Asset Pricing with Endogenously Uninsurable Tail Risks

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asset pricing with uninsurable idiosyncratic risks

- Challenges for asset pricing models
  - generate a high volatility/predictability of pricing kernel
  - produce a large exposure of firms' cash flow to aggregate shocks
- A theory of asset pricing based on uninsurable idiosyncratic risks
  - risk sharing is endogenously restricted due to lack of commitment
- Takeaway: agency frictions+ general equilibrium amplify risk premia
  - agency frictions disciplined with data on individual labor earnings

framework and results

- Embed optimal contracting in a general-equilibrium framework
  - diversified owners of firms insure workers using long-term compensation contracts
  - contracts restricted by limited commitment of both parties
- Key results
  - firm side limited commitment: tail risks in earnings
  - consumption-share of capital owners is procyclical and more persistent than output
  - cashflow exposure due to endogenous operating leverage

relation to literature

- Exogenously incomplete markets and asset pricing
    - Mankiw (1986), Constantinides and Duffie (1996), Constantinides and Ghosh(2015), Schmidt (2015), Krueger and Lustig (2010)
- Limited commitment
    - Kehoe and Levine (1993), Lustig, Syverson and Van Nieuwerburgh (2011), Ai and Li (2015)
    - Asset pricing: Alvarez and Jermann (2000, 2001), Chien and Lustig (2009)
- Aggregate stock returns and cyclical factor shares
    - Greenwald, Lettau, and Ludvigson (2014), Favilukis and Lin (2015)

setup

physical environment

- **Time:** Discrete, infinite horizon, t=0,1,2 . . .
- **Demography:** Two groups agents: Workers, Owners
- **Preferences:** Homogeneous preferences of Epstein-Zin type
- **Technology:** Firm j's output: $y_{j,t} = Y_t s_{j,t}$
- **Shocks:**
    - $\log Y_{t+1} - \log Y_t = g_{t+1} \sim$ Markov
    - $\log s_{j,t+1} - \log s_{j,t} = \varepsilon_{j,t+1} \sim$ i.i.d across j
    - distribution of $\varepsilon_j | g$ depends on g; $\mathbb{E}\left[e^{\varepsilon}\right] = 1$

contracting environment

- Capital owners diversified across firms
- Long-term labor compensation contracts $\mathbf{C} = \{C_t(g^t, \varepsilon^t)\}_t$
- Firm and worker values

$$\textbf{Firm: } V_t[\mathbf{C}|g^t,\varepsilon^t] = y_t - C_t + \mathbb{E}_t\left[\underbrace{\Lambda_{t,t+1}}_{Owners' SDF} V_{t+1}\right]$$

$$\textbf{Worker: } U_t\left[\mathbf{C}|g^t,\varepsilon^t\right] = \left((1-\beta)\, C_t^{1-\frac{1}{\psi}} + \beta \mathbb{M}_t\left[U_{t+1}\right]^{1-\frac{1}{\psi}}\right)^{\frac{1}{1-\frac{1}{\psi}}}$$

where $\mathbb{M}_t\left[U_{t+1}\right] \equiv \left(\mathbb{E}_t\left[U_{t+1}\right]^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$ is the CE operator

agency frictions

- Firm side limited commitment

$$V_t[\mathbf{C}|g^t, \varepsilon^t] \geq 0 \quad \text{for all histories}$$

- Worker side limited commitment

$$U_t[\mathbf{C}|g^t, \varepsilon^t] \geq \underline{u}(g_t)y_t \quad \text{for all histories}$$

recursive equilibrium

- Optimal contract recursive in promised value to worker U and y
- For aggregates need to keep track of Φ: joint distribution of U, y
- Given SDF $\Lambda(g'|\Phi, g)$ and law of motion, $\Phi' = \Gamma(g'|\Phi, g)$ firms
    - choose compensation $C(U, y)$ and $U'(g', \varepsilon'|U, y)$ s.t. limited commitment
- Markets clear

$$x(\Phi, g)Y + \int C(U, y) d\Phi(U, y) = Y$$

- $\Lambda(g'|g, \Phi)$ consistent with $x(\Phi, g)$ and $\Gamma(g'|g, \Phi)$ consistent with optimal contract

theoretical insights

optimal contract generates tail risk

- With full commitment
  - capital owners fully insure workers against idiosyncratic shocks

- With limited commitment
  - for sufficiently adverse productivity shock, NPV constraint binds
  - further falls in productivity lead to drops in labor compensation

