Asset Pricing with Endogenously Uninsurable Tail Risk[*]

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This paper studies asset pricing and labor market dynamics when idiosyncratic risk to human capital is not fully insurable. Firms use long-term contracts to provide insurance to workers, but neither side can fully commit; furthermore, owing to costly and unobservable retention effort, worker-firm relationships have endogenous durations. Uninsured tail risk in labor earnings arises as a part of an optimal risk-sharing scheme. In equilibrium, exposure to the tail risk generates higher aggregate risk premia and higher return volatility. Consistent with data, firm-level labor share predicts both future returns and pass-throughs of firm-level shocks to labor compensation.

Key words: Equity premium puzzle, dynamic contracting, tail risk, limited commitment

1   Introduction

A key challenge for macro asset pricing theories is to account for the large magnitude of equity premia and their substantial variations over time and across firms. In this paper, we provide an asset pricing model with imperfect risk sharing to address these patterns in risk premia. Uninsured *tail* or downside risk in labor earnings arises as an outcome of optimal risk-sharing arrangements in presence of limited commitment and moral hazard. Time variation in that tail risk drives aggregate equity premium and the volatility of stock market returns. The model is also consistent with firm-level labor share predicting both future returns and pass-throughs of firm-level shocks to labor compensation. Overall, the paper provides a unified view of labor market risk and asset prices within a general equilibrium optimal contracting framework.

The setup consists of two types of agents: firm owners and workers. Firm owners are well diversified and use long-term compensation contracts to provide insurance to workers who face idiosyncratic shocks to their human capital. Two agency frictions distinguish our paper from a standard representative agent asset pricing model. First, neither firm owners nor workers can commit to contracts that yield continuation values lower than their outside options. Second, owing to costly and unobservable retention effort, worker-firm relationships have endogenous durations. We embed these contracting frictions in a general equilibrium setting with aggregate shocks to study their labor market and asset pricing implications.

Downside risk in labor earnings, a key feature of the data, is driven by firm-side limited commitment and further amplified by the presence of moral hazard. Compensation contracts providing perfect risk sharing would insure workers against idiosyncratic labor productivity shocks. But when firms cannot commit to matches that yield negative net present value, large drops in labor productivity are accompanied by reductions in worker earnings. In addition, because retention effort is not observable, firms have a lower incentive to keep workers when the firm-worker match is less profitable. Thus, separation risk is elevated after adverse productivity shocks.

In the general equilibrium, exposure to downside risk drives several of our asset pricing results. First, it generates a stochastic discount factor that is more volatile than that in an otherwise identical economy without agency frictions. A necessary condition is recursive utility with preference for early resolution of uncertainty and persistent and countercyclical idiosyncratic risk to worker human capital. During recessions, the anticipation of lack of risk sharing in the future raises workers' current marginal utilities. The optimal risk-sharing scheme compensates by allocating a higher share of aggregate output from firm owners to workers. Therefore, the labor share moves inversely with aggregate output. The countercyclicality of labor share translates into a procyclical consumption share of all

unconstrained investors, including firm owners. This amplifies risk prices. In our quantitative analysis, owing to agency frictions, we find that Sharpe ratios are more than doubled.

The high volatility of the stochastic discount factor is further amplified by the moral hazard in firms' choice of retention effort. Higher expected returns during recessions lower valuation and weaken firms' incentive to retain workers, resulting in countercyclical separations. This feature of our model supplements a large macro labor literature that argues that discount rate variations are central in driving unemployment fluctuations. In our model, higher separations exacerbate tail risk and therefore the need for firm owners to provide insurance against aggregate shocks. This further raises equilibrium discount rates and amplifies risk prices.

Second, without relying on heteroskedastic aggregate shocks, our model produces substantial predictable variations in the risk premium, especially over long horizons. The dynamics of the pricing kernel depend on the fraction of firms that are likely to hit their limited commitment constraint. This introduces persistent variations in the volatility of the stochastic discount factor and makes returns predictable. In our model, regressions of returns for a claim to aggregate consumption on price-dividend ratios gives R-squares that are significant and increasing in horizon. Time variation in discount rates also amplifies the response of asset prices to aggregate shocks and further elevates the market equity premium.

Lastly, the above economic mechanism results in a significant heterogeneity in the cross section of expected equity returns and pass-through of firm-level shocks to labor compensation. Under the optimal contract, payments to workers insures them against aggregate productivity shocks making the residual capital income procyclical and thus more exposed to aggregate shocks. This delivers a form of operating leverage at the firm level. Firms that have experienced adverse idiosyncratic shocks have a higher fraction of their value promised to workers and are therefore more sensitive to aggregate shocks. As a result, they have lower valuation ratios and higher expected returns. Furthermore, firms with larger obligations to workers are more likely to hit the firm-side limited commitment constraint and are more likely to lower wage payments in response to an adverse idiosyncratic shock. We test these implications using CRSP/Compustat panel data and show that firm-level measures of labor share predict both future returns and pass-throughs of firm-level shocks to labor compensation.

Related literature This paper builds on the literature on limited commitment. Kehoe and Levine (1993) and Alvarez and Jermann (2000) develop a theory of incomplete markets based on one-sided limited commitment. Kocherlakota (1996) examines the implications of limited commitment on consumption risk sharing. Alvarez and Jermann (2001) and Chien and Lustig (2010) study the asset pricing implications of such environments. Rampini and

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Viswanathan (2010, 2013) develop a theory of capital structure based on limited commitment. Rampini and Viswanathan (2017) study the trade-off between intertemporal financing needs and insurance across states in a household insurance problem with limited commitment.

Most of the above theory builds on the Kehoe and Levine (1993) framework and implies that agents who experience large positive income shocks have an incentive to default because they have better outside options. As a result, positive income shocks cannot be insured, while downside risk in income is perfectly insured. To be consistent with data on labor earnings which show that downside earning risks are uninsured, we model two-sided lack of commitment as in Thomas and Worrall (1988) and augment it with moral hazard. The firm-side limited commitment problem in our model has a similar structure to those studied in Bolton et al. (2019) and Ai and Li (2015) and Ai et al. (forthcoming). In addition, we add aggregate shocks and focus on the general equilibrium effects of the firm-side limited commitment and moral hazard that have not been studied before.[1]

Our paper is related to asset pricing models with incomplete markets. Krueger and Lustig (2010) provide theoretical conditions under which the presence of idiosyncratic risk is irrelevant for the market price of aggregate risks. Mankiw (1986) and Constantinides and Duffie (1996) demonstrate how countercyclical volatility in incomes amplifies aggregate risk premia in the general equilibrium. Schmidt (2015) and Constantinides and Ghosh (2014) calibrate such incomplete markets models to recent administrative data on earnings and show that higher moments of labor income shocks require a significant risk compensation. For tractability, the Constantinides and Duffie (1996) framework requires an assumption of independently distributed shocks to income growth, which rules out any insurance or trading of financial assets in equilibrium and imposes that exposure to aggregate shocks is same for all households.[2] In contrast to the above papers, we take an optimal contracting approach to micro-found incomplete markets and use empirical evidence on labor earnings dynamics to restrict the choice of the parameters governing agency frictions. Our model allows investors to access a rich set of state-contingent payoffs. We explicitly characterize history dependence in labor earnings risk and exposure to aggregate shocks.

Our paper is also related to the literature on asset pricing with heterogeneous preferences, for instance, Guvenen (2009), Garleanu et al. (2012), Garleanu and Panageas (2015), Veronesi (2019), Gomez (2019), and Santos and Veronesi (2020). See Panageas (2019) for a recent review of this literature. These papers share a feature that effective risk aversion is a wealth-

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[1]Recently, several papers such as Tsuyuhara (2016), Abraham et al. (2017), and Lamadon (2016) study versions of long-term wage contracts with moral hazard. Lamadon (2016) allows for richer features such as worker and firm complementarities, and on-the-job search. However, none of these papers allow for aggregate risks or study asset pricing.

[2]Heaton and Lucas (1996) and Storesletten et al. (2007) are among the few papers that depart from the no-trade equilibrium to study risk premia in quantitative models with exogenously incomplete markets.

weighted average of individual risk aversions and generate time-varying equity premia when wealth shares fluctuate with aggregate shocks. In order to generate a sufficiently volatile stochastic discount factor, these settings still need to rely on high levels and large differences in risk aversion along with substantial movements in the wealth distribution. In contrast, our model uses homogeneous preferences with low risk aversion. Our model generates substantial movements in the volatility of stochastic discount factor because agency frictions amplify the risk exposure of the marginal agents who provide efficient insurance to rest of the economy.

Theoretical predictions of our model are also consistent with a recent literature that emphasizes the importance of labor-share dynamics in understanding asset prices. Our operating leverage results connect to insights in Danthine and Donaldson (2002) and Berk and Walden (2013). More recently, Favilukis and Lin (2016b) use models with sticky wages to demonstrate how countercyclical movements in labor shares help explain equity and credit risk premia in production economies. Our model's implication that variations in labor shares can account for a large fraction of aggregate stock market variations is consistent with the evidence in Greenwald et al. (2016, 2020) and Lettau et al. (2014).

Our computational method builds on Krusell and Smith (1998). Using techniques contributed by the dynamic contracting literature, such as Atkeson and Lucas (1992), we represent equilibrium allocations recursively by using a distribution of promised values as a state variable. However, in contrast to those papers, our environment has aggregate shocks, and the distribution of promised values responds to such shocks, even in an ergodic steady state.

The paper is organized as follows. In section 2, we describe the physical as well as the contracting environment, and formalize a recursive competitive equilibrium with long-term contracts. In section 3, we discuss the optimal contract. In section 4, we derive the asset pricing implications that arise from agency frictions and general equilibrium considerations. Finally, in sections 5 and 6, we present quantitative implications after calibrating to several aggregate and cross-sectional facts. Section 7 concludes.

2 Model

We start with the physical and contracting environment.

2.1 Setup

Demographics and endowments We consider a discrete time economy with $t = 0, 1, \ldots$. There are two groups of agents: a unit measure of firm owners and a unit measure of workers. Members of both groups have Epstein-Zin preferences with a common risk aversion γ and a common intertemporal elasticity of substitution (IES) ψ. In each period, workers die with

probability $1-\kappa$, and a similar measure of new workers are born. This specification guarantees that the total measure of workers equals one at all times. Upon birth, a worker is endowed with one unit of human capital. Firm owners are endowed with a diversified portfolio of equity shares in all firms.

Production and human capital Production is organized within N firms. We assume that N is large so that firms are perfectly competitive. In any period, a worker is either unemployed or matched to a firm. A worker produces output only in the firm in which he is employed. If employed in period t, worker i with human capital $h_{i,t}$ produces output

$$y_{i,t} = Y_t h_{i,t},$$

where Y_t is the aggregate productivity. We assume $Y_0 = 1$, and for $t \geq 1$,

$$\ln Y_{t+1} = \ln Y_t + g_t,$$

where g_t is a finite state Markov process with a one-step transition matrix $\{\pi\left(g'|g\right)\}_{g,g'}$.

The evolution of worker human capital in the next period depends on whether the worker is employed or unemployed. The law of motion for the human capital of worker i who remains employed with firm j in period $t+1$ is

$$h_{i,t+1} = h_{i,t} e^{\eta_{j,t+1} + \varepsilon_{i,t+1}}, \tag{1}$$

where conditioning on the aggregate Markov state g_t, the firm component $\eta_{j,t}$ is i.i.d. across firms but common to all workers in a firm; the worker-specific shock $\varepsilon_{i,t}$ is i.i.d. across workers; and $\eta_{j,t}$ and $\varepsilon_{i,t}$ are mutually independent. We use $f\left(\eta, \varepsilon | g\right)$ for the conditional density of (η, ε) and normalize so that $\mathbb{E}\left[e^{\varepsilon_{i,t}} | g_t\right] = 1$ and $\mathbb{E}\left[e^{\eta_{j,t}} | g_t\right] = 1$. We use $z_{i,j,t} = (\eta_{j,t}, \varepsilon_{i,t})$ for match-specific shocks for the worker-firm pair (i,j) at time t. The human capital of a worker not matched with a firm—that is, a worker who becomes (or remains) unemployed in period $t+1$—depreciates deterministically according to

$$h_{i,t+1} = \lambda h_{i,t}, \tag{2}$$

where the parameter $\lambda < 1$ describes human capital obsolescence. In each period, unemployed workers receive unemployment benefit $bY_t h_{i,t}$, where b is a constant.

Matching and separation A match between a worker and a firm can end in two ways: stochastically upon the arrival of a separation shock, or voluntarily by the firm or the worker. Firms can influence the probability of separation by exerting costly effort. We interpret such effort as a proxy for investments in organization capital that allow firms to retain workers

and help them accumulate human capital on the job. We denote the effort for keeping a worker at time t by θ_t and assume that the cost of effort per unit of output is specified by a function $A(\theta)$ with the first three derivatives strictly positive all $\theta \in (0,1)$. It is without loss of generality to denominate effort in probability units because the cost of effort is captured by the functional form of $A(\theta)$. In period $t+1$, conditioning on the survival of the match, both firms and workers can unilaterally initiate a separation. We denote such a voluntary separation decision by an indicator function δ_{t+1}, with $\delta_{t+1} = 0$ for separation.

Upon separation, a worker enters into unemployment. In each period, an unemployed worker receives an employment opportunity with probability $\chi \in (0,1)$. An employment opportunity enables a worker to access a labor market where firms offer long-term contracts. In addition to unemployed workers, newborn workers also have an employment opportunity. A worker with an employment opportunity can choose to establish a match with the firm that offers the most favorable contract. We assume that there is no cost for posting vacancies and all firms can compete for new workers.

Contracts Let τ denote the beginning of an employment relationship between a firm and a worker. A employment contract offered by a firm to a newly employed worker at time τ specifies: (i) net transfers or compensation from the firm to the worker $\{C_t\}_{t=\tau}^{\infty}$, (ii) firm's effort for keeping the match $\{\theta_t\}_{t=\tau}^{\infty}$, and (iii) match termination decisions $\{\delta_t\}_{t=\tau}^{\infty}$ for the duration of the match. Formally, an employment contract offered in period τ by firm j to worker i with human capital $h_{i,\tau}$ specifies $\{C_t, \theta_t, \delta_t\}_{t=\tau}^{\infty}$ as functions of aggregate and match-specific histories:

$$\mathscr{C}_{i,j,\tau} \equiv \left\{ C_{i,j,t}\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t\right), \theta_{i,j,t}\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t\right), \delta_{i,j,t}\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t\right) \right\}_{t=\tau}^{\infty}. \tag{3}$$

We use the convention that superscript t denotes the history of shocks up to time t: $g^t = \{g_1, g_2, \cdots, g_t\}$, and superscript $\tau \to t$ denotes the history of shocks from time τ to t: $z_{i,j}^{\tau \to t} = \{z_{i,j,\tau+1}, z_{i,j,\tau+2}, \ldots, z_{i,j,t}\}$, with $z_{i,j}^{\tau \to \tau} = \emptyset$ representing the trivial history.

We have made two simplifying assumptions in the choice of the contract space. First, we restrict our attention to employment contracts and do not allow payments from firms to workers who are not matched with the firm. In appendix A, we show that because of two-sided limited commitment—an agency friction that we introduce below—firms are unable to insure unemployed workers or workers employed by other firms, even if they are allowed to offer insurance contracts to these workers.

Second, we assume that the terms of a new employment contract depend on worker-specific history only through the human capital at the time of employment. This is true as long as workers with an employment opportunity have no pre-existing obligations or claims

and can extract full surplus from a new match. In our setup, limited commitment means that any pre-existing contracts can be costlessly reneged, while perfect competition among firms implies that the best contract indeed gives workers all the surplus. Therefore, it is without loss of generality to index new contracts with the human capital of the worker at the time when the match is formed. To simplify notion and exposition, we impose both of the above restrictions at the outset.

Firm value Let $\left\{\Lambda_t\left(g^t\right)\right\}_t$ denote the stochastic process for state prices; that is, $\Lambda_t\left(g^t\right)$ is the price at history g^t of one unit of consumption goods measured in period-0 consumption numeraire. For $t \geq \tau$, let $V_t\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t | \mathscr{C}_{i,j,\tau}\right)$ be the time-t present value of the cash-flow stream generated by a worker i currently matched to firm j under the employment contract $\mathscr{C}_{i,j,\tau}$. Dropping the explicit dependence on $\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t\right)$, the value of the employment contract $\mathscr{C}_{i,j,\tau}$ to a firm in period $t \geq \tau$ can be recursively constructed using

$$V_t\left(\mathscr{C}_{i,j,\tau}\right) = y_{i,t}\left[1 - A\left(\theta_{i,j,t}\right)\right] - C_{i,j,t} + \kappa\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\theta_{i,j,t}\delta_{i,j,t+1}V_{t+1}\left(\mathscr{C}_{i,j,\tau}\right)\right].$$

In the above equation, the flow profit for the firm equals the output of the worker net of compensation and retention cost. In the next period, the match continues with probability $\kappa\theta_{i,j,t}$, and $\delta_{i,j,t+1}$ is the indicator function for the decision to voluntarily terminate. The future cash flows are discounted using state prices $\{\Lambda_t\}$.

Worker utility Let $U^*\left(h, g^t\right)$ be the highest utility a worker with human capital h can achieve after receiving an opportunity to match with a firm at aggregate history g^t. The utility for an unemployed worker i with human capital $h_{i,t}$ at time t, denoted by $\overline{U}\left(h_{i,t}, g^t\right)$, is recursively constructed using

$$\overline{U}\left(h_{i,t}, g^t\right) = \left[(1-\beta)\left(by_{i,t}\right)^{1-\frac{1}{\psi}} + \beta\overline{\mathbb{M}}\left(h_{i,t}, g^t\right)^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{4}$$

where $by_{i,t}$ is the flow consumption from unemployment benefits provided by the government.[3] The term $\overline{\mathbb{M}}\left(h_{i,t}, g^t\right)$ is the certainty equivalent of the next-period utility: with probability $1-\chi$, unemployed workers stay unemployed with continuation utility $\overline{U}\left(h_{i,t+1}, g^{t+1}\right)$, and with probability χ, they are matched with a new firm and receive utility $U^*\left(h_{i,t+1}, g^{t+1}\right)$. Combining both of these possibilities,

$$\overline{\mathbb{M}}\left(h_{i,t}, g^t\right) = \left(\kappa\mathbb{E}_t\left[(1-\chi)\overline{U}\left(h_{i,t+1}, g^{t+1}\right)^{1-\gamma} + \chi U^*\left(h_{i,t+1}, g^{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}.$$

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[3]See appendix A for a discussion of why an unemployed worker cannot obtain any insurance, from either the previous employer or any other firm and, as a result, consumes only the value of unemployment benefits.

For $t \geq \tau$, let $U_t \left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t | \mathscr{C}_{i,j,\tau} \right)$ be the utility of a matched worker i at time t under the employment contract $\mathscr{C}_{i,j,\tau}$. It satisfies the recursion

$$U_t \left(\mathscr{C}_{i,j,\tau} \right) = \left[(1 - \beta) \left(C_{i,j,t} \right)^{1 - \frac{1}{\psi}} + \beta \mathbb{M}_t \left(\mathscr{C}_{i,j,\tau} \right)^{1 - \frac{1}{\psi}} \right]^{\frac{1}{1 - \frac{1}{\psi}}}, \tag{5}$$

where

$$\mathbb{M}_t \left(\mathscr{C}_{i,j,\tau} \right) = \left(\kappa \mathbb{E}_t \left[\theta_{i,j,t} \delta_{i,j,t+1} U_{t+1} \left(\mathscr{C}_{i,j,\tau} \right)^{1-\gamma} + (1 - \theta_{i,j,t} \delta_{i,j,t+1}) \overline{U} \left(h_{i,t+1}, g^{t+1} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}.$$

The computation of the certainty equivalent $\mathbb{M}_t \left(\mathscr{C}_{i,j,\tau} \right)$ accounts for the fact that the match between work i and firm j can be terminated exogenously with probability $1 - \theta_{i,j,t}$ or endogenously by setting $\delta_{i,j,t+1} = 0$.

Agency frictions   We impose two types of agency frictions. First, neither firms nor workers can commit. At the beginning of each period t, before production takes place, firms and workers have an opportunity to terminate their match by setting $\delta_{i,j,t} = 0$. Once a match is dissolved, the worker is unemployed, and the firm has the option of keeping open the vacancy or hiring a new worker. Second, firms' choices of effort $\theta_{i,j,t}$, which determine the probability that the match will continue to the next period, are observable neither to workers nor to any other firms. We show later that our specification of the moral hazard problem provides a tractable way to generate equilibrium separations and non-trivial labor market dynamics.[4]

The presence of agency frictions imposes incentive compatibility constraints on the feasibility of a contract $\mathscr{C}_{i,j,\tau}$. Perfect competition on the labor market and no cost for keeping or posting vacancies imply that the value of a firm's option of terminating a match is zero. Thus, incentive compatibility with respect to the firm-side limited commitment requires that the present value of any employment contract must be non-negative at all times. That is, for all match specific histories, either $V_t \left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t | \mathscr{C}_{i,j,\tau} \right) \geq 0$, or $V_t \left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t | \mathscr{C}_{i,j,\tau} \right) < 0$ and the firm voluntarily terminates the match by setting $\delta_{i,j,t} \left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t \right) = 0$. Thus, for all $\left(z_{i,j}^{\tau \to t}, g^t \right)$,

$$\delta_{i,j,t} V_t \left(\mathscr{C}_{i,j,\tau} \right) \geq 0. \tag{6}$$

Similarly, a worker always has the option of terminating a match and becoming unemployed to obtain utility $\overline{U}_t \left(h_{i,t}, g^t \right)$. Therefore, the worker-side limited commitment implies that at any match-specific history $z_{i,j}^{\tau \to t}$, either the worker continues the employment relationship and obtains a utility that is higher than his outside option, or he unilaterally

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[4]For a similar specification of the moral hazard problem between firms and workers, see Lamadon (2016).

terminates the match. Therefore, for all $\left(z_{i,j}^{\tau \to t}, g^t\right)$,

$$\delta_{i,j,t}\left[U_t\left(\mathscr{C}_{i,j,\tau}\right) - \overline{U}\left(h_{i,t}, g^t\right)\right] \geq 0. \tag{7}$$

Finally, the fact that retention effort is not observable implies that the choice of θ must be incentive compatible from the firm's perspective. This requires that for any match-specific history $\left(z_{i,j}^{\tau \to t}, g^t\right)$,

$$V_t\left(\mathscr{C}_{i,j,\tau}\right) \geq y_{i,t}\left[1 - A\left(\tilde{\theta}\right)\right] - C_{i,j,t} + \kappa\tilde{\theta}\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\delta_{t+1}V_{t+1}\left(\mathscr{C}_{i,j,\tau}\right)\right] \tag{8}$$

for all $\tilde{\theta}$.[5]

Feasibility and efficiency   We now define feasible and privately efficient contracts. These definitions take as given a stochastic process for state prices $\left\{\Lambda_t\left(g^t\right)\right\}_t$ and values of newly employed workers $\left\{U^*\left(\cdot, g^t\right)\right\}_t$.

Definition 1. *A contract $\mathscr{C}_{i,j,\tau}$ offered by a firm to a newly employed worker with human capital $h_{i,\tau}$ in period τ is feasible given $\left\{\Lambda_t\left(g^t\right), U^*\left(\cdot, g^t\right)\right\}_t$, if it satisfies limited commitment constraints (6) and (7) and incentive compatibility constraints (8).*

Definition 2. *A contract $\mathscr{C}_{i,j,\tau}$ offered by a firm to a newly employed worker with human capital $h_{i,\tau}$ in period τ is privately efficient given $\left\{\Lambda_t\left(g^t\right), U^*\left(\cdot, g^t\right)\right\}_t$, if it is feasible, and there does not exist an alternative feasible contract $\hat{\mathscr{C}}$ such that $V_\tau\left(h, \emptyset, g^\tau | \hat{\mathscr{C}}\right) > V_\tau\left(h, \emptyset, g^\tau | \mathscr{C}_{i,j,\tau}\right)$ and $U_\tau\left(h, \emptyset, g^\tau | \hat{\mathscr{C}}\right) \geq U_\tau\left(h, \emptyset, g^\tau | \mathscr{C}_{i,j,\tau}\right)$.*

A competitive equilibrium with long-term contracts needs to specify values of newly employed workers $\left\{U^*\left(\cdot, g^t\right)\right\}_t$, and equilibrium state prices $\left\{\Lambda_t\left(g^t\right)\right\}_t$. The value of a newly matched worker $U^*\left(\cdot, g^t\right)$ is determined by workers' optimal choice of contract on the competitive labor market. Given the process $\left\{\Lambda_t\left(g^t\right)\right\}_t$, at any aggregate history g^τ, the function $U^*\left(\cdot, g^\tau\right)$ solves

$$U^*\left(h, g^\tau\right) = \max_{\mathscr{C}}\left\{U_\tau\left(h, \emptyset, g^\tau | \mathscr{C}\right) : \mathscr{C} \text{ is privately efficient given } \left\{\Lambda_t\left(g^t\right), U^*\left(\cdot, g^t\right)\right\}_t\right\}. \tag{9}$$

Because firms can always choose to keep open a vacancy, any contract offered must satisfy the condition $V_\tau\left(\mathscr{C}_{i,j,\tau}\right) \geq 0$. Among all firms that offer employment contracts, a worker chooses to match with the firm that offers the most favorable terms. An implication of equation (9) is that at the beginning of each match, competitive labor markets drive firm value to zero.

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[5]We rely on the standard result in dynamic mechanism design: there is no profitable deviation in the dynamic environment if and only if one-step deviations are not profitable.

Finally, firm owners' optimal consumption and savings decisions impose a restriction on the a relationship between state prices $\left\{ \Lambda_t \left(g^t \right) \right\}_t$ and firm owners' equilibrium consumption. Because the firm owners hold the equity of all firms, they are well diversified and their consumption and portfolio choices depend only on aggregate quantities. Given a stochastic process for firm-owner consumption $\left\{ X_t \left(g^t \right) \right\}_t$, the utility of firm owners is given recursively by

$$W_t \left(g^t \right) = \max \left\{ (1 - \beta) X \left(g^t \right)^{1 - \frac{1}{\psi}} + \beta \mathbb{N}_t \left(g^t \right)^{1 - \frac{1}{\psi}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}},$$

where the certainty equivalent $\mathbb{N}_t \left(g^t \right) = \left(\mathbb{E}_t W_{t+1} \left(g^{t+1} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$. As is standard in asset pricing models with recursive utility, the optimality condition for firm owner's consumption and investment choice implies that the stochastic discount factor must satisfy

$$\frac{\Lambda_{t+1} \left(g^{t+1} \right)}{\Lambda_t \left(g^t \right)} = \beta \left[\frac{X_{t+1} \left(g^{t+1} \right)}{X_t \left(g^t \right)} \right]^{-\frac{1}{\psi}} \left[\frac{W_{t+1} \left(g^{t+1} \right)}{\mathbb{N}_t \left(g^t \right)} \right]^{\frac{1}{\psi} - \gamma}. \tag{10}$$

It is important to note that our formulation does not assume any form of exogenously incomplete market. From equation (10), firm owners have access to complete markets, and their intertemporal rate of substitution is a valid stochastic discount factor. Because long-term contracts can replicate aggregate and idiosyncratic state-contingent payoffs between firm owners and workers, the same condition holds true for workers unless an incentive compatibility constraint is binding.[6] In the absence of those agency frictions, all workers in our setup have access to a complete set of state-contingent payoffs via insurance contracts, and our setup becomes isomorphic to a standard representative agent complete markets model.

