

# Risk Preferences and The Macro Announcement Premium

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This paper develops a revealed preference theory for the equity premium around macroeconomic announcements. Stock returns realized around pre-scheduled macroeconomic announcements, such as the employment report and the FOMC statements, account for 55% of the market equity premium. We provide a characterization theorem for the set of intertemporal preferences that generate a positive announcement premium. Our theory establishes that the announcement premium identifies a significant deviation from time-separable expected utility and provides asset-market-based evidence for a large class of non-expected models. Our results provide conditions under which asset prices may rise prior to some macro-announcements and exhibit a pre-announcement drift.

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# 1 Introduction

In this paper, we develop a revealed preference theory for the risk premium for pre-scheduled macroeconomic announcements. We demonstrate that the premium around macroeconomic announcements provides asset-market-based evidence that establishes the importance of incorporating non-expected utility analysis in macro and asset pricing models.

Macro announcements, such as the release of the employment report and the FOMC statements, resolve uncertainty about the future course of the macroeconomy and therefore asset prices react to these announcements instantaneously. Empirically, a large fraction of the market equity premium is realized within a small number of trading days with significant macroeconomic announcements. During the 1961-2014 period, the cumulative excess returns of the S&P 500 index on the thirty days per year with significant macroeconomic news announcements averaged 3.36%, which accounts for 55% of the total annual equity premium during this period (6.19%). The average return on days with macroeconomic announcements is 11.2 basis points (bps) which is significantly higher than the 1.27 bps average return on non-announcement days. High-frequency-data-based evidence shows that much of this premium is realized in very short announcement windows, or a few trading hours prior to the announcement (pre-announcement drift).

To understand the above features of financial markets, we develop a theoretical model that allows uncertainty to resolve before the realizations of macroeconomic shocks and characterize the set of intertemporal preferences for the representative consumer under which an announcement premium arises.

Our main result is that resolutions of uncertainty are associated with realizations of equity premium if and only if the investor's intertemporal preference can be represented by a certainty equivalence functional that increases with respect to second-order stochastic dominance, a property we define as generalized risk sensitivity. This theorem has two immediate implications. First, the intertemporal preference has a time-separable expected utility representation if and only if the announcement premium is zero for all assets. Second, announcement premia can only be compensation for generalized risk sensitivity and cannot be compensation for the risk aversion of the Von Neumann–Morgenstern utility function.

The macro announcement premium, therefore, provides an asset-market-based evidence that identifies a key aspect of investors' preferences not captured by time-separable expected utility. Non-expected utilities such as the recursive utility (Kreps and Porteus [50], Epstein and Zin [28]), the maximin expected utility (Gilboa and Schmeidler [33]), the robust control model (Hansen and Sargent [42]), and the smooth ambiguity model (Klibanoff, Marinacci,

and Mukerji [48]), among others, are widely applied in asset pricing studies to enhance the model-implied market price of risk. We show that generalized risk sensitivity is the key property of these models that distinguishes their asset pricing implications from expected utility. The large magnitude of the announcement premium in the data can be interpreted as a strong empirical evidence for a broad class of non-expected utility models.

From an asset pricing perspective, the stochastic discount factor under non-expected utility generally has two components: the intertemporal marginal rate of substitution that appears in standard expected utility models and an additional term that can be interpreted as the density of a probability distortion. We demonstrate that the probability distortion is a valid stochastic discount factor for announcement returns. In addition, under differentiability conditions, generalized risk sensitivity is equivalent to the probability distortion being pessimistic, that is, it assigns higher weights to states with low continuation utility and lower weights to states with high continuation utility. Our results imply that the empirical evidence for the announcement premium is informative about the relative importance of the two components of the stochastic discount factor and therefore provides a considerable discipline for asset pricing models.

The long-run-risks-based asset pricing literature emphasizes the importance of compensation for risks associated with news about future (see e.g. Bansal and Yaron [9] and Hansen, Heaton, and Li [37]). This literature typically uses a conveniently parameterized Epstein and Zin [28] utility with preference for early resolution of uncertainty. Studies that quantitatively measure preference for early resolution of uncertainty find that resolutions of uncertainty are often associated with substantial welfare improvement in calibrated long-run risks models, for example, Ai [1] and Epstein, Farhi, and Strzalecki [25]. Our results imply that generalized risk sensitivity is precisely the class of preferences that requires compensation for news about future continuation utilities. While within the class of Epstein and Zin [28] utility, generalized risk sensitivity is equivalent to preference for early resolution of uncertainty, more general specifications of preferences allow news about the future to generate a large risk premium for equity without assuming any preference for early resolution of uncertainty.

Our theoretical framework also provides an explanation for the difference between the timing of the realization of the premiums for FOMC announcements and that for other macro-announcements. Using high-frequency data, Lucca and Moench [54] document a pre-announcement drift for FOMC announcements, but not for other macro announcements. That is, equity premiums start to materialize in the hours before the official FOMC announcements, but there is no such pattern in other announcements. Our theorem implies

the existence of a pre-announcement drift if investors receive informative signals in the hours before the official announcements. This interpretation of the data is therefore consistent with the empirical evidence provided by Cieslak, Morse, and Vissing-Jorgensen [18], who document systematic informal communication of Fed officials with the media and financial sector. We present a continuous-time model to account for the pre-announcement drift in FOMC announcements and the lack of it in other macro-announcements.

We also study the implications of time-non-separable utilities for the macro announcement premium. We establish that the external habit model of Campbell and Cochrane [14] generates a zero announcement premium, and the internal habit model of Constantinides [19] and Boldrin, Christiano, and Fisher [12] produces a negative announcement premium. The consumption substitutability model of Dunn and Singleton [23] and Heaton [44] is consistent with a positive announcement premium, although this feature of the utility function smooths the marginal utility process and does not account for the asset market data as highlighted by Gallant, Hansen, and Tauchen [32].

**Related literature** Our paper builds on the literature that studies decision making under non-expected utility. We adopt the general representation of dynamic preferences of Strzalecki [67]. Our framework includes most of the non-expected utility models in the literature as special cases. We show that examples of dynamic preferences that satisfy generalized risk sensitivity include the maxmin expected utility of Gilboa and Schmeidler [33], the dynamic version of which is studied by Chen and Epstein [17] and Epstein and Schneider [26]; the recursive preference of Kreps and Porteus [50] and Epstein and Zin [28]; the robust control preference of Hansen and Sargent [41, 42] and the related multiplier preference of Strzalecki [66]; the variational ambiguity-averse preference of Maccheroni, Marinacci, and Rustichini [55, 56]; the smooth ambiguity model of Klibanoff, Marinacci, and Mukerji [48, 49]; and the disappointment aversion preference of Gul [35]. We also discuss the relationship between our notion of generalized risk sensitivity and related decision theoretic concepts studied in the above papers, for example, uncertainty aversion and preference for early resolution of uncertainty.

A vast literature applies the above non-expected utility models to the study of asset prices and the equity premium. We refer the readers to Epstein and Schneider [27] for a review of asset pricing studies with the maxmin expected utility model, Ju and Miao [45] for an application of the smooth ambiguity-averse preference, Hansen and Sargent [39] for the robust control preference, Routledge and Zin [62] for an asset pricing model with disappointment aversion, and Bansal and Yaron [9] and Bansal [5] for the long-run risk models that build

on recursive preferences. Skiadas [64] provides an excellent textbook treatment of recursive-preferences-based asset pricing theory.

Different from the calibration methodology used in the above papers, our paper takes a revealed preference approach. Earlier work on the reveal preference analysis for expected utility includes Green and Srivastava [34] and Epstein [29]. More recently, Kubler, Selden, and Wei [51] and Echenique and Saito [24] developed asset-market-based characterizations of the expected utility model. None of the above papers focus on the macro announcement premium and generalized risk sensitivity.

Quantitatively, our findings are consistent with the literature that identifies large variations in marginal utilities from the asset market data, for example, Hansen and Jagannathan [38], Bansal and Lehmann [6], and Alvarez and Jermann [2, 3]. Our theory implies that most of the variations in marginal utility must come from generalized risk sensitivity and not from risk aversion in expected utility models.

The above observation likely has sharp implications for the research on macroeconomic policies. Several recent papers study optimal policy design problems in non-expected utility models. For example, Farhi and Werning [31] and Karantounias [47] analyze optimal fiscal policies with recursive preferences, and Woodford [68], Karantounias [46], Hansen and Sargent [43], and Kwon and Miao [53, 52] focus on preferences that fear model uncertainty. In the above studies, the nonlinearity in agents' certainty equivalence functionals implies a forward-looking component of variations in their marginal utilities that affects policy makers' objectives. Our results imply that the empirical evidence for the announcement premium can be used to gauge the magnitude of this deviation from expected utility, and to quantify the importance of robustness in the design of macroeconomic policies.

Our empirical results are related to the previous research on stock market returns on macroeconomic announcement days. The previous literature documents that stock market returns and Sharpe ratios are significantly higher on days with macroeconomic news releases in the United States (Savor and Wilson [63]) and internationally (Brusa, Savor, and Wilson [13]). Lucca and Moench [54] find similar patterns and document a pre-FOMC announcement drift. Mueller, Tahbaz-Salehi, and Vedolin [58] document an FOMC announcement premium on the foreign exchange market and attribute it to compensation to financially constrained intermediaries.

The rest of the paper is organized as follows. We document some stylized facts about the equity premium for macroeconomic announcements in Section 2. In Section 3, we present two simple examples to illustrate how the announcement premium can arise in models that

deviate from expected utility. We present our theoretical results and discuss the notion of generalized risk sensitivity in Section 4. We present a continuous-time model in Section 5 to quantitatively account for the evolution of the equity premium around macroeconomic news announcement days. Section 6 concludes.

## 2 Stylized facts

To demonstrate the significance of the equity premium for macro-announcements and to highlight the difference between announcement days and non-announcement days, we focus on a relatively small set of pre-scheduled macroeconomic announcements released at the monthly or lower frequency. Within this category, we select the top five announcements ranked by investor attention by Bloomberg users. This procedure yields, on average, thirty announcement days per year in the period of 1961-2014. We summarize our main findings below and provide details of the data construction in the data appendix.

1. A large fraction of the market equity premium is realized on a relatively small number of trading days with pre-scheduled macroeconomic news announcements.

We report the average market excess returns on macro announcement days and non-announcement days during the 1961-2014 period in Table 1. In this period, on average, thirty trading days per year have significant macroeconomic news announcements. At the daily level, the average stock market excess return is 11.21 basis points (bps) on announcement days and 1.27 bps on days without major macro announcements. As a result, the cumulative stock market excess return on the thirty announcement days averages 3.36% per year, accounting for about 55% of the annual equity premium (6.19%) during this period.

In Table 2, we report the average market excess return on announcement days (0) and the same moments for the market return on the day before (-1) and the day after (+1) announcement days. The difference between the mean return on announcement days and that on non-announcement days is statistically and economically significant with a t-statistic of 3.36. This evidence is consistent with the previous literature, for example, Savor and Wilson [63].

2. Most of the premiums for FOMC announcements are realized in the several hours prior to the announcements. The premiums for other macro announcements are realized at announcement.

