A Quantitative Model of Dynamic Moral Hazard

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May 19, 2019

Abstract

We develop a dynamic general equilibrium model of firms with moral hazard, which arises because some productivity shocks driving firm dynamics are privately observed by firm managers only. We characterize the optimal contract and its implications for firm size, growth, and managerial pay-performance sensitivity, which allow us to quantify the severity of the moral hazard problem and its impact. Our estimation shows that the magnitude of unobservable shocks causing moral hazard is relatively small and accounts for about 10% of the total variation of firm output. Nonetheless, moral-hazard induced incentive pay is quantitatively significant and accounts for 52% of managerial compensation. Our welfare analysis suggests that eliminating moral hazard results in about 3.4% increase in aggregate output of the whole economy.

Keywords: Dynamic Moral Hazard, CEO Compensation, General Equilibrium, Firm Dynamics

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1 Introduction

A large body of literature in finance and economics emphasizes the importance of moral hazard in determining managerial compensation, firm investment, and firm growth. The vast majority of this literature focuses only on its *qualitative* implications. In this paper, we develop a *general equilibrium* model of dynamic moral hazard to analyze the *quantitative* impact of moral hazard on both individual firm behavior and the aggregate economy.

We incorporate moral hazard in a neoclassical production economy with heterogeneous firms. In our model, firm output is determined by both managers' investment decisions and productivity shocks. Because output is only a noisy signal of investment, managers have an incentive to substitute investment for private benefit. We allow firm productivity to be driven by both observable and unobservable shocks, and we use information from both time-series and the cross section to identify their magnitudes in order to quantify the impact of moral hazard.

The key feature that distinguishes our model from the previous literature is the incorporation of both observable and unobservable productivity shocks. For theoretical convenience, most of the existing moral hazard models assume that all shocks are unobservable to the principal. However, observable productivity shocks may account for a large fraction of heterogeneity across firms. Because the key tradeoff in moral hazard models is between risk sharing and incentive provision, the magnitude of observable shocks relative to unobservable shocks is the main determinant of the quantitative impact of moral hazard.

We derive two analytical results which allow us to use empirical evidence on the cross-sectional distribution and time-series dynamics of firms to identify the relative importance of observable and unobservable shocks. First, we show that the severity of the moral hazard problem decreases with the manager's share in the firm, defined as the ratio of the manger's payoff (formally, his continuation utility under the optimal contract) to firm size. When a larger fraction of the firm's cash flow is promised to the manager, the incentive to divert cash flow is lower. Consequently, pay-performance sensitivity declines and firm investment rate increases.

Second, we establish that managers' share in the firm decreases after positive observable shocks but increases after positive unobservable shocks. Risk sharing requires that positive *observable* productivity shocks, which increase the total size of the firm, should lead to a reduction in the manager's share in the firm. The need for incentive provision implies that positive *unobservable* productivity shocks are followed by an increase in manager share in the firm.

The above theoretical insights allow us to identify the magnitude of observable and unobservable shocks from the data. In particular, if the size of unobservable shocks is relatively small, the need for risk sharing dominates, and managers' equity share in the firm must be decreasing in firms size. As a result, investment is inversely related to firm size and pay-performance sensitivity is positively related to size. In contrast, a large magnitude of unobservable shocks requires high powered incentives, and implies a flat or even positive relationship between investment and size and a non-monotonic relationship between pay-performance sensitivity and firm size. In addition, quantitatively, the magnitude of pay-performance sensitivity is also informative about the relative magnitude of observable and unobservable shocks and provides an over-identifying restriction in our estimation.

Building on our theoretical results, we estimate the magnitude of unobservable shocks using the Simulated Method of Moments (SMM) by exploiting moment conditions of the joint distribution of firm size, investment and managerial compensation. We find that in order to account for the strong inverse relationship between firm growth and size and the positive relationship between pay-performance sensitivity and size observed in the data, the magnitude of unobservable shocks must be relatively small compared with observable shocks. Our estimates suggest that only about 10% of the total variation in firm output is attributed to unobservable shocks. The difference in variances of observable and unobservable shocks is strongly statistically significant.