In addition, a more general contract space that allows payments from all firms to all workers does not change any conclusions. To avoid cumbersome notation, we have restricted attention to simple employment contracts, where payments are described only between firms and their current employees. In Appendix A, we show that due to the two-sided limited commitment and perfect competition among firms, the simple employment contract described in (3) is in fact optimal in the larger contracting space in which firms are allowed to offer insurance contracts to unrelated workers. Intuitively, defaulting on a contract between a firm and its employer results in separation and human capital loss, which serves as commitment device to sustain limited risk sharing between employer firms and their employees. However, defaulting on a contract between an unrelated firm and worker does not exclude either of the parties from entering into insurance contracts with others. Lemmas 1 and 2 in Appendix A show that under fairly general conditions, this lack of exclusion rules out any insurance

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[6]This is formally shown later in proposition 2.

between firms and unrelated workers.[7]

Equilibrium state prices, workers' outside valuations, and optimal contracts for each worker-firm pair depend on past histories of aggregate as well as firm- and worker-level idiosyncratic shocks. In general, this means that one needs to keep track of the distribution of worker characteristics such as human capital and terms of contracts within firms and across firms in the economy. This can quickly become unmanageable. However, thanks to the functional form choices on preferences and technology, we can define a competitive equilibrium with long-term contracts that is recursive in an appropriately constructed parsimonious set of state variables.

2.2 Recursive Competitive Equilibrium

To define a recursive competitive equilibrium, we follow the dynamic contracting literature (for instance, Thomas and Worrall 1988 or Atkeson and Lucas 1992) and express the contracting problem using promised utility as a state variable. We show that in our model, the homotheticity properties of preferences and technology and the random walk assumption of exogenous shocks imply that an individual worker's history can be summarized by a one-dimensional state variable u, which equals the current-period continuation utility divided by the current-period worker output. The aggregate history can be summarized by a vector of aggregate state variable $S \equiv (g, \phi, B)$. Here, g is the growth rate of aggregate productivity, ϕ is a one-dimensional measure that summarizes agent types, and B is the fraction of total output consumed by unemployed workers. That we ultimately need to keep track of only a one-dimensional distribution as a state variable is a key computational step for our quantitative analysis.

Normalized variables Before stating the normalized version of the optimal contracting problem, we define firms' value function $V(y, U, S)$ to be the maximum value of $V_t\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t | \mathscr{C}\right)$ that can be achieved by any contract $\mathscr{C}$ such that it is feasible and provides the worker a utility of at least U; that is, $U_t\left(h_{i,\tau}, z_{i,j}^{\tau \to t}, g^t | \mathscr{C}\right) \geq U$. Homogeneity of preferences and technology implies that the value function $V(y, U, S)$ satisfies

$$V(y, U, S) = v\left(\frac{U}{y}, S\right) y \tag{11}$$

for some normalized value function v. This motivates the concept of normalized promised utility $u \equiv \frac{U}{y}$. In addition, the highest utility a worker can achieve from a new match $U^*(h, g^\tau) = u^*(S) y$ and the worker's utility in unemployment $\overline{U}(h, g^t) = \overline{u}(S) y$, where

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[7]Chien and Lustig (2010) and Rampini and Viswanathan (2010, 2013) also assume a form of non-exclusion in limited commitment models.

$u^*(S)$ and $\overline{u}(S)$ are functions of aggregate states. It is possible to prove that the value function $v(u, S)$ must be strictly decreasing in u.[8] Therefore, equation (9) implies that $u^*(S)$ has to satisfy

$$u^*(S) = \max\{u : v(u, S) \geq 0\}. \tag{12}$$

In addition, (4) requires the following relationship between $\overline{u}(S)$ and $u^*(S)$:

$$\overline{u}(S) = \left[(1-\beta) b^{1-\frac{1}{\psi}} + \beta\lambda\overline{m}(S)^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{13}$$

with $\overline{m}(S) \equiv \left(\kappa\sum_{g'} \pi(g'|g)\left[e^{(1-\gamma)g'}\left\{(1-\chi)\overline{u}(S')^{1-\gamma} + \chi u^*(S')^{1-\gamma}\right\}\right]\right)^{\frac{1}{1-\gamma}}$.

Recursive optimal contracting Let $\Gamma_\phi(g', S)$ and $\Gamma_B(g', S)$ be the laws of motion for the endogenous aggregate states ϕ and B so that $S' \equiv (g', \phi', B') = (g', \Gamma_\phi(g', S), \Gamma_B(g', S))$. Let $\{\Lambda(S', S)\}_{g'}$ be the set of one-period-ahead Arrow security prices, and let $\zeta' = (g', \eta', \varepsilon')$ be the vector of next period aggregate and match-specific shocks. The Markov transition matrix $\{\pi(g'|g)\}_{g,g'}$ together with the conditional density $f(\eta, \varepsilon|g)$ define a probability distribution for ζ' conditional on g, which we denote as $\Omega(d\zeta'|g)$.

The normalized firm value $v(u, S)$ satisfies the following Bellman equation:

$$v(u, S) = \max_{c, \theta, \{u'(\zeta'), \delta'(\zeta')\}_{\zeta'}} \begin{array}{c} 1 - c - A(\theta) + \\ \kappa\theta \int \Lambda(S', S) e^{g'+\eta'+\varepsilon'} \delta'(\zeta') v(u'(\zeta'), S') \Omega(d\zeta'|g), \end{array} \tag{14}$$

subject to

$$u = \left[(1-\beta) c^{1-\frac{1}{\psi}} + \beta m^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{15}$$

$$\delta'(\zeta') v(u'(\zeta'), S') \geq 0, \quad for \quad all \quad \zeta', \tag{16}$$

$$\delta'(\zeta') \left[u'(\zeta') - \lambda\overline{u}(S')\right] \geq 0, \quad for \quad all \quad \zeta', \tag{17}$$

$$A'(\theta) = \kappa \int \Lambda(S', S) e^{g'+\eta'+\varepsilon'} \delta'(\zeta') v(u'(\zeta'), S') \Omega(d\zeta'|g), \tag{18}$$

where $m = \left\{\kappa \int e^{(1-\gamma)(g'+\eta'+\varepsilon')} \left[\theta\delta'(\zeta') u'(\zeta')^{1-\gamma} + (1 - \theta\delta'(\zeta')) \lambda\overline{u}(S')^{1-\gamma}\right] \Omega(d\zeta'|g)\right\}^{\frac{1}{1-\gamma}}$.

Equation (15) is the promise-keeping constraint ensuring that the current compensation and effort choices, together with the choices for future continuation values, deliver the utility u that is promised to the worker. Inequalities (16) and (17) are the recursive counterparts of the limited commitment constraints (6) and (7). Equation (18) is the first-order necessary condition for firms' choice of retention effort. Because the cost function $A(\theta)$ is strictly convex

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[8]See appendix B.1 for details on existence and monotonicity of the function v.

in θ, first-order conditions (18) are equivalent to (8) and therefore, as we prove in appendix B, necessary and sufficient for incentive compatibility. We label the above maximization problem as *P1*.

Aggregation   Let $x_t\left(g^t\right) = \frac{X_t\left(g^t\right)}{Y_t\left(g^t\right)}$ be the normalized consumption of the firm owners. Given a consumption function $x\left(S\right)$, firm owners' normalized utility, which we denote as $w\left(S\right)$, can be constructed from

$$w\left(S\right) = \left[\left(1-\beta\right)x\left(S\right)^{1-\frac{1}{\psi}} + \beta n\left(S\right)^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{19}$$

with the certainty equivalent $n\left(S\right) = \left\{\sum_{g'}\pi\left(g'|g\right)e^{\left(1-\gamma\right)g'}w\left(S'\right)^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}$. Using the normalized notation, the stochastic discount factor (SDF) $\Lambda\left(S',S\right)$ is given by

$$\Lambda\left(S',S\right) = \beta\left[\frac{x\left(S'\right)e^{g'}}{x\left(S\right)}\right]^{-\frac{1}{\psi}}\left[\frac{w\left(S'\right)e^{g'}}{n\left(S\right)}\right]^{\frac{1}{\psi}-\gamma}. \tag{20}$$

Finally, we describe the construction of the aggregate state variable ϕ, which we will refer to as the "summary measure." Let $\Phi_j\left(du,dh\right)$ denote the joint distribution of (u,h) for workers in firm j and $\Phi_0\left(dh\right)$ the distribution of human capital of unemployed workers. In general, $\{\Phi_j\}_{j=0}^N$ is a state variable in the construction of a recursive equilibrium because the resource constraint,

$$Y\int bh\Phi_0\left(dh\right) + Y\sum_{j=1}^N\int\int\left[c\left(u,S\right) + A(\theta,S)\right]h\Phi_j\left(du,dh\right) + Yx(S) = Y\sum_{j=1}^N\int\int h\Phi_j\left(du,dh\right), \tag{21}$$

depends on $\{\Phi_j\}_{j=0}^N$. Let $c\left(u,S\right)$ be the policy function for worker compensation in the problem *P1*. The total compensation to all workers

$$Y\sum_{j=1}^N\int\int c\left(u,S\right)h\Phi_j\left(du,dh\right) = Y\int c\left(u,S\right)\sum_{j=1}^N\left[\int h\Phi_j\left(dh|\,u\right)\right]\Phi_j\left(du\right),$$

where we decompose the joint distributions into a marginal distribution and a conditional distribution: $\Phi_j\left(du,dh\right) = \Phi_j\left(dh|\,u\right)\Phi_j\left(du\right)$. We define the summary measure by $\phi\left(du\right) \equiv \sum_{j=1}^N\int h\Phi_j\left(dh|\,u\right)$ for all u. For a given h, the term $\sum_{j=1}^N\Phi_j(du,dh)$ is the joint distribution of (u,h) across all firms, and thus $\phi(du)$ is the average human capital of employed workers of type u. We define the total compensation to all unemployed workers normalized by aggregate productivity as $B = \int bh\Phi_0\left(dh\right)$. Using the fact that total output equals $Y\int\phi\left(du\right)$, the

resource constraint can be written as

$$B + \int \left[c\left(u, S\right) + A(\theta(u, S)) \right] \phi\left(du\right) + x\left(S\right) = \int \phi\left(du\right). \qquad (22)$$

The above procedure reduces the $N + 1$ two-dimensional distributions $\{\Phi_j\}_{j=0}^{N}$ into a one-dimensional measure ϕ and a scalar B. This greatly simplifies our analysis.

Recursive competitive equilibrium Equilibrium can be constructed in two steps. In the first, we obtain policy functions $c\left(u, S\right)$, $\theta\left(u, S\right)$, $\{u'\left(u, S, \zeta'\right), \delta'\left(u, S, \zeta'\right)\}_{\zeta'}$ by solving problem *P1*. In the second, we use the policy functions to construct the laws of motion for the endogenous state variables Γ_ϕ and Γ_B.

The summary measure ϕ has a continuous density on $[\lambda \overline{u}\left(S\right), u^*\left(S\right)]$ which describes the human capital of all currently employed workers and a mass point on $u^*\left(S\right)$ for newly employed workers. The law of motion $\phi' = \Gamma_\phi\left(g', S\right)$ specifies the summary measure in the next period for each possible realization of g' as a function of current state S. The density of the continuous part of ϕ' is

$$\phi'(d\tilde{u}) = \kappa \int \theta\left(u, S\right) \left[\int e^{\eta' + \varepsilon'} f\left(\eta', \varepsilon'|g'\right) \delta\left(\zeta'\right) I_{\{u'(u, S, \zeta') \in d\tilde{u}\}} d\eta' d\varepsilon' \right] \phi\left(du\right) \qquad (23)$$

$\forall \tilde{u} \in [\lambda \overline{u}\left(S'\right), u^*\left(S'\right)]$, where I is the indicator function. The mass point of ϕ' at $u^*\left(S'\right)$ is given by

$$\phi'\left(\{u^*\left(S'\right)\}\right) = \left(1 - \kappa\right) + \kappa \chi \lambda \frac{B}{b}. \qquad (24)$$

In the above expression, $1 - \kappa$ is the total amount of human capital of newborn workers, a measure $1 - \kappa$ of whom arrive in each period with one unit of human capital. The second term is the amount of human capital of workers who will move to employment from the current unemployed pool. The term B is total unemployment benefit in the current period and $\frac{B}{b}$ is the total human capital of all unemployed workers. A fraction κ of them survive to the next period, their human capital decpreciates at rate $1 - \lambda$, and a fraction χ exit the unemployment pool.

The law of motion $\Gamma_B\left(g', S\right)$ maps $\left(g', S\right)$ to B', which is the total unemployment benefit in the next period in state g' and is given by

$$B' = \kappa \lambda \left[B(1 - \chi) + b \int \int \left(1 - \theta\left(u, S\right) \delta\left(\zeta'\right)\right) f\left(\eta', \varepsilon'|g'\right) \phi\left(du\right) d\eta' d\varepsilon' \right], \qquad (25)$$

where the first term $B(1 - \chi)$ accounts for all unemployed workers in the current period who will stay unemployed in the next period, and the second term accounts for workers who transit from the currently employed pool to unemployment in the next period.

Definition 3. *A recursive competitive equilibrium consists of stochastic discount factor* $\{\Lambda(S', S)\}$, *workers' value from unemployment* $\bar{u}(S)$, *the value from a new match* $u^*(S)$, *firm values* $v(u, S)$ *and policy functions* $\left(c(u, S), \theta(u, S), \{u'(u, S, \zeta'), \delta'(u, S, \zeta')\}_{\zeta'}\right)$, *consumption share of firm owners* $x(S)$, *and laws of motion* Γ_ϕ *and* Γ_B *such that (i) the stochastic discount factor satisfies (20); (ii) the firm value function and the policy functions solve problem P1; (iii) the laws of motion for aggregate states* ϕ *and* B *satisfy (23), (24), and (25); (iv) values for new and unemployed workers satisfy (12) and (13); and (v) the resource constraint (22) holds.*

3 The Optimal Contract

In this section, we provide a characterization of the optimal contract by discussing the properties of policy functions to problem *P1*. The policy functions of firm retention effort $\theta(u, S)$ and the termination decision $\delta'(u, S, \zeta')$ determine the labor market dynamics in our model, while the choices of consumption $c(u, S)$ and the next-period continuation utility $u'(u, S, \zeta')$ are responsible for the model's implications on risk sharing. We start with the labor–market related policy functions.

First, there are no voluntary terminations under the optimal contract. In the absence of complementarity between firm and worker productivities, an adverse shock to human capital makes the worker equally unproductive in all firms and proportionally lowers his eligible unemployment benefit. Separations, which lead to human capital losses, therefore lower worker utility without benefiting firms. The optimal contract avoids such inefficient separations by setting $\delta(u, S, \zeta') = 1$ for all ζ'. In our setup, although the possibility of separation serves as a punishment device and sustains some risk sharing between firms and their employees, it is never specified as an equilibrium outcome under the optimal contract.

Next, firms' retention policy function $\theta(u, S)$ is decreasing in u. Incentive compatibility constraint (18) requires that the marginal cost $A'(\theta)$ of retaining the worker equal its marginal benefit, the present value of the cash flow that the worker brings to the firm, $\kappa \int \Lambda(S', S) e^{g' + \eta' + \varepsilon'} v(u'(u, S, \zeta'), S') \Omega(d\zeta'|g)$. Because firm value is a decreasing function of the utility promised to workers, these marginal benefits are also decreasing in u. Thus, it is harder to induce a higher retention effort when the promised utility to worker is high. The following lemma summarizes the above discussion of $\delta(u, S, \zeta')$ and $\theta(u, S)$.

Proposition 1. *In any equilibrium in which the stochastic discount factor and the law of motion for aggregate state variables satisfy condition (6) in appendix B, for all* (u, S, ζ'), $\delta'(u, S, \zeta') = 1$, *and the policy function for retention effort,* $\theta(u, S)$ *is decreasing in* u *for all* S.

Proof. See appendix B. □

More generally, the above proposition implies that separation rates are higher when the value of a worker to the firm is low. This may be due to either a lower future surplus from the worker (that is, lower levels of $v\left(u'\left(u, S, \zeta'\right), S'\right)$) or a higher discount rate (that is, lower values of Λ). Therefore, the specification of the moral hazard problem—in particular, the incentive compatibility constraint (18)—generates countercyclical unemployment. Also, the implication that $\delta'\left(u, S, \zeta'\right) = 1$ is useful for the tractability of the model. It allows us to replace the firm- and work-side limited commitment constraints in equations (16) and (17) by

$$v\left(u'\left(\zeta'\right), S'\right) \geq 0, \tag{26}$$

$$u'(\zeta') \geq \lambda \overline{u}\left(S'\right). \tag{27}$$

We now turn to the implications of the optimal contract for risk sharing. With full commitment, firms can perfectly insure workers against idiosyncratic shocks. Therefore, workers' continuation utilities $y e^{\varepsilon' + \eta'} u'\left(u, S, \zeta'\right)$ do not respond to these shocks and are equalized across all possible realizations of (η', ε'). When 0 is a possible realization of η' and ε', this optimal risk-sharing condition can be written as

$$u'\left(u, S, \zeta'\right) = e^{-(\varepsilon' + \eta')} u'\left(u, S, g', 0, 0\right), \forall \zeta'. \tag{28}$$

Thus, under perfect risk sharing, the elasticity of normalized utility with respect to idiosyncratic shocks is -1.

Under limited commitment, equation (28) cannot hold for all values of (η', ε'). Because a worker can always separate voluntarily, the promised utility under the optimal contract cannot be lower than what he receives upon a voluntary termination, $\lambda \overline{u}\left(S'\right)$. Clearly, for large and positive realizations of $\varepsilon' + \eta'$, the full risk sharing policy in (28) would imply that $e^{-(\varepsilon' + \eta')} u'\left(u, S, g', 0, 0\right) < \lambda \overline{u}\left(S'\right)$ and violate the worker-side limited commitment (27). As a result, given the current state (u, S), there is a threshold level $\overline{\varepsilon}(u, S, g')$ for every g', such that for all $\varepsilon' + \eta' \geq \overline{\varepsilon}(u, S, g')$, the worker-side limited commitment constraint binds and

$$u'\left(u, S, \zeta'\right) = \lambda \overline{u}\left(S'\right). \tag{29}$$

Conversely, the firm-side limited commitment imposes an upper bound on $u'\left(u, S, \zeta'\right)$. Because $u^*\left(S'\right)$ is the highest utility a worker can achieve and $v\left(u^*\left(S'\right), S'\right) = 0$, any promised utility higher than $u^*\left(S'\right)$ results in a negative firm value. Large and negative realizations of $\varepsilon' + \eta'$ therefore imply that the full risk sharing policy (28) would lead to $e^{-(\varepsilon' + \eta')} u'\left(u, S, g', 0, 0\right) > u^*\left(S'\right)$ and thus violate the firm-side limited commitment

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(26). Under the optimal contract, there is a threshold function $\underline{\varepsilon}(u, S, g')$, such that for all $\varepsilon' + \eta' < \underline{\varepsilon}(u, S, g')$, the firm-side limited commitment constraint has to bind, and

$$u'\left(u, S, \zeta'\right) = u^*\left(S'\right). \tag{30}$$

In the interior, $\underline{\varepsilon}(u, S, g') < \varepsilon' + \eta' < \overline{\varepsilon}(u, S, g')$, none of the above constraints bind, and the intertemporal marginal rate of substitution of all agents has to equalize. In the following proposition, we summarize the properties of consumption and continuation utility policies.

Proposition 2. *In any equilibrium in which the stochastic discount factor and the law of motion for aggregate state variables satisfy condition (6) in appendix B, there exist threshold levels $\underline{\varepsilon}(u, S, g')$ and $\overline{\varepsilon}(u, S, g')$ with $\underline{\varepsilon}(u, S, g') < \overline{\varepsilon}(u, S, g')$, such that for all $\varepsilon' + \eta' > \overline{\varepsilon}(u, S, g')$, $u'(u, S, \zeta')$ is given by (29), and for all $\varepsilon' + \eta' < \underline{\varepsilon}(u, S, g')$, $u'\left(u, S, \zeta'\right)$ satisfies (30). For all $\varepsilon' + \eta' \in [\underline{\varepsilon}(u, S, g'), \overline{\varepsilon}(u, S, g')]$, $u'\left(u, S, \zeta'\right)$ is strictly decreasing in $\varepsilon' + \eta'$ and satisfies*

$$\left[\frac{x\left(S'\right)}{x\left(S\right)}\right]^{-\frac{1}{\psi}} \left[\frac{w\left(S'\right)}{n\left(S\right)}\right]^{\frac{1}{\psi} - \gamma} \left(1 + \frac{\iota\left(u, S\right)}{\theta\left(u, S\right)}\right) = e^{-\gamma(\eta' + \varepsilon')} \left[\frac{c\left(u'\left(u, S, \zeta'\right), S'\right)}{c\left(u, S\right)}\right]^{-\frac{1}{\psi}} \left[\frac{u'\left(u, S, \zeta'\right)}{m\left(u, S\right)}\right]^{\frac{1}{\psi} - \gamma}, \tag{31}$$

where $\iota\left(u, S\right) > 0$ is given in appendix B.

Proof. See appendix B. $\qquad\square$

The above proposition has several implications. First, large and positive realizations of ε' and η' imply that $u'\left(u, S, \zeta'\right)$ must be set to a constant and cannot respond to further increases in $\eta' + \varepsilon'$. As a result, the level of continuation utility, $ye^{\varepsilon' + \eta'} u'\left(u, S, \zeta'\right)$, must increase with positive productivity shocks. High promised values are met with higher future compensation. This feature of our setting is similar to that in Harris and Holmstrom (1982), Kehoe and Levine (1993), and Alvarez and Jermann (2000).

Second, in contrast to these papers in which workers are perfectly insured against downside risk, the limited commitment constraint on the firm side implies that sufficiently negative realizations of $\eta' + \varepsilon'$ also cannot be hedged. A sequence of negative worker- or firm-specific productivity shocks lowers worker output. Keeping an extremely unproductive worker is a negative net present value undertaking for the firm, since the cash flow produced by the worker is not enough to pay for his promised compensation. In addition to lower retention effort as mentioned above, lack of commitment from the firm side requires reductions in future worker compensation in order to provide incentives for the firm to continue the match. As we will demonstrate in subsequent sections, this feature is key for our model to generate volatile asset prices along with tail risk in labor earnings.

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Third, equation (31) in proposition 2 implies that the intertemporal marginal rate of substitution has to be equal for all agents in the economy unless the limited commitment constraints are binding. This includes firm owners as well as a subset of workers. Equation (31) also highlights the difficulty in generating a high volatility of stochastic discount factor without directly assuming exogenous market segmentation. Given preference parameters, the volatility of the SDF is determined entirely by the risk exposure of the consumption of marginal investors. If a heterogeneous agent-based model such as ours generates a stochastic discount factor that is more volatile than a representative agent model, then it must also provide an explanation for why the additional risk exposure in marginal investors' consumption is not insured away through conditions like equation (31). In the next section, we demonstrate conditions under which agency frictions generates downside risk in labor earnings and amplify the volatility of the stochastic discount factor in equilibrium.

4   Agency Frictions and Asset Pricing

In this section, we highlight how agency frictions affect aggregate and cross-sectional asset returns. General equilibrium linkages between tail risk in labor earnings and the pricing kernel are key for agency frictions to amplify risk premia. We start with an "irrelevance" result in the spirit of Krueger and Lustig (2010) that provides conditions under which agency frictions are irrelevant for both the price of aggregate risk and aggregate labor market dynamics. We then analyze a special case of our model to isolate the mechanism that amplifies the volatility of the stochastic discount factor and distinguish it from alternatives in the literature. We also derive a set of testable predictions of our model mechanism, which are later confronted with the data.

4.1   An Irrelevance Result

Krueger and Lustig (2010) show that if the aggregate endowment growth is i.i.d. and the distribution of idiosyncratic shocks $f(\eta, \varepsilon \mid g)$ is independent of the aggregate states, then uninsurable idiosyncratic risk does not affect the price of aggregate shocks in a wide set of incomplete markets models. To formalize a version of their result in our setting with contracting frictions, we start with the following definition.

Definition 4. *Given the economy described in section 2.2, an equivalent deterministic economy with a modified discount rate is an otherwise identical economy except that the aggregate growth rate is set to zero and the time discount factor β is modified to $\beta \left(\mathbb{E} \left[e^{(1-\gamma)g'} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$.*

In the following proposition, we provide conditions under which a recursive competitive

equilibrium in the stochastic economy can be constructed from the equilibrium of an equivalent deterministic economy with a modified discount rate.

Proposition 3. *(Krueger and Lustig) Suppose that g_t is i.i.d. over time and $f(\eta, \varepsilon | g)$ does not depend on g. If there exists an equilibrium in the equivalent deterministic economy with a modified discount rate, then there exists an equilibrium of the stochastic economy described in section 2.2 with the stochastic discount factor*

$$\Lambda\left(S', S\right) = \frac{1}{\hat{R}\left(\phi, B\right)} \frac{e^{-\gamma g'}}{\mathbb{E}\left[e^{(1-\gamma)g'}\right]}, \tag{32}$$

where $\hat{R}(\phi, B)$ is the risk-free interest rate in the equivalent deterministic economy with a modified discount rate.

Proof. See section A in the online appendix (not for publication). □

With i.i.d aggregate growth rates, the stochastic discount factor in the section 2.2 economy with full commitment and no moral hazard equals $\beta e^{-\gamma g'}$.[9] This is also the stochastic discount factor for a representative agent economy in which the growth rate of aggregate consumption is g_t. Equation (32) states that the stochastic discount factor in the economy with agency frictions differs only by a multiplicative constant. Therefore, agency frictions affect the risk-free interest rate but are irrelevant for the pricing aggregate risks. Proposition 3 imposes no other restriction on the distribution of idiosyncratic risk and, in particular, allows $f(\eta, \varepsilon | g)$ to contain a fat tail as long as it is the same across all realizations of g. We show in appendix B that the optimal contract in the equivalent deterministic economy with a modified discount rate can be used to construct the optimal contract in the stochastic economy by simply adjusting for aggregate growth and that the consumption share of firm owners in the stochastic economy equals that in the equivalent deterministic economy.

4.2 Aggregate Implications

Proposition 3 tells us that for agency frictions to have an impact on aggregate risk premia, we must deviate from its assumptions of i.i.d. growth and a time-invariant distribution of idiosyncratic shocks. In the rest of this section, we use a special case of our model to analyze such a departure. The special case highlights the interaction between agency frictions, risk in labor earnings, and the market price of aggregate risk.

We proceed by making several simplifying assumptions. Many of these assumptions are designed to isolate features and implications that are novel to our setting and to help us

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[9]In absence of agency frictions, the consumption share of firm owners $x(S)$ is constant, and equations (19) and (20) simplify to $\Lambda\left(S', S\right) = \beta e^{-\gamma g'}$.

obtain closed-form solutions for equilibrium returns. We relax these assumptions later in the quantitative section, where we use numerical methods to solve the general model described in section 2.2.

Assumption 1. *Aggregate shocks $g_t \in \{g_L, g_H\}$ with $g_L < g_H$. From period one on, the transition probability from state g to state g' satisfies $\pi(g'|g) = 1$ if $g' = g$. Each firm has a single worker and $\eta = 0$. Let the distribution $f(\varepsilon|g = g_H)$ be degenerate, and the distribution $f(\varepsilon|g = g_L)$ be a negative exponential with parameter ξ.[10]*

This assumption includes the main departures from proposition 3. To capture the persistence of aggregate shocks, we assume that booms ($g_t = g_H$) and recessions ($g_t = g_L$) are permanent. To parsimoniously model countercyclical idiosyncratic shocks, we impose no idiosyncratic shocks in booms and negatively exponentially distributed shocks in recessions. The assumption that firm-level shocks $\eta = 0$ is without loss of generality, since proposition 2 shows that the optimal contract depends only on $\varepsilon + \eta$. In what follows, we interpret ε as both a firm- and a worker-level shock.

Assumption 2. *Preferences satisfy $\gamma \geq \psi = 1$.*

The crucial part here is that $\gamma \geq \psi$. The assumption of unit elasticity of intertemporal substitution is merely for tractability.

Assumption 3. *Workers can fully commit.*

As shown in proposition 2, uninsurable risk in the left tail of labor earnings comes from the firm-side limited commitment and separations. In section 6.1, we show the that worker-side limited commitment has little quantitative impact on the equity premium but matters for accounting for patterns in earning dynamics. Hence, here we abstract from the lack of commitment on the worker side.