  $\Longrightarrow$ tail risks in earnings

optimal contract generates tail risk

- With full commitment
  - workers' share inversely proportional to shocks

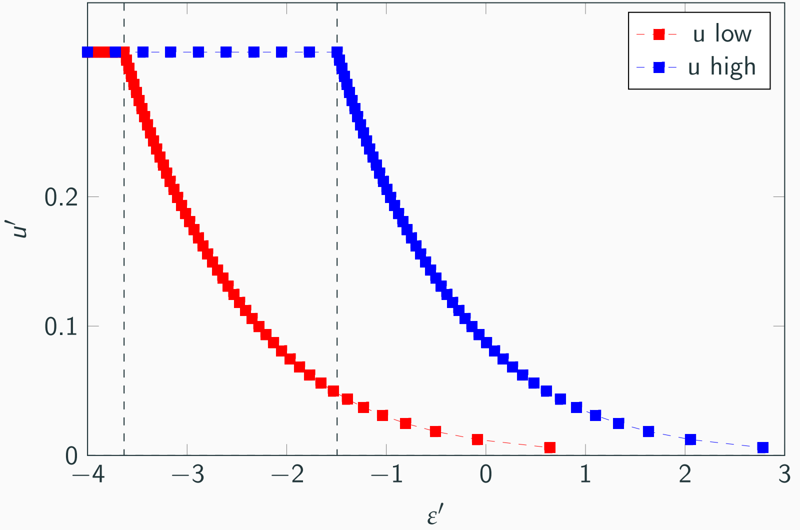
$$u'\left(u, g', \varepsilon' \middle| \phi, g\right) \propto e^{-\varepsilon'}$$

- With limited commitment
  - lack of insurance for sufficiently adverse shocks

$$u'\left(u, g', \varepsilon' \middle| \phi, g\right) = \bar{u}(g', \phi'), \quad \forall \varepsilon' \leq \bar{\varepsilon}(u|g', g, \phi)$$

$\bar{u}$ implicitly defined: $v(\bar{u}(g, \phi)) = 0$

normalized continuation values



higher risk premia

- How do aggregate shocks affect
  - marginal investors or capital owners' consumption
  - firm valuations
- Interaction of optimal contracting with general equilibrium
  - intuition using a simple example
  - quantitative results for the general case later

a simple example

- Shocks

  (a) persistent aggregate risk

$$g_t = g_1 \text{ for all } t \geq 1$$

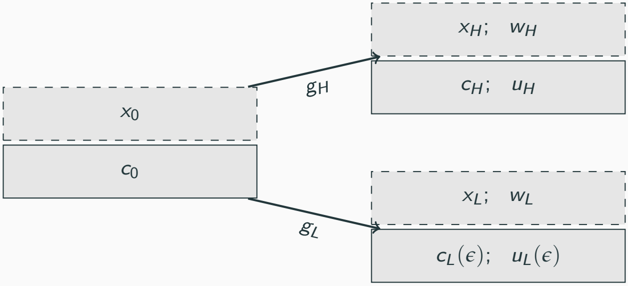
  (b) countercylical idiosyncratic risk

$$\varepsilon | g_L \sim \text{ negative exponential } (\lambda) \quad \varepsilon | g_H \sim \text{degenerate}$$

- Two period optimal contracting

$$C_t = \alpha y_t \text{ for all } t \geq 2$$

simple example: time line and notation



procyclical consumption share of capital owners

Risk aversion $= \gamma$, IES$=\psi$

Proposition

(i) Expected utility: If $\gamma = \frac{1}{\psi}$ then $x_H < x_L$

(ii) Recursive utility: There exists $\gamma^* \in (\frac{1}{\psi}, 1 + \lambda)$ such that $\gamma > \gamma^*$ implies $x_H > x_L$

equilibrium allocation

- optimal risk sharing

$$\underbrace{\left(\frac{x_L}{x_H}\right)^{-\frac{1}{\psi}}\left(\frac{w_L}{w_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of capital owner}}=\underbrace{\left(\frac{e^{\bar{\varepsilon}}c_L\left(\bar{\varepsilon}\right)}{c_H}\right)^{-\frac{1}{\psi}}\left(\frac{e^{\bar{\varepsilon}}u_L\left(\bar{\varepsilon}\right)}{u_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of marginal worker}}$$