Based on our structural estimation, we evaluate the quantitative impact of moral hazard in general equilibrium. In our dynamic model, moral hazard reduces efficiency for several reasons. First, incentive provision requires managerial compensation to respond to unobservable shocks and, therefore, reduces risk sharing. Second, moral hazard leads to back-loaded compensation policies and, hence, affects the intertemporal allocation of managerial compensation. Third, moral hazard distorts firms' investment policies and lowers the steady-state capital in the economy. Our estimates suggest that incentive provision is quite costly — incentive pay accounts for about 52% of the total CEO compensation. Our decomposition analysis reveals that about two thirds of incentive pay is due to limited risk sharing and one third is the compensation for distortions in the intertemporal allocation of CEO compensation. We also evaluate the aggregate impact of investment distortions and find that eliminating moral hazard increases the total output of the economy by about 3.4%.

Several novel features of our model are important for the purpose of quantifying the impact of moral hazard. First, we allow for observable shocks, which are the key determinant of the severity of moral hazard. Second, we adopt the constant relative risk aversion preferences. Most of the continuous-time contracting models assume risk neutrality or constant absolute risk aversion for tractability. The constant relative risk aversion preference allows us to quantify the tradeoff between incentive provision and risk sharing. The optimal contract is fully determined up to an ordinary differential equation and can be efficiently solved for the purpose of estimation. Third, the dynamic general equilibrium setup allows us to exploit the implication of the optimal dynamic contract for the cross-sectional distribution of firm characteristics to identify the structural model parameters. In addition, general equilibrium is essential for understanding the welfare implications of moral hazard. A reduction in moral hazard is associated with higher levels of investment and capital accumulation. In a partial equilibrium, taking prices as given, firm profit is linear in capital (Hayashi (1982)). However, total output is decreasing return to scale with respect to capital at the aggregate level. Thus, in order to evaluate the impact of moral hazard across all firms, it is important to account for the decreasing return to scale in the aggregate production function, which requires a general equilibrium setup.

Related literature Our theoretical framework builds on the literature on optimal dynamic contracting, especially continuous-time models. The continuous-time methodology allows semi-closed form solutions and makes it possible for us to estimate the model. The optimal contracting problem in our set-up is related to Sannikov (2008), DeMarzo and Sannikov (2006), DeMarzo, Fishman, He, and Wang (2012), and Zhu (2013). To study the quantitative implications of moral hazard, different from the above literature, we consider a general equilibrium setup with neoclassical production technologies and risk-averse preferences.

Within the continuous-time contracting literature, our paper is mostly related to models that link moral hazard to pay-to-performance sensitivity, for example, He (2009), Hoffmann and Pfeil (2010), Edmans, Gabaix, Sadzik, and Sannikov (2012), Li (2017), and Di Tella and Sannikov (2016), Hackbarth, Rivera, and Wong (2018), and papers that study the impact of moral hazard on firm dynamics, for example, Hartman-Glaser, Lustig, and Xiaolan (2019) and Chi and Jin (2017).[1]

Several recent papers study the quantitative impact of dynamic agency using structural estimation or calibration. Based on an estimated firm dynamics model with frictions, Nikolov and Schmid (2016) study how information friction influences firms' capital structure and investment policy. Nikolov and Whited (2014) focuses on the relationship between agency conflicts and cash accumulation. Nikolov, Schmid, and Steri (Forthcoming) examine the determinants of corporate liquidity management. Xiaolan (2014) studies the quantitative

[1]Recent papers that study moral hazard in continuous-time setup include Hartman-Glaser, Mayer, and Milbradt (2018), Hartman-Glaser, Piskorski, and Tchistyi (2012), Feng and Westerfield (2018), Feng (2018), Leung (2017), Williams (2011), Piskorski and Westerfield (2016), Biais, Mariotti, Plantin, and Rochet (2007), Gryglewicz, Mayer, and Morellec (2018).

impact of limited commitment on firm-worker risk sharing. Sun and Xiaolan (2019) structurally estimate a dynamic agency model of firm financing. Different from their papers, our paper focuses on the identification of the relative magnitude of observable and unobservable shocks and the estimation of the quantitative impact of moral hazard on aggregate output.