Assumption 4. *Effort is costly only in period one, in which case $A(\theta) = a\left[\ln\left(\frac{1}{1-\theta}\right) - \theta\right]$ for some $a > 0$.*

The parameter a in function $A(\theta)$ measures the severity of the moral hazard problem, with $a = 0$ corresponding to the case in which effort is costless and moral hazard is irrelevant.

Assumption 5. *For $t = 2, 3, \ldots$, both employed and unemployed workers produce output and consume α fraction of their output: $C_t = \alpha y_t$.*

From period 2 on, workers consume a fixed fraction of their output. This assumption captures that because of lack of full risk sharing, workers' consumption exposed to

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[10]The form of the negative exponential distribution is described in appendix C.

Figure 1: Timing of the simple model
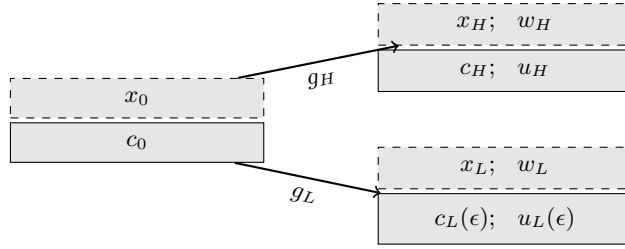
idiosyncratic shocks in future recessions. We assume that unemployed workers lose $1 - \lambda$ fraction of their human capital but keep producing output. They are otherwise subject to the same law of motion of human capital as employed workers.

In figure 1, we plot an event tree for the simple economy. Let the firm owners' consumption share in period 0 be x_0, and let workers' initial promised utility be u_0. We assume all workers have the same promised utility u_0; therefore, there is a unique u_0^* that is consistent with the aggregate resource constraint. In comparative static exercises, we study optimal contracting with an arbitrary u_0, even though in equilibrium the measure of agents at u_0 might be zero unless $u_0 = u_0^*$. We let $x_H \equiv x(g_H)$ and $x_L \equiv x(g_L)$, and let $w_H \equiv w(g_H)$ and $w_L \equiv w(g_L)$. For an arbitrary initial promised utility u_0, we use $\theta_H(u_0) \equiv \theta(u'(u_0, g_H), g_H)$ and $\theta_L(u_0, \varepsilon) \equiv \theta(u'(u_0, g_L, \varepsilon), g_L)$ to denote the effort choice and likewise for compensation policy $\{c_H(u_0), c_L(u_0, \varepsilon)\}$, $\{v_H(u_0), v_L(u_0, \varepsilon)\}$ for firms' value function at nodes H and L, respectively. The policy functions for compensation, firm effort, and the value functions at node H do not depend on ε since there is no idiosyncratic shock at node H. The following proposition provides conditions under which agency frictions amplify the equity premium and generate countercyclical unemployment.

Proposition 4. *(Aggregate Implications) Under assumptions 1–5, for expected utility preferences, i.e., $\gamma = 1$, firm owners' consumption share is countercyclical; that is, $x_H < x_L$. For general recursive utility, there exists a $\hat{\gamma} \in [1, 1 + \xi)$ such that if $\gamma > \hat{\gamma}$, then (i) firm owners' consumption share is procyclical; that is, $x_H > x_L$ and (ii) separation rates are countercyclical; that is, $\theta_H(u_0) > \theta_L(u_0, \varepsilon)$ for all (u_0, ε).*

Because the consumption Euler equation must hold for the unconstrained firm owners, amplification in the market price of risk relative to a representative agent model is equivalent to firm owner's consumption share being procyclical. The first part of proposition 4 implies that countercylical idiosyncratic risk by itself is not sufficient for amplifying the volatility of the equilibrium stochastic discount factor. Independent of the risk aversion γ, the optimal contract generates uninsurable tail risk (proposition 2). However, under expected utility, the pricing kernel is less volatile than the pricing kernel in an otherwise identical economy with

full commitment.

Countercyclical idiosyncratic risk means that relative to booms, a larger fraction of worker-firm pairs are constrained in recessions. Because constrained firms cut compensation, in the aggregate, there is a higher fraction of resources available for firm owners during a recession. Since goods markets need to clear, these resources are allocated between the firm owners and the unconstrained workers by equating their intertemporal marginal rates of substitution. With expected utility, this amounts to equalizing the growth rates of consumption of the firm owners and the unconstrained workers. Therefore, for $\gamma = 1 = \frac{1}{\psi}$, the consumption share for firm owners increases in a recession, resulting in $x_L > x_H$. This makes aggregate asset prices less volatile.

The second implication of proposition 4 is that keeping the intertemporal elasticity of substitution fixed, a large enough risk aversion results in a procyclical consumption share for firm owners. Why does the cyclicality of x flip signs when risk aversion becomes larger relative to inverse of the intertemporal elasticity of substitution? Persistent recessions that are associated with a lack of risk sharing in the future imply lower continuation values for all workers. As risk aversion exceeds the inverse of the intertemporal elasticity of substitution, contemporaneous marginal utilities are decreasing functions of continuation utility.[11] Optimal risk sharing, which requires equating marginal rates of substitution between firm owners and unconstrained workers, is now achieved by transferring resources away from the firm owners in recessions. Proposition 4 says that for sufficiently high risk aversion, this incentive is strong enough to dominate the effect of market clearing and delivers procylical consumption shares for firm owners.

The last part of proposition 4 says that separation rates are higher in recessions relative to booms. In our model, labor income has two sources of tail risk. First, the distribution of productivity shock ε has a left tail. As shown in proposition 2, under firm-side limited commitment, this tail risk cannot be fully insured within optimal labor compensation contracts. Second, workers become unemployed with probability θ and lose a fraction $1 - \lambda$ of human capital in each period until they are matched with a new firm.

The countercyclicality of unemployment risk asserted in part (ii) of proposition 4 is a direct consequence of incentive compatibility under moral hazard. Without moral hazard, to efficiently deliver promised utility to workers, firms will typically choose a lower separation rate in recessions, when human capital depreciation and consumption reduction are more costly for workers. With moral hazard, such arrangements are no longer incentive compatible,

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[11]Ai and Bansal (2018) define the class of preferences under which marginal utility decreases with continuation utility as generalized risk-sensitive preferences. Generalized risk sensitivity is the key property of preferences captured by the assumption $\gamma > \frac{1}{\psi}$ that is responsible for the procyclical consumption share in our model.

and firms equalize the marginal cost of retention effort to the present value of profits that a worker can create without considering the cost of separation to workers. Since valuation ratios in recessions are lower relative to booms, firms exert less effort to retain workers, leading to countercyclical separation rates.

The effects of limited commitment and those of moral hazard reinforce each other to amplify the volatility of the stochastic discount factor. Limited commitment amplifies risk prices because optimal contracts insure workers against adverse aggregate shocks which makes firm owners' consumption more risky. Higher separations in recessions magnify the downside risk in labor earnings and hence the need for insurance. Thus, higher separation risk leads to more procyclical consumption for marginal agents, and the resulting higher discounting in turn leads to lower worker valuations, lower retention effort from firms, and more separations.

Contrasting the mechanism to alternatives proposed in the literature   Proposition 4 contrasts our setup with several exogenously incomplete market models—for example, Constantinides and Duffie (1996), Constantinides and Ghosh (2014), and Schmidt (2015), as well as setups that impose exogenous market segmentation such as Basak and Cuoco (1998) or Guvenen (2009). In these papers, agents are not allowed to offer any insurance contracts to one another. In our model, agents are allowed access to a rich set of state-contingent payoffs, the only restriction being incentive compatibility constraints. In the Constantinides and Duffie (1996)–style models, all agents are marginal investors in risky assets, and hence countercyclical uninsurable risk in consumption automatically translates into a more volatile pricing kernel. In our simple example, because market incompleteness is determined by agency frictions, agents with adverse idiosyncratic shocks are constrained and therefore are not marginal. Hence, higher idiosyncratic volatility by itself does not increase the market price of risk.

Alvarez and Jermann (2001) and Chien and Lustig (2010) derive asset pricing implications in a setting with one-sided limited commitment. This corresponds to a version of our model where firms can fully commit but workers cannot. Such environments produce high equity premia when more workers are constrained in adverse aggregate states. The worker-side limited commitment constraint binds for worker-firm pairs that receive large positive idiosyncratic productivity shocks. Because constrained workers need to be compensated with higher current consumption, by market clearing, the consumption for unconstrained agents drops, raising their marginal utilities. To amplify the risk premium, such a model would necessarily require more positive skewness in labor earnings in recessions relative to booms; an implication that is inconsistent with the key feature of labor market risk that we highlight in the introduction. In addition, quantitatively, uninsurable tail risk on the downside are much more powerful than on the upside in amplifying the volatility of the stochastic discount

factor. The workings of the simple example explain how a combination of firm-side limited commitment with recursive utility jointly deliver downside risk in labor earnings and higher risk premia.

Proposition 4 also distinguishes our model from Favilukis and Lin (2016b), and other papers that use sticky wages to explain the high equity premium. In these models, markets are complete and labor compensation contracts do not affect the pricing kernel. These models produce higher equity premia through an "operating leverage" channel: labor compensation is less sensitive to aggregate shocks, and this amplifies the risk exposure of capital income. Since operating leverage affects only the volatility of cash flows, these models need to assume a high level of risk aversion to match aggregate Sharpe ratios. In contrast to models with exogenous wage rigidity, in our setup, risk premia are amplified primarily through the effect of agency frictions on the volatility of the stochastic discount factor and not because of a higher volatility of dividends.[12] We return to this implication in our quantitative analysis in section 6.1.

4.3   Cross-Sectional Implications

In addition to the implications for aggregate risk prices and aggregate unemployment dynamics, our model has predictions for the cross section of returns and labor earnings. We outline these implications here, and in section 6.3 we formally test them using panel data on firm-level returns and firm-level labor shares.

In our model, heterogeneity in firms is summarized by a single state variable u. High-u firms promise a larger fraction of current and future cash flow to workers than low-u firms. Thus, u can be interpreted as "labor leverage." For a firm with labor share u_0, define the elasticity of wage payments with respect to idiosyncratic shocks as $\Upsilon(u_0) = \mathbb{E}\left[\frac{\partial \ln[e^\varepsilon c_L(u_0,\varepsilon)]}{\partial \varepsilon}\right]$. The term $e^\varepsilon c_L(u_0,\varepsilon)$ is the level of compensation to a worker with initial promised utility u_0 at node L. Next, define the valuation risk exposure or beta of a firm indexed by u_0 as $\mathcal{B}(u_0) = \left(\frac{v_H(u_0)}{\mathbb{E}[e^\varepsilon v_L(u_0,\varepsilon)]}\right)$. Below, we provide two comparative static results with respect to u_0.

Proposition 5. *(Cross-Sectional Implications) Under assumptions 1–5, (i) $\frac{\partial}{\partial u_0}\Upsilon(u_0) > 0$ and (ii) there exists a $\hat{\gamma} \in [1, 1+\xi)$ such that $\forall \gamma > \hat{\gamma}$, $\exists \hat{u}$, where $\hat{u}$ is defined by $\underline{\varepsilon}(\hat{u}, g_L) = \ln\frac{1+\xi}{\xi}$, such that $\forall u_0 < \hat{u}$, $\frac{\partial}{\partial u_0}\mathcal{B}(u_0) > 0$.*

Part (i) of the proposition says that the average elasticity of compensation with respect to idiosyncratic shock ε is increasing in promised utility u_0. Firms that promised a

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[12]In our model, the claim on aggregate dividends also has a higher price-to-dividend ratio in booms relative to recessions. In section B of the online appendix (not for publication), we show that under assumptions 1–5, $\exists\,\hat{\gamma} \in [1, 1+\xi)$ such that $\gamma > \hat{\gamma}$ implies $\frac{v_H(u_0^*)}{E\left[e^\varepsilon v_L(u_0^*,\varepsilon)\right]} > 1$.

higher fraction of cash flows to workers are more likely to be constrained. Whenever the limited commitment constraint binds, perfect risk sharing is no longer possible, and worker compensation responds to idiosyncratic productivity shocks. Thus, labor shares predict firm-level wage pass-throughs. In section 6.3, we show that consistent with the above implication of our model, payments to workers in firms with higher labor leverage are more sensitive to firm-level idiosyncratic shocks.

Part (ii) of the proposition summarizes our model's implications for the cross section of equity returns. Compensation contracts insure workers against aggregate shocks, which makes the residual dividends more risky. In our model, firms with high u_0 have low market-to-book ratios and high labor leverage. In the cross section, the operating leverage effect is stronger for high u_0 firms. These firms promise a large fraction of their cash flow to workers, bear more aggregate risk, and compensate investors by delivering higher expected returns. In section 6.2, we use panel data on firm-level measures of labor obligations and equity prices to show that low market-to-book ratio and high labor leverage firms indeed have higher expected returns.

From a worker's perspective, proposition 5 implies that the exposure of their consumption is increasing in promised utility, which is a measure of their wealth. This implication further contrasts our setup with those with only worker-side limited commitment. In these settings, (rich) agents who have experienced a history of positive shocks are more likely to be constrained, and (poor) agents who have experienced a history of negative shocks are typically not constrained. For these models to generate an amplified equity premium, the marginal rate of substitution of the unconstrained agents necessarily needs to be more volatile than that of an average agent. Therefore, they imply that the unconstrained poor agents' risk exposure to the stock market must be higher than that of the constrained wealthy agents, an implication that is inconsistent with the empirical evidence on stockholding patterns by wealth and income.[13]

5 Quantitative Analysis

5.1 Numerical Algorithm

Policy functions and state prices depend on the infinite-dimensional state variable ϕ. The distribution ϕ shows up directly in the market clearing condition and indirectly as an argument in the stochastic discount factor when we describe the optimal contracting problem *P1*. We use a numerical procedure similar to that in Krusell and Smith (1998) and replace the distribution ϕ with suitable summary statistics. We assume that agents compute future

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[13]See for instance Malloy et al. (2009).

state prices by projecting the stochastic discount factor on the space spanned by $\{g_t, x_t\}$ and use $x_{t+1} = \Gamma_x(x_t, g_t, g_{t+1})$ as a forecasting rule for x_t. Our choice of the forecasting rule is numerically efficient because given a law of motion for x, the stochastic discount factor is completely pinned down.[14]

Using the forecasting function Γ_x, we compute the stochastic discount factor $\Lambda(g', x, g)$. With $\Gamma_x(x, g, x')$ and $\Lambda(g', x, g)$, we solve the Bellman equation for the optimal contracting problem using an endogenous grid method and value function iteration. In appendix D, we describe a procedure that uses a grid on $\underline{\varepsilon}(u, S, g')$, which is the threshold for the idiosyncratic shock such that the firm-side limited commitment constraint binds, to tractably solve the contracting problem $P1$. After approximating the policy functions, we simulate a panel of agents and use the simulated data to update the law of motion Γ_x. We repeat this procedure until the function Γ_x converges. Appendix D describes the detailed steps and related diagnostics.

5.2 Calibration

Model parameters are divided into two sets: (i) parameters governing the stochastic process for aggregate shocks and (ii) parameters governing labor market flows and the distribution of idiosyncratic shocks to workers' human capital.

Aggregate shocks A period is a quarter. We time aggregate outcomes and report annual moments. We assume that the aggregate productivity process $\{g_t\}_t$ is a sum of a two-state Markov chain and a homoskedastic i.i.d. Gaussian component:[15]

$$\ln Y_{t+1} - \ln Y_t = g_{t+1} + \sigma_{\mathcal{E}} \mathcal{E}_t.$$

The state space for the Markov chain is $\{g_H, g_L\}$. We refer to states with $g = g_H$ as "booms" and states with $g = g_L$ as "recessions." The aggregate shock process $\{g_t, \mathcal{E}_t\}_t$ is calibrated as in Ai and Kiku (2013). They jointly estimate the values for $\{g_H, g_L\}$, the Markov transition matrix, and the volatility parameter $\sigma_{\mathcal{E}}$ from post-war aggregate consumption data. Our calibration implies an average duration of 12 years for booms and four years for recessions. The parameters for aggregate shocks are listed in the top part of table 1.

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[14]The market clearing condition equation (22) implies that x is linear in a $c(u, S)$ weighted average of the distribution ϕ. It summarizes information in ϕ by assigning relatively more weight to values of u that have a larger effect on aggregate resources. This choice contrasts our algorithm to that in Krusell and Smith (1998), who use the first moment of the distribution of wealth as a summary statistic.

[15]To better fit the autocorrelation of aggregate consumption growth, we use a more flexible process than the one listed section 2.1. Equilibrium prices and the optimal contract satisfy a homogeneity property, and the presence of i.i.d $\mathcal{E}$ shocks does not increase the state space for the value and policy functions.

Labor market flows and evolution of human capital   We calibrate the parameters that govern labor market flows and the evolution of human capital using transition rates between employment status, estimates of earning losses after separation, cross-sectional moments of labor earnings distributions, and other aggregate moments such as the mean and volatility of total labor compensation relative to aggregate consumption. Below, we specify our functional form choices and discuss the identification of key parameters by pairing them with the most relevant moments.

We set $\kappa = 1\%$ to obtain an average working life of 25 years. To better match the observed aggregate unemployment dynamics, we use a richer specification for the cost of retention function $A(\theta, g) = e^{a_0[\theta - a_{1,g}]}$. The scale of the cost function is normalized such that overall the costs are negligible relative to the total output. We interpret a separation in the model as a transition to the state of long-term unemployment (12 months and beyond). The parameters for $A(\cdot)$, and $\{\chi, \lambda, b\}$ are pinned down by the transition rates from employment to long-term unemployment in booms and recessions, the duration of long-term unemployment, the average earnings losses upon separation, and the estimate of the flow value of unemployment. To compute the flows in and out of long-term unemployment, we use data from the Current Population Survey summarized in table 1 of Shibata (2015). For earnings losses on separation, we use information from Davis and von Wachter (2011), who estimate the present value of earning losses due to job separations. We target the consumption equivalent of the flow value of unemployment to be 65% of pre-separation wage earnings.[16] The parameters and moments related to labor flows are listed in the middle panel of table 1.

Workers' human capital is affected by worker- and firm-level idiosyncratic shocks $\varepsilon + \eta$. We assume $\varepsilon = \alpha\varepsilon_W$ and $\eta = (1 - \alpha)\varepsilon_F$, where ε_W and ε_F are i.i.d. according to a continuous density $f(\cdot|g)$. To capture the feature that the (negative) skewness of labor earnings is cyclical, we model the distribution $f(\cdot|g)$ to be a Gaussian distribution in booms and a mixture distribution of a Gaussian and a fat-tailed distribution with a negative exponential form in recessions. We assume that both the Gaussian distributions as well as the negative exponential distribution satisfy a normalization such that the exponential of the draw has a unit mean. These restrictions imply $f(\cdot|g)$ is paramterized by the following: the standard deviation of the Gaussian distribution for booms σ_H; the standard deviation of the Gaussian distribution for recessions σ_L; the intensity parameter for the negative exponential distribution ξ; and the mixture weight $\rho \in (0, 1)$, which is the probability of drawing from the negative exponential distribution in recessions.

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[16]The empirical labor literature has a wide range of values for the flow value of unemployment. Shimer (2008) uses the unemployment insurance replacement rate of 40%, Rudanko (2011), and Mulligan (2012) add the value of home production and leisure and target a higher number of 85%, and Hagedorn and Manovskii (2008) use an even higher estimate of about 95%.

We set the parameter α to match the within- and across-firm variations in labor earnings as reported in Song et al. (2015) and calibrate the parameters $\{\sigma_H, \sigma_L, \rho, \xi\}$ to match the cyclical properties of the moments of labor earnings calculated using the Panel Study of Income Dynamics (PSID).[17] We restrict the sample to households where the "head of household" is a male whose working age is between 15 and 64 and who reports at least 500 hours of work in two consecutive years. Our measure of earnings is the regression residual of post-tax labor earnings on observable characteristics: age of the head, the age square, family size, and education level of the head. To obtain our target moments, we compute the cross-sectional standard deviation and Kelly skewness for log earnings growth, which are then averaged separately for "boom years" and "recession years."[18] In the bottom part of table 1, we report the parameter values and moments related to the earnings distribution.

Our model closely matches the standard deviations of the earnings growth in booms and recessions. We obtain a Kelly skewness of -3% in booms and -10% in recessions, as compared with -3.2% and -9%, respectively, in the PSID.[19]

All parameters affect the aggregate labor share. In our model, the employed workers' consumption as a fraction of aggregate consumption is countercyclical. It has a mean of 70%, a standard deviation of 3%, and an autocorrelation of 0.58. These moments are consistent with the data of aggregate labor compensation. We use national income and product accounts (NIPA) to compute the ratio of aggregate labor compensation to aggregate consumption and then detrend the series. For the sample 1947–2015, the mean labor share in consumption is 75%, the standard deviation is 2.94%, and the autocorrelation is 0.88.

6 Results

We discuss the implications for asset pricing and labor market dynamics.

6.1 Aggregate Asset Prices

In table 2, we summarize aggregate asset pricing moments. The baseline calibration is under the column labeled "Model-Baseline," and the column labeled "Model-No Frictions" is the version without limited commitment and moral hazard. We report the properties of returns

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[17]The PSID is a longitudinal household survey of U.S. households with a nationally representative sample of over 18,000 individuals. Information on these individuals and their descendants has been collected continually, including data covering employment, income, wealth, expenditures, health, education, and numerous other topics. The PSID data were collected annually during the period 1968–97 and biennially after 1997.

[18]We treat 1980–82, 1991–92, 2000–01, and 2007–09 as recession years and the remaining ones as boom years.

[19]In a previous version, we also reported results for an alternative calibration that targeted moments from Guvenen et al. (2014) and produced similar asset pricing results. Compared to the Guvenen et al. (2014) data, the PSID allows us to control for transfers from the government and lifecycle earning patterns that we abstract from in our setup.

Table 1: PARAMETERS

Parameters	Values	Targeted moments	Values
Aggregate Risk			
g_H, g_L	0.35%, -0.15%	Mean, std of consumption growth	1.08%, 2.14%
$\pi(g_H|g_H)$	0.99	Duration of booms	12 yrs
$\pi(g_L|g_L)$	0.95	Duration of recessions	4 yrs
$\sigma_{\mathcal{E}}$	1.2%	Autocorr of consumption growth	0.44
Labor Market			
$a_{1,H}, a_{1,L}$	.995, .9925	Annualized separations rates	2%, 3%
χ	8%	Long-term unemployment duration	3 years
λ	96%	PV of earning losses on separation	30%
b	1	Flow value of unemployment	40-95%
κ	0.99	Duration of working life	25 years
Idiosyncratic Risk			
α	82%	Across firm wage variation	40%
σ_L, σ_H	7.0%, 8.0%	Std. of labor earnings change in booms and recessions	32%, 31%
τ, ρ	4.155, 2%	Kelly skewness of labor earnings change in booms and recession	-3.2%, -8.9%
Other parameters			
β, ψ, γ	0.989, 2, 5	Discount factor, IES, risk aversion	

Notes: All reported moments are annualized. The NIPA sample for aggregate consumption is 1930-2007. We follow the estimation procedure in Ai and Kiku (2013). The CPS transition rates are computed using the monthly average of workers' transitions over 12-month intervals between January 1976 and July 2014. Davis and von Wachter (2011) use longitudinal Social Security records from 1974 to 2008. The earnings losses are computed using job displacements defined as in long-tenure men, 50 years or younger, in mass-layoff events at firms with at least 50 employees. The earnings losses are accumulated for 20 years at a discount rate of 5% and are expressed as a percentage of displaced workers' average annual predisplacement earnings. The flow value of unemployment is relative to wages and in the range of estimates in Shimer (2008), Rudanko (2011), and Hagedorn and Manovskii (2008). The within- and between- firm wage variation is taken from table 6 in Song et al. (2015). We use the PSID for periods 1968-2014. The sample selection is explained in the text.

on both a claim to aggregate consumption $Y_t \int \phi_t(du)$ and a claim to aggregate corporate dividends $x_t Y_t$. Our model generates a high equity premium and a low risk-free interest rate with a risk aversion $\gamma = 5$ and an IES $\psi = 2$. Without assuming any financial leverage, the equity premium on the claim to corporate dividends is about 3.67% per year in the baseline model. The average debt-to-equity ratio for publicly traded U.S. firms is about 50%.[20] Accounting for financial leverage, our model implies a market equity premium of 5.5%, which is close to the historical average excess return of 6.06% on the U.S. aggregate stock market index. In contrast, the equity premium on the unlevered corporate dividends is 0.62% per year in the first-best economy without limited commitment and moral hazard.

The premium on a risky asset is proportional to the covariance between the stochastic discount factor and its return. Our model generates a high equity premium for two reasons. First, agency frictions amplify the unconditional volatility of the stochastic discount factor. As explained in proposition 4, the insurance motives against persistent countercyclical tail risk in labor earnings imply a procyclical consumption share of the marginal investors. A more volatile stochastic discount factor is reflected in higher Sharpe ratios. Using the mean and the standard deviation of excess returns from table 2, the Sharpe ratio on the claim to aggregate dividends in the baseline is 48.5%, which is more than twice as large as that in the case with no frictions.

The second reason for the high equity premium is the large volatility of stock returns. In our model, stock returns are volatile because agency frictions generate fluctuations in the volatility of the stochastic discount factor over time. The general equilibrium implications of the agency problem introduce a new channel that raises the volatility of the stochastic discount factor in recessions relative to booms. The reason is the presence of the distributional state variable ϕ, whose slow-moving dynamics are summarized in persistent changes in the firm owners' share of aggregate consumption x_t. Prolonged recessions are associated with increasingly lower levels of the firm owner's consumption share. This implies that small changes in x_t translate into large variations in $\frac{x_{t+1}}{x_t} e^{g_{t+1}}$, which is the consumption growth rate of the firm owners. In equilibrium, the amplified volatility of the firm owner's consumption is compensated by a higher risk premium. The second effect of low x_t in recessions is a higher discounting of the future match surplus. This lowers firms' incentives to retain workers and exacerbates the moral hazard problem. Agents anticipate more separations and a higher downside earnings risk, which feeds back into a higher risk premia. On the other hand, in booms, the level of x_t is high, and the volatility and discounting effects are diminished.

This asymmetry results in countercyclical risk prices, high return volatility, and predictability of market returns by valuation ratios. The model delivers a 7.40% standard

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[20]See Graham et al. (2015) for details on measurement of corporate leverage.

deviation of the return on the unlevered claim to corporate dividends, which is about three times higher than its counterpart in the economy with full commitment and no moral hazard. Given a low volatility of aggregate consumption and an only moderately volatile risk-free rate, most of the increase in the volatility of the market return is accounted for by the time-varying equity premium.

Table 2: AGGREGATE ASSET PRICING IMPLICATIONS

Moments	Model		Data
	Baseline	No Frictions	
Excess return on consumption			
mean	3.59%	0.62%	-
std.	7.40%	2.86%	-
Excess return on dividends			
mean	3.67%	0.62%	6.06%
std.	7.61%	2.86%	19.8%
Std of log SDF			
booms	19.15%	17.83%	38.00%
recessions	35.70%	27.80%	66.00%
Risk free rate			
mean	2.81%	5%	0.56%
std.	2.86%	0.85%	2.89%

Notes: All moments are annualized. In the "Model" column, the claim to consumption is $Y_t \int \phi_t(du)$. The the claim to dividends is $x_t Y_t$ and assumes zero financial leverage. The column labeled "No Frictions" is the first best economy, i.e., without limited commitment and moral hazard with same parameters for preferences and technology as the baseline. The column labeled "Data" column computes market return as value-weighted returns from CRSP stock index and adjusted for CPI inflation. Estimates of debt-to-equity for publicly traded U.S. firms range from 40%-50%. The risk-free rates are computed as in the appendix of Beeler and Campbell (2012). The estimates for Sharpe ratios on the market return in booms and recessions are from Lustig and Verdelhan (2012).

