29%
$Vol\left[\Delta \ln C\right]$	1.53%	0.58%	2.42%	0.97%
$Vol\left[\Delta \ln I\right]$	5.25%	13.74%	6.04%	16.07%
$E\left[u\right]$	81.3%	78.8%	78.7%	76.5%
$Vol\left[u\right]$	3.68%	4.99%	4.75%	5.40%
$E\left[\ln R_I - \ln R_f\right]$	0.60%	0.68%	0.66%	0.83%
$Vol\left[\ln R_I - \ln R_f\right]$	0.37%	0.23%	0.52%	0.29%

Table 4 documents the first and second moments of macroeconomic quantities and interest rates in recession and non-recession periods in the data and in our model with financial shocks. Both in the model and in the data, recession is classified as two consecutive quarters of decline in real GDP.

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