Our paper is also related to the broader literature on structural estimation in corporate finance. For example, Taylor (2010, 2013) focuses on estimating learning models of CEO pay and CEO turnover. Li, Whited, and Wu (2016) estimate a model with limited commitment to quantify the importance of collateral versus taxes in firms' capital structure decisions. Li and Whited (2016) estimate an adverse selection model to study the pattern of capital reallocation over business cycles.

The rest of the paper is organized as follows. We describe the setup of our model in Section 2. We provide the solution to the optimal contracting problem and lay out our identification strategy in Section 3. Section 4 describes our structural estimation and presents quantitative results. Section 5 concludes.

2 Setup of the Model

In this section, we introduce our equilibrium model of investment and managerial compensation with moral hazard.

2.1 Preferences and Technology

Time is infinite and continuous. There is a continuum of firms indexed by j. The output of firm j, denoted y_j, is produced from capital (K_j) and labor (N_j) using a standard Cobb-Douglas production technology: $y_j = zK_j^{\alpha}N_j^{1-\alpha}$, where α is the capital share. The operating profit of firm j is defined as

$$\pi\left(K_j\right) = \max\left\{zK_j^{\alpha}N_j^{1-\alpha} - WN_j\right\}, \tag{1}$$

where W is the equilibrium wage. We assume that the market for unskilled labor, N, is perfectly competitive, which implies that the profit function is linear in capital, i.e., $\pi\left(K_j\right) = AK_j$, where A is the equilibrium marginal product of capital.

Capital accumulation requires special skills and can only be done by a manager. The law of motion of capital is given by:

$$dK_{j,t} = K_{j,t} \left[\left(i_{j,t} - \delta \right) dt + \sigma^T dB_{j,t} \right], \tag{2}$$

where $i_{j,t} = \frac{I_{j,t}}{K_{j,t}}$ is the investment-to-capital ratio, δ is the depreciation rate, $dB_{j,t}$ is a vector of firm-specific Brownian motion shocks and σ is a vector of the corresponding volatilities.

In our setup, firm capital, $K_{j,t}$, is publicly observable but investment decisions, $I_{j,t}$, are known only to the manager. Firm owners cannot infer the actual amount of investment from the observable capital stock because a part of the Brownian motion shock is assumed to be unobservable. In particular, we assume that (suppressing the time subscript):

$$\sigma^T dB_j = \sigma_U dB_{U,j} + \sigma_O dB_{O,j}, \tag{3}$$

where the Brownian motion $B_{U,j}$, is unobservable to all but the manager who operates the firm, and the Brownian motion $B_{O,j}$ is public information.

At any time t, firm owners face the following budget constraint:

$$D_{j,t} + C_{j,t} + h\left(\frac{I_{j,t}}{K_{j,t}} \right) K_{j,t} = AK_{j,t}, \tag{4}$$

where $D_{j,t}$ is the amount of dividends, $C_{j,t}$ is managerial consumption, and $h\left(\frac{I_{j,t}}{K_{j,t}}\right) K_{j,t}$ is the total cost of investment. We use a standard quadratic adjustment cost function, $h(i) = i + \phi (i - i^*)^2$. We assume that consumption and investment decisions are privately observable by managers. Moral hazard arises because managers can always substitute investment for their own private consumption. The degree of moral hazard is determined by the relative magnitude of unobservable versus observable shocks, $\frac{\sigma_U^2}{\sigma_O^2}$.

We assume that managers are risk-averse and that they are subject to health shocks at Poisson rate κ. Once hit by a health shock, the manager exits the economy and the capital of the firm that he runs evaporates. The time-t continuation utility of the manager is given by:

$$U_t = \left\{ E_t \left[\int_0^\infty e^{-(\beta+\kappa)s} \left(\beta + \kappa \right) C_{t+s}^{1-\gamma} ds \right] \right\}^{\frac{1}{1-\gamma}}, \tag{5}$$

where β is the discount rate of the manager, and γ is the coefficient of relative risk aversion. Firm owners are assumed to be well diversified and maximize the present value of dividends,

$$E_0 \left[\int_0^\infty e^{-(r+\kappa)t} D_t dt \right],$$

where r is the equilibrium interest rate.