In Table 3, we report the point estimates with standard errors for average hourly excess returns around announcements. The announcement window 0 corresponds to the announcement in a one-hour window including the announcement. The hourly returns typically peak at announcement, as reflected in row 1 of the table. The mean return during the announcement hour is economically important: 6.46 bps with a standard error of 2.71. The difference in mean excess returns in announcement hours compared to non-announcement hours, as in the daily returns data, is significant with a t-statistic of 2.06. In the case of FOMC announcements, consistent with Lucca and Moench [54], the mean returns prior to the announcement window are statistically significant (see row 2 of Table 3); this pre-announcement drift is not reflected in other macro announcements, as shown in row 3 of Table 3. In Figure 1, we plot the average hourly stock market excess returns for FOMC announcements (top panel) and those for other macro announcements (bottom panel) in the hours around the announcements. There is a "pre-announcement drift" for FOMC announcements, but not for other macro announcements. The premiums for non-FOMC announcements mainly realize at announcement.<sup>1</sup>

In addition, Lucca and Moench [54] document that there is no statistically significant pre-FOMC announcement drift for Treasury bonds in the 1994-2011 period, while Savor and Wilson [63] presented evidence for a moderate level of announcement premium for Treasury bonds, which averages about *3bps* on announcement days during the longer sample period of 1961-2009.

### 3 A two-period model

In this section, we set up a two-period model and discuss two simple examples to illustrate conditions under which resolutions of uncertainty are associated with realizations of the equity premium.

#### 3.1 Model setup

We consider a representative-agent economy with two periods (or dates), 0 and 1. There is no uncertainty in period 0, and the period-0 aggregate endowment is a constant,  $C_0$ . Aggregate endowment in period 1, represented by  $C_1$ , is a random variable. We assume a finite number

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<sup>1</sup>The evidence reported in Table 3 is robust to using half-hourly windows as opposed to hourly windows.

Figure 1: **Hourly return around announcements**

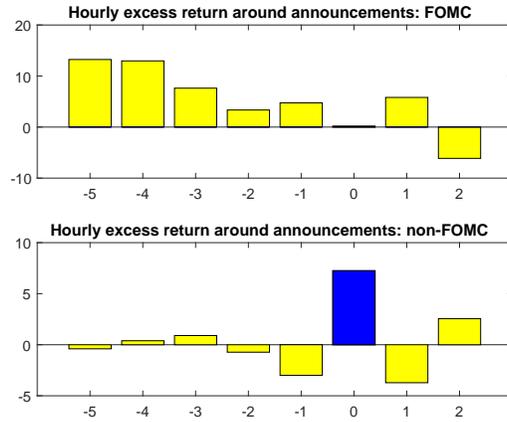


Figure 1 plots the average hourly excess returns around macro announcements in the period of 1997-2013. Time 0 is the hour before and including the announcement time, and time "k" is k hours after announcements. The top panel is for FOMC announcement and the bottom panel includes all other macro announcements.

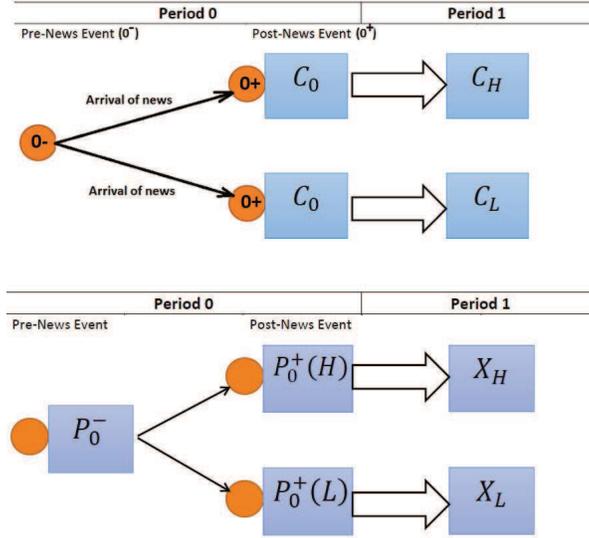
of states:  $n = 1, 2, \dots, N$ , and  $\{C_1(n)\}_{n=1,2,\dots,N}$  denotes the possible realizations of  $C_1$ . The probability of each state is  $\pi(n) > 0$  for  $n = 1, 2, \dots, N$ .

Period 0 is further divided into two subperiods. In period  $0^-$ , before any information about  $C_1$  is revealed, the pre-announcement asset market opens, and a full set of Arrow-Debreu securities is traded. The asset prices at this point are called pre-announcement prices and are denoted as  $P^-$ . Note that  $P^-$  cannot depend on the realization of  $C_1$ , which is unknown at this point.

In period  $0^+$ , the agent receives a news announcement,  $s$ , that carries information about  $C_1$ . Immediately after the arrival of  $s$ , the post-announcement asset market opens. Asset prices at this point, which are called post-announcement prices, depend on  $s$  and are denoted as  $P^+(s)$ . In period 1, the payoff of the Arrow-Debreu securities is realized, and  $C_1$  is consumed. In Figure 2, we illustrate the timing of the information and consumption (top panel) and that of the asset markets (bottom panel) assuming that  $N = 2$  and that the news announcement  $s$  fully reveals  $C_1$ .

The announcement return of an asset, which we denote as  $R_A(s)$ , is defined as the return of a strategy that buys the asset before the pre-scheduled announcement and sells right

Figure 2: **Consumption and asset prices in the two-period model**



afterwards:

$$R_A(s) = \frac{P^+(s)}{P^-}. \quad (1)$$

The risk-free announcement return is defined as the return on an asset with  $P^+(s) = 1$  for all  $s$ . In our setup, because consumption at  $0^-$  and consumption at  $0^+$  are perfect substitutes, the risk-free announcement return must be one. We say that the asset requires a positive announcement premium if  $E[R_A(s)] > 1$ . We also define the post-announcement return conditioning on announcement  $s$  as:  $R_P(X|s) = \frac{X}{P^+(s)}$ . Clearly, the total return of the asset from period  $0^-$  to period 1 is  $R(X) = R_A(s) R_P(X|s)$ .

Two assumptions in our model deserve further clarifications. First, the assumption that news arrives within period 0 is equivalent to a specification that consumption at  $0^-$  and consumption at  $0^+$  are perfect substitutes. This feature of the model implies that the risk-free announcement return – the return on an asset that delivers one unit of consumption goods unconditionally at  $0^+$  – is 0. Allowing consumption at  $0^+$  to differ from that at  $0^-$  implies a nonzero risk-free announcement return but does not affect any of our analysis below as long as consumption at  $0^+$  does not depend on the content of the news; in continuous-time models with Brownian motions shocks, this assumption becomes inconsequential, as we will show in Section 5. The key element of our assumption here is that the arrival of news announcements is not associated with a resolution of uncertainty about current-period consumption, but is associated with that of future consumption.

Second, the assumption that claims to Arrow securities that pay off at time 1 can be traded at both at  $0^-$  and  $0^+$  effectively completes the market at time  $0^+$ , even though we do not explicitly allow agents to trade at time  $0^-$  a full set of Arrow securities that pay off at time  $0^+$ . Because the announcement fully reveals the state, after announcement  $s$ , the only Arrow security that can have a strictly positive price is the one that pays off in exactly the same state.

We note two important properties of the news announcement in our model. First, news announcements affect the conditional distribution of future consumption, but rational expectations imply that surprises in the news must average to zero by the law of iterated expectation. Second, we make the simplifying assumption that consumption does not instantaneously respond to the announcement though asset prices can. This assumption captures the well-established empirical finding of Hall [36] and Parker and Julliard [60], among others, that consumption does not contemporaneously co-vary with stock markets returns; it seems to vary sluggishly with lagged stock returns. Bansal and Shaliastovich [7], in addition, document that even large movements in the stock prices are not associated with significant immediate adjustments in aggregate consumption. The lack of contemporaneous covariance between stock returns and consumption implies that the contribution of the consumption covariance with asset returns over very short intervals, if any, is too small to impact the announcement premiums, which as discussed in Section 2, are realized over daily and hourly intervals.

The assumption that consumption does not respond to announcement is also appropriate from the theoretical perspective, because in standard production-based models, the endogenous response of consumption to announcements is generally quantitatively small and yields a negative announcement premium. The response of consumption is quantitatively small because risk-averse agents dislike large consumption adjustments, particularly over daily or hourly intervals. The announcement premium that results from the immediate response of consumption to news is in general negative, because it is hard to adjust output instantaneously upon announcements. As shown in Beaudry and Portier [10] and Beaudry and Portier [11], without changes in output, consumption and Tobin's  $q$  (and therefore asset returns) move in opposite directions upon news regardless of whether the income or substitution effect dominates. This channel therefore contributes negatively to the announcement premium. Our assumption allows us to focus on the relevant aspect of preferences, as show in Theorem 2 and in the parametric example of Section 5.2, that generates a positive announcement premium.

### 3.2 Expected utility<sup>2</sup>

We first consider the case in which the representative agent has expected utility:  $u(C_0) + \beta E[u(C_1)]$ , where  $u$  is strictly increasing and continuously differentiable. For simplicity, we assume that  $s$  fully reveals  $C_1$  in this example and the one in the following section, although our general result in Section 3.3 does not depend on this assumption. The pre-announcement price of an asset with payoff  $X$  is given by:

$$P^- = E \left[ \frac{\beta u'(C_1)}{u'(C_0)} X \right]. \quad (2)$$

In period  $0^+$ , because  $s$  fully reveals the true state, the agent's preference is represented by

$$u(C_0) + \beta u(C_1(s)). \quad (3)$$

Therefore, for any  $s$ , the post-announcement price of the asset is

$$P^+(s) = \frac{\beta u'(C_1(s))}{u'(C_0)} X(s). \quad (4)$$

Clearly, the expected announcement return is  $E[R_A(s)] = \frac{E[P^+(s)]}{P^-} = 1$ . There can be no announcement premium on any asset under expected utility.

### 3.3 An example with uncertainty aversion

Consider an agent with the constraint robust control preferences of Hansen and Sargent [40]:

$$\begin{aligned} & u(C_0) + \beta \min_m E[mu(C_1)] \\ & \text{subject to : } E[m \ln m] \leq \eta \\ & E[m] = 1. \end{aligned} \quad (5)$$

The above expression also can be interpreted as the maxmin expected utility of Gilboa and Schmeidler [33]. The agent treats the reference probability measure, under which the equity premium is evaluated (by econometricians), as an approximation. As a result, the agent takes into account a class of alternative probability measures, which are represented by the

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<sup>2</sup>We use the term "expected utility" to mean utility functions that are additively separable with respect to both time and states.

density  $m$ , close to the reference probability measure. The inequality  $E[m \ln m] \leq \eta$  requires that the relative entropy of the alternative probability models to be less than  $\eta$ .

In this case, the pre-announcement price of an asset with payoff  $X$  is:

$$P^- = E \left[ m^* \frac{\beta u'(C_1)}{u'(C_0)} X \right], \quad (6)$$

where  $m^*$  is the density of the minimizing probability for (5) and can be expressed as a function of  $s$ :

$$m^*(s) = \frac{e^{-\frac{u(C_1(s))}{\theta}}}{E \left[ e^{-\frac{u(C_1)}{\theta}} \right]}. \quad (7)$$

The positive constant in the above expression,  $\theta$ , is determined by the binding relative entropy constraint  $E[m^* \ln m^*] = \eta$ .

In period  $0^+$ , after the resolution of uncertainty, the agent's utility reduces to (3). As a result, the post-announcement price of the asset is the same as that in (4). Therefore, we can write the pre-announcement price as:

$$P^- = E [m^*(s) P^+(s)]. \quad (8)$$

Because  $m^*$  is a decreasing function of the date-1 utility,  $u(C_1)$ , it is straightforward to prove the following claim.