Time variation in the risk premium also generates the predictability of future excess returns by price-to-dividend ratios, an empirical regularity documented by several papers including Campbell and Shiller (1988), Fama and French (1988), and Hodrick (1992). In table 3, we report the results of predictability regressions in our model and those in the data. We regress excess stock market returns measured at one-to-twelve quarter horizons on the log price-to-dividend ratio at the start of the measuring period. The "Data" column reports coefficients and R^2 of these regressions using the SP500 returns over the period 1947–2015, where the data construction follows Beeler and Campbell (2012). We report the same regression results using model-simulated data in the "Model-Baseline" column. Overall, the model produces regression coefficients and R^2 that are consistent with those in the data. We also match the pattern that predictability is higher for longer-horizon returns. As a

comparison, the first-best case in the column "Model-No Frictions" has a very low R^2.

Our calibration does not explicitly target moments in return predictability regressions. In fact, compared with the data, our model has a higher R^2 in predictability regressions. Leading asset pricing models, such as the habit model of Campbell and Cochrane (1999) and the long-run risk model of Bansal and Yaron (2004), typically assume that dividend growth contains an extra stochastic component that is orthogonal to shocks in aggregate consumption, which implies that a significant component stock price movements is unpredictable. For parsimony, our model does not make this assumption and therefore generates a higher predictability relative to the data.

The moments of risk-free interest rate in our model are fairly in line with standard asset pricing models. The volatility of the risk-free rate in our baseline model is 2.86% per annum. There is a wide range of estimates for this moment in the literature depending on estimation details and sample choices. The volatility of risk-free interest rate in our model is slightly higher than standard asset pricing models such as Campbell and Cochrane (1999) and Bansal and Yaron (2004). However, it is consistent with recent papers that construct "ex ante" measures of the risk-free rate; see, for example, Schorfheide et al. (2018) and Beeler and Campbell (2012).

Table 3: AGGREGATE RETURN PREDICTABILITY

Horizon	Model				Data	
(quarters)	Baseline		No Frictions			
	β	R^2	β	R^2	β	R^2
2	-0.356	0.157	-0.381	0.001	-0.062	0.042
4	-0.580	0.251	-0.739	0.001	-0.113	0.075
8	-0.788	0.329	-1.409	0.002	-0.190	0.119
12	-0.860	0.345	-2.029	0.003	-0.236	0.142
16	-0.871	0.328	-2.600	0.003	-0.277	0.166

Notes: The coefficients and R^2 of the regressions $\sum_{j=1}^{J}(r_{t+j} - r_{f,t+j}) = \alpha + \beta(pd_t) + \epsilon_{t+j}$. The column labeled "Model-Baseline" uses data simulated by the baseline calibration. The column labeled "Model-No Frictions" is the first best economy, i.e., without limited commitment and moral hazard with same parameters for preferences and technology as the baseline. The column labeled "Data" follows the construction in Beeler and Campbell (2012).

Model benchmarking In this section, we compare our results to nested cases that capture important benchmarks in the literature. The comparisons highlight why our model generates risk premia that are high and countercylical relative to setups such as those with one-sided limited commitment or exogenous wage rigidity when we require consistency with the aggregate and distributional data on labor earnings across models. Table 4 summarizes the findings.

Assume that firms can fully commit. This version of our model is similar to Alvarez and Jermann (2001) or Chien and Lustig (2010), who study the asset pricing implications of worker-side limited commitment. We keep all other features of the model unchanged, including the assumption that workers obtain all the surplus from new matches and the specification of the moral hazard problem. The results are under the column labeled "Worker-Side" in table 4.

The risk premium on the aggregate endowment claim and the volatility of returns are lower in the model with only-worker-side limited commitment. The intuition for this result can be explained as follows. First, the tightness of the worker-side limited commitment constraint does not significantly change over time. In the model, the worker-side limited commitment constraint binds for workers that receive sufficiently positive idiosyncratic shocks. However, the right tail of the distribution of idiosyncratic shocks is similar in booms and recessions. This is because our calibration is disciplined by the feature of the data that the standard deviation and the right skewness of labor earnings are almost acyclical. Second, worker-side limited commitment generates uninsurable upside risk in labor earnings. Even with recursive utility, this does not produce quantitatively significant effects on marginal utilities.

In terms of the labor market moments, we find that the model with only-worker-side limited commitment misses the large negative Kelly skewness of labor earnings in recessions and other measures of tail risk, which in our baseline model is generated by the firm-side limited commitment constraint. In addition, the lack of time variation in discount rates mitigates the cyclicality of separation rates through the moral hazard channel.

Next we compare our model to a version of Favilukis and Lin (2016b). Their model features a complete-markets stochastic discount factor and exogenous wage rigidity that generates countercyclical labor shares. We capture the Favilukis and Lin (2016b) mechanism in our setup by assuming that the aggregate dividend process follows $\tilde{x}(g_t)Y_t$, where $\tilde{x}(g_H) > \tilde{x}(g_L)$. We keep all other parameters of the model unchanged and discipline the choice of $\tilde{x}(g_H)$ and $\tilde{x}(g_L)$ by calibrating them to match the mean and standard deviation of labor shares of 67% and 2%, respectively, as in Favilukis and Lin (2016b). We then price the resulting $\tilde{x}(g)Y$ claim using a stochastic discount factor that is derived from a representative agent economy version of our model.

The "Exogenous Wage Rigidity" version of the model delivers a low equity premium of 0.681% and a small volatility of excess returns of 3.09%. These values are only slightly higher than those in our first-best case reported in table 2 under the column labeled "No Frictions." The volatility of aggregate labor share in the data is small and this limits the ability of models relying exclusively on operating leverage to generate high risk prices. In addition to wage rigidity, Danthine and Donaldson (2002) allow for exogenous movements in factor shares.

However, their setup uses standard log preferences and movement in factor shares cannot generate significant variations in the conditional volatility of the stochastic discount factor. As a result, their model cannot account for the coexistence of high volatility of returns and low volatility of dividends and the risks-free rate (Campbell and Shiller, 1988). In contrast to models with exogenous wage rigidity, in our setup, as we have shown above, risk premia are amplified primarily through the effect of agency frictions on the volatility of the stochastic discount factor. Under recursive preferences, the endogenously generated time-varying factor shares translate into persistent movements in the market price of risk and are key to produce return predictability and large variations in price-to-dividend ratios.

Our baseline generates a significantly higher premium. Agency frictions in our model amplify the volatility of the stochastic discount factor as well as the risk exposure of the aggregate dividend claim. For example, in table 2 under the column labeled "Model-Baseline," we see that while the risk premium on the aggregate consumption claim is 3.59%, the premium on the claim to corporate dividends is 3.67%. The small difference in these risk premia highlights that the amplification is primarily due to a more volatile stochastic discount factor and the role of the cash flow volatility channel is small.

Modeling the mixture distribution is necessary to match the extent and cyclicality of tail risk observed in labor earnings and, at the same time, deliver an approximately acyclical standard deviation of earnings growth as observed in the PSID. To highlight its importance, in table 4 under the column labeled "No Mixture," we report two calibrations without assuming a mixture distribution: (i) $\sigma_H = \sigma_L$ and (ii) $\sigma_H < \sigma_L$.

In the case where the distribution of idiosyncratic risk is independent of the aggregate state—that is, $\sigma_H = \sigma_L = 8.3\%$—we find that the asset pricing implications are almost similar to the first-best case, consistent with the Krueger and Lustig (2010) intuition outlined in section 4.1. In the case $\sigma_L > \sigma_H$, it is possible to make σ_L sufficiently higher than σ_H so that the implied volatility of the stochastic discount factor is similar to the baseline calibration. With $\sigma_L = 10.3\%$ and $\sigma_H = 8\%$, we are able to get an equity premium on the unlevered aggregate consumption claim of 3.2%. However, we find that the earnings growth distribution has (counterfactually) countercyclical standard deviation, 38% in recessions and 30% in booms, and almost no cyclicality in Kelly skewness.

Discount rates in general equilibrium models can be constructed from the marginal rate of substitution of marginal investors. Although all agents who do not face a binding limited commitment constraint are marginal investors in our model, it is more convenient to construct the SDF from firm owners' consumption, because they are never constrained.[21] To further

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[21]Firm owners are not the only investors whose marginal rate of substitution is a valid stochastic discount factor. This is true for all unconstrained workers. In our baseline calibration, less than 5% of workers are constrained in any given quarter.

illustrate the mechanism for the high volatility of SDF in our model, we report the volatility of firm owner's consumption in all four versions of the model in table 4.

First, the standard deviation of consumption growth for firm owners is 10% per year. In the data, it is difficult to reliably measure the consumption of wealthy stockholders. Using the sample from Consumer Expenditure Survey (CEX), Wachter and Yogo (2007) report that the median standard deviation of consumption growth for the bottom 50% of stock-holding households is 7.26% and that of the top 25% is about 11.38% per year. Firm owners' consumption is more procyclical than aggregate consumption. As we explain in section 4, this extra risk exposure in not insured away in equilibrium, because workers' marginal utility is high in recessions owing to uninsurable idiosyncratic risk, and it is optimal for firm owners to bear more aggregate risks than workers.

Second, the pattern of the volatility of consumption growth for firm owners echoes the pattern of the volatility of stochastic discount factor across models. The inability for other versions of the model to generate a high volatility of SDF can be attributed to the lack of risk exposure of firm owners' equilibrium consumption. The version of our model with countercyclical second moment ($\sigma_L > \sigma_H$) also generates a high volatility of consumption growth for firm owners and a high volatility of SDF, but at the expense of excessive countercyclical standard deviation in labor earnings growth.

6.2   Cross Section of Expected Returns

Value premium   Stocks with low valuation ratios (value stocks) earn higher average returns than stocks with high valuation ratios (growth stocks). The difference in the mean returns of value and growth stocks is robust to various ways of constructing the valuation ratio—for example, as the ratio of the market value of the firm to its book value or as the ratio of the market price of the stock to earnings per share; see Fama and French (1992) and Fama and French (1993).

Our model generates a value premium. The price-to-earnings ratio and expected returns are functions of the state variable u_t, which summarizes the fraction of future cash flows that is promised to workers. Firms with high-u workers have a high operating leverage and a low valuation ratio. Proposition 5 states that such firms should have a higher expected return. To compare our model implications with data, we sort stocks into three portfolios ranked by earnings-to-price ratios.[22]   The mean high-minus-low return is 6.27% per year, with a t-statistic of 5.01. The same portfolio-sorting procedure in the data simulated from the model generates a value premium of 4.66% per year.

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[22]The return series for these portfolios is obtained from Kenneth French's website and covers the period 1956—2016.

Table 4: COMPARISON TO OTHER BENCHMARKS

Moments	Data	Models				
		Baseline	Worker-Side	Exogenous Wage Rigidity	No Mixture	
					$\sigma_H = \sigma_L$	$\sigma_H < \sigma_L$
Excess return on consumption						
mean	-	3.59%	1.16%	0.62%	1.03%	3.20%
std.	-	7.40%	2.43%	2.86%	3.16%	6.75%
Excess return on dividends						
mean	6.06%	3.67%	0.91%	0.68%	1.04%	3.27%
std.	19.8%	7.61%	2.52%	3.09%	3.36%	6.94%
Std of log SDF						
booms	38.00%	19.34%	13.75%	17.83%	9.43%	18.95%
recessions	66.00%	35.7%	23.00%	27.80%	21.22%	36.26%
Firm owners consumption						
std.	7% to 11%	10%	4.54%	3.84%	3.82%	7.72%
Risk free rate						
mean	0.56%	2.81%	5.07%	4.73%	4.33%	3.21%
std.	2.89%	2.86%	1.44%	0.39%	0.77%	2.28%

Notes: All moments are annualized. The column labeled "Data" column computes market return as value-weighted returns from CRSP stock index and adjusted for CPI inflation. Estimates of debt-to-equity for publicly traded U.S. firms range from 40%-50%. For the firm owner consumption growth in the "Data" column, we use Wachter and Yogo (2007) estimates of standard deviation of consumption growth for the stock-holding households. In the "Model" columns, the claim to consumption is $Y_t \int \phi_t(du)$. The the claim to dividends is $x_t Y_t$ and assumes zero financial leverage. For all cases, technology and preferences parameters are the same as the baseline. The column labeled "Worker-Side" relaxes constraint $v(u, S) \geq 0$. The column labeled "Exogenous Wage Rigidity" uses the first-best stochastic discount factor, in the row "Excess returns on $x_t Y_t$" we price an unlevered claim to corporate dividends with the cash flow on this claim is modeled as $\tilde{x}(g)Y$ with more details in the main text. In the column labeled "No Mixture", we set the mixture probability of drawing from the negative exponential ρ to zero. For subcolumn labeled "$\sigma_H = \sigma_L$" the value for std. in booms and recessions is set to 8.3% and for the subcolum labeled of "$\sigma_H < \sigma_L$", the value for $\sigma_L = 10.3\%$ and the value of $\sigma_H = 8.3\%$.

In our model, firms with a history of negative idiosyncratic shocks have higher expected return. A similar effect is documented by Bondt and Thaler (1985) as "long-term reversal." In our model, long-term reversal and value premium are due to the same economic mechanism, and hence they are highly correlated. Consistent with this implication of our theory, Fama and French (1996) show that the returns on value-growth portfolios and long-term reversal sorted portfolios are highly correlated.

Labor leverage and the cross section of expected returns   A more direct test of the model mechanism is the connection between the value premium and firm-level obligations to workers. We use the merged CRSP/Compustat panel to test this implication.

We focus on publicly traded firms in the Compustat database and regress excess returns on a firm's equity, which are defined as the difference between equity returns and the three-month T-bill rate, on firm-level labor shares and time fixed effects.

$$\text{Excess Return}_{f,t+1} = \alpha_r + \beta_r \times \text{LaborShare}_{f,t} + \lambda_{rt}. \tag{33}$$

Following Donangelo et al. (2016), labor share for firm f at period t is constructed using

$$\text{LaborShare}_{f,t} = \frac{XLR_{ft}}{OPID_{f,t} + XLR_{f,t} + \Delta INV_{f,t}}, \tag{34}$$

where XLR is the total wage bill, OPID is operating profit before interest and depreciation, and INV is change in inventories. Whenever XLR is not available, we construct an extended labor share (ELS) using the procedure described in Donangelo et al. (2016). In table 5, we report our results both with labor share, under the column labeled "Using LS," and with extended labor share, under the column labeled "Using ELS." Consistent with our model, labor share predicts expected returns, and the point estimate for β_r is positive and significant.[23] These findings are consistent with and complementary to other studies such as Donangelo et al. (2016), who document returns on labor share–sorted portfolios and estimate versions of (33), as well as Favilukis and Lin (2016a), who use wage rigidity as a proxy for labor leverage at the industry level and show that labor leverage predicts cross-industry expected returns.

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[23]The estimates are robust to including various control variables such as leverage and total assets in the regression (33). See section C in the online appendix (not for publication).

Table 5: FIRM-LEVEL RETURNS AND LABOR SHARES

Coefficients	Using LS	Using ELS
Labor share	1.38	1.25
	(0.41)	(0.19)
Time fixed effects	Yes	Yes
no. of obs.	15170	83611
no. of entities	1645	9591

Notes: The sample consist of firm-year observations from CRSP/Compustat merged files for the years 1968-2016. In the column labeled "Using LS" we use labor share computed using (34), and in the column labeled "Using ELS" we use the procedure described in Donangelo et al. (2016) and construct "extended labor share." In both specifications, labor shares are standardized and twice lagged, and standard errors are clustered at firm level.

6.3 Labor Market Implications

In this section, we focus on the implications for aggregate and cross-sectional labor market dynamics.

Discount rates and unemployment risks The incentive compatibility condition (18) links unemployment risk to worker valuations that are influenced by discount rate variations. In our model, prolonged recessions are states with high expected returns and low present values of cash flows from workers. Because firms' retention effort is not observable, they have a lower incentive to keep workers in times of low valuations. Several papers in the recent literature emphasize the link between discount rates and unemployment; see, for example, Hall (2017), Kehoe et al. (2019) and Borovicka and Borovickova (2018). In contrast to these papers, the variation in discount rates in our setting is driven by general equilibrium implications of contracting frictions, and our model is consistent with broad patterns in aggregate and cross-sectional asset returns.

In our model, average separation rates are countercyclical: 3% per year in recessions and 2% per year in booms. In the presence of separations, part of the tail risk in labor earnings is driven partly by the extensive margin when workers transition from employment to long-term unemployment. We decompose large earnings drops—that is, reductions in individual earnings of more than 20%—into two categories: separations and within-employment compensation cuts. In our calibration, 48.5% of large earnings drops are due to separation and the remaining 51.5% are due to a binding firm-side limited commitment constraint. This pattern is consistent with Guvenen et al. (2014), who document that workers in the left tail of the income distribution are more likely to experience a large drop in earnings, and claim that a nonnegligible fraction of the drop is due to unemployment risk.

The separation risk is also quantitatively important in accounting for the volatility of the stochastic discount factor. Shutting down separations—that is, under a calibration with $\theta = 1$—and keeping all other parameters unchanged lowers the annualized risk premium on the aggregate consumption claim from 3.6% to 2.14%. We interpret this as both channels being salient and quantitatively relevant in accounting for the equity premium.

Exposures to idiosyncratic and aggregate shocks Propositions 4 and 5 have direct implications for how idiosyncratic and aggregate shocks are insured in the presence of agency frictions. Owing to to firm-side limited commitment, workers with adverse histories are more exposed to idiosyncratic shocks in recessions. The optimal contract compensates this lack of insurance by providing such workers an additional hedge against aggregate shocks. Thus, the consumption of workers with adverse histories would have a relatively higher exposure to idiosyncratic shocks and a lower exposure to aggregate shocks.

To test whether firms with larger obligations to workers provide less insurance against idiosyncratic shocks, we measure the pass-through of firm-level shocks to their wage payments and check whether these pass-throughs systematically vary with the firm-level labor share. We estimate the regression

$$\Delta \log \text{WageBill}_{f,t+1} = \alpha_w + \beta_{w0}\, \text{LaborShare}_{f,t} + \beta_{w1}\Delta \log \text{Sales}_{f,t}$$
$$+ \gamma_w \Delta \log \text{Sales}_{f,t} \times \text{LaborShare}_{f,t} + \lambda_{wt}, \qquad (35)$$

where $\text{WageBill}_{f,t+1}$ is the total wage bill of firm f in year $t+1$ and $\text{LaborShare}_{f,t}$ is as defined in equation (34). Our sample includes all firms in Compustat for the period 1959-2017.

We report our regression results in table 6, where standard errors are in parentheses. Consistent with our model's implication of imperfect risk sharing, the point estimate of the pass-through coefficient β_1 is positive but less than one.[24] Furthermore, the interaction term $\gamma_w > 0$ and is statistically significant. This confirms the conclusion of proposition 5 that firms with higher labor leverage have a higher pass-through coefficient. In section C of the online appendix (not for publication), we estimate a version of (35) where we split the sales growth into a negative sales growth part and positive sales growth part. We find that consistent with the model, the interaction term is driven mainly by the negative part of sales growth.

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[24]Guiso et al. (2005) also estimate the extent of insurance within the firm using administrative-level matched employer-employee data and similar regressions.

Table 6: FIRM-LEVEL WAGE PASS-THROUGHS AND LABOR SHARES

Coefficients	Using LS	Using ELS
LogSales	0.4159	0.3187
	(0.0422)	(0.0276)
LaborShare	-0.0726	-0.1648
	(0.007)	(0.0061)
LaborShare × LogSales	0.3871	0.3538
	(0.0776)	(0.0517)
Time fixed effects	Yes	Yes
no. of obs	40289	117128
no. of entities	4028	12806

Notes: The sample consist of firm-year observations from Compustat for the years 1959-2016. We follow Donangelo et al. (2016) in the construction of firm labor share, the results of which are reported in the column labeled "Using LS", and the construction of extended labor share, the results of which are reported in the column labeled "Using ELS." In both specifications, labor shares are twice lagged, and standard errors are clustered at the firm level.

7   Conclusion

We present an asset pricing model where risk premia are amplified by agency frictions. Under the optimal contract, sufficiently adverse shocks to worker productivity are uninsured. In general equilibrium, exposure to downside tail risk results in a more volatile stochastic discount factor and time variations in discount rates. These features of the pricing kernel yield quantitatively large and volatile risk premia and generate a substantial cross-sectional variation in returns across firms. Our model is also consistent with firm-level measures of labor share that predict both future returns and pass-throughs of firm-level shocks to wage payments.

An interesting extension of our setup would be to allow for a storage technology, such as physical capital, along with related agency frictions such as hidden savings for workers or the ability of firm owners to additionally use capital as collateral. This will open up a host of new predictions about aggregate business cycle fluctuations, firm-level and aggregate asset prices as well as capital misallocation in the cross section of firms. A recent paper by Tong and Ying (2020) builds on our setup and studies asset pricing implications of limited commitment in production economies.

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Online Appendix for *Asset Pricing with Endogenously Uninsurable Tail Risk*

A   Additional details for restrictions on the space of contracts

In this section, we show that given the equilibrium we construct in the main text, there are no incentives for firms to offer any insurance to workers that are not currently matched with the firm. We build the argument in several steps. We first show that firms do not have incentives to offer any unemployment insurance to workers after separation. Then we show that the restricted employment contract that we construct in the main text of the paper is in fact optimal in a larger contracting space where all firms are allowed to offer insurance to all workers.

A.1   Insurance provision to unemployed workers

First, consider an optimal contracting problem of a firm that offers payments $\left\{\widetilde{C}_{t+s}\right\}_{s=0}^{\infty}$ to an unemployed worker subject to two-sided limited commitment. Let $\widetilde{V}(U, y, S)$ be the value of the insurance contract to a firm as a function of worker output y, promised utility U for a given aggregate state S. Following the steps in the main text, the above contracting problem can be expressed as

$$\widetilde{V}(U, y, S) = max_{\widetilde{C}, \left\{\widetilde{U}'(g')\right\}_{g'}} \left\{ -\widetilde{C} + \kappa \sum_{g'} \pi(g'|g) \Lambda(S', S)(1 - \chi) \widetilde{V}\left(\widetilde{U}'(g'), \lambda y, S'\right) \right\}$$

subject to

$$\left\{ (1 - \beta)\left[by + \widetilde{C}\right]^{1 - \frac{1}{\psi}} + \beta \left(\kappa \sum_{g'} \pi(g'|g)\left[(1 - \chi)\widetilde{U}'(g')^{1-\gamma} + \chi\left(u^*(S')\lambda y\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \right)^{1 - \frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \geq U,$$

$$(36)$$

$$\widetilde{U}'(g') \geq \bar{u}(S')\lambda y, \; for \; all \; g', \tag{37}$$

$$\widetilde{V}\left(\widetilde{U}'(g'), \lambda y, S'\right) \geq 0, \; for \; all \; g'. \tag{38}$$

where functions $u^*(S)$ and $\bar{u}(S)$ are defined in equations (13) and equation (12) respectively.

The optimal contract chooses the current period payment to the worker $\widetilde{C}$ and a menu of continuation utilities $\left\{\widetilde{U}'(g')\right\}_{g'}$ to maximize the net present value of the contract to the

firm. The human capital of an unemployed worker depreciates deterministically at rate $1 - \lambda$; therefore, in the absence of idiosyncratic shocks, the continuation utility is only a function of aggregate shock g'. To understand the expression for the continuation payoff, note that in the next period, with probability χ, the worker stay unemployed, in which case the value of the contract is $\widetilde{V}\left(\widetilde{U}'(g'), \lambda y, S'\right)$. With probability $1 - \chi$, the worker receives an opportunity to match with a firm, which can be a different firm or the same firm who is providing the insurance. In either case, because of competition, the value of the continuation contract after the worker finds an employment opportunity give the worker a continuation value of $u^*(S)\lambda y$ and a value of zero to the firm.

Inequality (36) is the promise keeping constraint. The worker receives by as unemployment benefit and a transfer of $\widetilde{C}$ from the insurance firm. In the next period, with probability $1 - \chi$, the worker stays unemployed and receives promised utility $\widetilde{U}'(g')$. With probability χ, the worker has an opportunity to match with another firm and receives $u^*(S')\lambda y$. Inequality (37) is the limited commitment constraint for workers: promised utility under the insurance contract has to be higher than the utility associated with consuming unemployment benefit as workers always have an option to default on the contract offered by the insurance firm and to consume the unemployment benefit thereafter. Because workers' human capital depreciate at rate $1 - \lambda$, the utility associated with consuming unemployment benefit is $\bar{u}(S')\lambda y$ in the next period. Finally, inequality (37) is the limited commitment constraint for the firm which requires the net present value of the insurance contract to be non-negative for the firm.

The lemma below provides a sufficient condition for the absence of unemployment insurance offered by firms.

Lemma 1. *Suppose λ is small enough, in particular, for all S and S',*

$$\lambda \leq \left[\frac{x(S')}{x(S)}\right]\left[\frac{w(S')}{n(S)}\right]^{\psi\gamma - 1}. \tag{39}$$

Then, at $U = \bar{u}(S)y$, we must have $\widetilde{C}(U, y, S) = 0$, $\widetilde{U}'(U, y, S, g') = \bar{u}(S')\lambda y$, and $\widetilde{V}(U, y, S) = 0$.

Proof. As in the main text, the optimal contracting problem can be normalized. Homogeneity of the problem implies $\widetilde{V}(U, y, S) = \widetilde{v}(u, S)y$ for some $\widetilde{v}$, where $u = \frac{U}{y}$. Using normalized value and policy functions, we can write the optimal contracting problem in the normalized

form. Define the T operator as

$$T\widetilde{v}(u,S) = max_{c,\{u'(g')\}_{g'}} \left\{ -\widetilde{c} + \kappa \sum_{g'} \pi\left(g'|g\right) \Lambda\left(S',S\right) \chi e^{g'} \widetilde{v}\left(u'\left(g'\right),S'\right) \right\} \tag{40}$$

$$s.t. \left\{ (1-\beta)\left[b+\widetilde{c}\right]^{1-\frac{1}{\psi}} + \beta\left(\lambda m\right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-1/\psi}} \geq u \tag{41}$$

$$m = \left\{ \kappa \sum_{g'} \pi\left(g'|g\right) \left[(1-\chi)u'\left(g'\right)^{1-\gamma} + \chi u^*\left(S'\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \right\} \tag{42}$$

$$u'\left(g'\right) \geq \bar{u}\left(S'\right), for \ \ all \ \ g' \tag{43}$$

$$\widetilde{v}\left(u'\left(g'\right),S'\right) \geq 0, for \ all \ \ g'. \tag{44}$$

Under standard discounting assumptions, T is a contraction on the set of bounded continuous functions and $\widetilde{v}(u,S)$ is the unique fixed point of the T operator. The conclusion of the above lemma is therefore equivalent to the following property of the normalized optimal contracting problem, that is, for $u = \bar{u}(S)$,

$$\widetilde{c}(u,S) = 0; \ u'\left(u,S,g'\right) = \bar{u}\left(S'\left(g'\right)\right) \ for \ all \ g'; \ and \ \widetilde{v}(u,S) = 0. \tag{45}$$

Consider the constrained maximization problem (40). Let μ be the Lagrangian multiplier for the constraint (41). The first order conditions are:

$$1 = \mu\left(1-\beta\right) \left(\frac{c}{u}\right)^{-\frac{1}{\psi}},$$

$$\Lambda\left(S',S\right) \frac{d}{du}\widetilde{v}\left(u'\left(g'\right),S'\right) + \mu\beta e^{-\gamma g'} \left(\frac{\lambda m}{u}\right)^{-\frac{1}{\psi}} \left(\frac{u'\left(g'\right)}{m}\right)^{-\gamma} \geq 0, \ for \ all \ g' \tag{46}$$

and "=" holds if $\widetilde{v}\left(u'\left(g'\right)|S'\right) > 0$. The envelope condition implies $\frac{d}{du}\widetilde{v}(u,S) = \mu$. Combining the above conditions, and using the expression for the stochastic discount factor in (20), the optimality condition (46) can be written as

$$\left[\frac{x\left(S'\right)}{x\left(S\right)}\right]^{-\frac{1}{\psi}} \left[\frac{w\left(S'\right)}{n\left(S\right)}\right]^{\frac{1}{\psi}-\gamma} \leq \lambda^{-\frac{1}{\psi}} \left[\frac{b+\widetilde{c}\left(\bar{u}\left(g'\right),S'\right)}{b+\widetilde{c}\left(u,S\right)}\right]^{-\frac{1}{\psi}} \left[\frac{u'\left(g'\right)}{m}\right]^{\frac{1}{\psi}-\gamma}, \tag{47}$$

and "=" holds if $\widetilde{v}\left(u'\left(g'\right)|S'\right) > 0$. Because (44) is a standard convex programming problem, (47) is both necessary and sufficient for optimality.