- Market clearing

$$x_H + c_H = 1$$

$$x_L + \int e^{\varepsilon} c_L\left(\varepsilon\right) f\left(\varepsilon \middle| g_L\right) = 1$$

intuition: expected utility

- optimal risk sharing **with expected utility**

$$\underbrace{\left(\frac{x_L}{x_H}\right)^{-\frac{1}{\psi}}}_{\text{MU of capital owner}} = \underbrace{\left(\frac{e^{\bar{\varepsilon}} c_L\left(\bar{\varepsilon}\right)}{c_H}\right)^{-\frac{1}{\psi}}}_{\text{MU of marginal worker}}$$

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$$\underbrace{\frac{x_L}{x_H}}_{\text{consumption of capital owner}} = \underbrace{\frac{e^{\bar{\varepsilon}} c_L\left(\bar{\varepsilon}\right)}{c_H}}_{\text{consumption of marginal worker}}$$

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- In booms there is no idiosyncratic risk

$$c_H = 1 - x_H$$

- In recessions marginal agent consumes more than average

$$e^{\bar{\varepsilon}} c_L\left(\bar{\varepsilon}\right) > 1 - x_L$$

intuition: expected utility

- optimal risk sharing **with expected utility**

$$\underbrace{\frac{x_L}{x_H}}_{\text{consumption of capital owner}} > \underbrace{\frac{1-x_L}{1-x_H}}_{\text{consumption of average worker}}$$

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$\Longrightarrow$ countercylical consumption share of capital owner

$$x_L > x_H$$

intuition: expected utility

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intuition: recursive utility

- optimal risk sharing more generally

$$\underbrace{\left(\frac{x_L}{x_H}\right)^{-\frac{1}{\psi}}\left(\frac{w_L}{w_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of capital owner}} = \underbrace{\left(\frac{e^{\bar{\varepsilon}}c_L\left(\bar{\varepsilon}\right)}{c_H}\right)^{-\frac{1}{\psi}}\left(\frac{e^{\bar{\varepsilon}}u_L\left(\bar{\varepsilon}\right)}{u_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of marginal worker}}$$

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- New force: tail risks in the *future* affect current marginal utilities

$$u_L\left(\bar{\varepsilon}\right)\to 0 \text{ as } \gamma\to 1+\lambda$$

intuition: recursive utility

- optimal risk sharing more generally

$$\underbrace{\left(\frac{x_L}{x_H}\right)^{-\frac{1}{\psi}} \left(\frac{w_L}{w_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of capital owner}} = \underbrace{\left(\frac{e^{\bar{\varepsilon}} c_L\left(\bar{\varepsilon}\right)}{c_H}\right)^{-\frac{1}{\psi}} \left(\frac{e^{\bar{\varepsilon}} u_L\left(\bar{\varepsilon}\right)}{u_H}\right)^{\frac{1}{\psi}-\gamma}}_{\text{MU of marginal worker}}$$

- New force: tail risks in the *future* affect current marginal utilities

$$u_L\left(\bar{\varepsilon}\right) \to 0 \text{ as } \gamma \to 1+\lambda$$

$\implies$ procyclical consumption share of capital owner

$$x_H > x_L$$

optimal contract generates operating leverage

- Aggregate dividends more procyclical than output
  - follows from procyclical consumption share of capital owners

- Individual firm's risk exposure increases in obligations to workers
  - negative shocks predict higher expected returns
  - consistent with cross-sectional evidence on equity returns

Proposition

For $\gamma > \gamma^* \in (\frac{1}{\psi}, 1+\lambda)$, we have $\frac{d}{du_0}\left[\frac{v_H(u_0)}{E[v_L(\varepsilon, u_0)]}\right] > 0$

quantitative analysis

calibration

- Aggregate shocks:

$$\ln Y_{t+1} - \ln Y_t = g_{t+1} + \sigma(g_{t+1})\eta_{t+1}$$

  g_t is a two state Markov process and η_t is i.i.d standard Gaussian
  - match moments of post-war aggregate consumption data
- Idiosyncratic shocks: flexible mixture distributions, separately for booms and recession
  - match cross-sectional moments of labor earnings conditional on aggregate shock using Guvenen et al. (2014) and PSID
- Existing workers exit the work force at rate $\kappa > 0$ per period
- Workers initial utility and outside option proportional to first best utility