**Claim 1.** *Suppose that the post-announcement price,  $P^+(s)$ , is a strictly increasing function of  $C_1$ , then  $P^- < E[P^+(s)]$ . As a result, the announcement premium for the asset is strictly positive.*

The intuition behind the above result is clear. Because uncertainty is resolved after the announcement, asset prices are discounted using marginal utilities. Under expected utility, the pre-announcement price is computed by using the probability-weighted marginal utilities, and therefore the pre-announcement price must equal the expected post-announcement prices and there can be no announcement premium under rational expectation. Under the robust control preference, the pre-announcement price is not computed by using the reference probability, but rather by using the pessimistic probability that overweighs low-utility states and underweighs high-utility states as shown in equation (7). As a result, uncertainty aversion applies extra discounting for payoffs positively correlated with utility, and therefore the asset market requires a premium for such payoffs relative to the risk-free returns.

Because the probability distortion,  $m^*$ , discounts announcement returns, we will call it

the announcement stochastic discount factor (SDF), or A-SDF, to distinguish it from the standard SDF in intertemporal asset pricing models that are derived from agents' marginal rate of intertemporal substitution of consumption. In our model, there is no intertemporal consumption decision before or after the announcement. The term  $m^*$  reflects investors' uncertainty aversion and identifies the probability distortion relative to rational expectation.

The announcement in the above setup should be defined as the resolution of macroeconomic uncertainty associated with a revelation of public information. Both pre-scheduled macroeconomic announcements and informal communications about their contents before the scheduled announcements can lead to a resolution of macroeconomic uncertainty. In our setup, there is no need to distinguish pre-scheduled announcements from communications before the scheduled announcements. The same model as we discussed above will generate a pre-announcement drift, that is, realizations of announcement premiums before the scheduled announcements, under the assumption that communications occur before the officially scheduled announcements.

### 3.4 A simple characterization of announcement premium

Our goal is to characterize the set of preferences that is consistent with a positive announcement premium. While the main theorem of the paper is stated for a fully dynamic model with an infinite dimensional probability space, it is useful to discuss the intuition behind our results in the two-period setup with discrete probability spaces. We make two assumptions to simplify our discussion, although none of them is needed for our general theorem in the next section. First, we assume a finite state space with equal probabilities. That is,  $\pi(s) = \frac{1}{N}$  for  $s = 1, 2, \dots, N$ . Second, like in the above examples, we assume that the announcement at time  $0^+$  fully reveals the true state.

Because there is no uncertainty after announcement at time  $0^+$ , we assume that the agent ranks consumption streams according to a time-separable utility and denote utility in state  $s$  as  $V_s = u(C_0) + \beta u(C_{1,s})$ . At time  $0^-$ , prior to the announcement, the agent ranks uncertainty outcomes according to a general certainty equivalence functional  $\mathcal{I}[V]$ , where  $\mathcal{I}$  maps random variables into the real line.<sup>3</sup> Because there are  $N$  states of the world, we use the vector notation  $V = [V_1, V_2, \dots, V_N]$  and think of  $\mathcal{I}$  as a mapping from from the  $N$ -dimensional Euclidean space to the real line, i.e.,  $\mathbf{R}^N \rightarrow \mathbf{R}$ . Our examples in Sections 3.2 and 3.3 corresponds to the special cases in which  $\mathcal{I}$  is the expectation operator and the

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<sup>3</sup>We follow Strzalecki [67] and call  $\mathcal{I}$  the certainty equivalent. However, it is important to note that  $\mathcal{I}[V]$  is measured in utility terms and not in consumption terms.

maximin expected utility operator, respectively.

Clearly, the post-announcement price of any asset is the same as computed in (4). To compute pre-announcement prices, we assume that  $\mathcal{I}$  is continuously differentiable. From the date  $0^-$  perspective, the agent's marginal utility with respect to  $C_0$  is

$$\frac{\partial}{\partial C_0} \mathcal{I}[V] = \sum_{s=1}^N \frac{\partial}{\partial V_s} \mathcal{I}[V] \cdot u'(C_0),$$

and the marginal utility with respect to  $C_{1,s}$  is

$$\frac{\partial}{\partial C_{1,s}} \mathcal{I}[V] = \frac{\partial}{\partial V_s} \mathcal{I}[V] \cdot \beta u'(C_{1,s}).$$

The pre-announcement price of an asset can be computed as the marginal utility weighted payoffs:

$$P^- = \sum_{s=1}^N \frac{\frac{\partial}{\partial C_{1,s}} \mathcal{I}[V]}{\frac{\partial}{\partial C_0} \mathcal{I}[V]} X_s = E \left[ m^*(s) \beta \frac{u'(C_1(s))}{u'(C_0)} X(s) \right], \quad (9)$$

where

$$m^*(s) = \frac{1}{\pi(s)} \frac{\frac{\partial}{\partial V_s} \mathcal{I}[V]}{\sum_{s=1}^N \frac{\partial}{\partial V_s} \mathcal{I}[V]}. \quad (10)$$

Clearly, the asset pricing equation (8) holds with the A-SDF  $m^*$  defined by (10).

The A-SDF  $m^*$  in equation (10) must be a density as it sums up to one and can be interpreted as a probability distortion. Intuitively, a nonnegative announcement premium for all payoffs that are increasing functions of continuation utility is equivalent to  $m^*$  being pessimistic in the sense that it overweighs states with lower utility. Mathematically, the latter condition is equivalent to the partial derivatives,  $\frac{\partial}{\partial V_s} \mathcal{I}[V]$ , being negatively comonotone with respect to  $V$ , that is, for all  $s$  and  $s'$ ,

$$\frac{\partial}{\partial V_s} \mathcal{I}[V] \geq \frac{\partial}{\partial V_{s'}} \mathcal{I}[V] \text{ if and only if } V_s \leq V_{s'}. \quad (11)$$

The above condition is known to be equivalent to Schur concavity, and under the assumption of finite states with equal probabilities, is equivalent to monotonicity with respect to second-order stochastic dominance (see Marshall, Arnold, and Olkin [57] and Muller and Stoyan [59]). In the following section, we formally develop the above results in a fully dynamic model with a continuous probability space without making the simplifying assumptions of fully-revealing announcements and finite states with equal probabilities.

## 4 Risk preferences and the announcement premium

### 4.1 A dynamic model with announcements

**Preferences** The setup of our model follows Strzalecki [67] but extends his framework to allow for announcements. Let  $S$  be a non-atomic measurable space, and  $\Sigma$  be the associated Borel  $\sigma$ -field. Let  $(\Omega, \mathcal{F}) = (S, \Sigma)^{2T}$  be the product space. We index the  $2T$  copies of  $(S, \Sigma)$  by  $j = 0^+, 1^-, 1^+, 2^-, \dots, T-1^+, T^-$  with the interpretation that  $t^-$  is the pre-announcement period at time  $t$  and  $t^+$  is the post announcement period at time  $t$ . A typical element in  $\Omega$  is therefore denoted  $\omega = \{s_0^+, s_1^-, s_1^+, s_2^-, \dots, s_{T-1}^+, s_T^-\}$ . Let  $z_{t-1}^+ = \{s_0^+, s_1^-, s_1^+, s_2^-, \dots, s_{t-1}^+, s_{t-1}^-\}$  and  $z_t^- = \{s_0^+, s_1^-, s_1^+, s_2^-, \dots, s_{t-1}^+, s_t^-\}$  denote the history of the realizations of states until  $t-1^+$  and that until  $t^-$ , respectively. Let  $\mathcal{F}_{t-1}^+ = \sigma(z_{t-1}^+)$ , and  $\mathcal{F}_t^- = \sigma(z_t^-)$  be the  $\sigma$ -fields generated by the history of realizations, for  $t = 1, 2, \dots, T$ . The filtration  $\{\mathcal{F}_{t-1}^+, \mathcal{F}_t^-\}_{t=1}^T$  represents public information. We use  $\mathbf{Z}$  to denote the set of all histories, and let  $z \in \mathbf{Z}$  denote a generic element of  $\mathbf{Z}$ .

We endow the measurable space  $(\Omega, \mathcal{F})$  with a non-atomic probability measure  $P$ , under which the distribution of  $\{s_0^+, s_1^-, s_1^+, s_2^-, \dots, s_{T-1}^+, s_T^-\}$  is stationary. The interpretation is that  $P$  is the probability measure under which all expected returns are calculated. We assume that consumption takes value in  $\mathbf{Y}$ , an open subset of  $\mathbf{R}$ . Let  $L^2(\Omega, \mathcal{F}, P)$  be the Hilbert space of square integrable real valued random variables defined on  $(\Omega, \mathcal{F}, P)$ . A consumption plan is an  $\{\mathcal{F}_t^-\}_{t=1}^T$  adapted process  $\{C_t\}_{t=1}^T$ , such that  $C_t$  is a  $\mathbf{Y}$ -valued square integrable random variables for all  $t$ . We denote  $\mathcal{C}$  the space of all such consumption plans, and a typical element in  $\mathcal{C}$  is denoted as  $\mathbf{C} \in \mathcal{C}$ . The above setup allows us to model announcements, that is, revelations of public information associated with realizations of  $\{s_{t-1}^+\}_{t=1}^T$  separately from the realizations of consumption. As a result, our theory will be able to separate the property of preferences that requires premiums for assets with payoff correlated with resolutions of uncertainty from the property of preferences that demands excess returns for assets that comove with realizations of consumption.

Below, we describe a recursive procedure to construct a system of conditional preferences  $\{\succsim_z\}_{z \in \mathbf{Z}}$  on  $\mathcal{C}$ , such that  $\mathbf{C} \succsim_z \mathbf{C}'$  if  $V(\mathbf{C}|z) \geq V(\mathbf{C}'|z)$ . As in Strzalecki [67] we focus on intertemporal preferences that can be represented recursively by

$$V = u(C) + \beta \mathcal{I}[V']. \quad (12)$$

Formally, the representative agent's dynamic preference is defined by a triple  $\{u, \beta, \mathcal{I}\}$ , where

$u : \mathbf{Y} \rightarrow \mathbf{R}$  is a strictly increasing Von Neumann–Morgenstern utility function, and  $\beta \in (0, 1]$  is the subjective discount rate. The certainty equivalence functional  $\mathcal{I}$  is a family of functions,  $\{\mathcal{I}[\cdot|z]\}_{z \in \mathbf{Z}}$ , such that  $\forall z \in \mathbf{Z}$ ,  $\mathcal{I}[\cdot|z] : L^2(\Omega, \mathcal{F}, P) \rightarrow \mathbf{R}$  is a (conditional) certainty equivalence functional that maps continuation utilities into the real line. Given  $\{u, \beta, \mathcal{I}\}$ , the agent’s utility function is constructed recursively as follows.