To prove (45), note that set of functions that are concave in its first argument and satisfy $\widetilde{v}\left(\bar{u}\left(S\right),S\right) = 0$ is a closed subset in the set of bounded continuous functions. To prove that the unique fixed of T satisfies $\widetilde{v}\left(\bar{u}\left(S\right),S\right) = 0$, we start by assuming $\widetilde{v}(u,S)$ is concave in

the first argument and satisfies $\widetilde{v}\left(\bar{u}\left(S\right),S\right)=0$, and we need to show that $T\widetilde{v}$ satisfies the same properties.

Because $\widetilde{v}\left(u,S\right)$ is concave in its first argument, condition (47) together with the promise keeping constraint (41) are sufficient for optimality. Under assumption (39), the proposed policy functions in (45) satisfy the first order condition (47). In addition, the promise keeping constraint is satisfied by the definition of $\bar{u}\left(S\right)$ in (13). Therefore, the policy functions (45) are optimal. Clearly, under the proposed policy functions, $T\widetilde{v}\left(\bar{u}\left(S\right),S\right)=0$. The fact that $T\widetilde{v}\left(\bar{u}\left(S\right),S\right)$ must be concave follows from standard argument (see, for example, Ai and Li (2015)). □

The above lemma has two implications. First, under condition (39), in equilibrium, a firm cannot earn a positive profit by offering a non-trivial insurance contract to any unemployed worker. To see this, note that the value function $\widetilde{V}\left(U,y,S\right)$ must be strictly decreasing in U. Because the utility provided by the unemployment benefit is the lower bound of the utility that an unemployed worker can achieve, we must have $U\geq\bar{u}\left(s\right)y$ under any non-trivial insurance contract. Therefore, $\widetilde{V}\left(U,y,S\right)\leq0$, i.e. no firm can make a positive profit by deviating from the trivial insurance contract.

Second, employer firms cannot offer any severance pay to a worker upon unemployment. To see this, consider an augmented contract space $\mathscr{C}\cup\widetilde{\mathscr{C}}$ with $\widetilde{\mathscr{C}}$ specifying payments to worker after separation. From the history at which the worker is unemployed, the firm's value under any contract with non-trivial payment to the worker cannot exceed $\widetilde{V}\left(U,y,S\right)$ defined in (38). An augmented contract with non-trivial severance pay will give unemployed workers a value higher than the autarky value of consuming the unemployment benefits $\bar{u}\left(S'\right)y'$. Thus by the same argument as in the previous paragraph, any such arrangement will imply $\widetilde{V}\left(U,y,S\right)<0$, which violates the firm-side limited commitment.

Intuitively, it is the joint assumption of two-sided limited commitment and perfect competition on firm side that rule out unemployment insurance in equilibrium. The income of unemployed workers are front loaded. In our model, as human capital depreciates, so does the unemployment benefit. To provide any non-trivial intertemporal consumption smoothing to unemployed workers, a firm would need to backload its payment. The limited commitment on firm side ($\widetilde{v}\left(\bar{u}\left(S\right),S\right)\geq0$) implies that firms cannot commit to backloaded payments unless they can expect some profit in the future. However, there is no profit to be made in an insurance contract with an unemployed worker: the worker will need continued payment as long as he is unemployed; once the worker is employed, limited commitment on worker side and perfect competition between firms mean that the worker will extract all surplus in the new match and cannot commit to pay back the unemployment insurance provider. The fact that workers extract all surplus in a new employment contract is the key feature of our

model that rules out equilibrium private unemployment insurance.

Finally, from a quantitative point of view, condition (39) is a fairly weak assumption on λ. In an economy without aggregate risk, it is equivalent to $\lambda \leq 1$. In our calibration, $\lambda = 0.96$ at the quarterly level, and (39) is certainty satisfied.

A.2   Insurance provision to other workers

Here we show that the employment contract that we construct in the main text of the paper is in fact optimal in a larger contracting space where all firms are allowed to offer insurance to all workers. To do so, we follow several steps. In step 1, we describe a dynamic game in which firms compete for workers by offering long-term contracts where all firms are allowed to pay all workers subject to incentive compatibility. In step 2, we describe an equilibrium strategy in the above game where contracts only involve non-trivial payments from firms to their employees. In step 3, we show that the proposed contract is optimal in a Subgame Perfect Nash Equilibrium (SPNE) of the game.

STEP 1: Here, we describe a game where all firms are allowed to offer contracts to all workers. We first introduce some terminologies and notations. We define $\{\iota_{i,t}\}_{t=0}^{\infty}$ to be the stochastic process that records the birth, death, and unemployment shocks experienced by worker i.[25] In addition, upon receiving an opportunity to match, a worker randomizes among all firms that offer the most favorable contract. We use $\upsilon_{i,t}$ to denote the outcome of the randomization device, with $\upsilon_{i,t} = j$ if firm j is chosen by worker i in period t. We use $\zeta_{i,j}(t) = (g_t, \eta_{j,t}, \varepsilon_{i,t}, \iota_{i,t}, \upsilon_{i,t})$ to denote time t shocks for a firm-worker pair and $\zeta_{i,j}^t = \left(g^t, \eta_j^t, \varepsilon_i^t, \iota_i^t, \upsilon_i^t\right) = \{g_s, \eta_{j,s}, \varepsilon_{i,s}, \iota_{i,s}, \upsilon_{i,s}\}_{s=0}^t$ to denote the history of shocks of a firm-worker pair up to time t.[26] Because all workers are endowed with one unit of human capital at birth, given the history of shocks, $\zeta_{i,j}^t$, we can recover $h_{i,t}$ and $y_{i,t}$ for all t using equations (1) and (2), which we denote as $h_t(\zeta_{i,j}^t)$.

A contract offered by firm j to worker i specifies the net transfers from the firm to the

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[25] To keep the convention that $\{\iota_{i,t}\}_{t=0}^{\infty}$ are exogenous shocks not influenced by agents' actions, we can assume that they are i.i.d. random variables uniformly distributed on $[0, 1]$. If worker i is employed in period t, the outcome of the match with the employer firm is decribed by $I_{\left\{\iota_{i,t+1} \leq \theta_{i,j,t}\right\}}$, where I is the indicator function. That is, $I_{\left\{\iota_{i,t+1} \leq \theta_{i,j,t}\right\}} = 0$ if the worker separates from the current firm and become unemployed in period $t + 1$ and $I_{\left\{\iota_{i,t+1} \leq \theta_{i,j,t}\right\}} = 1$ if the worker continues the match with his current employer in period $t + 1$. Consistent with the setup of our model, the probability of the survival of the match is $\theta_{i,j,t}$. Similarly, for worker i who is unemployed in period t, $I_{\left\{\iota_{i,t+1} \leq 1-\chi\right\}} = 0$ if the worker continue to stay unemployed in period $t + 1$, and $I_{\left\{\iota_{i,t+1} \leq 1-\chi\right\}} = 1$ if the worker receives an employment opportunity to match with a firm in period $t + 1$.

[26] The usage of ζ is consistent with the main text in the sense that it is a vector of aggregate and idiosyncratic shocks in period t. However, unlike in the main text of the paper, here, worker i and firm j do not necessarily have an employment relationship. In addition, the shock structure in this more general setup is richer — for example, it contains unemployment shocks, $\iota_{i,t}$ — in order to allow for a larger contracting space.

worker and the retention effort , $\mathscr{C}_{i,j} \equiv \left\{ C_{i,j,t}\left(\zeta_{i,j}^{t}\right), \theta_{i,j,t}\left(\zeta_{i,j}^{t}\right) \right\}_{t=0}^{\infty}$, as functions of the history of shocks.[27] Clearly, our setup implies that $\theta_{i,j,t}\left(\zeta_{i,j}^{t}\right) = 0$ unless worker i is matched with firm j at history $\zeta_{i,j}^{t}$. We use $\mathscr{C} = \langle \mathscr{C}_{i,j}\rangle_{i,j}$ as the collection of contracts offered by all firms to all workers. We suppress the decision for keeping the firm-worker match $\delta_{i,j,t}$ and assume from the outset that a firm-worker match is never voluntarily separated. As we have shown in proposition 1 in the main text of the paper, this is without loss of generality.

We consider a repeated game in which in each period t, all firms offer longer term contracts to all workers and denote a contract offered by firm j as $\langle \mathscr{C}_{i,j}\rangle_i$. After all firms make offers, workers make decisions on which contract(s) to accept. A worker is free to default on previous contracts at any history. Default on the contract offered by the employer firm results in a separation of the match and termination of all future cash transfers. Default on a contract offered by an unrelated firm results in termination of all future cash flows and nothing else. In addition, a worker who has an employment opportunity can choose the firm that offers the most attractive employment contract to match. If indifferent, he randomizes. Finally, firms are free to default on their contract at any point in time. A default on the firm side results in a separation of the match (if this is a contract with an employee) and termination of all future cash transfers.

In a typical period t, given the action of all firms, $\mathscr{C}$, a firm's payoff is calculated as the present value of all cash flows generated by contracts with all workers. A worker's payoff is the present value of utility that the worker receives under all accepted contracts with all firms.

STEP 2: Here we construct a SPNE of the game specified above using the optimal employment contract that we describe in the main text of the paper. To decribe an equilibrium contract, we can without loss of generality focus on contracts that will be accepted at all times and all histories, because if part of the contract is not accepted at some history in equilibrium, we can simply rename the contract so that it prescribes zero transfer between the firm and the worker after that history. As a result, even though firms are allowed to offer a different contract in every period, in the construction of the SPNE of the game, we can focus on the case where the same contract (that will be accepted at all future histories and all times) is offered in every period.

In every period, firms offer contracts to three different types of workers: workers employed by them, workers employed by other firms, and unemployed workers. To decribe an SPNE strategy, we first specify the contract offered to an employee based on the optimal employment contract we describe in the paper. Given the pricing kernel $\Lambda\left(S', S\right)$ and the equilibrium value of workers with a job opportunity, $yu^{*}\left(S\right)$, firms' value function defined in (11) is the unique

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[27]Here, we use the same notation $\mathscr{C}_{i,j}$ as in the main text to denote contracts in a larger space.

fixed point of the following T operator:

$$TV(U, y, S) = max_{C, \theta, \{U'(\zeta')\}} \left\{ (y - C) + \kappa\theta \int \Lambda(S', S) V\left(U'(\zeta'), ye^{g' + \eta' + \varepsilon'}, S'\right) \Omega(d\zeta'|g) \right\}$$

$$s.t. \left\{ (1 - \beta) C^{1 - \frac{1}{\psi}} + \beta \left(\kappa \int \left[\theta U'(\zeta')^{1-\gamma} + (1 - \theta) \left[\bar{u}(S') ye^{g' + \eta' + \varepsilon'} \right]^{1-\gamma} \right] \Omega(d\zeta'|g) \right)^{\frac{1}{1-\gamma}} \right\} \geq U$$

$$U'(\zeta') \geq \bar{u}(S') ye^{g' + \eta' + \varepsilon'} \tag{48}$$

$$V\left(U'(\zeta'), ye^{g' + \eta' + \varepsilon'}, S'\right) \geq 0. \tag{49}$$

We denote the policy functions associated with the above dynamic programming program as $C(U, y, S)$ and $\{U'(U, y, S, \zeta')\}_{\zeta'}$.

As is standard in the dynamic contracting literature, in any period t, given a vector of initial state variables, (U, y, S), the continuation contract from period t that specifies payment to employed workers in all future dates and states can be constructed recursively from the policy functions of the above dynamic contracting problem. We denote an employment contract with initial condition (U, y, S) as $\mathscr{C}(U, y, S)$. To specify the continuation contract from any history $\zeta_{i,j}^t$, we only need a procedure to construct the initial state variables $\left(U_{i,t}\left(\zeta_{i,j}^t\right), y_{i,t}\left(\zeta_{i,j}^t\right), S_t\left(g^t\right) \right)$ at that history. The construction of the exogenous state variables $y_{i,t}\left(\zeta_{i,j}^t\right)$ and $S_t\left(g^t\right)$ are straightforward and are described in the main text of the paper. We use the following procedure to construct the promised uitility at history $\zeta_{i,j}^t$. Let $\zeta_{i,j}^\tau < \zeta_{i,j}^t$ be the closest history that preceeds $\zeta_{i,j}^t$ such that at $\zeta_{i,j}^\tau$, the worker has an employment opportunity to match with a firm. Set $U_{i,\tau}\left(\zeta_{i,j}^\tau\right) = u^*(S_\tau) y_\tau\left(\zeta_{i,j}^\tau\right)$. Given this initial promised utility at history $\zeta_{i,j}^\tau$, we use the history of shocks between $\zeta_{i,j}^\tau$ and $\zeta_{i,j}^t$ and the policy function from (49) to construct the promised utility at $\zeta_{i,j}^t$, $U_{i,t}\left(\zeta_{i,j}^t\right)$. Below is our proposed SPNE strategy.

- Offer the contract $\mathscr{C}\left(U_{i,t}\left(\zeta_{i,j}^t\right), y_{i,t}\left(\zeta_{i,j}^t\right), S_t\left(g^t\right)\right)$ to worker i if worker i is currently an employee.

- Promise to offer $\mathscr{C}\left(U_{i,\tau}\left(\zeta_{i,j}^\tau\right), y_{i,\tau}\left(\zeta_{i,j}^\tau\right), S_\tau\left(g^\tau\right)\right)$ at any future history $\zeta_{i,j}^\tau$ where the worker has an employment opportunity, where $U_{i,\tau}\left(\zeta_{i,j}^\tau\right) = u^*(S_\tau) y_\tau\left(\zeta_{i,j}^\tau\right)$.

- Offer a trivial contract, that is, a contract with zero transfers between the firm and the worker at all future contingencies, if worker i is not currently anemployee. Here the worker can either be unemployed or employed by another firm.

With a slight abuse of terminology, we will call the above contracts employment contracts.[28]

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[28]The notion of employment contract here is the same as the one defined in the main text of the paper but

STEP 3: Here we provide a formal proof that the above described employment contracts constitute an SPNE in the game we describe in Step 1. We first summarize our results in the following lemma.

Lemma 2. *The employment contracts described above is an SPNE.*

Proof. Because at any history $\zeta_{i,j}^t$, a worker can either be employed by a firm, or unemployed but have an employment opportunity to match with a firm, or unemployed and do not have an employment opportuniy in the current period, to establish that the employment contract is an SPNE, we need to show that given all other firms' strategy, none of the following deviations can yield a higher profit for the firm without violating any of the incentive compatibility constraints: (i) a different contract to an employed worker; (ii) a different contract to a worker who is employed by another firm; (iii) a different contract to a worker who is previously unemployed but has an employment opportunity in the current period; (iv) a different contract to an unemployed worker who remains unemployed in the current period; (v) a combination of the above.

First, because the employment contact solves the optimal contracting problem (49), firms cannot obtain a higher profit by offering a different contract to an employee.

Second, no firm can obtain a higher profit by offering a non-trivial insurance contract to a workers who is currently working for another firm. We prove this claim by contradiction. Suppose at $\zeta_{i,j}^t$, a firm has a profitable deviation by offering a nontrivial insurance contract to a worker who is currently employed by another firm, the policy functions and the associated value functions for the insurance contract must solve the following optimal contracting problem:

$$\widetilde{V}(U, y, S) = max_{\widetilde{C}, \{\widetilde{U}'(\zeta')\}_{z'}} \left\{ -\widetilde{C} + \kappa \int \Lambda(S', S) \widetilde{V}\left(\widetilde{U}'(\zeta'), y'(\zeta') | S'\right) \Omega\left(d\zeta' | S\right) \right\}$$

$$subject\ to\ \left\{ (1-\beta)\left[C(U, y) + \widetilde{C}\right]^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}\left[\widetilde{U}'(\zeta')^{1-\gamma} | S\right]\right)^{\frac{1}{1-\gamma}} \right\}^{1-\frac{1}{\psi}} \geq U, \tag{50}$$

$$\widetilde{U}'(\zeta') \geq U'(U, y, \zeta', S'), \tag{51}$$

$$\widetilde{V}\left(\widetilde{U}'(\zeta'), y'(\zeta') | S'\right) \geq 0. \tag{52}$$

where $C(U, y)$ and $\{U'(U, y, \zeta', S')\}$ are the policy functions of the optimal contracting problem in (49). In the objective function of the optimal contracting problem, (52), $\widetilde{C}$ is the net payment from the firm to the unrelated worker. Inequality (50) is the promise keep constraint. If the worker accept the contract, his utility is given by

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extended to a larger contracting space that allows the specification of payment between firms and unrelated workers.

$$\left\{(1-\beta)\left[C\left(U,y\right)+\widetilde{C}\right]^{1-\frac{1}{\psi}}+\beta\left(\mathbb{E}\left[\widetilde{U}'\left(\zeta'\right)|S\right]\right)^{\frac{1}{1-\gamma}}\right\},$$ where the current period consumption includes the payment from the current employer, $C\left(U,y\right)$ as well as the transfer from the unrelated firm, $\widetilde{C}$. Equation (51) is the limited commitment constraint for the worker. Because the worker can always default on the contract offered by the unrelated firm and obtain the utility under the employment contract $U'\left(U,y,\zeta',S'\right)$, in order to prevent the worker from default, the promised utility for the next period, $\widetilde{U}'\left(\zeta'\right)$, must be at least as high as what the worker can obtain under the employment contract, $U'\left(U,y,\zeta',S'\right)$. Inequality (52) in the firm-side limited commitment constraint. Because the insurance contract is a profitable deviation, we must have $\widetilde{V}\left(U,y,S\right)>0$. To arrive at a contradiction, we define $\check{C}\left(U,y,S\right)=C\left(U,y,S\right)+\widetilde{C}\left(U,y,S\right)$ and $\check{U}\left(U,y,S,\zeta'\right)=\widetilde{U}\left(U,y,S,\zeta'\right)$. Note that $\left\{\check{C}\left(U,y,S\right),\left[\check{U}\left(U,y,S,\zeta'\right)\right]\right\}$ is a feasible policy for (49) with $\check{V}\left(U,y,S\right)=V\left(U,y,S\right)+\widetilde{V}\left(U,y,S\right)$ as the value function. However, $\check{V}\left(U,y,S\right)>V\left(U,y,S\right)$, which contracts $V\left(U,y,S\right)$ being the optimal solution to (49).

Third, given that all firms offer the optimal contract that provides the highest utility to workers when worker obtains an employment opportunity, no firm can make a higher profit by deviating from this strategy. Fourth, as we show in lemma 1, firms cannot make a positive profit by offering a non-trivial insurance contract to unemployed workers. Finally, combining all of the above arguments, it is clear that a combinations of deviations will not be profitable either. $\qquad\square$

B Proof for propositions 1 and 2

B.1 Characterization of equilibrium

In this section, to prepare for the proofs for propositions 1, and 2, we provide a set of necessary and sufficient conditions that characterize the equilibrium. We first state a lemma that establishes that the equality constraint (18) can be replaced by an inequality constraint so that the optimal contracting problem *P1* is a standard convex programming problem.

Lemma 3. *Suppose $A'\left(\theta\right)$, $A''\left(\theta\right)$, and $A'''\left(\theta\right)>0$ for all $\theta\in\left(0,1\right)$. The policy functions for the optimal contracting problem P1 in the main text can be constructed from the solution*

to the a convex programming problem described below

$$v\left(u,S\right)=\max_{c,\theta,\{u'(\zeta'),\delta'(\zeta')\}_{\zeta'}}\left\{\begin{array}{c}1-c-A\left(\theta\right)+\\ \kappa\theta\int\Lambda\left(S',S\right)e^{g'+\eta'+\varepsilon'}\left[\delta'\left(\zeta'\right)v\left(u'\left(\zeta'\right),S'\right)\right]\Omega(d\zeta'|g),\end{array}\right\} \tag{53}$$

$$s.t:\quad u\leq\left[\left(1-\beta\right)c^{1-\frac{1}{\psi}}+\beta m^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{54}$$

$$\delta'\left(\zeta'\right)v\left(u'\left(\zeta'\right),S'\right)\geq 0,\ for\ all\ \zeta', \tag{55}$$

$$\delta'\left(\zeta'\right)\left[u'(\zeta')-\lambda\bar{u}\left(S'\right)\right]\geq 0,\ for\ all\ \zeta', \tag{56}$$

$$A'(\theta)\leq\kappa\int\Lambda\left(S',S\right)e^{g'+\eta'+\varepsilon'}\delta'\left(\zeta'\right)v\left(u'\left(\zeta'\right),S'\right)\Omega(d\zeta'|g), \tag{57}$$

where $m=\left\{\kappa\int e^{(1-\gamma)(g'+\eta'+\varepsilon')}\left[\theta\delta'\left(\zeta'\right)u'\left(\zeta'\right)^{1-\gamma}+\left(1-\theta\delta'\left(\zeta'\right)\right)\lambda\bar{u}\left(S'\right)^{1-\gamma}\right]\Omega(d\zeta'|g)\right\}^{\frac{1}{1-\gamma}}.$

Proof. We label the above-stated maximization problem as *P2*. The assumption that $A'\left(\theta\right)$ is strictly convex means that (54)–(57) describe a convex set with a nonempty interior and the objective function (53) is concave. Thus, problem *P2* is a convex programming problem. Suppose the stochastic discount factor and the law of motion of the aggregate state variables jointly satisfy the following condition:

Assumption 6. *For some* $\epsilon>0$*, and for all* (S)*,*

$$\sum\pi\left(g'|g\right)\Lambda\left(S',S\right)e^{g'}<1-\epsilon. \tag{58}$$

Given assumption 6, standard arguments from Stokey et. al (1989) imply that there is a unique v in the space of bounded continuous functions that satisfies (53). In addition, v is continuous, strictly decreasing, strictly concave and differentiable in the interior. We denote the optimal policy functions for *P2* by $\left\{c\left(u,S\right),\theta\left(u,S\right),\{\delta'\left(u,S,\zeta'\right),u'\left(u,S,\zeta'\right)\}_{\zeta'}\right\}$. We first show that policy function for separations satisfies $\delta'\left(u,S,\zeta'\right)=1$ for all ζ'.

Suppose there exists some (u,S) such that with strictly positive probability, $\delta'\left(\tilde{\zeta}'\right)=0$. Consider an alternative set of policy functions denoted by hats:

$$\hat{c}\left(u,S\right)=c(u,S),\quad\hat{\theta}\left(u,S\right)=\theta(u,S),\quad\hat{\delta}'\left(u,S,\zeta'\right)=1\ for\ all\ \zeta'$$

$$\hat{u}'\left(u,S,\zeta'\right)=I_{\{\delta'(u,S,\zeta')=1\}}\times u\left(u,S,\zeta'\right)+I_{\{\delta'(u,S,\zeta')=0\}}\times\left(\lambda\bar{u}\left(S'\right)+\epsilon\right)$$

for some $\epsilon>0$ such that $\lambda\bar{u}\left(S'\right)+\epsilon<u^{*}\left(S'\right)$, where $u^{*}\left(S\right)$ is such that $v\left(u^{*}\left(S\right),S\right)=0$. Because the value function is strictly decreasing, we have $v\left(\hat{u}'\left(u,S,\tilde{\zeta}'\right),S'\right)>0$ for $\tilde{\zeta}'$ where $\delta'\left(\tilde{\zeta}'\right)=0$ Then it is easy to verify that see that the hat policy functions satisfy (54)–(57) and achieve a higher value for the objective in equation (53) and therefore cannot be optimal.

Thus,

$$\delta'\left(u, S, \zeta'\right) = 1 \; for \; \; all \; \; \zeta' \tag{59}$$

We next show that optimal choices for *P2* are feasible for problem *P1*. Optimal policies for *P2* satisfy a set of first-order necessary conditions. In particular, let $\iota \geq 0$ be the Lagrange multiplier of the constraint (57), first-order conditions with respect to θ after imposing (59) implies

$$\iota A''\left(\theta\right) = \frac{\beta}{1-\beta} c^{\frac{1}{\psi}} m^{\gamma - \frac{1}{\psi}} \frac{1}{1-\gamma} \int e^{(1-\gamma)(\eta' + \varepsilon')} \left\{ u'\left(\zeta'\right)^{1-\gamma} - \lambda \bar{u}\left(S'\right)^{1-\gamma} \right\} \Omega(d\zeta'|g). \tag{60}$$

The limited commitment constraint on worker side, equation (56) along with (59) implies that right-hand side of (60) must be strictly positive. Therefore, $\iota > 0$ and (57) must holds with equality at the optimum.

Let ι_u be the Lagrange multiplier of the promise keeping constraint (54), the first-order condition with respect to c implies

$$\iota_u = \frac{1}{1-\beta} \left(\frac{c}{u}\right)^{\frac{1}{\psi}} > 0. \tag{61}$$

Thus, inequality (54) must also hold with equality at the optimum. As a result, the optimal policy for *P2* satisfy all of the constraints for *P1* and as the constraint set for *P2* larger, the optimal policies to *P2* also attain the maximum for *P1*. $\qquad \square$

The first-order necessary conditions for *P2* imply that the above policy functions must satisfy

1. $\forall \eta' + \varepsilon' \in [\underline{\varepsilon}(u, S, g'), \; \bar{\varepsilon}(u, S, g')]$, $u'(u, S, \zeta')$ satisfy

$$\Lambda\left(S', S\right) = \frac{\beta e^{-\gamma(g' + \eta' + \varepsilon')}}{1 + \frac{\iota(u,S)}{\theta(u,S)}} \left[\frac{c\left(u'\left(u, S, \zeta'\right), S'\right)}{c\left(u, S\right)}\right]^{-\frac{1}{\psi}} \left[\frac{u'\left(u, S, \zeta'\right)}{m\left(u, S\right)}\right]^{\frac{1}{\psi} - \gamma}. \tag{62}$$

2. $\forall \eta' + \varepsilon' \geq \bar{\varepsilon}(u, S, g')$ and $\forall \eta' + \varepsilon' \leq \underline{\varepsilon}(u, S, g')$,

$$u'\left(u, S, \zeta'\right) = \lambda \bar{u}\left(S'\right), \tag{63}$$

$$u'\left(u, S, \zeta'\right) = u^*\left(S'\right). \tag{64}$$

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3. The Lagrange multiplier $\iota(u, S)$ satisfies

$$\iota(u, S) = \frac{1}{A''(\theta(u, S))} \frac{\beta}{1 - \beta} c(u, S) m(u, S) \times \frac{1}{1 - \gamma}$$
$$\times \left\{ \int e^{(1-\gamma)(\eta' + \varepsilon')} \left\{ u'(u, S, \zeta')^{1-\gamma} - \lambda \bar{u}(S')^{1-\gamma} \right\} \Omega\left(d\zeta' \,\middle|\, g\right) \right\}. \tag{65}$$

The policy functions must satisfy the equality constraints of the problem *P1*

$$A'(\theta(u, S)) = \kappa \int \Lambda(S', S) e^{g' + \eta' + \varepsilon'} v\left(u'(s'), S'\right) \Omega\left(\zeta'\right), \tag{66}$$

$$u = \left[(1 - \beta) c^{1 - \frac{1}{\psi}} + \beta m(u, S)^{1 - \frac{1}{\psi}} \right]^{\frac{1}{1 - \frac{1}{\psi}}}. \tag{67}$$

The following lemma states that conditions (62) - (67) are both necessary and sufficient for optimality.