model fit

Moments	Data		Model	
	SSA	PSID	SSA	PSID
Std. of 1 yr earnings growth in booms	0.53	0.31	0.55	0.33
Std. of 1 yr earnings growth in recessions	0.54	0.32	0.59	0.33
log 95 - log 5 earnings in booms	3.06	1.71	3.79	2.18
log 95 - log 5 earnings in recessions	3.05	1.62	3.85	2.28
log 90 - log 10 earnings in booms	2.12	1.254	2.89	1.70
log 90 - log 10 earnings in recessions	2.14	1.158	2.96	1.72
log 75 - log 25 earnings in booms	0.98	0.61	1.52	0.92
log 75 - log 25 earnings in recessions	0.98	0.57	1.45	0.89
Kelly skewness, 1 yr earnings growth booms	0.5%	0.0%	0.0%	1.6%
Kelly skewness, 1 yr earnings growth recessions	-8.9%	-5.9%	-4.9%	-3.0%
Average labor share in aggregate consumption	70%		69.6%	68.4%
Std. of labor share	5%		4.8%	5.13%

results

asset pricing

We set risk aversion to 5 and IES to 2.

1. Equity premia is high
2. Returns are volatile and predictable
3. Expected returns vary in the cross section

asset pricing: equity premia

Equity premia with agency frictions is high because SDF is more volatile

Moments	Full commt.	Limited commt.	
		SSA	PSID
Equity premium on Y_t	0.62%	3.39%	3.49%
Volatility of returns on Y_t	2.22%	9.27%	10.39%
Equity premium on $x_t Y_t$	0.62%	3.79%	3.76%
Volatility of returns on $x_t Y_t$	2.22%	10.76%	11.51%
Sharpe ratio bounds	0.27, 0.15	0.50, 0.27	0.47, 0.28
Average risk free rate	4.55%	1.55%	1.48%
Volatility of risk free rate	0.44%	2.21%	3.01%

return volatility and predictability

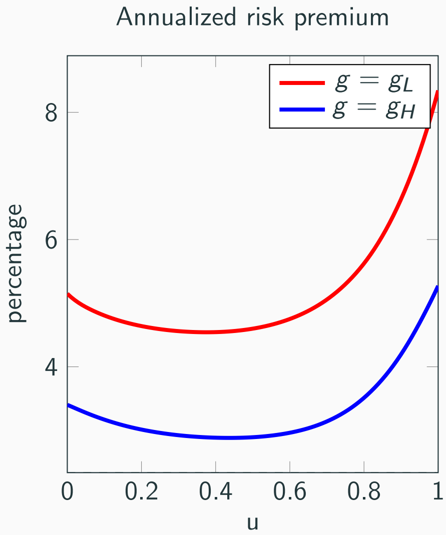
Return volatility and predictability: time-varying discount rates due to dynamics of x_t

Horizon J	Model		Data	
(years)	β	R^2	β	R^2
1	-0.22	0.07	-0.12	0.09
2	-0.37	0.12	-0.20	0.15
3	-0.47	0.15	-0.27	0.20
4	-0.53	0.16	-0.32	0.23
5	-0.58	0.17	-0.41	0.27

Table 1: Return predictability: $\sum_{j=1}^{J}(r_{t+j} - r_{f,t+j}) = \alpha + \beta(pd_t) + \epsilon_{t+j}$