- At the terminal time  $T$ ,  $V(\mathbf{C}|z_T^-) = u(C_T)$ .
- For  $t = 1, 2, 3, \dots, T$ , given  $V(\mathbf{C}|z_t^-)$ , in period  $t - 1$  after the signal  $s_{t-1}$  is revealed,  $V(\mathbf{C}|z_{t-1}^+)$  is calculated according to:

$$V(\mathbf{C}|z_{t-1}^+) = u(C_{t-1}) + \beta \mathcal{I}[V(\mathbf{C}|z_t^-)|z_{t-1}^+]$$

- For  $t = 1, 2, 3, \dots, T$ , given a continuation utility  $V(\mathbf{C}|z_{t-1}^+)$ , in period  $t - 1$  before the signal  $s_{t-1}$  is received,  $V_{z_{t-1}^-}(\mathbf{C})$  is defined as

$$V(\mathbf{C}|z_{t-1}^-) = \mathcal{I}[V(\mathbf{C}|z_{t-1}^+)|z_{t-1}^-].$$

Here there is no intertemporal consumption before and after the signal  $s_{t-1}$  is received, and we simply use the certainty equivalence functional  $\mathcal{I}$  to aggregate utility across states.

In Appendix A, we show that the above representation incorporates the following dynamic preferences under uncertainty and provide expressions for the A-SDF implied by these preferences:

1. The recursive utility of Kreps and Porteus [50] and Epstein and Zin [28].
2. The maxmin expected utility of Gilboa and Schmeidler [33]. The dynamic version of this preference is studied in Epstein and Schneider [26] and Chen and Epstein [17].
3. The variational preferences of Maccheroni, Marinacci, and Rustichini [55], the dynamic version of which is studied in Maccheroni, Marinacci, and Rustichini [56].
4. The multiplier preferences of Hansen and Sargent [39] and Strzalecki [66].
5. The second order expected utility of Ergin and Gul [30].
6. The smooth ambiguity preferences of Klibanoff, Marinacci, and Mukerji [48] and Klibanoff, Marinacci, and Mukerji [49].

7. The disappointment aversion preference of Gul [35].

**Asset markets** We consider a representative agent economy where agents' preferences are as described in the last section, and the aggregate endowment is given by  $\{Y_t\}_{t=1}^T \in \mathcal{C}$ . Asset market opens at each history  $z \in \mathbf{Z}$ . We interpret the realizations of  $\{s_t^+\}_{t=1}^T$  as announcements, as they carry information about future consumption but is not associated with realizations of current period consumption. Markets at period  $t^-$  are called pre-announcement markets. Here, agents can trade a vector of  $J+1$  returns:  $\{R_j(\cdot | z_t^-)\}_{j=0,1,\dots,J}$ , where  $\{R_j(s_t^+ | z_t^-)\}_{s_t^+}$  represents a state-contingent return traded at history  $z_t^-$  that pays off  $R_j(s_t^+ | z_t^-)$  at all subsequent history  $\{(z_t^-, s_t^+)\}_{s_t^+}$ . Similarly, agents can trade a vector of post-announcement returns,  $\{R_j(\cdot | z_t^+)\}_{j=0,1,\dots,J}$  on post-announcement markets. We adopt the convention that  $R_0(\cdot | z)$  is a one-step risk-free return at history  $z \in \mathbf{Z}$  and write it as  $R_0(z)$  whenever convenient.

Given the recursive nature of the preferences, the optimal consumption-portfolio choice problem of the agent can be solved by backward induction. For  $t = 1, 2, \dots, T$ , we use  $V_t^-(W)$  and  $V_t^+(W)$  to denote the agent's continuation utility as a function of wealth at history  $z_t^-$  and  $z_t^+$ , respectively and call them value functions. In the last period  $T$ , agents simply consume their total wealth, and therefore  $V_T^-(W) = u(W)$ . For  $t = 1, 2, \dots, T-1$ , we denote  $\xi = [\xi_0, \xi_1, \xi_2, \dots, \xi_J]$  as the vector of investment in the post-announcement asset market and write the corresponding consumption-portfolio choice problem as:

$$\begin{aligned} V_t^+(W_{z_t^+}) &= \max_{C, \xi} \left\{ u(C) + \beta \mathcal{I} \left[ V_{t+1}^-(W_{z_t^+, s_{t+1}^-}) \middle| z_t^+ \right] \right\} \\ C + \sum_{j=0}^J \xi_j &= W_{z_t^+} \\ W_{z_t^+, s_{t+1}^-} &= \sum_{j=0}^J \xi_j R_j(s_{t+1}^- | z_t^+), \quad \text{all } s_{t+1}^-. \end{aligned}$$

Similarly, the optimal portfolio choice problem on the pre-announcement market is

$$\begin{aligned} V_t^-(W_{z_t^-}) &= \max_{\zeta} \mathcal{I} \left[ V_t^+(W_{z_t^-, s_t^+}) \middle| z_t^- \right] \\ W_{z_t^-, s_t^+} &= W_{z_t^-} - \sum_{j=0}^J \zeta_j + \sum_{j=0}^J \zeta_j R_j(s_t^+ | z_t^-), \end{aligned}$$

where  $\zeta = [\zeta_0, \zeta_1, \zeta_2, \dots, \zeta_J]$  is a vector of investment in announcement returns.

We assume that for some initial wealth level,  $W_0$  and a sequence of returns  $\left\{ \{R_j(\cdot | z)\}_{j=0,1,\dots,J} \right\}_{z \in \mathbf{Z}}$ , an interior competitive equilibrium with sequential trading exists where all markets clear. For simplicity, we started with returns directly in our description of

the equilibrium with the understanding that returns can always be constructed from prices. Below we provide a formal definition of announcement premium.

**Definition 1.** (*Announcement premium*)

The announcement premium for asset  $j$  at history  $z_t^-$  is defined as

$$E [R_j (\cdot | z_t^-) | z_t^-] - R_0 (z_t^-).$$

## 4.2 The announcement SDF

To relate the announcement premium to the properties of the certainty equivalence functional,  $\mathcal{I}[\cdot]$ , we first provide some definitions. The certainty equivalence functional  $\mathcal{I}$  is said to be monotone with respect to first-order (second-order) stochastic dominance if  $X_1 \geq_{FSD} X_2$  ( $X_1 \geq_{SSD} X_2$ ) implies  $\mathcal{I}[X_1|z] \geq \mathcal{I}[X_2|z]$  a.s.. It is strictly monotone with respect to first-order (second-order) stochastic dominance if  $X_1 >_{FSD} X_2$  ( $X_1 >_{SSD} X_2$ ) implies  $\mathcal{I}[X_1|z] > \mathcal{I}[X_2|z]$  a.s., where  $\geq_{FSD}$  and  $\geq_{SSD}$  stand for first- and second-order stochastic dominance, respectively.<sup>4</sup> In what follows, we assume that  $\mathcal{I}$  is normalized, that is,  $\mathcal{I}[X|z] = X$  a.s. whenever  $X$  is a measurable function of  $z$ .

Conceptually, the properties of asset prices impose restrictions on the derivatives of the utility functions. Our theoretical exercise amounts to recovering the properties of the utility functions from their derivatives. In our setup, the certainty equivalence functional is defined on the infinite dimensional space  $L^2(\Omega, \mathcal{F}, P)$ . We therefore need a notion of differentiability in infinite dimensional spaces, which we formalize as follows.

**Definition 2.** (*Fréchet Differentiable with Lipschitz Derivatives*) The certainty equivalence functional  $\mathcal{I}$  is Fréchet differentiable if  $\forall z \in \mathbf{Z}, \forall X \in L^2(\Omega, \mathcal{F}, P)$ , there exists a unique continuous linear functional,  $D\mathcal{I}[X|z] \in L^2(\Omega, \mathcal{F}, P)$ , such that for all  $\Delta X \in L^2(\Omega, \mathcal{F}, P)$ ,

$$\lim_{\|\Delta X\| \rightarrow 0} \frac{|\mathcal{I}[X + \Delta X|z] - \mathcal{I}[X|z] - \int D\mathcal{I}[X|z] \cdot \Delta X dP|}{\|\Delta X\|} = 0.$$

A Fréchet differentiable certainty equivalence functional  $\mathcal{I}$  is said to have a Lipschitz derivatives if  $\forall X, Y \in L^2(\Omega, \mathcal{F}, P), \forall z \in \mathbf{Z}, \|D\mathcal{I}[X|z] - D\mathcal{I}[Y|z]\| \leq K \|X - Y\|$  for some constant  $K$ .<sup>5</sup>

<sup>4</sup>The definitions of first- and second-order stochastic dominance are standard and are provided in the appendix.

<sup>5</sup>The standard definition of Fréchet differentiability requires the existence of the derivative as a continuous

We first present a result for the existence of A-SDF.

To simplify notations, we let  $V(s_t^+ | z_t^-) \equiv V_t^+(W_{z_t^-, s_t^+})$  denote the value of the representative agent's continuation utility at announcement  $s_t^+$  following history  $z_t^-$ . The following theorem provides an existence result for the A-SDF.

**Theorem 1.** (*Existence of A-SDF*)<sup>6</sup>

Suppose both  $u$  and  $\mathcal{I}$  are Lipschitz continuous, Fréchet differentiable with Lipschitz continuous derivatives. Suppose that  $\mathcal{I}$  is strictly monotone with respect to first-order stochastic dominance, then in any interior competitive equilibrium with sequential trading,  $\forall z_t^- \in \mathbf{Z}$ , the risk-free announcement return  $R_0(z_t^-) = 1$ . In addition, there exists a nonnegative measurable function  $m^* : \mathbf{R} \rightarrow \mathbf{R}^+$  such that

$$E[m^*(V(\cdot | z_t^-)) \{R_j(\cdot | z_t^-) - 1\} | z_t^-] = 1 \text{ for all } j = 0, 1, 2, \dots, J. \quad (13)$$

Under the regularity condition (B.15) in Appendix B.2,  $E[m^*(V(\cdot | z_t^-)) | z_t^-] = 1$  and (13) can be written as:

$$E[m^*(V(\cdot | z_t^-)) R_j(\cdot | z_t^-) | z_t^-] = 1 \text{ for all } j = 0, 1, 2, \dots, J. \quad (14)$$

To provide a precise statement about the sign of the announcement premium, we focus our attention on payoffs that are co-monotone with continuation utility. Let  $\{f(s_t^+ | z_t^-)\}_{s_t^+}$  be an asset traded at history  $z_t^-$  with payoff contingent on the announcement  $s_t^+$ . The payoff  $f$  is said to be *co-monotone with continuation utility* if

$$[f(s | z_t^-) - f(s' | z_t^-)] [V(s | z_t^-) - V(s' | z_t^-)] \geq 0 \text{ for all } s, s' \text{ almost surely.} \quad (15)$$

Intuitively, co-monotonicity captures the idea that the payoff,  $f$ , is an increasing function of the continuation utility,  $V$ . The following theorem formalizes our discussion in Section 3.4 and provides a necessary and sufficient condition for the announcement premium.

**Theorem 2.** (*Announcement Premium*) Under the assumptions of Theorem 1,

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linear functional. Because we focus on functions defined on the Hilbert space  $L^2(\Omega, \mathcal{F}, P)$ , we apply the Riesz representation theorem and denote  $D\mathcal{I}[X|z]$  as the representation of the derivative in  $L^2(\Omega, \mathcal{F}, P)$ .

<sup>6</sup>To avoid overly technical conditions, we assume the Fréchet differentiability of  $\mathcal{I}$  in Theorems 1 and 2. The proofs in the Appendix go through under weaker conditions. In particular, we do not need the Fréchet derivative of  $\mathcal{I}[V]$  to be unique, we only need the projection of the gradient,  $D\mathcal{I}[V]$ , onto  $L^2(\Omega, \sigma(V), P)$  to be unique. This latter condition allows for multiple prior preferences and robust control preferences that do not satisfy Fréchet differentiability.