Lemma 4. *Suppose there exist an SDF $\Lambda(S', S)$, a worker's value from unemployment, $\bar{u}(S)$, and a law motion for aggregate state variables that satisfy assumption 6. Suppose that given $\Lambda(S', S)$, $\bar{u}(S)$, and the law of motion for state variables, policy functions for problem P2 satisfy (59), the optimality conditions (62)-(65), and the equality constraints (66)-(67). In addition, $\frac{c(u, S)}{u}$ is nondecreasing in u for all S. Let $v(u, S)$ be the unique fixed point of the operator T:*

$$Tv(u, S) = \frac{1 - c(u, S) - A(\theta(u, S)) +}{\kappa\theta(u, S) \int \Lambda(S', S) e^{g' + \eta' + \varepsilon'} v\left(u'(u, S, \zeta'), S'\right) \Omega\left(d\zeta' \,\middle|\, g\right)}. \tag{68}$$

Then, the policy functions together with the value function $v(u, S)$ solve the problem P2.

Proof. Suppose there exists a set of policy functions that satisfy conditions (62)-(67). Given condition (6), the operator defined in (68) is a contraction, and we can construct the value function $v(u, S)$ from the policy functions as the unique fixed point of (68). The first-order conditions (62)-(64) imply that the value function constructed above must satisfy

$$\frac{\partial}{\partial u} v(u, S) = -\frac{1}{1 - \beta} \left(\frac{c(u, S)}{u} \right)^{\frac{1}{\psi}}. \tag{69}$$

Because $\frac{c(u, S)}{u}$ is nondecreasing in u, $\frac{\partial}{\partial u} v(u, S)$ must be nonincreasing, that is, $v(u, S)$ is a concave function of u. As a result, given $v(u, S)$, the first-order conditions, (62)-(67) can be shown to be equivalent to the set of first-order conditions for the programming problem *P2*, which is necessary and sufficient for optimality. Therefore, the above constructed value functions and policy functions must solve the optimal contracting problem *P2*, as needed. $\square$

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Given the above discussion, it is straightforward to provide a characterization for the equilibrium price and quantities using optimality conditions. We summarize these conditions in the following lemma. The proof is omitted as it follows directly from lemma 3 and lemma 4.

Lemma 5. *The equilibrium prices and quantities can be summarized as a set of functions:* $x(S), c(u,S), \theta(u,S), \iota(u,S), \{\bar{\varepsilon}(u,S,g'), \underline{\varepsilon}(u,S,g')\}_{g'}, \{\delta(u,S,\zeta'), u'(u,S,\zeta')\}_{\zeta'};$ *worker' outside option* $\bar{u}(S)$ *and initial utility at employment* $u^*(S)$; *a law of motion of* ϕ *and* B; *a SDF and a firm value function* $v(u,S)$, *such that the SDF is consistent with capital owner's consumption, that is,* $\Lambda(S',S)$ *and* $x(S)$ *satisfy equation (20), where the capital owner's utility,* $w(S)$ *is constructed from* $x(S)$ *using equation (19); the value function and policy functions satisfy (59), and the optimality conditions (62)-(67); the outside option* $\bar{u}(S)$ *satisfies (13),* $u^*(S)$ *satisfies* $v(u^*(S),S) = 0$ *for all* S, *and the law of motion of the aggregate state variables satisfy (23) and (25).*

We now prove proposition 1 and 2.

B.2   Proofs of propositions 1 and 2

In lemma 3, we have already proved that $\delta(u,S,\zeta') = 1$ for all ζ' is optimal for problem $P2$ and lemma 5 asserts that the same policy rule is optimal for problem $P1$ too.

We next provide the characterization for the policy functions $u'(u,S,\zeta')$ and then $\theta(u,S)$. Given assumption 6 and lemma 3, standard arguments from Stokey et. al (1989) imply that the value function v for the optimal contracting problem (14) is continuous, strictly decreasing, strictly concave and differentiable in the interior. Because the value function is strictly decreasing, the limited commitment constraint (16) can be written as $u'(s') \leq u^*(S')$ for all s', where $u^*(S)$ is defined by equation (12). Therefore, the first-order condition with respect to continuation utility and the envelope condition for the programming problem (53) together imply that one of the following three cases have to true:

1. In the interior, equation (31) holds.

2. The worker-side limited commitment constraint binds, $u'(u,S,\zeta') = \lambda\bar{u}(S')$, and,

$$\left[\frac{x(S')}{x(S)}\right]^{-\frac{1}{\psi}} \left[\frac{w(S')}{n(S)}\right]^{\frac{1}{\psi}-\gamma} \left(1 + \frac{\iota(u,S)}{\theta(u,S)}\right) \geq e^{-\gamma(\eta'+\varepsilon')} \left[\frac{c(u'(u,S,\zeta'),\phi',B')}{c(u,S)}\right]^{-\frac{1}{\psi}} \left[\frac{u'(u,S,\zeta')}{m(u,S)}\right]^{\frac{1}{\psi}-\gamma} \tag{70}$$

3. The firm-side limited commitment costraint binds, $u'(s') = u^*(S')$,

$$\left[\frac{x(S')}{x(S)}\right]^{-\frac{1}{\psi}} \left[\frac{w(S')}{n(S)}\right]^{\frac{1}{\psi}-\gamma} \left(1 + \frac{\iota(u,S)}{\theta(u,S)}\right) \leq e^{-\gamma(\eta'+\varepsilon')} \left[\frac{c(u'(u,S,\zeta'),\phi',B')}{c(u,S)}\right]^{-\frac{1}{\psi}} \left[\frac{u'(u,S,\zeta')}{m(u,S)}\right]^{\frac{1}{\psi}-\gamma} \tag{71}$$

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Define $\mathcal{E} = \{\eta' + \varepsilon' : \ equation\ (31)\ holds\}$. Also, let

$$\underline{\varepsilon}\left(u, S, g'\right) = \inf \mathcal{E}, \quad \bar{\varepsilon}\left(u, S, g'\right) = \sup \mathcal{E}. \tag{72}$$

Let $l_u\left(u, S\right)$ be the Lagrange multiplier for the promise-keeping constraint of the programing problem (53), then

$$\frac{\partial}{\partial u} v\left(u, S\right) = l_u\left(u, S\right) = \frac{1}{1-\beta}\left(\frac{c\left(u, S\right)}{u}\right)^{\frac{1}{\psi}}, \tag{73}$$

where the first equality is the envelope theorem, and the second equality is the first-order condition, (61). Because v is concave, the above condition implies that $c\left(u, S\right)$ must be strictly increasing in u. Thereore, the optimality condition (31) implies that on $\mathcal{E}$, $u'\left(u, S, \zeta'\right)$ must be strictly decreasing in $\eta' + \varepsilon'$. Clearly, the strict monotonicity of $u'\left(u, S, \zeta'\right)$ implies that $u'\left(u, S, \zeta'\right) = \lambda \bar{u}\left(S'\right)$ if $\eta' + \varepsilon' = \underline{\varepsilon}\left(u, S, g'\right)$ and $u'\left(u, S, \zeta'\right) = u^*\left(S'\right)$ if $\eta' + \varepsilon' = \bar{\varepsilon}\left(u, S, g'\right)$.

First, $\forall \eta' + \varepsilon' > \underline{\varepsilon}\left(u, S, g'\right)$, we must have $u'\left(u, S, \zeta'\right) = \lambda \bar{u}\left(S'\right)$. Otherwise, none of the equations, (31), (70), or (71) can hold. Similarly, $\forall \eta' + \varepsilon' < \bar{\varepsilon}\left(u, S, g'\right)$, we must have $u'\left(u, S, \zeta'\right) = u^*\left(S'\right)$.

Second, to complete the proof of part 1 and 2 of proposition 2, we need to show that $\forall \eta' + \varepsilon' \in \left(\underline{\varepsilon}\left(u, S, g'\right), \bar{\varepsilon}\left(u, S, g'\right)\right)$, condition (31) must hold. It is enough to show $u'\left(u, S, \zeta'\right) \in \left(\lambda \bar{u}\left(S'\right), u^*\left(S'\right)\right)$. This can be proved by contradiction. Suppose $\eta' + \varepsilon' \in \left(\underline{\varepsilon}\left(u, S, g'\right), \bar{\varepsilon}\left(u, S, g'\right)\right)$ and $u'\left(u, S, \zeta'\right) = \lambda \bar{u}\left(S'\right)$, then the fact that equation (31) holds at $\bar{\varepsilon}\left(u, S, g'\right)$ implies that (note that $\eta' + \varepsilon' < \bar{\varepsilon}\left(u, S, g'\right)$)

$$\left[\frac{x(S')}{x(S)}\right]^{-\frac{1}{\psi}} \left[\frac{w(S')}{n(S)}\right]^{\frac{1}{\psi}-\gamma} \left(1 + \frac{\iota(u,S)}{\theta(u,S)}\right) < e^{-\gamma(\eta'+\varepsilon')} \left[\frac{c\left(\lambda \bar{u}\left(S'\right), \phi', B'\right)}{c\left(u, S\right)}\right]^{-\frac{1}{\psi}} \left[\frac{\lambda \bar{u}\left(S'\right)}{m\left(u, S\right)}\right]^{\frac{1}{\psi}-\gamma},$$

which is a contradiction to condition (70). Similarly, one can show that $u'\left(u, S, \zeta'\right) = u^*\left(S'\right)$ cannot be true either.

To prove the second part of proposition 1, note that because the value function is strictly concave in u, the Lagrange multiplier $\iota_u\left(u, S\right)$ must be strictly increasing in u. The first-order condition with respect to $u'\left(u, S, \zeta'\right)$ in the programming problem (53) then implies that $u'\left(u, S, \zeta'\right)$ must be strictly increasing in u as well. Given constraint (18), the monotonicity of $\theta\left(u, S\right)$ with respect to u then follows directly from the $u'\left(u, S, \zeta'\right)$ is increasing with respect to u and the fact that $v\left(u', S'\right)$ is strictly decreasing in u'.

C Proofs of propositions 4 and 5

This section provides the proofs for propositions 4 and 5. In subsection C.1, we provide closed-form solutions for the equilibrium prices and quantities for the simple economy. We

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prove proposition 4 in subsection C.2 and proposition 5 in subsection C.3.

C.1 Equilibrium in the simple economy

Notation We first introduce some notation. In the simple model in Section 4, we assume that the worker-specific shock follows a negative exponential distribution. The density of a negative exponential distribution is given by $f(\varepsilon|g_L) = \xi e^{\xi(\varepsilon - \varepsilon_{MAX})}$, for $\varepsilon \leq \varepsilon_{MAX}$, and $f(\varepsilon|g_L) = 0$ otherwise, where ξ and ε_{MAX} are the parameters of the distribution. For later reference, we note that the moments of $f(\varepsilon|g_L)$ can be easily computed as $\int_{-\infty}^{\varepsilon} e^{\theta t} f(t|g_L) \, dt = \frac{\xi}{\xi + \theta} e^{-\xi \varepsilon_{MAX} + (\theta + \xi)\varepsilon}$, for $\xi + \theta > 0$. Clearly, the assumption $\mathbb{E}[e^{\varepsilon}] = 1$ amounts to a parameter restriction that $\varepsilon_{MAX} = \ln \frac{1+\xi}{\xi}$. We will impose this restriction and call the above distribution a negative exponential distribution with parameter ξ.

As explained in the main text of the paper, we represent policy functions and value functions as functions of the period-0 promised utility u_0. For an arbitrary u_0, we use $u_H(u_0) \equiv u'(u_0, g_H)$, and $u_L(u_0, \varepsilon') \equiv u'(u_0, g_L, \varepsilon')$ to denote the normalized promised utility for a worker with initial promised utility u_0 at nodes H and L, respectively. We use $c_0(u_0)$ for workers' consumption policy at nodes 0. The rest of the policy and value functions are the same as defined in the main text. We also denote $\underline{\varepsilon}_L(u_0) \equiv \underline{\varepsilon}(u_0, g_L)$ as the lowest level of realization of the ε_1 shock such that the limited commitment constraint does not bind at node L. In addition, let u_H^{FB} and u_L^{FB} denote the utility-to-consumption ratio of an agent who consumes the aggregate consumption in state g_H and g_L, respectively. That is, they are the normalized utility associated with full risk sharing. The first best levels, u_H^{FB} and u_L^{FB} are determined by $u_H^{FB} = (e^{g_H} u_H^{FB})^{\beta}$ and $u_L^{FB} = (e^{g_L} u_L^{FB})^{\beta}$. We use u_L^{CD} to denote the normalized utility of an agent in an economy without risk sharing. That is, it is utility-consumption ratio of an agent who consumes y_t every period:

$$u_L^{CD} = \left(\int \left[e^{\{\varepsilon' + g_L\}} u_L^{CD} \right]^{1-\gamma} f(\varepsilon'|g_L) d\varepsilon \right)^{\frac{\beta}{1-\gamma}}. \tag{74}$$

It is straightforward to show that as $\gamma \to 1 + \xi$, $u_L^{CD} \to 0$. We solve the general equilibrium in the simple economy by backward induction.

Below, we first solve the value functions and policy functions at nodes H and L in period 1. In the second step, we analyze the optimal contracting problem in period 0 for an arbitrary promised utility u_0. Finally, we impose market clearing to solve for the equilibrium stochastic discount factor.

Value functions at nodes H and L To solve the optimal contracting problem in period 1, note that our assumption that from period 2 and on, all workers consume an α fraction of their output implies that firm value functions in period 2, after normalized by worker output,

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take a simple form: $v_2\left(u', g_H\right) = v_2\left(u', g_L\right) = \frac{1-\alpha}{1-\beta}$. This allows us to derive a closed-form solution for the value functions and consumption policies for period 1 at node H and L, respectively. We summarize our results in the following lemma and refer readers to section B of the online appendix (not for publication) for the detailed derivation.

Lemma 6. *(Value function in period 1)*

The firm's value function at nodes H and L are give by

$$
\begin{aligned}
v\left(u, g_H\right) &= 1 - c\left(u, g_H\right) + \frac{\beta}{1-\beta} x_H - a \ln\left[1 + \frac{\beta x_H}{a\left(1-\beta\right)}\right], \quad and & (75)\\
v\left(u, g_L\right) &= 1 - c\left(u, g_L\right) + \frac{\beta}{1-\beta} x_L - a \ln\left[1 + \frac{\beta x_L}{a\left(1-\beta\right)}\right], & (76)
\end{aligned}
$$

respectively, where the consumption policies are given by $c\left(u, g_H\right) = \left(\alpha e^{g_H} u_H^{FB}\right)^{-\frac{\beta}{1-\beta}} u^{\frac{1}{1-\beta}}$, and $c\left(u, g_L\right) = \left(\alpha \Upsilon e^{g_L} u^{CD}\right)^{-\frac{\beta}{1-\beta}} u^{\frac{1}{1-\beta}}$, where the parameter Υ is defined as

$$
\Upsilon = \left\{ \int_{-\infty}^{\infty} e^{(1-\gamma)\varepsilon'} f\left(\varepsilon' \,\middle|\, g_L\right) d\varepsilon' \right\}^{\frac{1}{1-\gamma}}. \tag{77}
$$

The policy functions for effort choice do not depend on u. We denote $\theta_H = \theta\left(u, g_H\right)$, and $\theta_L = \theta\left(u, g_L\right)$, and

$$
\theta_H = 1 - \frac{a}{a + \frac{\beta}{1-\beta} x_H}, \quad \theta_L = 1 - \frac{a}{a + \frac{\beta}{1-\beta} x_L}. \tag{78}
$$

At node L, limited commitment on firm side requires that $v_L\left(u\right) \geq 0$. Therefore, by equation (76), the maximum amount of consumption that the firm can promise to deliver to a worker at node L is $1 - A\left(\theta_L\right) + \theta_L \frac{\beta}{1-\beta} x_L$, which we will denote as c_L^{MAX}. Recall that for a worker with initial promised utility u_0, $\underline{\varepsilon}_L\left(u_0\right)$ is the lowest level of realization of the ε' shock such that the limited commitment constraint does not bind at node L. We must have, for all u_0,

$$
c_L\left(u_0, \underline{\varepsilon}_L\left(u_0\right)\right) = 1 + \frac{\beta}{1-\beta} x_L - a \ln\left[1 + \frac{\beta x_L}{a\left(1-\beta\right)}\right]. \tag{79}
$$

We now turn to the optimal contracting problem as node 0.

Optimal contracting at node 0   We develop our results in several lemmas. The key to characterize the policy functions for the optimal contracting problem at node 0 is the consumption and promised utility for the marginal worker in period 1 at node L. Here, the marginal worker is defined as the one with the lowest level of realization of ε_1 shock such that the limited commitment constraint does not bind at node L, i.e., $\varepsilon_1 = \underline{\varepsilon}_L\left(u_0\right)$. Our first lemma uses the optimal risk sharing condition (31) to relate the marginal rate of substitution of a marginal worker to that of the capital owners.

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Lemma 7. *(FOC for the marginal agent)*

Given the consumption share of the capital owners, x_H and x_L, for all u_0, the normalized consumption of the marginal worker with $\varepsilon_1 = \underline{\varepsilon}_L(u_0)$ must satisfy:

$$\frac{c_H(u_0)}{e^{(1+\tau)\underline{\varepsilon}_L(u_0)}c_L(u_0, \underline{\varepsilon}_L(u_0))}\left[\frac{u_L^{FB}k(\theta_H)}{\Upsilon u_L^{CD}k(\theta_L)}\right]^{\tau} = \frac{x_H}{x_L}, \tag{80}$$

where we denote $\tau = \frac{\beta(\gamma-1)}{1+(1-\beta)(\gamma-1)}$, and $k(\theta) = \left[\theta + (1-\theta)\lambda^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$.

Next, we provide a lemma that links the consumption of a marginal worker to the expected consumption of an average worker at node L.

Lemma 8. *(Expected worker consumption at node L)*

Given the consumption share of the capital owners, x_H and x_L, the expected consumption of a worker with promised utility u_0 at node L is given by:

$$E\left[e^{\varepsilon'}c_L\left(u_0, \varepsilon'\right)\right] = e^{(1+\tau)\underline{\varepsilon}_L(u_0)}c_L\left(u_0, \underline{\varepsilon}_L(u_0)\right)\Phi\left(\underline{\varepsilon}_L(u_0)\right), \tag{81}$$

for all u_0, where the function $\Phi(\varepsilon)$ is defined as

$$\Phi(\varepsilon) = \frac{\xi}{\xi - \tau}e^{-\tau\varepsilon_{MAX}} - \frac{\xi(1+\tau)}{(1+\xi)(\xi-\tau)}e^{-\xi\varepsilon_{MAX}+(\xi-\tau)\varepsilon}. \tag{82}$$

lemma 7 is the optimal risk sharing condition that equalizes the marginal rate of substitution of workers and capital owners across the two states in period 1. The next lemma provides another first-order condition that links the marginal rate of substitution of capital owners and workers across time. Together lemma 7 and lemma 9 below completely characterize optimal risk sharing conditions.

Lemma 9. *(Optimal risk sharing)*

Optimal risk sharing requires that for all u_0,

$$\left[\frac{x_H}{c_H(u_0)}\right]^{1+(1-\beta)(\gamma-1)} = \left[\frac{x_0}{c_0(u_0)}\right]\left[\frac{\bar{n}_0(x_H, x_L)}{\bar{m}_0(u_0)}\right]^{\gamma-1}, \quad where \tag{83}$$

$$\bar{n}_0(x_H, x_L) = \left[\pi\left(e^{(1+\beta)g_H}x_H^{(1-\beta)}\left(u_H^{FB}\right)^{\beta}\right)^{1-\gamma} + (1-\pi)\left(e^{(1-\gamma)g_L}x_L^{(1-\beta)}\left(u_L^{FB}\right)^{\beta}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \quad and,$$

$$\bar{m}_0(u_0) = \left[\begin{array}{c} \pi\left(e^{(1+\beta)g_H}c_H^{1-\beta}\left(u_H^{FB}k(\theta_H)\right)^{\beta}\right)^{1-\gamma} + \\ (1-\pi)e^{(1-\gamma)g_L}\left[e^{(1+\tau)\underline{\varepsilon}_L(u_0)}c_L(u_0,\ \underline{\varepsilon}_L(u_0))\right]^{(1-\beta)(1-\gamma)}\left\{\frac{1}{\alpha}m_L\right\}^{\beta(1-\gamma)}\Psi\left(\underline{\varepsilon}_L(u_0)\right) \end{array}\right]^{\frac{1}{1-\gamma}},$$

$$\tag{84}$$

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where $\Psi(\varepsilon)$ is given by:

$$\Psi(\varepsilon) = \left\{ \frac{\xi}{\xi - \tau} e^{-\tau \varepsilon_{MAX}} - \frac{\xi(1 - \gamma + \tau)}{(\xi - \tau)(\xi + 1 - \gamma)} e^{-\xi \varepsilon_{MAX} + (\xi - \tau)\underline{\varepsilon}_L(u_0)} \right\}. \tag{85}$$

General Equilibrium   A unit measure of a single type of workers and market clearing at node 0, node H, and node L implies that u_0^* solves $c_0(u_0^*) = 1 - x_0$, $c_H(u_0^*) = 1 - x_H$, and $E\left[e^{\varepsilon'} c_L(u_0^*, \varepsilon')\right] = 1 - x_L$, respectively. Note that equation (79) implies

$$c_L(u_0^*, \underline{\varepsilon}_L(u_0^*)) = 1 + \frac{\beta}{1 - \beta} x_L - a \ln\left[1 + \frac{\beta x_L}{a(1 - \beta)}\right]. \tag{86}$$

Using the market clearing at node L and equation (81) in lemma 8, we have $1 - x_L = e^{(1+\tau)\underline{\varepsilon}_L(u_0^*)} c_L(u_0, \underline{\varepsilon}_L(u_0^*)) \Phi(\underline{\varepsilon}_L(u_0^*))$, which, after combining with (86), gives

$$e^{(1+\tau)\underline{\varepsilon}_L(u_0^*)} \Phi(\underline{\varepsilon}_L(u_0^*)) = \frac{1 - x_L}{1 + \frac{\beta}{1-\beta} x_L - a \ln\left[1 + \frac{\beta x_L}{a(1-\beta)}\right]}. \tag{87}$$

Equations (86) and (87) together define $c_L(u_0, \underline{\varepsilon}_L(u_0^*))$ and $\underline{\varepsilon}_L(u_0^*)$ as functions of x_L. With a bit abuse of notation, we denote these functions as $c_L(x_L)$ and $\underline{\varepsilon}(x_L)$.

Focusing on type-u_0^* agents, using lemma 8, we can replace the term $e^{(1+\tau)\underline{\varepsilon}_L(u_0^*)} c_L(u_0^*, \underline{\varepsilon}_L(u_0^*))$ in equation (80) by the following

$$e^{(1+\tau)\underline{\varepsilon}_L(u_0^*)} c_L(u_0^*, \underline{\varepsilon}_L(u_0^*)) = (1 - x_L) \Phi(\underline{\varepsilon}(x_L))^{-1}. \tag{88}$$

Therefore, the first order condition (80) can be written as

$$\Phi(\underline{\varepsilon}(x_L)) \left[\frac{u_L^{FB} k(\theta_H)}{\Upsilon u_L^{CD} k(\theta_L)}\right]^\tau = \frac{x_H}{x_L} \frac{1 - x_L}{1 - x_H}. \tag{89}$$

Also, we use the marketing clearing condition to replace c_H by $1 - x_H$ and use (88) to replace $e^{(1+\tau)\underline{\varepsilon}_L(u_0^*)} c_L(u_0^*, \underline{\varepsilon}_L(u_0^*))$. We define workers' certainty equivalent as a function of x_H, x_L, and ε using (84)

$$\bar{m}_0(x_H, x_L, \varepsilon) = \left\{ \begin{array}{l} \pi\left[e^{(1+\beta)g_H}(1 - x_H)^{(1-\beta)}\left(u_H^{FB} k(\theta_H)\right)^\beta\right]^{1-\gamma} \\ + (1 - \pi)\left[e^{(1+\beta)g_L}\left[\frac{1-x_L}{\Phi(\varepsilon)}\right]^{(1-\beta)}\left[\Upsilon u_L^{CD} k(\theta_L)\right]^\beta\right]^{1-\gamma} \Psi(\varepsilon) \end{array} \right\}^{\frac{1}{1-\gamma}}. \tag{90}$$

This allows us to write the first order condition (83) as functions of x_H and x_L:

$$\left[\frac{x_H}{1-x_H}\right]^{1+(1-\beta)(\gamma-1)} = \left[\frac{x_0}{1-x_0}\right]\left[\frac{\bar{n}_0(x_H,x_L)}{\bar{m}_0(x_H,x_L,\underline{\varepsilon}(x_L))}\right]^{\gamma-1}. \tag{91}$$

Give an initial condition of x_0, equations (89) and (91) can be jointed solved for x_H and x_L. Other equilibrium quantities can then be constructed analogously.

C.2 Proof of proposition 4

1. From the definition of u_L^{CD} in (74), it is clear that as $\gamma \to 1+\xi$, $u_L^{CD} \to 0$. Consider equation (89). It is straightforward to verify that $\Phi(\varepsilon)$ is strictly positive and bounded (see equation (82)). Also, both $k(\theta_H)$ and $k(\theta_L)$ are bounded. Therefore, as $\gamma \to 1+\xi$ the left hand side converges to ∞, and we must have $\frac{x_H}{x_L} \to \infty$. By continuity, there exists $\hat{\gamma} \in (1, 1+\xi)$ such that $\frac{x_H}{x_L} > 1$ if and only if $\gamma > \hat{\gamma}$, as needed.

   In addition, if $\gamma = 1$, then $\tau = 0$. Using the definition of $\Phi(\varepsilon)$, $\Phi(\varepsilon) = 1 - \frac{1}{(1+\xi)}e^{-\xi(\varepsilon_{MAX}-\varepsilon)} < 1$. Therefore, we must have $\frac{x_H}{x_L} < 1$.

2. The economy without moral hazard is a special case in which the parameter for cost of effort, $a = 0$. We use $\theta_H(a)$ and $\theta_L(a)$ to denote policy functions with the understanding that they are policy functions of the moral hazard economy if $a > 0$, and they stand for policy functions in the economy without moral hazard if $a = 0$. Using our result from Part 1 of the proof, as $\gamma \to 1+\xi$, $\frac{x_H}{x_L} \to \infty$. Because both x_H and x_L are bounded between $[0,1]$, we must have $x_L \to 0$. Therefore, $\theta_L(a) \to 0$ by equation (78). Also, equation (91) implies that as $\gamma \to 1+\xi$, $\bar{m}_0(x_H(a), x_L(a), \underline{\varepsilon}(x_L(a))) \to 0$; therefore, $x_H(a) \to 0$ as well. Therefore, as $\gamma \to 1+\xi$, $\theta_H(a) \to 1 - \frac{a}{a+\frac{\beta}{1-\beta}x_H^*}$. Consider equation (89), for an arbitrary a, $\left[\frac{k(\theta_H(a))}{k(\theta_L(a))}\right]^\tau = \left[\frac{\theta_H(a)+(1-\theta_H(a))\lambda^{1-\gamma}}{\theta_L(a)+(1-\theta_L(a))\lambda^{1-\gamma}}\right]^{\frac{\tau}{1-\gamma}}$. Suppose $a > 0$, then as $\gamma \to 1+\xi$, there exist $\epsilon > 0$ such that

$$\left[\frac{\theta_H(a)+(1-\theta_H(a))\lambda^{1-\gamma}}{\theta_L(a)+(1-\theta_L(a))\lambda^{1-\gamma}}\right]^{\frac{\tau}{1-\gamma}} \to \left[\frac{1-\frac{a}{a+\frac{\beta}{1-\beta}x_H^*(a)}+\left(1-\frac{a}{a+\frac{\beta}{1-\beta}x_H^*(a)}\right)\lambda^{-\xi}}{\lambda^{-\xi}}\right]^{-\frac{1}{\xi}\frac{\beta\xi}{1+\xi(1-\beta)}} > 1+\epsilon.$$

   In addition, equation (87) implies that as $\gamma \to 1+\xi$, $x_L \to 0$, and therefore, $\underline{\varepsilon}_L(a) \to \varepsilon^*$ for all a, where ε^* is such that $e^{(1+\tau)\varepsilon^*}\Phi(\varepsilon^*) = 1$. Therefore, with $a > 0$, for γ close enough to $1+\xi$, we must have $\Phi(\underline{\varepsilon}_L(a))\left[\frac{u_L^{FB}k(\theta_H(a))}{\Upsilon u_L^{CD}k(\theta_L(a))}\right]^\tau > \Phi(\underline{\varepsilon}_L(0))\left[\frac{u_L^{FB}}{\Upsilon u_L^{CD}}\right]^\tau$. Equation (89) implies that for γ close enough to $1+\xi$, $\frac{x_H}{x_L} > \frac{\tilde{x}_H}{\tilde{x}_L}$ because as $\gamma \to 1+\xi$, $x_L \to 0$ and $x_H \to x_H^*$ has a limit.