cross-section of equity premia

Returns vary across firms because of operating leverage



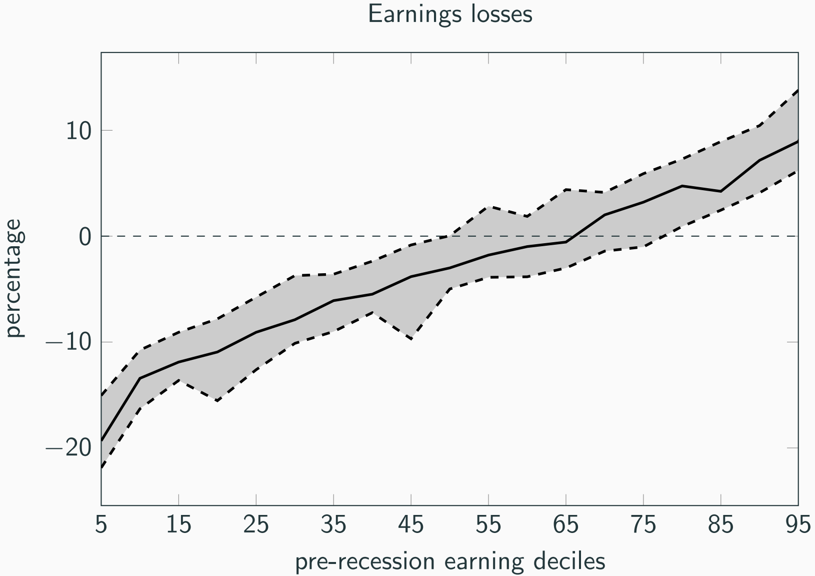
earnings and wealth exposure

- Optimal contract: history dependence in individual earning dynamics

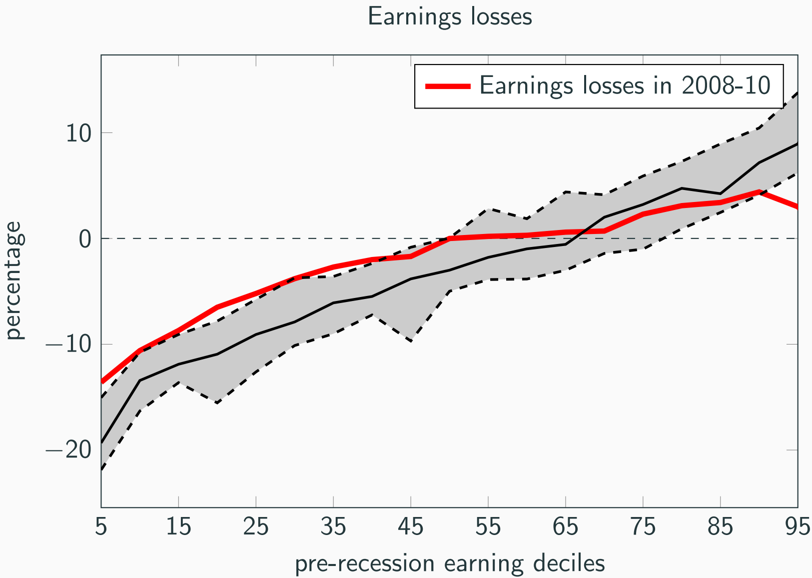
- Exposure to aggregate shocks varies in the cross-section

- Two interesting exposures: earnings, consumption-replicating portfolios
  - poor workers suffer greater losses in recessions
  - poor workers are less exposed to stock markets

earnings exposure



earnings exposure



wealth exposures

- Implementation
  - firm-specific security, claims to agg. endowment and risk-free bond
- Risk exposure of workers' compensation package
  - wealth $\equiv$ market value of compensation-replicating portfolio
  - exposure to aggregate risk calculated as

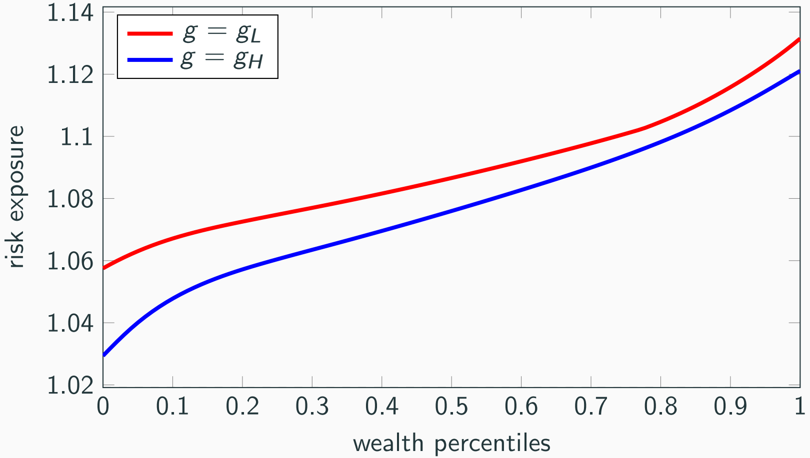
$$\Delta(u|g) = \frac{\text{wealth in booms}}{\text{wealth in recessions}}$$

- Finding: adverse idiosyncratic shocks

  $\implies$ lower exposure to agg. shocks and higher exposure to firm specific shocks

wealth exposures

$$\Delta(u|g) = \frac{\text{wealth in booms}}{\text{wealth in recessions}}$$



conclusions

- A theory of asset pricing with discipline from cross-sectional earnings data
- Agency frictions + GE give rise to higher market price of risks and operating leverage
- limited stock market participation and heterogeneous exposure to aggregate risks