1. The A-SDF  $m^*(V) = 1$  for all  $V$  if and only if  $\mathcal{I}$  is the expectation operator.
2. The following conditions are equivalent:
  - (a) The certainty equivalence functional  $\mathcal{I}$  is monotone with respect to second-order stochastic dominance.
  - (b) The A-SDF  $m^*(V)$  is a non-increasing function of continuation utility  $V$ .
  - (c) The announcement premium is nonnegative for all payoffs that are co-monotone with continuation utility.

Theorem 2 is our revealed preference characterization of the announcement premium. The presence of the announcement premium imposes restrictions on preferences because according to Theorem 1, the A-SDF that prices announcement returns is constructed from marginal utilities. Therefore, data on announcement returns imposes restrictions on the marginal utilities of investors, and marginal utilities can be integrated to obtain the utility function itself. Like with any revealed preference exercise, richer data allows more precise statements about preferences. Here, the assumption of the non-atomic probability space is important, as it allows us to construct a rich enough set of test assets and use the pricing information on these assets to infer the properties of investors' utility functions.

The key insight of Theorem 2 is that the announcement premium is informative about how agents aggregate continuation utilities to compute their certainty equivalence.

It is clear from the example of expected utility in Section 3.2 that there can be no announcement premium under expected utility. The first part of the above theorem implies that the converse of the statement is also true: if we have enough test assets and the announcement premiums for all test assets are zero, we can infer that the representative agent must be an time-separable expected utility maximizer.

The second part of the theorem provides a necessary and sufficient condition for positive announcement premiums. In particular, if the announcement premium for all payoffs that are co-monotone with continuation utility is positive, then we can conclude that the certainty equivalence functional  $\mathcal{I}$  must be monotone with respect to second-order stochastic dominance.

To conclude that  $\mathcal{I}$  is increasing in second-order stochastic dominance, we only need announcement premium to be nonnegative for a relatively small class of assets, that is, assets that satisfy the co-monotonicity condition in (15). However, if we already know that  $\mathcal{I}$  is increasing in second-order stochastic dominance, it is straightforward to show that

announcement premium must be nonnegative for a much larger set of assets. In particular, any payoff of the form  $f(s|z_t^-) + \varepsilon$ , where  $E[\varepsilon|z_t^-, s] = 0$ , must require a nonnegative announcement premium. This observation is useful because in asset pricing implications, many payoffs are not measurable functions of continuation utility.<sup>7</sup>

### 4.3 Generalized risk-sensitive preferences

Theorem 2 motivates the following definition of generalized risk sensitivity.

**Definition 3.** (*Generalized Risk Sensitivity*)

*An intertemporal preference  $\{u, \beta, \mathcal{I}\}$  is said to satisfy (strictly) generalized risk sensitivity, if the certainty equivalence functional  $\mathcal{I}$  is (strictly) monotone with respect to second-order stochastic dominance.*

Under the assumptions of Theorem 1, generalized risk sensitive preference is precisely the class of preferences that requires a nonnegative risk compensation for all assets with announcement payoff co-monotone with investors' continuation utility.

Intuitively, generalized risk sensitivity is a “concavity” property of the certainty equivalence functional. The decision theory literature has studied related properties of the certainty equivalence functional, for example, uncertainty aversion (Gilboa and Schmeidler [33]), and the preference for early resolution of uncertainty (Kreps and Porteus [50]). To clarify the notion of generalized risk sensitivity, in this section, we discuss its relationship with the above decision theoretic concepts. Throughout, we will assume that the intertemporal preference,  $\{u, \beta, \mathcal{I}\}$ , is normalized and satisfies the assumptions of Theorem 1. Also, we assume that either  $u(\mathbf{Y}) = \mathbf{R}$  or  $u(\mathbf{Y}) = \mathbf{R}^+$  as in Strzalecki [67].

**Generalized risk sensitivity and uncertainty aversion** As in Strzalecki [67], we define uncertainty aversion as the quasi-concavity of the certainty equivalence functional  $\mathcal{I}$ :

**Definition 4.** (*Uncertainty aversion*)

*An intertemporal preference  $\{u, \beta, \mathcal{I}\}$  is said to satisfy uncertainty aversion, if the certainty equivalence functional  $\mathcal{I}$  is quasi-concave, that is,  $\forall X_1, X_2 \in L^2(\Omega, \mathcal{F}, P)$ ,  $\mathcal{I}[\lambda X_1 + (1 - \lambda) X_2] \geq \min\{\mathcal{I}[X_1], \mathcal{I}[X_2]\}$ .*

We make the following observations about the relationship between uncertainty aversion and generalized risk sensitivity. We provide the formal proofs in Appendix C.1.

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<sup>7</sup>We thank an anonymous referee for pointing this out.

1. *Quasi-concavity of  $\mathcal{I}$  is sufficient, but not necessary, for generalized risk sensitivity.*

A direct implication of the above result is that all uncertainty-averse preferences can be viewed as different ways to formalize generalized risk sensitivity. Under the assumptions of Theorem 1, they all require a nonnegative announcement premium (for all assets with payoffs co-monotone with continuation utility). These preferences include the maxmin expected utility of Gilboa and Schmeidler [33], the second-order expected utility of Ergin and Gul [30], the smooth ambiguity preferences of Klibanoff, Marinacci, and Mukerji [48], the variational preferences of Maccheroni, Marinacci, and Rustichini [55], the multiplier preferences of Hansen and Sargent [39] and Strzalecki [66], and the confidence preferences of Chateauneuf and Faro [16].

In Appendix C.1, we provide a proof for the sufficiency of quasi-concavity for generalized risk sensitivity. To illustrate that quasi-concavity is not necessary, in the same appendix, we also provide an example that satisfies generalized risk sensitivity, but not quasi-concavity.

2. *If  $\mathcal{I}$  is of the form  $\mathcal{I}[V] = \phi^{-1}(E[\phi(V)])$ , where  $\phi$  is a continuous and strictly increasing function, then generalized risk sensitivity is equivalent to quasi-concavity, which is also equivalent to the concavity of  $\phi$ .*

The certainty equivalence function of many intertemporal preferences takes the above form, for example the the second-order expected utility of Ergin and Gul [30] and the recursive preferences of Kreps and Porteus [50] and Epstein and Zin [28]. For these preferences, generalized risk sensitivity is equivalent to the concavity of  $\phi$ .

3. *We assume that the  $\phi$  function in the representation (16) below is continuous and strictly increasing. Within this class of smooth ambiguity-averse preferences, uncertainty aversion is equivalent to generalized risk sensitivity.*

The smooth ambiguity-averse preference of Klibanoff, Marinacci, and Mukerji [48, 49] can be represented in the form of (12) with the following choice of the certainty equivalence functional:

$$\mathcal{I}[V] = \phi^{-1} \left\{ \int_{\Delta} \phi(E^x[V]) d\mu(x) \right\}. \quad (16)$$

We use  $\Delta$  to denote a set of probability measures indexed by  $x$ ,  $P_x$ . The notation  $E^x[\cdot]$  stands for the expectation under the probability  $P_x$ , and  $\mu(x)$  is a probability measure over  $x$ . In Appendix C, we show that generalized risk sensitivity is equivalent to the concavity of  $\phi$ , which is also equivalent to uncertainty aversion.

### Generalized risk sensitivity and preference for early resolution of uncertainty

In this section, we discuss the relationship between generalized risk sensitivity and preference for early resolution of uncertainty. We leave the asset pricing implications of this relationship to the next section, but focus on the decision theoretical aspect here. Our definition of preference for early resolution follows directly Strzalecki [67]. We first introduce a binary relation,  $\geq_t$  on a subspace of  $\mathcal{C}$  (see also definition 1 of Strzalecki [67]).

Let  $\check{C} : (S, \Sigma) \rightarrow \mathbf{Y}$  be a measurable function. We use  $\check{C}_t^-$  to denote the  $\mathcal{F}_t^-$  measurable consumption plan that depends only on  $s_t^-$  through  $\check{C}(s_t^-)$ :  $\check{C}_t^-(s_0^+, s_1^-, s_1^+, s_2^-, \dots, s_{T-1}^+, s_T^-) = \check{C}(s_t^-)$ . Similarly,  $\check{C}_t^+$  is constructed from  $\check{C}$  by setting  $\check{C}_t^+(s_0^+, s_1^-, s_1^+, s_2^-, \dots, s_{T-1}^+, s_T^-) = \check{C}(s_t^+)$ . In the following lemma, we also use  $y_j \in \mathbf{Y}$  to represent constant consumption plans that are measurable with respect to the trivial  $\sigma$ -field  $\{\emptyset, \Omega\}$ .

#### Definition 5. (Early Resolution)

Let  $\mathbf{C}, \mathbf{C}' \in \mathcal{C}$ , then  $\mathbf{C} \geq_t^- \mathbf{C}'$  if there exists  $\{y_j\}_{j \neq t+1}$  such that  $C_j = C'_j = y_j$ , for  $j = 1, 2, \dots, t, t+2, \dots, T$ , and  $C_{t+1} = \check{C}_t^+$ ,  $C'_{t+1} = \check{C}_t^-$ .

Intuitively,  $\mathbf{C}$  and  $\mathbf{C}'$  are consumption plans that have no uncertainty other than in period  $t+1$ . They provide identical paths of consumption, except that  $C_{t+1}$  depends on the realization of the state  $s_t^+$ , whereas  $C'_{t+1}$  depends on  $s_t^-$ . In other words, under plan  $\mathbf{C}'$ , uncertainty in  $C_{t+1}$  is not known until  $t+1^-$ , and under plan  $\mathbf{C}$ , uncertainty in  $C_{t+1}$  is known earlier, at  $t^+$ . Preference for early resolution of uncertainty is defined as follows.

#### Definition 6. (Preference for Early resolution of Uncertainty)

A system of conditional preferences  $\{\succsim_z\}_{z \in \mathbf{Z}}$  is said to satisfy preference for early resolution of uncertainty if  $\forall \mathbf{C}, \mathbf{C}' \in \mathcal{C}$ ,  $\mathbf{C} \geq_t^- \mathbf{C}'$  implies  $\mathbf{C} \succsim_{z_t^-} \mathbf{C}'$ .

We summarize our main results on the relationship between preference for early resolution of uncertainty and generalized risk sensitivity as follows. The formal proofs for these statements can be found in Appendix C.2 of the paper.

1. *Concavity of the certainty equivalence functional  $\mathcal{I}$  is sufficient for both generalized risk sensitivity and preference for early resolution of uncertainty.*

Note that concavity implies quasiconcavity and therefore generalized risk sensitivity. Theorem 2 of Strzalecki [67] also implies that these preferences satisfy preference for early resolution of uncertainty. As a result, Theorems 2 and 3 of Strzalecki [67] imply

that the variational preference of Maccheroni, Marinacci, and Rustichini [55] satisfies both generalized risk sensitivity and prefers early resolution of uncertainty.

2. If  $\mathcal{I}$  of the form  $\mathcal{I}[V] = \phi^{-1}(E[\phi(V)])$  or is the smooth ambiguity preference,  $\mathcal{I}[V] = \int_{\Delta} \phi(E^x[V]) d\mu(x)$ , where  $\phi$  is strictly increasing and twice continuously differentiable, then generalized risk sensitivity implies preference for early resolution of uncertainty if either of the following two conditions hold.