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3. By Part 1 of the proposition, for γ large enough, $x_H > x_L$. The fact that $\theta_H > \theta_L$ follows from equation (78).

C.3 Proof of proposition 5

Firm risk pass through   Fixing u_0, equation (109) implies that $\forall \varepsilon' \geq \underline{\varepsilon}(u_0)$, $\frac{d\ln[e^\varepsilon c_L(u_0,\varepsilon)]}{d\varepsilon} = -\tau$. For $\varepsilon' < \underline{\varepsilon}(u_0)$, the limited commitment constraint binds, and $e^\varepsilon c_L(u_0,\varepsilon) = e^\varepsilon c_L(u_0, \underline{\varepsilon}(u_0))$. Therefore, $\frac{d\ln[e^\varepsilon c_L(u_0,\varepsilon)]}{d\varepsilon} = 1$. Combining the above two equations, we have $E\left[\frac{\partial \ln[e^\varepsilon c_L(u_0,\varepsilon)]}{\partial \varepsilon}\right] = \int_{-\infty}^{\underline{\varepsilon}_L(u_0)} f(\varepsilon'|g_L)\,d\varepsilon'\tau + \int_{\underline{\varepsilon}_L(u_0)}^{\varepsilon_{MAX}} f(\varepsilon'|g_L)\,d\varepsilon' = e^{-\xi(\varepsilon_{MAX}-\underline{\varepsilon}_L(u_0))} - \tau\left[1 - e^{-\xi(\varepsilon_{MAX}-\underline{\varepsilon}_L(u_0))}\right]$. Clearly, the average elasticity is increasing in $\underline{\varepsilon}_L(u_0)$. Using the optimal risk sharing conditions (99) and (83), we can show that $\underline{\varepsilon}_L(u_0)$ is an increasing function of u_0.

Cross section of expected returns   To characterize the dependence of $\frac{v_H(u_0)}{E[e^\varepsilon v_L(u_0,\varepsilon)]}$, note that in general, $c_H(u_0) = \frac{x_H}{x_L}\left[\frac{\xi u_L^{CD} k(\theta_L)}{u_L^{FB} k(\theta_L)}\right]^\tau e^{(1+\tau)\underline{\varepsilon}_L(u_0)} c_L(u_0, \underline{\varepsilon}_L(u_0))$ by lemma 7, and $E[e^\varepsilon c_L(u_0,\varepsilon)] = e^{(1+\tau)\underline{\varepsilon}_L(u_0)} c_L(u_0, \underline{\varepsilon}_L(u_0))\Phi(\underline{\varepsilon}_L(u_0))$ by lemma 8. Because at $\varepsilon = \underline{\varepsilon}_L(u_0)$, the limited commitment constraint, $v_L(u_0,\varepsilon) = 0$ binds, $c_L(u_0, \underline{\varepsilon}_L(u_0)) = 1 + \frac{\beta}{1-\beta}x_L - a\ln\left[1 + \frac{\beta x_L}{a(1-\beta)}\right]$ by (79). To simplify notation, we denote $\Gamma_H = 1 + \frac{\beta}{1-\beta}x_H - a\ln\left[1 + \frac{\beta x_H}{a(1-\beta)}\right]$ and $\Gamma_L = 1 + \frac{\beta}{1-\beta}x_L - a\ln\left[1 + \frac{\beta x_L}{a(1-\beta)}\right]$. We then write $\frac{v_H(u_0)}{E[e^\varepsilon v_L(u_0,\varepsilon)]}$ as $\frac{v_H(u_0)}{E[e^\varepsilon v_L(u_0,\varepsilon)]} = \frac{\Gamma_H - \phi e^{(1+\tau)\underline{\varepsilon}_L(u_0)}}{\Gamma_L\{1 - e^{(1+\tau)\underline{\varepsilon}_L(u_0)}\Phi(\underline{\varepsilon}(u_0))\}}$, where we denote $\phi = \frac{x_H}{x_L}\left[\frac{\xi u_L^{CD} k(\theta_L)}{u_L^{FB} k(\theta_L)}\right]^\tau \Gamma_L$ to simplify notation. By proposition 2, $\underline{\varepsilon}(u_0)$ is a strictly increasing function of u_0. Therefore, we complete the proof for proposition 5 by noticing that

$$\frac{\partial}{\partial \underline{\varepsilon}} \frac{\Gamma_H - \phi e^{(1+\tau)\underline{\varepsilon}}}{\{1 - e^{(1+\tau)\underline{\varepsilon}}\Phi(\underline{\varepsilon})\}} > 0. \tag{92}$$

It is possible to show that the above inequality holds for γ large enough (but smaller than $1 + \xi$ so that worker utility is well defined). We refer the readers to lemma 11 in section B of the online appendix (not for publication) for the details of proof.

D   Computational Algorithm

We describe our computation algorithm. The algorithm consists of an "outer loop", in which we iterate over the law of motion for aggregate states and an associated stochastic discount factor, and an "inner loop", in which we solve for the optimal contract. Below are the steps of our numerical procedure.

1. Initialize the law of motion of x, $\Gamma_x(g, x, g')$. We use a log-linear functional form:

$$\log x' = a(g, g') + b(g, g') \log x. \tag{93}$$

Given the law of motion of x, the SDF $\Lambda(x, g, g')$ is calculated using

$$\Lambda(x, g, g') = \beta \left[\frac{x'(g'|g, x) e^{g'}}{x} \right]^{-\frac{1}{\psi}} \left[\frac{w(x', g') e^{g'}}{n(g, x)} \right]^{\frac{1}{\psi} - \gamma},$$

where $w(g, x)$ and $n(g, x)$ are derived from equation (19).

2. The inner loop consists of using $\Gamma_x(g, x, g')$ and $\Lambda(x, g, g')$, to solve the value function $v(u, g, x)$, the worker-outside value $\overline{u}(g, x)$ and value of a new job $u^*(g, x)$ along with the policy functions $c(u, g, x)$, $\theta(u, g, x)$ and $u'(u, g, x, \zeta')$ that solve the optimal contracting problem *P1*. We solve Bellman equation by a modified value function iteration as applying a standard value function iteration is complicated by the presence of the occasionally binding constraints (16) and (17). Our procedure borrows elements from endogenous grid method of Carroll (2006). Please see below "Details of Inner Loop."

3. To check the accuracy in computing the optimal contract, we compute Euler equation errors. Fixing u, x, g and the aggregate state next period g', we draw 1000 idiosyncratic shocks $\varepsilon' + \eta'$ such that both agent and firm-side limited commitment constraints are not binding. We then use the maximum absolute log10 ratio of worker's MRS to owners' MRS across these shocks as our measure of Euler Equation Error. We repeat this procedure for different (u, x, g) and g' combinations with values of (u, x) that are not on the grid points where the value function is solved. The Euler equation errors computed this way range between -3 and -4, which suggests that our approximation is reasonable.

4. We now describe the outer loop where we use optimal policies to simulate the model and update Γ_x. Please see below the paragraph "Details of the simulation procedure"

5. Up to now, we have described a procedure to simulate forward the economy. This allows us to compute the market clearing $\{x_{t+1}^{MC}\}_{t=0}^{\infty}$ as follows:

$$x_{t+1}^{MC} = \sum_{m=1}^{N+2} \phi_{t+1}[m] - \sum_{m=1}^{N+2} c(\hat{u}[m](t+1)|g_{t+1}, x_{t+1})\phi_{t+1}[m] - B_{t+1}. \tag{94}$$

Given the sequence of $\{g_t\}_{t=1}^{T}$, we simulate the economy forward for T periods to obtain $\{x_t^{MC}\}_{t=0}^{T}$. We divide the sample into four cases: $g_H \to g_H$, $g_H \to g_L$, $g_L \to g_H$, $g_L \to g_L$ and use regression to update the law of motion of x. We go back to step

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1 to iterate. Note the under the above procedure, given the sequence of $\{g_t\}_{t=1}^T$, the sequence of x_{t+1} that is used for computing decision rules is complete determined by (94). In the simulation, we assume that x_{t+1} follows the perceived law of motion, based on which agent make their decisions. We use the market clearing condition to update the actual law of motion of x and iterate.

6. We divide the sample into four cases: $g_H \to g_H$, $g_H \to g_L$, $g_L \to g_H$, $g_L \to g_L$ and use regressions (93) to update the law of motion of x. We go back to step 1 to iterate until the unconditional R^2 approaches 99.9%.

Details of the Inner Loop

1. Guess $v(u, g, x)$ and $c(u, g, x)$. These imply functions $u^*(g, x)$ and $\bar{u}(g, x)$ using equations (12) and (13). We denote $c(u^*(g, x)|g, x)$ and $c(\lambda\bar{u}(g, x), g, x)$ by $c^*(g, x)$ and $\bar{c}(g, x)$.

2. Let $\{\underline{\varepsilon}(u, S, g'), \bar{\varepsilon}(u, S, g')\}_{g'}$ be the thresholds for $\eta' + \varepsilon'$ such that constraint (16) and (17) bind for a worker with state u, aggregate states (S) and next period for aggregate shock $g' = g_L$. Define a grid $\underline{\mathcal{E}}_L \times \mathcal{X} \equiv \{(\varepsilon_{L,0}, x_0), (\varepsilon_{L,1}, x_0), \ldots, (\varepsilon_{L,n\mathcal{E}}, x_{n\mathcal{X}})\}$ with the understanding that $\underline{\varepsilon}_L(j)$ and $x(j)$ are the entries in the jth element of the grid $\underline{\mathcal{E}}_L \times \mathcal{X}$ with $j \in \{1, 2, \ldots, n\mathcal{E} \times n\mathcal{X}\}$.

3. For all $j \in \{1, 2, \ldots, n\mathcal{E} \times n\mathcal{X}\}$, we solve for $\{\underline{\varepsilon}_{g'}(j), \bar{\varepsilon}_{g'}(j)\}_{g'}$ that are consistent with $\underline{\varepsilon}_L(j)$ and the guess for functions v and c in step (a) using the following equations that need to hold for all g'

$$\frac{\Lambda(x(j), g, g')}{\Lambda(g_L, g, x(j))} = \frac{e^{-\gamma(\underline{\varepsilon}_{g'}(j))}}{e^{-\gamma(\underline{\varepsilon}_{g_L}(j))}} \left[\frac{c^*(g', \Gamma_x(x(j), g, g'))}{c^*(g_L, \Gamma_x(x(j), g, g'))}\right]^{-\frac{1}{\psi}} \left[\frac{u^*(g', \Gamma_x(x(j), g, g'))}{u^*(g_L, \Gamma_x(x(j), g, g'))}\right]^{\frac{1}{\psi}-\gamma}$$

and

$$\frac{\Lambda(x(j), g, g')}{\Lambda(g_L, g, x(j))} = \frac{e^{-\gamma(\bar{\varepsilon}_{g'}(j))}}{e^{-\gamma(\bar{\varepsilon}_{g_L}(j))}} \left[\frac{\bar{c}(g', \Gamma_x(x(j), g, g'))}{\bar{c}(g_L, \Gamma_x(x(j), g, g'))}\right]^{-\frac{1}{\psi}} \left[\frac{\bar{u}(g', \Gamma_x(x(j), g, g'))}{\bar{u}(g_L, \Gamma_x(x(j), g, g'))}\right]^{\frac{1}{\psi}-\gamma}.$$

4. Now we construct the policy function $u'(\zeta', j)$. First, $\forall \eta' + \varepsilon' < \underline{\varepsilon}_{g'}(j)$ use (63) and $\forall \eta' + \varepsilon' > \bar{\varepsilon}_{g'}(j)$ use (64); for $\eta' + \varepsilon' \in (\underline{\varepsilon}_{g'}(j), \bar{\varepsilon}_{g'}(j))$ use

$$\frac{e^{-\gamma(\underline{\varepsilon}_{g'}(j))}}{e^{-\gamma(\eta'+\varepsilon')}} \left[\frac{c^*(g', \Gamma_x(x(j), g, g'))}{c(u')}\right]^{-\frac{1}{\psi}} \left[\frac{u^*(g', \Gamma_x(x(j), g, g'))}{u'}\right]^{\frac{1}{\psi}-\gamma} = 1$$

to solve out for u'.

5. We compute $c(j)$, $\theta(j)$ and $\iota(j)$ using equations (62), 65, and (66), where certainty equivalent $m(j)$ only depends on $\{u'(s',g)\}_{s'}$ and $\{\overline{u}(g',\Gamma_x(x(j),g,g'))\}_{g'}$.

6. Finally, we use the promise keeping constraint (15) to back out $u(j)$ that is consistent with $c(j)$ and $\{u'(s',g)\}_{s'}$ and we use the objective function of the firm, the right hand side of (14) to obtain v_j

7. The guess for $v(u,g,x)$ and $c(u,g,x)$ are updated by interpolating values $\{u_j,v_j\}$ and $\{u_j,c_j\}$. We then iterate until the value function and consumption functions both converge with a tolerance of $1e-7$ under a sup norm.

Details for the simulation procedure: Let $\phi(t)$ denote the summary measure at time t. In simulations, we approximate the continuous distribution $\phi(t)$ by a finite-state distribution as follows. We choose $u_1^{(t)}, u_2^{(t)}, \cdots, u_{N+1}^{(t)}$, where $u_1^{(t)} = \lambda \overline{u}(g_t,x_t)$ and $u_{N+1}^{(t)} = u^*(g_t,x_t)$. A density ϕ is characterized by a set of grid points $\{\hat{u}[n](t)\}_{n=1}^{N+3}$ and corresponding weights $\{\phi[n](t)\}_{n=1}^{N+3}$ such that (i) $\hat{u}[1]$ and $\hat{u}[N+1]$ are the boundaries where the limited commitment constraint binds: $\hat{u}[1] = \lambda\overline{u}(g_t,x_t)$ and $\hat{u}[N+1] = u^*(g_t,x_t)$; $\hat{u}[N+2] = u^*(g_t,x_t)$ is the restarting utility; (ii) $\{\hat{u}[n]\}_{n=2,3,\cdots N}$ are the interior points: $\hat{u}[j] \in (u_{j-1}, u_j)$, for $j = 2,3\cdots N$, are chosen appropriately to minimize the approximation error.; (iii) $\phi[1]$ and $\phi[N+1]$ are the total amount of human capital owned by agents with a binding limited commitment constraint at $\hat{u}[1]$ and $\hat{u}[N+1]$, respectively; (iv) $\{\phi[n]\}_{n=2,3,\cdots N}$ are the human capital owned by agents in the interior; (iv) the mass on $\phi[N+2]$ is the human capital of agents who (re)start at $u^*(g,x)$, this include both the newly employed and the new born. (v) the mass $\phi[N+3]$, which is the total human capital owned by workers in the the unemployed pool.

1. Start with an initial distribution of u, denoted $\{\phi_0(u)\}$. Having solved x_0, use the law of motion of $u'(u,g,x,\zeta')$ to compute ϕ_1. Here we describe a general procedure to solve for $\{\phi[n](t+1); \hat{u}[n](t+1); x_{t+1}\}_{n=1}^{N+3}$ and B_{t+1} given $\{\phi[n](t); \hat{u}[n](t); x_t\}_{n=1}^{N+3}$ and B_t. Note that the assumed law of motion gives a natural candidate for x_{t+1}. We denote $x_{t+1} = \Gamma(x_t, g_t, g_{t+1})$.

2. First, we approximate the distribution $s \sim f(\varepsilon + \eta|g)$ by a finite dimensional distribution such that $\sum_k^K f_g[j] = 1$ and $\sum_k^K e^{s_k} f_g[j] = 1$, for $g = g_H, g_L$.

3. Given $\{\phi[n](t), \hat{u}[n](t)\}_{n=1}^{N+3}$ for period t, conditioning on the realization of aggregate state g_{t+1}, for each $n = 1, 2, \cdots, N+2$, we compute $\{\phi_{t+1}[n,k]\}_{n,k}$.

$$\phi_{t+1}[n,k] = \kappa\theta(\hat{u}[n](t), g_t, x_t) f_{g_{t+1}}[k] \phi_t[n] e^{s_k}, \quad k = 1, 2 \ldots, K.$$

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4. We now compute $\{\phi_{t+1}[m]\}_m$ for the next period.

$$\phi_{t+1}[1] = \sum_{n=1}^{N+2}\sum_{k=1}^{K}\phi_{t+1}[n,j]\, I_{\{u'(\hat{u}[n](t),g_t,x_t,g_{t+1},s_k)\le\lambda\bar{u}(g_{t+1},x_{t+1})\}},$$

$$\phi_{t+1}[2] = \sum_{n=1}^{N+2}\sum_{k=1}^{K}\phi_{t+1}[n,k]\, I_{\left\{u'(\hat{u}[n](t),g_t,x_t,g_{t+1},s_k)\in\left(u_1^{(t+1)},u_2^{(t+1)}\right)\right\}},$$

$$\phi_{t+1}[m] = \sum_{n=1}^{N+2}\sum_{k=1}^{K}\phi_{t+1}[n,k]\, I_{\left\{u'(\hat{u}[n](t),g_t,x_t,g_{t+1},s_k)\in\left[u_{m-1}^{(t+1)},u_m^{(t+1)}\right)\right\}},\quad m=3,\dots N$$

$$\phi_{t+1}[N+1] = \sum_{n=1}^{N+2}\sum_{k=1}^{K}\phi_{t+1}[n,k]\, I_{\{u'(\hat{u}[n](t),g_t,x_t,g_{t+1},s_k)\ge u^*(g_{t+1},x_{t+1})\}},$$

$$\phi_{t+1}[N+2] = 1 - \kappa + \kappa\lambda\chi\phi_t[N+3]$$

$$\phi_{t+1}[N+3] = \kappa\lambda\left\{\sum_{n=1}^{N+2}[1-\theta(\hat{u}[n](t),g_t,x_t)]\,\phi_t[n] + [1-\chi]\,\phi_t[N+3]\right\}$$

5. We need to update the vector normalized utilities $\{\hat{u}[n](t+1)\}_{n=1}^{N+2}$. Clearly, we should have $\hat{u}[1](t+1) = \lambda\bar{u}(g_{t+1},x_{t+1})$, $\hat{u}[N+1](t+1) = u^*(g_{t+1},x_{t+1})$ and $\hat{u}[N+2](t+1) = u^*(g_{t+1},x_{t+1})$. For $m = 2,\dots,N$, we choose $\hat{u}[m](t+1) \in \left[u_{m-1}^{(t+1)},u_m^{(t+1)}\right)$ such that the resource constraint holds exactly for $u \in \left[u_{m-1}^{(t+1)},u_m^{(t+1)}\right)$. That is, we pick $\hat{u}[m](t+1)$ to be the solution (denoted $\hat{u}$) to

$$\sum_{n=1}^{N+2}\sum_{k=1}^{K}\phi_{t+1}[n,k]\,c\left(u'(\hat{u}[n](t),g_t,x_t,g_{t+1},\varepsilon_j),g_{t+1},x_{t+1}\right)I_{\left\{u'(\hat{u}[n](t),g_t,x_t,g_{t+1},\varepsilon_k)\in\left[u_{m-1}^{(t+1)},u_m^{(t+1)}\right)\right\}}$$
$$= c(\hat{u},g_{t+1},x_{t+1})\phi_{t+1}[m].$$

6. Finally, the total unemployment benefit consumed by all unemployed workers is $B_{t+1} = b\phi_{t+1}[N+3]$.

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Online Appendix Not for Publication for *Asset Pricing with Endogenously Uninsurable Tail Risk*

A Proof of proposition 3

Proposition 3 follows directly from the lemma below, which provides the details for the construction of the equilibrium in the stochastic economy from a given equilibrium of the deterministic economy with a modified discount rate.

Lemma 10. *Suppose g_t is i.i.d. over time and $f\left(\cdot\,|\,g\right)$ does not depend on g. Suppose there exists an equilibrium in the equivalent deterministic economy with modified discount rate. An equilibrium of the stochastic economy can be constructed as follows.*

i) *The SDF is given by equation (32) in proposition 3.*

ii) *Workers' value from unemployment and the value of a new job are given by:*

$$\bar{u}\left(S\right)=\widehat{\bar{u}}\left(\phi,B\right),\quad u^{*}\left(S\right)=\widehat{u}^{*}\left(\phi,B\right),$$

respectively, where $\widehat{\bar{u}}\left(\phi,B\right)$ and $\widehat{u}^{*}\left(\phi,B\right)$ are the corresponding equilibrium quantities in the equivalent deterministic economy with a modified discount rate.

iii) *The consumption share of capital owners is*

$$x\left(S\right)=\hat{x}\left(\phi,B\right),$$

where $\hat{x}\left(\phi,B\right)$ is the capital owner's consumption share in the equivalent deterministic economy with a modified discount rate.

iv) *The value function and policy functions of the optimal contracting problem are given by*

$$v\left(u,S\right)=\hat{v}\left(u,\phi,B\right),\;\;c\left(u,S\right)=\hat{c}\left(u,\phi,B\right),$$
$$\theta\left(u,S\right)=\hat{\theta}\left(u,\phi,B\right),\;\;u'\left(u,S,g',\varepsilon'\right)=\hat{u}'\left(u,\phi,B,\varepsilon'\right).$$

v) *The law of motion for aggregate state variables $\left(\phi,B\right)$ is the same as that in the equivalent deterministic economy with a modified discount rate.*

Proof. To prove that the proposed allocations and prices constitutes an equilibrium, we use lemma 5 to verify the equilibrium conditions. First, we show that the proposed stochastic

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discount factor is consistent with capital owners' consumption and utility process. Given capital owner's consumption and utility in the stochastic economy, using equation (20),

$$\Lambda\left(S',S\right) = \beta e^{-\gamma g'}\left[\frac{\hat{x}\left(\phi',B'\right)}{\hat{x}\left(\phi,B\right)}\right]^{-\frac{1}{\psi}}\left[\frac{w\left(\phi',B'\right)}{\left(\mathbb{E}\left[e^{(1-\gamma)g'}\hat{w}^{1-\gamma}\left(\phi',B'\right)\right]\right)^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}$$

The utility $w\left(\phi',B'\right)$ is deterministic and the above can be written as

$$\Lambda\left(S',S\right) = \beta e^{-\gamma g'}\left[\frac{\hat{x}\left(\phi',B'\right)}{\hat{x}\left(\phi,B\right)}\right]^{-\frac{1}{\psi}}\left(\mathbb{E}\left[e^{(1-\gamma)g'}\right]\right)^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} = \hat{\beta}\left[\frac{\hat{x}\left(\phi',B'\right)}{\hat{x}\left(\phi,B\right)}\right]^{-\frac{1}{\psi}}\frac{e^{-\gamma g'}}{\mathbb{E}\left[e^{(1-\gamma)g'}\right]}. \quad (95)$$

Given the consumption policy in the deterministic economy, the SDF in the deterministic economy reduces to a risk-free discount rate $R\left(\phi,B\right)$ with

$$\frac{1}{R\left(\phi,B\right)} = \hat{\beta}\left[\frac{x\left(\phi',B'\right)}{x\left(\phi,B\right)}\right]^{-\frac{1}{\psi}}. \quad (96)$$

Combing equations (95) and (96), it is clear that the SDF defined in (32) is consistent with capital owners' consumption in the stochastic economy.

Next, we show that the proposed value function and policy functions also solve the optimal contracting problem in the stochastic economy. It is enough to show that the value function of the deterministic economy is also a fixed point of the Bellman operator implied by the optimal contracting problem *P1*. Given that the two economies have the same workers' outside options $\bar{u}\left(S\right)$ and that $\hat{\beta} = \beta\left(\mathbb{E}\left[e^{(1-\gamma)g'}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$, it is easy to see that constraints (15), (16), and (17) are identical in both economies. Given $v\left(u,\phi,B\right)$ the term

$$\int \Lambda\left(S',S\right)e^{g'+\eta'+\varepsilon'}v\left(u'\left(s'\right),S'\right)\Omega\left(d\zeta'|g\right)$$

can be written as

$$\frac{1}{R\left(\phi,B\right)}\sum\pi\left(g'\right)\int\frac{e^{-\gamma g'}}{\mathbb{E}\left[e^{(1-\gamma)g'}\right]}e^{g'+\varepsilon'+\eta'}v\left(u'\left(\varepsilon',\eta'\right),\phi',B'\right)f\left(\varepsilon'+\eta'\right)d\varepsilon'd\eta'$$

$$= \frac{1}{R\left(\phi,B\right)}\int\frac{\mathbb{E}\left[e^{(1-\gamma)g'}\right]}{\mathbb{E}\left[e^{(1-\gamma)g'}\right]}e^{\varepsilon'+\eta'}v\left(u'\left(\varepsilon',\eta'\right),\phi',B'\right)f\left(\varepsilon'+\eta'\right)d\varepsilon'd\eta'$$

$$= \frac{1}{R\left(\phi,B\right)}\int e^{\varepsilon'+\eta'}v\left(u'\left(\varepsilon',\eta'\right),\phi',B'\right)f\left(\varepsilon'+\eta'\right)d\varepsilon'd\eta',$$

which is identical to that in the deterministic economy. Therefore, $v\left(u,S\right) = \hat{v}\left(u,\phi,B\right)$ is also the value function of the optimal contracting problem in the stochastic economy.

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Finally, conditions 3 and 4 in lemma 5 also hold, because these requirements are identical in the deterministic economy and the stochastic economy. This completes the proof. □

B   Details of the simple model

Proof for Lemma 6

Proof. Here, we only provide details for the deviation of the value function at node H. The value function at node L can be computed in the same way. At node H, the optimal contracting problem is written as

$$v(u, g_H) = \max_{c, \theta, u'} \left\{ 1 - c - A(\theta) + \theta \frac{1}{R_H} e^{g_H} v_2(u', g_H) \right\} \tag{97}$$
$$\text{subject to}: \ u = c^{1-\beta} \left(e^{g_H} u' \right)^{\beta}$$
$$u' = \alpha u_H^{FB}$$
$$v_{2,H}(u') \geq 0$$
$$A'(\theta) = \frac{1}{R_H} e^{g_H} v_{2,H}(u')$$

where we use $v_2(u', g_H)$ for the value function in period 2. Because there is no aggregate uncertainty in period 2, we replace the stochastic discount factor by a risk-free discount rate, $\frac{1}{R_H}$. The absence of idiosyncratic shocks and the fact that workers consume α fraction of their output imply that workers' utility is α times the utility of a representative consumer, that is $u' = \alpha u_H^{FB}$. Note also, because the firm always receive $1 - \alpha$ fraction of y_t after period 2, the limited commitment constraint $v_2(u', g_H) \geq 0$ does not bind.

To derive a close-form solution for $v_H(u)$, we first note that $v_2(u', g_H) \geq 0 = \frac{1-\alpha}{1-\beta}$. From period 2 and on, capital owner's consumption and firms' cash flow are both proportional to aggregate output. Under the assumption of unit elasticity, the price-to-dividend ratio of the firm's cash flow is $\frac{1}{1-\beta}$. Because the firm receive $1 - \alpha$ fraction of y_t, the ratio of firm value normalized by y_t is $\frac{1-\alpha}{1-\beta}$.

Second, because capital owner's consumption share is x_H in period 1 and $1 - \alpha$ in period 2, the discount factor is $\frac{1}{R_H} = \beta \left[\frac{1-\alpha}{x_H} e^{g_H} \right]^{-1}$. Therefore, the value function can be written as

$$v(u, g_H) = 1 - c - A(\theta) + \theta \frac{\beta}{1-\beta} x_H. \tag{98}$$

The consumption policy can be backed out from the promise-keeping constraint $u = c^{1-\beta} \left(e^{g_H} u' \right)^{\beta}$. In addition, given then functional form of $A(\theta)$, the optimal effort θ can be solved from the incentive constraint, $A'(\theta) = \frac{\beta}{1-\beta} x_H$, which gives (78). Replacing θ in (98) with the optimal policy, we obtain the representation of the value function in (75).