- (a)  $u(\mathbf{Y}) = \mathbf{R}$  and there exists  $A \geq 0$  such that  $-\frac{\phi''(a)}{\phi'(a)} \in [\beta A, A]$  for all  $a \in \mathbf{R}$ .  
(b)  $u(\mathbf{Y}) = \mathbf{R}^+$  and  $\beta \left[ -\frac{\phi''(k+\beta a)}{\phi'(k+\beta a)} \right] \leq -\frac{\phi''(a)}{\phi'(a)}$  for all  $a, k \geq 0$ .

The above two conditions are the same as Conditions 1 and 2 of Strzalecki [67]. Intuitively, they require that the Arrow-Pratt coefficient of the function  $\phi$  does not vary too much. In both cases, generalized risk sensitivity implies the concavity of  $\phi$ . By Theorem 4 of Strzalecki [67], either of the above conditions implies preference for early resolution of uncertainty.

Because the recursive utility with constant relative risk aversion and constant IES can be represented in the form of (12), with

$$u(C) = \frac{1}{1 - \frac{1}{\psi}} C^{1 - \frac{1}{\psi}}, \quad \mathcal{I}[V] = \phi^{-1}(E[\phi(V)]), \quad (17)$$

where  $\phi(x) = \left[ \frac{1 - \frac{1}{\psi}}{1 - \gamma} x \right]^{\frac{1 - \gamma}{1 - 1/\psi}}$ , it follows from Condition (b) that  $\mathcal{I}$  is quasi-concave and therefore requires a nonnegative announcement premium if and only if  $\gamma \geq \frac{1}{\psi}$ . That is, for this class of preferences, preference for early resolution of uncertainty and generalized risk sensitivity are equivalent.

3. In general, preference for early resolution of uncertainty is neither sufficient nor necessary for generalized risk sensitivity.

In Appendix C, we provide an example of a generalized risk-sensitive preference that violates preference for early resolution of uncertainty, as well as an example of an utility function that prefers early resolution of uncertainty, but does not satisfy generalized risk sensitivity.

4. Generalized risk sensitivity and indifference toward the timing of resolution of

uncertainty implies the following "maxmin expected utility" representation:

$$\mathcal{I}[V] = \inf_{m \in M(F_V)} \int mV dP. \quad (18)$$

In the above expression,  $F_V$  stands for the distribution of  $V$ , and  $M(F_V)$  is a family of densities that depends on  $F_V$ . In the maxmin expected utility of Gilboa and Schmeidler [33], the set of priors is typically specified without referencing the distribution of  $V$ . Therefore, the above representation (18) contains preferences that are not allowed in Gilboa and Schmeidler [33]. If we further require  $\mathcal{I}$  to be quasi-concave, then Theorem 1 in Strzalecki [67] implies that  $M(F_V)$  cannot depend on  $F_V$ , and  $\mathcal{I}[V]$  is the maxmin expected utility in the sense of Gilboa and Schmeidler [33].

## 4.4 Asset pricing implications

**Risk compensation for news** The SDF for non-expected utility models of the form (12) can be written as  $m^*(s) \beta \frac{u'(C_1(s))}{u'(C_0)}$ , as in Equation (9).<sup>8</sup> Theorem 2 has two important implications for the properties of this SDF. First, generalized risk sensitivity is precisely the class of preferences under which  $m^*$  is a decreasing function of continuation utility and therefore enhances risk compensation. Second, generalized risk sensitivity can be identified from the empirical evidence about announcement returns. In fact, as we show below, the empirical evidence on the announcement premium can be used to gauge the quantitative importance of  $m^*$ .

The term  $m^*$  arises in the maxmin expected utility model as the minimizing probability distortion and in the Epstein and Zin [28] utility model when risk aversion does not equal to the inverse of IES. Whenever  $m^*$  decreases in continuation utility, it captures an additional source of risk compensation not present in the expected utility models: News that affects continuation utility requires a premium even when not correlated with contemporaneous consumption. Although this feature is often attributed to uncertainty aversion or preference for early resolution of uncertainty, the essential property here is generalized risk sensitivity. In maxmin expected utility models and recursive utility models, uncertainty aversion and preference for early resolution of uncertainty, respectively, imply an  $m^*$  that is decreasing with respect to continuation utility, because in these models, it constitutes a sufficient condition

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<sup>8</sup>For example, Hansen and Sargent [39] use a risk-sensitive operator to motivate the term  $m^*$  as a decreasing function of continuation utility. In this sense, our notion of generalized risk sensitivity generalizes the risk-sensitive operator of Hansen and Sargent [39].

for generalized risk sensitivity.

In general, preference for early resolution of uncertainty is neither necessary nor sufficient for generalized risk sensitivity. Ai [1] and Epstein, Farhi, and Strzalecki [25] provide calculations that show a quantitatively significant preference for early resolution of uncertainty in calibrated long-run risks models. Our results imply that although in the context of Epstein and Zin [28] preference, preference for early resolution of uncertainty is equivalent to generalized risk sensitivity, the fact that news about future, or long-run risks requires compensation does not depend on preference for early resolution in more general classes of preferences.

**Decomposition of returns by the timing of its realizations** In general, equity returns can be decomposed into an announcement return and a post-announcement return. Using the notations we setup in Section 3 of the paper, the return of an asset can be computed as:

$$R(X) = \frac{X}{P^-} = R_P(X|s) R_A(s),$$

where  $R_A(s)$  is the announcement return defined in (1), and  $R_P(X|s) = \frac{X}{P^+(s)}$  is the post-announcement return (conditioning on  $s$ ). The optimal portfolio choice problem on the post-announcement asset market implies that for each  $s$ , there exists  $y^*(C_1|s)$ , which is a function of  $C_1$ , such that

$$E[y^*(C_1|s) R_P(X|s)|s] = 1, \tag{19}$$

for all post-announcement returns. In our simple model in which  $s$  fully reveals  $C_1$ ,  $y^* = \frac{\beta u'(C_1)}{u'(C_0)}$ . In general,  $y^*$  depends on agents' intertemporal marginal rate of substitution on the post-announcement asset market. Combining Equations (11) and (19), and applying the law of iterated expectation, we have

$$E[m^* y^* \cdot R(X)] = 1.^9 \tag{20}$$

Equation (20) appears in many intertemporal asset pricing models. Equations (11) and (19) provide a decomposition of intertemporal returns into an announcement return and a post-announcement return and a decomposition of the SDF. We make the following comments.

1. Theorem 2 implies that the announcement premium must be compensation for

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<sup>9</sup>We show in Appendix B that this decomposition holds in the fully dynamic model. See equation (14).

generalized risk sensitivity and cannot be compensation for risk aversion associated with the Von Neumann–Morgenstern utility function  $u$ , because the A-SDF,  $m^*$ , depends only on the curvature of the certainty equivalence functional  $\mathcal{I}[\cdot]$ , and not on  $u$ . The announcement premium is determined by the properties of  $\mathcal{I}[\cdot]$ , whereas the post-announcement premium, which is not realized until the action of consumption is completed, reflects the curvature of  $u$ .

2. The entropy bounds of Bansal and Lehmann [6] and Alvarez and Jermann [3] provide some insights about the contribution of  $m^*$  to equity risk premiums. The entropy of any variable  $X$  is  $L(X) = \ln E[X] - E[\ln X]$ . The entropy bound implies that  $L(m^*) \geq E\left[\ln \frac{R_A}{R_0}\right]$  and  $L(m^*y) \geq E\left[\ln \frac{R}{R_f}\right]$ , where we use  $R_0$  for the risk-free announcement return and  $R_f$  for the cumulative risk-free return over the pre- and the post-announcement period. Using the average annual market return in Table 1,  $L(m^*) \geq 3.17\%$  per annum and  $L(m^*y) \geq 5.08\%$  per annum. The announcement returns are large and comprise about 55% of the total equity premium. This clearly suggests a large contribution of  $m^*$  to the market price of risk. On a daily basis, the equity premium on announcement days is 11.2 bps, while the average daily return in the entire sample period is 2.5 bps. The lower bound on the entropy of the SDF on announcement days is roughly five times of that on an average trading day. This evidence implies that  $L(m^*)$  is sizable and that models in which announcement returns are absent or small are misspecified from the perspective of the asset market data.
3. The Hansen-Jaganathan bound (Hansen and Jagannathan [38]) for the SDF leads to a similar conclusion. Equation (11) implies that for any announcement return,  $R_A$ ,  $\sigma[m^*] \geq \frac{1}{R_0} \frac{E[R_A] - R_0}{\sigma[R_A]}$ . Using the the Sharpe ratio for announcement-day returns reported in Table 1, we have:  $\sigma[m^*] \geq 9.85\%$  at the daily level. This bound is much tighter than the Hansen-Jaganathan bound derived from the annualized market returns for the SDF  $m^*y$  during the same period:  $\sigma[m^*y] \geq 2.55\%$ .

**Pre-FOMC announcement drift** As we have remarked before, the theoretical notion of announcement can be interpreted as pre-scheduled macro announcements or informal communications before the officially scheduled announcements. As a result, Theorem 2 is also a statement about the pre-announcement drift. That is, if the contents of the announcements are communicated to the public before the pre-scheduled announcements, then these communications will be associated with realizations of risk premiums under generalized risk sensitive. In the continuous-time example in the next section, we will

demonstrate our model’s implications for both the announcement premium and the pre-announcement drift.

## 5 Continuous-time examples

In this section, we use a continuous-time setup to discuss the implications of several examples of dynamic preferences for the announcement premium. The continuous-time setup highlights the distinction between the compensation for generalized risk sensitivity that is instantaneously realized upon announcements and the risk premium that investors receive incrementally as shocks to consumption materialize slowly over time. It also allows us to provide precise statements about the announcement premium implied by some time-non-separable utilities not allowed by our representation (12).

### 5.1 Consumption and information

We consider a continuous-time representative agent economy, in which the growth rate of aggregate consumption contains a predictable component,  $x_t$  and an i.i.d. component modeled by increments of a Brownian motion:

$$\frac{dC_t}{C_t} = x_t dt + \sigma dB_{C,t}.$$

We assume that  $x_t$  is a continuous-time AR(1) process (an Ornstein-Uhlenbeck process) not observable to the agents of the economy. The law of motion of  $x_t$  is

$$dx_t = a_x (\bar{x} - x_t) dt + \sigma_x dB_{x,t}, \tag{21}$$

where  $B_{C,t}$  and  $B_{x,t}$  are independent standard Brownian motions.

The representative agent in the economy can use two sources of information to update beliefs about  $x_t$ . First, the realized consumption path contains information about  $x_t$ , and second, at pre-scheduled discrete time points  $T, 2T, 3T, \dots$ , additional signals about  $x_t$  are revealed through announcements. For  $n = 1, 2, 3, \dots$ , we denote  $s_n$  as the signal observed at time  $nT$  and assume  $s_n = x_{nT} + \varepsilon_n$ , where  $\varepsilon_n$  is normally distributed with mean zero and variance  $\sigma_S^2$ .

Because the posterior distribution of  $x_t$  is Gaussian, it can be summarized by the first two moments. We define  $\hat{x}_t = E_t[x_t]$  as the posterior mean and  $q_t = E_t[(x_t - \hat{x}_t)^2]$  as the

posterior variance, respectively, of  $x_t$  given information up to time  $t$ . At time  $t = nT$ , where  $n$  is an integer, agent updates beliefs using Bayes' rule:

$$\hat{x}_{nT}^+ = \frac{1}{q_{nT}^+} \left[ \frac{1}{\sigma_S^2} s + \frac{1}{q_{nT}^-} \hat{x}_{nT}^- \right]; \quad \frac{1}{q_{nT}^+} = \frac{1}{\sigma_S^2} + \frac{1}{q_{nT}^-}, \quad (22)$$

where  $\hat{x}_{nT}^+$  and  $q_{nT}^+$  are the posterior mean and variance after announcements, and  $\hat{x}_{nT}^-$  and  $q_{nT}^-$  are the posterior mean and variance before announcements, respectively.

In the interior of  $(nT, (n+1)T)$ , the agent updates her beliefs based on the observed consumption process using the Kalman-Bucy filter:

$$d\hat{x}_t = a[\bar{x} - \hat{x}_t] dt + \frac{q(t)}{\sigma} d\tilde{B}_{C,t}, \quad (23)$$

where the innovation process,  $\tilde{B}_{C,t}$  is defined by  $d\tilde{B}_{C,t} = \frac{1}{\sigma} \left[ \frac{dC_t}{C_t} - \hat{x}_t dt \right]$ . The posterior variance,  $q(t)$ , satisfies the Riccati equation:

$$dq(t) = \left[ \sigma_x^2 - 2a_x q(t) - \frac{1}{\sigma^2} q^2(t) \right] dt. \quad (24)$$

## 5.2 Generalized risk sensitive preferences

**Preferences and the stochastic discount factor** We first consider a simple example of generalized risk sensitivity. We assume that the representative agent has a Kreps-Porteus utility with  $\gamma > \frac{1}{\psi}$  and specify the continuous-time preference as the limit of the discrete-time recursion in (12).<sup>10</sup> Over a small time interval,  $\Delta > 0$ ,

$$V_t = (1 - e^{-\rho\Delta}) u(C_t) + e^{-\rho\Delta} \mathcal{I}[V_{t+\Delta} | \hat{x}_t, q_t], \quad (25)$$

where  $u$  and  $\mathcal{I}[\cdot | \hat{x}_t, q_t]$  are given in equation (17). To derive closed-form solutions, we focus on the limiting case of  $\psi = 1$ , where  $u(C) = \ln C$  and  $\mathcal{I}[V] = \frac{1}{1-\gamma} \ln E[e^{(1-\gamma)V}]$ . This preference can also be interpreted as the multiplier robust control preference of Hansen and Sargent [39].

As in previous discrete-time examples, the stochastic discount factor over a small interval  $(t, t + \Delta)$  is given by

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<sup>10</sup>The continuous-time version of this preference is developed in Duffie and Epstein [22, 21].

$$SDF_{t,t+\Delta} = e^{-\rho\Delta} \frac{u'(C_{t+\Delta})}{u'(C_t)} \frac{e^{(1-\gamma)V_{t+\Delta}}}{E_t[e^{(1-\gamma)V_{t+\Delta}}]}. \quad (26)$$

Clearly, the term  $m_{t+\Delta}^* = \frac{e^{(1-\gamma)V_{t+\Delta}}}{E_t[e^{(1-\gamma)V_{t+\Delta}}]}$  is a density and can be interpreted as a probability distortion.

**Announcement premiums** We assume that the stock market is the claim to the following dividend process:

$$\frac{dD_t}{D_t} = [\bar{x} + \phi(x_t - \bar{x})] dt + \phi\sigma dB_{C,t}, \quad (27)$$

and allow the leverage parameter,  $\phi > 1$ , so that dividends are more risky than consumption, as in Bansal and Yaron [9].

In the interior of  $(nT, (n+1)T)$ , all state variables,  $C_t$ ,  $\hat{x}_t$ , and  $q_t$  in (26) are continuous functions of  $t$ . As a result, as  $\Delta \rightarrow 0$ ,  $SDF_{t,t+\Delta} \rightarrow 1$ , and the equity premium on any asset must converge to zero. In fact, using a first-order approximation, we show in Appendix D.1 that the equity premium over the interval  $(t, t + \Delta)$  must be proportional to the holding period  $\Delta$ :

$$\left[ \gamma\sigma + \frac{\gamma-1}{a_x + \rho} \frac{q_t}{\sigma} \right] \left[ \phi\sigma + \frac{\phi-1}{a_x + \rho} \frac{q_t}{\sigma} \right] \Delta. \quad (28)$$

In contrast, let  $t = T - \frac{1}{2}\Delta$ , so that the interval  $(t, t + \Delta)$  always contains an announcement. As  $\Delta \rightarrow 0$ ,  $e^{-\rho\Delta} \frac{u'(C_{t+\Delta})}{u'(C_t)} \rightarrow 1$ , but  $m_{t+\Delta}^*$  does not. Because the value function  $V_{t+\Delta}$  depends on the announcement, and  $E_t[e^{(1-\gamma)V_{t+\Delta}}]$  does not, the probability distortion does not disappear as  $\Delta \rightarrow 0$ . In Appendix D.1, we show that in the limit, the announcement premium can be approximated by:

$$\frac{\gamma-1}{a_x + \rho} \frac{\phi-1}{a_x + \rho} (q_T^- - q_T^+). \quad (29)$$

We make the following two observations.

1. As  $\Delta \rightarrow 0$ , the market equity premium vanishes without the presence of an announcement, but stays strictly positive in the presence of an announcement, as shown in Equations (28) and (29).

In Figure 3, we plot the average hourly market return around announcements. We choose standard parameters used in the long-run risks literature, the details of which

Figure 3: **Average hourly returns around announcements**

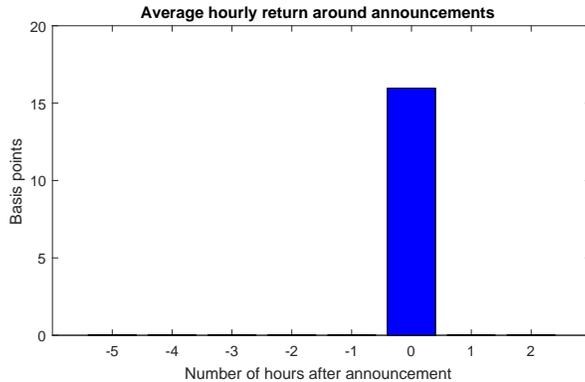


Figure 3 plots the model implied average hourly return around pre-scheduled announcements.

are provided in Appendix D.2. The premium realized during the announcement hour is 17 bps, whereas the average return during the non-announcement hours is much smaller by comparison.

- As shown in Equation (29), the magnitude of the announcement premium is proportional to the amount of uncertainty reduction,  $q_T^- - q_T^+$ , and is increasing in the persistence of the impact of the uncertainty. Increases in the persistence of  $x_t$ , which is inversely related to  $a_x$ , have two effects.<sup>11</sup> First, it implies that revelations of  $x_t$  have a stronger impact on continuation utility  $V_{t+\Delta}$  and therefore  $m_{t+\Delta}^*$ . Second, more persistence in the expected growth rate of cash flow is also associated with a stronger effect of the announcements on the price-to-dividend ratio of the equity. Together, they imply that the announcement premium must increase with the persistence of  $x_t$ . The above observation implies that the heterogeneity in the magnitude of the premium for different macro announcements can potentially be explained by the differences in their informativeness and the significance of their welfare implications.
- In our endowment economy model, although instantaneous consumption  $C_T$  does not respond to the announcement made at time  $T$ , future consumption does, as  $x_T$  is the expected consumption growth rate. Our results on the announcement premium and the pre-announcement drift below continue to hold in neoclassical production economies where  $x_t$  is interpreted as expected productivity growth and consumption is allowed to respond instantaneously to announcements about  $x_t$ . As we remark earlier, the

<sup>11</sup>The autocorrelation between  $x_t$  and  $x_{t+\Delta}$  is roughly  $1 - a_x\Delta$  for small values of  $\Delta$ .

instantaneous reaction of consumption to announcements will contribute to a small and negative premium, but the overall announcement will be positive as long as we allow for a significant generalized risk sensitivity in preferences.<sup>12</sup>

Figure 4: **Pre-FOMC announcement drift**

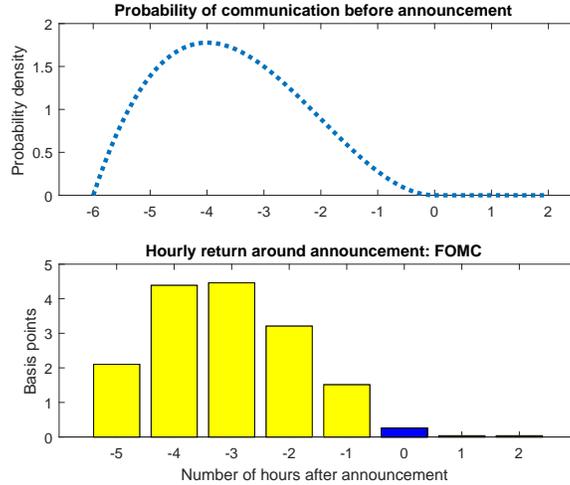


Figure 4 plots the probability density of communication before announcement (top panel) and the average hourly return around announcements (bottom panel).

**The pre-FOMC announcement drift** As we explain earlier in the paper, the announcement in our theory represents a resolution of macroeconomic uncertainty. It can be due to pre-scheduled macro announcements, or comes from informal communication from Fed officials to the public, as documented by Cieslak, Morse, and Vissing-Jorgensen [18]. Our model implies a pre-announcement drift in the presence of such communications.

In Figure 4, we plot the implication of our model on pre-FOMC announcement drift, assuming communications occur before announcements. For simplicity, we assume that communication, whenever occurs, fully reveals  $x_t$ , and we plot the probability density of communication at time  $t$  ( $y$ -axis) as a function of  $t$  ( $x$ -axis) in the top panel of Figure 4. In the bottom panel, we plot the model-implied average hourly market return around announcements. Note that the magnitude of the announcement premium is proportional to the probability of the occurrence of communication. The announcement premium peaks in

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<sup>12</sup>We have solved a model with neoclassical production technology and obtain similar results on announcement premium and pre-announcement drift. The results are available upon request.

the hours with the highest probability of communication, and converges to zero as  $t \rightarrow 0$ , because communication occurs with probability one before the pre-scheduled announcement time. This pattern of the pre-announcement drift implied by our model is very similar to its empirical counterpart in Figure 1.

To evaluate the dynamics of nominal bond returns and the announcement premiums in bond markets, we also solve a more extensive model related to Piazzesi and Schneider [61] and Bansal and Shaliastovich [8] with growth and inflation dynamics. The bond announcement premiums, like in the data, are 3 bps. In small samples (comparable to those used in earlier empirical work) the pre-announcement drift is statistically absent in bonds returns because the premiums are very small, but are present in equity markets, as in the data.<sup>13</sup>

### 5.3 Time-non-separable preferences

In this section, we analyze several examples of time-non-separable preferences that are not allowed by our representation (12). We continue to use the specification of consumption and information structure in Section 5.1. We assume that the representative agent ranks intertemporal consumption plans according to the following utility function:

$$E \left[ \int_0^\infty e^{-\rho t} u(C_t + bH_t) dt \right], \quad (30)$$

for some appropriately defined habit process  $\{H_t\}_{t=0}^\infty$ , which we specify below. The above representation includes the external habit model of Campbell and Cochrane [14], the internal habit model of Constantinides [19] and Boldrin, Christiano, and Fisher [12], and the consumption substitutability model (see Dunn and Singleton [23] and Heaton [44]) as special cases. We make the following observations and provide the detailed proofs in Appendix D.2 of the paper.