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The optimal contracting problem at node L has a similar structure:

$$v(u, g_L) = \max_{c, \theta, u'(\varepsilon')} \left\{ 1 - c - A(\theta) + \theta \frac{1}{R_L} \int_{-\infty}^{\infty} e^{g_L + \varepsilon'} v_2 \left(u'(\varepsilon'), g_L \right) f\left(\varepsilon' \middle| g_L \right) d\varepsilon' \right\}$$

$$subject\ to:\ u = c^{1-\beta} \left\{ \int_{-\infty}^{\infty} \left[e^{g_L + \varepsilon'} u'(\varepsilon') \right]^{1-\gamma} f\left(\varepsilon' \middle| g_L \right) d\varepsilon' \right\}^{\frac{\beta}{1-\gamma}}$$

$$u'(\varepsilon') = \alpha u_L^{CD}$$

$$v_{2,L} \left(u'(\varepsilon') \right) \geq 0$$

$$A'(\theta) = \frac{1}{R_L} e^{g_L} \int_{-\infty}^{\infty} v_{2,L} \left(u'(\varepsilon') \right) f\left(\varepsilon' \middle| g_L \right) d\varepsilon'.$$

The above problem can be greatly simplified by noting that $v_2 \left(u'(\varepsilon'), g_L \right) = \frac{1-\alpha}{1-\beta}$ and $u'(\varepsilon') = \alpha u_L^{CD}$ do not depend on ε'. Also, using the definition of in (77), $\left\{ \int_{-\infty}^{\infty} \left[e^{\varepsilon'} u'(\varepsilon') \right]^{1-\gamma} f\left(\varepsilon' \middle| g_L \right) d\varepsilon' \right\}^{\frac{1}{1-\gamma}} = \alpha u_L^{CD} \Upsilon$. The rest of the proof can be completed by following the same steps in the solution of (97). □

Proof for lemma 7

Proof. By proposition 2, the optimal risk sharing condition (31) must hold with equality for the marginal worker with the realization of $\underline{\varepsilon}_L(u_0)$ at node L. Comparing the optimal risk-sharing conditions for consumption at node H and at L, we have

$$\left[\frac{c_H(u_0)}{e^{\underline{\varepsilon}_L(u_0)} c_L(u_0, \underline{\varepsilon}_L(u_0))} \right]^{-1} \left[\frac{u_H(u_0)}{e^{\underline{\varepsilon}_L(u_0)} u_L(u_0, \underline{\varepsilon}_L(u_0))} \right]^{1-\gamma} = \left[\frac{x_H}{x_L} \right]^{-1} \left[\frac{w_H}{w_L} \right]^{1-\gamma}. \tag{99}$$

We can use the promise-keeping constraint to represent continuation utilities as functions of consumption. For capital owners,

$$w_H = x_H^{1-\beta} n_H^\beta, \quad where\ n_H = (1-\alpha) e^{g_H} u_H^{FB},$$
$$w_L = x_L^{1-\beta} n_L^\beta, \quad where\ n_L = (1-\alpha) e^{g_L} u_L^{FB}, \tag{100}$$

where the computation of continuation utility n_H and n_L uses the fact that capital owners are not exposed to idiosyncratic risks and that together they consume $1-\alpha$ fraction of aggregate output. Because workers are not exposed to idiosyncratic risks at node H and consume α fraction of aggregate output, their continuation utility at node H can be computed using

$$u_H(u_0) = [c_H(u_0)]^{1-\beta} m_H^\beta, \quad where\ m_H = \alpha u_H^{FB} e^{g_H} k(\theta_H), \tag{101}$$

where $k(\theta)$ is defined in lemma 7. At node L, workers consume αy_t for $t = 2, 3, \ldots$. In period 2, following node L, a worker stays employed with probability θ_L, in which case his output

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is $y_2 = y_1 e^{g_L + \varepsilon'}$. With probability $1 - \theta_L$, a worker loses $1 - \lambda$ fraction of human capital and his output is $y_2 = \lambda y_1 e^{g_L + \varepsilon'}$. Therefore, the certainty equivalent for a worker at node L is

$$
\begin{aligned}
m_L &= \left\{ \int_{-\infty}^{\infty} \left[e^{g' + \varepsilon'} \left(\theta_L \alpha u_L^{CD} + (1 - \theta_L) \lambda \alpha u_L^{CD} \right) \right]^{1-\gamma} f\left(\varepsilon' \middle| g_L \right) d\varepsilon' \right\}^{\frac{1}{1-\gamma}} \\
&= \alpha \Upsilon u_L^{CD} e^{g_L} k\left(\theta_L \right),
\end{aligned}
\tag{102}
$$

where we define $\Upsilon \in (0, 1)$ as in (77). Therefore, the normalized utility of the marginal agent at node L can be written as

$$
u_L\left(u_0, \underline{\varepsilon}_L\left(u_0 \right) \right) = \left[c_L\left(u_0, \ \underline{\varepsilon}_L\left(u_0 \right) \right) \right]^{1-\beta} m_L^{\beta}, \quad \text{where } m_L = \alpha \Upsilon u_L^{CD} e^{g_L} k\left(\theta_L \right).
\tag{103}
$$

Now we use expressions in (100) and (103) to replace the continuation utilities in (99) and simplify to get

$$
\left[\frac{c_H\left(u_0 \right)}{e^{[1+\tau]\varepsilon_L(u_0)} c_L\left(u_0, \ \underline{\varepsilon}_L\left(u_0 \right) \right)} \right]^{-\Omega} \left[\frac{\Upsilon u_L^{CD} k\left(\theta_L \right)}{u_L^{FB} k\left(\theta_H \right)} \right]^{-\beta(1-\gamma)} = \left[\frac{x_H}{x_L} \right]^{-\Omega},
\tag{104}
$$

where to simplify notation, we denote

$$
\Omega \equiv 1 + (1 - \beta)(\gamma - 1) > 0, \quad and \quad \tau \equiv \frac{\beta(\gamma - 1)}{\Omega}.
\tag{105}
$$

We can therefore obtain (80) by raising both sides of equation (99) to their $-\frac{1}{\Omega}$th power. $\quad\square$

Proof for lemma 8

Proof. Note that $\forall \varepsilon' \leq \underline{\varepsilon}_L\left(u_0 \right)$, the limited commitment constraint binds, and $c_L\left(u_0, \ \varepsilon' \right) = c_L\left(u_0, \ \underline{\varepsilon}_L\left(u_0 \right) \right)$. Therefore, the expected consumption of a worker with promised utility u_0 at node L can be computed as

$$
\int_{-\infty}^{\underline{\varepsilon}_L(u_0)} e^{\varepsilon'} c_L\left(u_0, \ \underline{\varepsilon}_L\left(u_0 \right) \right) f\left(\varepsilon' \middle| g_L \right) d\varepsilon' + \int_{\underline{\varepsilon}_L(u_0)}^{\varepsilon_{MAX}} e^{\varepsilon'} c_L\left(u_0, \ \varepsilon' \right) f\left(\varepsilon' \middle| g_L \right) d\varepsilon'.
\tag{106}
$$

To compute $c_L\left(u_0, \ \varepsilon' \right)$, note that the first order condition (31) implies that for all $\varepsilon' \geq \underline{\varepsilon}_L\left(u_0 \right)$,

$$
e^{-\gamma \varepsilon'} \left[c_L\left(u_0, \ \varepsilon' \right) \right]^{-1} \left[u_L\left(u_0, \varepsilon' \right) \right]^{1-\gamma} = e^{-\gamma \underline{\varepsilon}_L(u_0)} \left[c_L\left(u_0, \ \underline{\varepsilon}_L\left(u_0 \right) \right) \right]^{-1} \left[u_L\left(u_0, \ \underline{\varepsilon}_L\left(u_0 \right) \right) \right]^{1-\gamma}.
\tag{107}
$$

We can compute $u_L\left(u_0, \varepsilon' \right)$ as:

$$
u_L\left(u_0, \varepsilon' \right) = c_L^{1-\beta}\left(u_0, \ \varepsilon' \right) m_L^{\beta},
\tag{108}
$$

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where the expression of m_L is given in equation (102). Equations (107) and (108) together imply

$$e^{-\gamma\varepsilon'}\left[c_L\left(u_0,\varepsilon'\right)\right]^{-1+(1-\gamma)(1-\beta)} = e^{-\gamma\underline{\varepsilon}_L(u_0)}\left[c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right)\right]^{-1+(1-\gamma)(1-\beta)}.$$

Raising both sides of the above equation to the $-\frac{1}{\Omega}th$ power and using the definition of Ω and τ in (105), we have, for all $\varepsilon \geq \underline{\varepsilon}_L\left(u_0\right)$,

$$e^{\varepsilon'}c_L\left(u_0,\varepsilon'\right) = e^{-\tau\varepsilon'}e^{(1+\tau)\underline{\varepsilon}_L(u_0)}c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right). \tag{109}$$

Now, we compute the first term in the integral in (106) as:

$$\int_{-\infty}^{\underline{\varepsilon}_L(u_0)} e^{\varepsilon'}c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right)f\left(\varepsilon'\mid g_L\right)d\varepsilon' = c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right)\int_{-\infty}^{\underline{\varepsilon}_L(u_0)} e^{\varepsilon'}f\left(\varepsilon'\mid g_L\right)d\varepsilon'$$

$$= \frac{\xi}{1+\xi}e^{-\xi\varepsilon_{MAX}+(1+\xi)\underline{\varepsilon}_L(u_0)}c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right) \tag{110}$$

and the second term as

$$\int_{\underline{\varepsilon}_L(u_0)}^{\varepsilon_{MAX}} e^{\varepsilon'}c_L\left(u_0,\ \varepsilon'\right)f\left(\varepsilon'\mid g_L\right)d\varepsilon'$$

$$= e^{(1+\tau)\underline{\varepsilon}_L(u_0)}c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right)\int_{\underline{\varepsilon}_L(u_0)}^{\varepsilon_{MAX}} e^{-\tau\varepsilon'}f\left(\varepsilon'\mid g_L\right)d\varepsilon$$

$$= e^{(1+\tau)\underline{\varepsilon}_L(u_0)}c_L\left(u_0,\ \underline{\varepsilon}_L\left(u_0\right)\right)\frac{\xi}{\xi-\tau}\left[e^{-\tau\varepsilon_{MAX}} - e^{-\xi\varepsilon_{MAX}+(\xi-\tau)\underline{\varepsilon}_L(u_0)}\right] \tag{111}$$

We obtain equation (81) by summing up (110) and (111). □

Proof for lemma 9

Proof. The optimal risk sharing condition implies that

$$\left[\frac{c_H\left(u_0\right)}{c_0}\right]^{-1}\left[\frac{u_H\left(u_0\right)}{m_0}\right]^{1-\gamma} = \left[\frac{x_H}{x_0}\right]^{-1}\left[\frac{w_H}{n_0}\right]^{1-\gamma}. \tag{112}$$

Using equation (100),

$$w_H = x_H^{1-\beta}\left[(1-\alpha)e^{g_H}u_H^{FB}\right]^{\beta}, \ w_L = x_L^{1-\beta}\left[(1-\alpha)e^{g_L}u_L^{FB}\right]^{\beta}, \ and$$

$$n_0 = \left[\pi\left(e^{g_H}w_H\right)^{1-\gamma} + (1-\pi)\left(e^{g_L}w_L\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{113}$$

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To calculate workers' utility, use (101) to obtain

$$u_H(u_0) = [c_H(u_0)]^{1-\beta} \left[\alpha u_H^{FB} e^{g_H} k(\theta_H)\right]^\beta. \tag{114}$$

Using equations (113) and (114) to replace the relevant terms in (112), we obtain equation (83). It remains to calculate workers' certainty equivalent,

$$m_0 = \left\{ \pi \left[e^{g_H} u_H(u_0)\right]^{1-\gamma} + (1-\pi) \int_{-\infty}^{\infty} \left[e^{g_L+\varepsilon'} u_L(u_0, \varepsilon')\right]^{1-\gamma} f(\varepsilon'|g_L) d\varepsilon' \right\}^{\frac{1}{1-\gamma}}. \tag{115}$$

Note that for $\varepsilon' \geq \underline{\varepsilon}_L(u_0)$, using equation (107), we can write

$$
\begin{aligned}
\left[e^{\varepsilon'} u_L(u_0, \varepsilon')\right]^{1-\gamma} &= \left[e^{\varepsilon'} c_L(u_0, \varepsilon')\right] e^{-\gamma \underline{\varepsilon}_L(u_0)} \left[c_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{-1} \left[u_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{1-\gamma} \\
&= \left[e^{\varepsilon'} c_L(u_0, \varepsilon')\right] e^{-\gamma \underline{\varepsilon}_L(u_0)} \left[c_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{-1+(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)},
\end{aligned}
$$

where the second equality uses (103) to compute $u_L(u_0, \underline{\varepsilon}_L(u_0))$ as a function of consumption. Therefore,

$$
\begin{aligned}
& \int_{\underline{\varepsilon}_L(u_0)}^{\varepsilon_{MAX}} \left[e^{\varepsilon'} u_L(u_0, \varepsilon')\right]^{1-\gamma} f(\varepsilon'|g_L) d\varepsilon' \\
={}& \int_{\underline{\varepsilon}_L(u_0)}^{\varepsilon_{MAX}} \left[e^{\varepsilon'} c_L(u_0, \varepsilon')\right] f(\varepsilon'|g_L) d\varepsilon' \times e^{-\gamma \underline{\varepsilon}_L(u_0)} \left[c_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{-1+(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)} \\
={}& \frac{\xi}{\xi-\tau} \left[e^{-\tau \varepsilon_{MAX}} - e^{-\xi \varepsilon_{MAX}+(\xi-\tau)\underline{\varepsilon}_L(u_0)}\right] \times e^{(1-\gamma+\tau)\underline{\varepsilon}_L(u_0)} \left[c_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)} \\
={}& \frac{\xi}{\xi-\tau} \left[e^{-\tau \varepsilon_{MAX}} - e^{-\xi \varepsilon_{MAX}+(\xi-\tau)\underline{\varepsilon}_L(u_0)}\right] \left[e^{(1+\tau)\underline{\varepsilon}_L(u_0)} c_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)} \tag{116}
\end{aligned}
$$

where the second equality uses the same calculation as in (111) and the last equality uses the definition of τ to simplify. For $\varepsilon' < \underline{\varepsilon}_L(u_0)$, the firm-side limited commitment constraint binds, and

$$
\begin{aligned}
\left[e^{\varepsilon'} u_L(u_0, \varepsilon')\right]^{1-\gamma} &= e^{(1-\gamma)\varepsilon'} \left[u_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{1-\gamma} \\
&= e^{(1-\gamma)\varepsilon'} \left[c_L(u_0, \underline{\varepsilon}_L(u_0))\right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)},
\end{aligned}
$$

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where the second equality applies equation (103). Therefore,

$$
\int_{-\infty}^{\underline{\varepsilon}_L(u_0)} \left[e^{\varepsilon'} u_L \left(u_0, \varepsilon' \right) \right]^{1-\gamma} f \left(\varepsilon' \middle| g_L \right) d\varepsilon'
$$

$$
= \int_{-\infty}^{\underline{\varepsilon}_L(u_0)} e^{(1-\gamma)\varepsilon'} f \left(\varepsilon' \middle| g_L \right) d\varepsilon' \times \left[c_L \left(u_0, \ \underline{\varepsilon}_L \left(u_0 \right) \right) \right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)}
$$

$$
= \frac{\xi}{\xi + 1 - \gamma} e^{-\xi \varepsilon_{MAX} + (1-\gamma+\xi)\underline{\varepsilon}_L(u_0)} \times \left[c_L \left(u_0, \ \underline{\varepsilon}_L \left(u_0 \right) \right) \right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)}
$$

$$
= \frac{\xi}{\xi + 1 - \gamma} e^{-\xi \varepsilon_{MAX} + (\xi-\tau)\underline{\varepsilon}_L(u_0)} \times \left[e^{(1+\tau)\underline{\varepsilon}_L(u_0)} c_L \left(u_0, \ \underline{\varepsilon}_L \left(u_0 \right) \right) \right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)} \tag{117}
$$

where the last equality uses the definition of τ to simplify. Combining (116) and (117), we have

$$
\int_{-\infty}^{\infty} \left[e^{\varepsilon'} u_L \left(u_0, \varepsilon' \right) \right]^{1-\gamma} f \left(\varepsilon' \middle| g_L \right) d\varepsilon'
$$

$$
= \left[e^{(1+\tau)\underline{\varepsilon}_L(u_0)} c_L \left(u_0, \ \underline{\varepsilon}_L \left(u_0 \right) \right) \right]^{(1-\beta)(1-\gamma)} m_L^{\beta(1-\gamma)} \Psi \left(\underline{\varepsilon}_L \left(u_0 \right) \right), \tag{118}
$$

where $\underline{\varepsilon}_L \left(u_0 \right)$ is defined in (85). We obtain the expression (84) by combing (115) and (118). $\qquad \square$

Proof for the claim that Price-dividend ratio is procyclical Here we provide a proof for claim in footnote 12. Consider first firm value at node H, (75). Because there is no idiosyncratic shock at node H, there is only one type of firm, and $u = u_H$. Using the market clearing condition at node H, $1 - c_H = x_H$. Therefore, in equilibrium,

$$
\begin{aligned}
v_H \left(u_0^* \right) &= 1 - c_H \left(u_0^* \right) + \frac{\beta}{1-\beta} x_H - a \ln \left[1 + \frac{\beta x_H}{a \left(1 - \beta \right)} \right] \\
&= x_H + \frac{\beta}{1-\beta} x_H - a \ln \left[1 + \frac{\beta x_H}{a \left(1 - \beta \right)} \right] \\
&= \frac{1}{1-\beta} x_H - a \ln \left[1 + \frac{\beta x_H}{a \left(1 - \beta \right)} \right].
\end{aligned}
$$

At node L, firm value is given by $e^{\varepsilon} v_L \left(u_0^*, \varepsilon \right)$. Using equation (76),

$$
v_L \left(u_0^*, \varepsilon \right) = 1 - c_L \left(u_0^*, \varepsilon \right) + \frac{\beta}{1-\beta} x_L - a \ln \left[1 + \frac{\beta x_L}{a \left(1 - \beta \right)} \right].
$$

Note that

$$
E \left[e^{\varepsilon} c_L \left(u_0^*, \varepsilon \right) \right] = 1 - x_L
$$

by market clearing. Therefore,

$$
\begin{aligned}
E\left[e^{\varepsilon}v_L\left(u_0^*,\varepsilon\right)\right] &= 1 - E\left[e^{\varepsilon}c_L\left(u_0^*,\varepsilon\right)\right] + \frac{\beta}{1-\beta}x_L - a\ln\left[1 + \frac{\beta x_L}{a\left(1-\beta\right)}\right] \\
&= \frac{1}{1-\beta}x_L - a\ln\left[1 + \frac{\beta x_L}{a\left(1-\beta\right)}\right].
\end{aligned}
$$

Using our previous arguement, as $\gamma \to 1 + \xi$, $\frac{x_H}{x_L} \to \infty$. Therefore, for γ large enough, we must have $\frac{v_H\left(u_0^*\right)}{E\left[e^{\varepsilon}v_L\left(u_0^*,\varepsilon\right)\right]} > 1$, as needed.

Proof for inequality 11   The proof for the last part of proposition 5 requires inequality (92), which we formally state as a lemma below.

Lemma 11. *There exists $\tilde{\gamma} \in (1, 1+\xi)$ such that $\gamma > \tilde{\gamma}$ implies that for all $\varepsilon \in (-\infty, \varepsilon_{MAX})$,*

$$
\frac{\partial}{\partial\varepsilon}\left[\frac{\Gamma_H - \phi e^{(1+\tau)\varepsilon}}{1 - e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)}\right] > 0. \tag{119}
$$

Proof. We can compute (119) as :

$$
\begin{aligned}
&\frac{\partial}{\partial\varepsilon}\left[\frac{\Gamma_H - \phi e^{(1+\tau)\varepsilon}}{1 - e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)}\right] \\
=\ &\frac{-\phi e^{(1+\tau)\varepsilon}\left(1+\tau\right)\left[1 - e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)\right] + \left[\Gamma_H - \phi e^{(1+\tau)\varepsilon}\right]e^{(1+\tau)\varepsilon}\left[\left(1+\tau\right)\Phi\left(\varepsilon\right) + \Phi'\left(\varepsilon\right)\right]}{\left[1 - e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)\right]^2}.
\end{aligned}
$$

We focus on the numerator and simplify:

$$
\begin{aligned}
&-\phi e^{(1+\tau)\varepsilon}\left(1+\tau\right)\left[1 - e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)\right] + \left[\Gamma_H - \phi e^{(1+\tau)\varepsilon}\right]e^{(1+\tau)\varepsilon}\left[\left(1+\tau\right)\Phi\left(\varepsilon\right) + \Phi'\left(\varepsilon\right)\right] \\
=\ &\Gamma_H\left[\left(1+\tau\right)\Phi\left(\varepsilon\right) + \Phi'\left(\varepsilon\right)\right] - \phi\left[\left(1+\tau\right) + e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)\right]
\end{aligned}
$$

It is therefore enough to show

$$
\Gamma_H\left[\left(1+\tau\right)\Phi\left(\varepsilon\right) + \Phi'\left(\varepsilon\right)\right] - \phi\left[\left(1+\tau\right) + e^{(1+\tau)\varepsilon}\Phi\left(\varepsilon\right)\right] > 0 \tag{120}
$$

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Using the expression of $\Phi(\varepsilon)$, we can compute

$$
\begin{aligned}
(1+\tau)\Phi(\varepsilon)+\Phi'(\varepsilon) &= (1+\tau)\frac{\xi}{\xi-\tau}\left[e^{-\tau\varepsilon_{MAX}}-e^{-\lambda\varepsilon_{MAX}+(\xi-\tau)\varepsilon}\right] \\
&= (1+\tau)\frac{\xi}{\xi-\tau}e^{-\tau\varepsilon_{MAX}}\left[1-e^{-(\xi-\tau)\varepsilon_{MAX}+(\xi-\tau)\varepsilon}\right] \\
&= (1+\tau)\frac{\xi}{1+\xi}e^{-\tau\varepsilon_{MAX}}\frac{1+\xi}{\xi-\tau}\left[1-e^{-(\xi-\tau)\varepsilon_{MAX}+(\xi-\tau)\varepsilon}\right] \\
&= (1+\tau)e^{-(1+\tau)\varepsilon_{MAX}}\frac{1+\xi}{\xi-\tau}\left[1-e^{-(\xi-\tau)(\varepsilon_{MAX}-\varepsilon)}\right]>0,
\end{aligned}
$$

where the last line uses the fact $\varepsilon_{MAX}=\ln\frac{1+\xi}{\xi}$. Also, the second term in equation (120) can be written as

$$
\begin{aligned}
(1+\tau)+e^{(1+\tau)\varepsilon}\Phi'(\varepsilon) &= (1+\tau)\left[1-\frac{\xi}{1+\xi}e^{-\lambda\varepsilon_{MAX}+(1+\xi)\varepsilon}\right] \\
&= (1+\tau)\left[1-e^{-(1+\xi)(\varepsilon_{MAX}-\varepsilon)}\right].
\end{aligned}
$$

Therefore, to prove (120), it is enough to show that for all ε,

$$
\Gamma_H e^{-(1+\tau)\varepsilon_{MAX}}\frac{1+\xi}{\xi-\tau}\left[1-e^{-(\xi-\tau)(\varepsilon_{MAX}-\varepsilon)}\right]-\phi\left[1-e^{-(1+\xi)(\varepsilon_{MAX}-\varepsilon)}\right]>0.
$$

Because $\phi\to 0$ as $\gamma\to 1+\xi$, we have $\Gamma_H e^{-(1+\tau)\varepsilon_{MAX}}>\phi$ for γ close enough to $1+\xi$. We complete the proof by making the following observation

Define

$$
\begin{aligned}
f(\varepsilon) &= \frac{1+\xi}{\xi-\tau}\left[1-e^{-(\xi-\tau)(\varepsilon_{MAX}-\varepsilon)}\right] \\
g(\varepsilon) &= 1-e^{-(1+\xi)(\varepsilon_{MAX}-\varepsilon)},
\end{aligned}
$$

then $f(\varepsilon)>g(\varepsilon)$ for all $\varepsilon<\varepsilon_{MAX}$. To see this, note that $f(\varepsilon_{MAX})=g(\varepsilon_{MAX})=0$. Also, $f'(\varepsilon)<g'(\varepsilon)$ for all $\varepsilon<\varepsilon_{MAX}$, because

$$
\begin{aligned}
f'(\varepsilon) &= -(1+\xi)e^{-(\xi-\tau)(\varepsilon_{MAX}-\varepsilon)} \\
g'(\varepsilon) &= -(1+\xi)e^{-(1+\xi)(\varepsilon_{MAX}-\varepsilon)}.
\end{aligned}
$$

$\square$

Table 7: FIRM-LEVEL WAGE PASS-THROUGHS AND LABOR SHARES

Coefficients	Using LS	Using ELS
LogSales_plus	0.69	0.55
	(0.05)	(0.04)
LogSales_minus	0.13	0.12
	(0.06)	(0.04)
Labor share	-0.02	-0.12
	(0.01)	(0.01)
Labor share × LogSales_plus	0.00	0.1
	(0.12)	(0.06)
Labor share × LogSales_minus	0.76	0.55
	(0.12)	(0.08)
Time Fixed Effects	Yes	Yes

Notes: The sample consist of firm-year observations from COMPUSTAT/CRSP merged files for the years 1959-2016. In the column labeled "Using LS" we use labor share computed using (34), and in the column labeled "Using ELS" we use the procedure described in Donangelo et al. (2016) and construct "extended labor share." In both specifications, labor shares are standardized and twice lagged, and standard errors are clustered at firm level.

C   More details on wage-pass-through and returns in the cross section

In this section, we provide corroborating evidence for the empirical results in sections 6.2 and 6.3. As a robustness for specification (35), we estimate

$$\Delta \log \text{WageBill}_{f,t+1} = \alpha_w + \beta_{w0} \, \text{LaborShare}_{f,t} + \beta_{w1}^{+} \max\{\Delta \log \text{Sales}_{f,t}, 0\}$$
$$\beta_{w1}^{-} \min\{\Delta \log \text{Sales}_{f,t}, 0\} + \gamma_w^{+} \Delta \max\{\log \text{Sales}_{f,t}, 0\} \times \text{LaborShare}_{f,t}$$
$$+ \gamma_w^{-} \min\{\Delta \log \text{Sales}_{f,t}, 0\} \times \text{LaborShare}_{f,t} + \lambda_{wt}. \tag{121}$$

The firm-side limited commitment binds with adverse firm-level shocks. This would imply that γ_w^{-} or the coefficient on the negative part of sales growth should be positive and statistically significant. In table 7, we verify this.

Next we estimate a version of (33) with total assets and book leverage as further controls. In table 8, we verify that the coefficient on labor leverage remains positive and statistically significant.

Table 8: FIRM-LEVEL RETURNS AND LABOR SHARES

Coefficients	Using LS	Using ELS
Labor share	1.24	0.85
	(0.43)	(0.20)
Leverage	0.63	1.17
	(0.91)	(0.35)
log Assets	-1.06	-2.22
	(0.43)	(0.21)
Time Fixed Effects	Yes	Yes

Notes: The sample consist of firm-year observations from COMPUSTAT for the years 1959-2016. We follow Donangelo et al. (2016) in the construction of firm labor share, the results of which are reported in the column labeled "Using LS", and the construction of extended labor share, the results of which are reported in the column labeled "Using ELS." In both specifications, labor shares are twice lagged, and standard errors are clustered at the firm level. *Log Assets* is the logarithm of book value of assets and *Leverage* is defined as the ratio of long-term debt plus debt in current liabilities divided by total assets.