1. *The external habit model has zero announcement premium.*

Suppose  $-1 < b < 0$  and  $H_t$  is a habit process defined as

$$H_t = \left( 1 - \int_0^t \xi(t, s) ds \right) H_0 + \int_0^t \xi(t, s) C_s ds, \quad (31)$$

where  $\{\xi(t, s)\}_{s=0}^t$  is a nonnegative weighting function that satisfies the regularity conditions (D.11)-(D.13) in Appendix D.2. In the external habit model, the

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<sup>13</sup>This evidence is available upon request from the authors.

consumption,  $\{C_s\}_{s=0}^t$  in equation (31) is interpreted as the aggregate consumption that is exogenous to the choice of the agent. Our specification is therefore a generalization of the Campbell and Cochrane [14] model in continuous time. Because the habit stock  $H_t$  is exogenous, as in expected utility models, marginal utilities depend only on current-period consumption, and there can be no announcement premium.

2. *The internal habit model generates a negative announcement premium.*

We continue to assume  $-1 < b < 0$  and (31), except that  $\{C_s\}_{s=0}^t$  in equation (31) is interpreted as the agent's own consumption choice. This model is a generalized version of the Constantinides [19] model. As we show in the appendix, the marginal utility of  $C_t$  can be written as:

$$e^{-\beta t} \left\{ u'(C_t + bH_t) + bE \left[ \int_0^\infty e^{-\beta s} \xi(t+s, t) u'(C_{t+s} + bH_{t+s}) ds \middle| \hat{x}_t, q_t \right] \right\}. \quad (32)$$

We show in Appendix D.2 that (32) is an increasing function of  $\hat{x}_t$ . Therefore, the internal habit model implies a negative premium for any return positively correlated with  $\hat{x}_t$ . Intuitively, an increase in current consumption,  $C_t$ , lowers the future utility flow. Positive news about  $\hat{x}_t$  indicates higher consumption in the future and dampens the negative effect of current consumption on future utility. As a result, when  $b < 0$ , the marginal utility in (32) is increasing with respect to  $\hat{x}_t$ .

3. *The consumption substitutability model produces a positive announcement premium.*

Suppose the agent's preference is defined by (30) and (31) with  $b > 0$ . In this case, past consumption increases current period utility. It is straightforward to show that the marginal utility (32) is a decreasing function of  $\hat{x}_t$ . Therefore, the announcement premium is positive for any asset with a return positively correlated with  $\hat{x}_t$ . Note that even though the presence of consumption substitutability produces a positive announcement premium, its presence lowers agents' effective risk aversion and exacerbates the equity premium puzzle, as emphasized by Gallant, Hansen, and Tauchen [32].

## 6 Conclusion

Motivated by the fact that a large fraction of the market equity premium is realized on a small number of trading days with significant macroeconomic announcements, in this paper,

we provide a revealed preference analysis of the equity premium for macro announcements. We show that a positive announcement premium is equivalent to generalized risk sensitivity, that is, investors' certainty equivalence functional being increasing in second-order stochastic dominance. We demonstrate that generalized risk sensitivity is exactly the class of preferences that demands risk compensation for news that affects continuation utilities, or "long-run risks". As a result, our theoretical framework implies that the announcement premium can be interpreted as asset-market-based evidence for a broad class of non-expected utility models that have this feature.

Due to its high-frequency nature, continuous-time models are particularly convenient for the study of the announcement premium and the pre-announcement drift in, for example, the FOMC announcements. We show in a continuous-time model that the pre-announcement drift can arise in environments in which the information about the announcement is communicated to the market prior to the scheduled announcement.

Several related topics may provide promising directions for future research. A natural extension of the current paper would be to provide a characterization of generalized risk sensitivity in the continuous-time framework. Such conditions may bear interesting connections with the recent paper by Skiadas [65], who provides a continuous-time analysis of certainty equivalence functionals of non-expected utilities. Another idea worth careful exploration would be to evaluate if asset-market frictions related to liquidity or slow-moving capital, as emphasized by Duffie [20], may contribute/affect the announcement premium and the pre-announcement drift. Finally, our theory has several implications that may be tested empirically. For example, our analysis implies that the magnitude of the announcement premiums is determined by the informativeness of the announcement about the future course of the economy. In addition, there is a sizable literature that documents significant excess returns at the firm level around earnings announcements, for example, Chari, Jagannathan, and Ofer [15] and Ball and Kothari [4]. To the extent that these earnings announcements carry news about the macroeconomy, these facts can be consistent with our theory. Further exploration of this issue may also be an interesting direction for future research.

# Data Appendix

Here, we describe the details of our empirical evidence on the macroeconomic announcement premium.

**Data description** We focus on the top five macroeconomic news ranked by investor attention among all macroeconomic announcements at the monthly or lower frequency. They are unemployment/non-farm payroll (EMPL/NFP) and producer price index (PPI) published by the U.S. Bureau of Labor Statistics (BLS), the FOMC statements, the gross domestic product (GDP) reported by U.S. Bureau of Economic Analysis, and the Institute for Supply Management’s Manufacturing Report (ISM) released by Bloomberg.<sup>14</sup>

The EMPL/NFL and PPI are both published at a monthly frequency and their announcement dates come from the BLS website. The BLS began announcing its scheduled release dates in advance in 1961 which is also the start date for our EMPL/NFL announcements sample. The PPI data series start in 1971.<sup>15</sup> There are a total of eight FOMC meetings each calendar year and the dates of FOMC meetings are taken from the Federal Reserve’s web site. The FOMC statements begin in 1994 when the Committee started announcing its decision to the markets by releasing a statement at the end of each meeting. For meetings lasting two calendar days we consider the second day (the day the statement is released) as the event date. GDP is released quarterly beginning from 1997, which is the first year that full data are available, and the dates come from the BEA’s website.<sup>16</sup> Finally, ISM is a monthly announcement with dates coming from Bloomberg starting from 1997. The last year for which we collect data on all announcements is 2014.

**Equity return on announcement days** Table 1 reports the mean, standard deviation, and Sharpe ratio of the annual return of the market, and the same moments for the return on announcement days. The announcement returns are calculated as the cumulative

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<sup>14</sup>Both unemployment and non-farm payroll information are released as part of the Employment Situation Report published by the BLS. We treat them as one announcement.

<sup>15</sup>While the CPI data is also available from the BLS back to 1961, once the PPI starts being published it typically precedes the CPI announcement. Given the large overlap in information between the two macro releases much of the "news" content in the CPI announcement will already be known to the market at the time of its release. For this reason we opt in favor of using PPI.

<sup>16</sup>GDP growth announcements are made monthly according to the following pattern: in April the advance estimate for Q1 GDP growth is released, followed by a preliminary estimate of the same Q1 GDP growth in May and a final estimate given in the June announcement. Arguably most uncertainty about Q1 growth is resolved once the advance estimate is published and most learning by the markets will occur prior to this release. For this reason we will focus only on the 4 advance estimate release dates every year.

market returns on announcement days within a year. This is equivalent to the return of a strategy that long the market before the day of the pre-scheduled news announcements, hold it on the trading day with the news announcement, and sell immediately afterwards.

In Table 2, we compare the average daily stock market return on news announcement days, which we denote as  $t$ , that on the day before the news announcement ( $t - 1$ ), and that after the news announcement ( $t + 1$ ). We present our results separately for each of the five news announcement and for all news, where standard errors are shown in parentheses.<sup>17</sup> Excess market returns are taken from Kenneth French's web site. The vast majority of announcements are made on trading days. When this is not the case we assign the news release to the first trading day that follows the announcement.

**High frequency returns** In Figure 1, we plot the average stock market returns over 30-minute intervals before and after news announcements. Here we use high frequency data for the *S&P* 500 SPDR that runs from 1997 to 2013 and comes from the TAQ database. Each second the median price of all transactions occurring that second is computed.<sup>9</sup> The price at lower frequency intervals (for example 30-min) is then constructed as the price for the last (most recent) second in that interval when transactions were observed. The exact time at which the news are released are reported by Bloomberg. In Figure 1, the return at time 0 is the 30-minute news event return. Employment/Non-farm payroll, GDP and PPI announcements are made at 8:30 AM before the market begins. In these cases we will consider the 30-minute news event return to be the return between 4:00 PM (close of trading) of the previous day and 9:30 AM when the market opens on the day of the announcement. The 30-minute event return for ISM announcements, which are made at 10:00AM, covers the interval between 9:30 AM and 10:00 AM of the announcement day. Finally, the timing of the FOMC news release varies. We add 30 minutes to the announcement time to account for the press conference after the FOMC meeting.<sup>18</sup>

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<sup>17</sup>They are Newey-West standard errors (5-lags) of an OLS regression of excess returns on event dummies.

<sup>18</sup>For example if a statement is released at 14:15 PM, we add 30 minutes for the press conference that follows and then we round the event time to 15:00 PM.

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**Table 1**  
**Market Return on Announcement and Non-announcement Days**

	# days p. a.	daily prem.	daily std.	premium p.a.
Market	252	2.46 <i>bps</i>	98.2 <i>bps</i>	6.19%
Announcement	30	11.21 <i>bps</i>	113.8 <i>bps</i>	3.36%
No Announcement	222	1.27 <i>bps</i>	95.9 <i>bps</i>	2.82%

This table documents the mean and standard deviation of the market excess return during the 1961-2014 period. The column " # days p.a." is the average number of trading days per annum, the second column is the daily market equity premium on all days, that on announcement days, and that on days with no announcement. The column "daily std." is the standard deviation of daily returns. The column "premium p.a." is the cumulative market excess returns within a year, which is compute by multiplying the daily premium by the number of event days and converting it into percentage points.

**Table 2**  
**Average Daily Return around Announcements (Basis Points)**

	-1	0	+1
All Announcements	1.77 (2.86)	11.21 (2.96)	0.84 (3.22)
All w/o FOMC	0.69 (2.78)	9.28 (3.05)	0.99 (3.24)
No Announcement	— — —	1.27 (0.91)	— — —

This table documents the average daily return during the 1961-2014 period in basis points on event days (column "0"), that before event days (column "-1"), and that after event days (column "+1") with the standard error of the point estimates in parenthesis. The row "All announcements" is the average event day return on all announcement days; "All w/o FOMC" is the average event day return on all announcement days except FOMC announcement days; and "No announcements" is the average daily return on non-announcement days.

**Table 3**  
**Average hourly return around announcements**

	-5	-4	-3	-2	-1	0	1	+2
All Announcements	0.72 (1.45)	3.13 (1.37)	1.96 (0.93)	-0.21 (0.99)	-1.55 (1.28)	6.46 (2.71)	-2.75 (1.98)	1.84 (1.69)
FOMC	13.24 (4.95)	12.97 (5.12)	7.62 (2.56)	3.34 (2.21)	4.75 (1.60)	0.16 (4.08)	5.81 (5.99)	-6.13 (7.45)
All w/o FOMC	-0.40 (1.52)	0.39 (1.26)	0.90 (1.02)	-0.73 (1.10)	-3.00 (1.47)	7.25 (3.16)	-3.72 (2.03)	2.55 (1.34)

This table reports the average hourly return around announcements during the 1997-2013 period, with standard errors of the point estimates in parenthesis. The announcement time is normalized to zero.  $k$  stands for the hourly return between hour  $k - 1$  and hour  $k$ .