2.2 Profit Maximization

In our setup, high investment accelerates capital accumulation and increases output. However, because investment is not observable and managers have incentives to substitute investment for consumption, shareholders' investment plan can be implemented only if managers find it optimal to follow the plan. To induce investment and ensure that shareholders' and managers' interests are aligned, firm owners reward high output and punish low output. In a dynamic setting, these incentives are provided by conditioning future managerial compensation on past performance. Below, we formally describe the optimal contracting problem in our model.

A contract is a sequence of dividends, managerial compensation, and investment policies, $\left(\left\{D_{j,t}\left(K_j^t, B_{O,j}^t\right)\right\}, \left\{C_{j,t}\left(K_j^t, B_{O,j}^t\right)\right\}, \left\{i_{j,t}\left(K_j^t, B_{O,j}^t\right)\right\}\right)_{t=0}^{\infty}$ that depends on the history of the realization of observables, which we denote by $K_j^t = \{K_{j,t}\}_{s=0}^{t}$, $B_{O,j}^t = \{B_{O,j,s}\}_{s=0}^{t}$. To save notations, we write a contract as $\{D_t, C_t, i_t\}_{t=0}^{\infty}$. Given a contract $\{D_t, C_t, i_t\}_{t=0}^{\infty}$, if the manager follows the dividend payout policy, $\{D_t\}_{t=0}^{\infty}$, but chooses an alternative investment policy, $\{\tilde{\imath}_t\}_{t=0}^{\infty}$, his continuation utility at time t can be written as:

$$U_t\left(\{\tilde{\imath}_s\}_{s=0}^{\infty}\right) = \left\{E_t\left[\int_0^{\infty} e^{-(\beta+\kappa)s}\left(\beta+\kappa\right)\left(C_{t+s} + h\left(i_{t+s}\right)K_{t+s} - h\left(\tilde{\imath}_{t+s}\right)K_{t+s}\right)^{1-\gamma} ds\right]\right\}^{\frac{1}{1-\gamma}}. \tag{6}$$

That is, at time t, if the manager chooses a lower investment rate $(\tilde{\imath}_t)$ than what is specified under the contract (i_t), he can privately save $h\left(i_{t+s}\right)K_{t+s} - h\left(\tilde{\imath}_{t+s}\right)K_{t+s}$ units of capital and use it for consumption without being detected by the shareholder. Therefore, a contract $\{D_t, C_t, i_t\}_{t=0}^{\infty}$ is incentive compatible if the investment policy specified by the contract is optimal from the manager's perspective, that is, if:

$$U_t\left(\{i_s\}_{s=0}^{\infty}\right) \geq U_t\left(\{\tilde{\imath}_s\}_{s=0}^{\infty}\right), \quad \text{for all } t, \tag{7}$$

for all investment policies $\{\tilde{\imath}_s\}_{s=0}^{\infty}$.

We assume double-sided limited commitment of financial contracts, as in Ai, Kiku, and Li (2015).[2] We assume that upon default, managers can take away a fraction of the firm's assets but are forever excluded from the credit market.[3] That is, managers are not allowed

[2]Some type of limited commitment is required to make the moral hazard problem non-trivial. In the absence of frictions, the principal can typically implement the efficient allocation arbitrarily closely if she is allowed to apply extremely severe punishment on the agent (see Mirrlees (1974)). In reality, managers typically have a variety of outside options and can always choose to leave a firm. We formally model outside options by limited commitment.

[3]Similar specifications are used in Albuquerque and Hopenhayn (2004), Kiyotaki and Moore (1997), and Kehoe and Levine (1993).

to enter into any intertemporal risk sharing contracts after default. As we show in Appendix B, the utility of the manager upon default is a linear function of capital: $u_{MIN}K_t$. Hence, limited commitment on the manager side requires

$$U_t\left(\{i_s\}_{s=0}^{\infty}\right) \geq u_{MIN}K_t, \quad \text{for all } t \geq 0. \tag{8}$$

The expression for u_{MIN} is provided in the appendix. Any compensation plan that violates condition (8) may lead to the manager defaulting on the contract.

We also assume that shareholders cannot commit to negative net present value (NPV) projects. This constraint requires that the net present value of firms' cash flows stays positive at all times:

$$E_t\left[\int_t^{\infty} e^{-(\mathbf{r}+\kappa)(s-t)} D_s ds\right] \geq 0, \quad \text{for all } t \geq 0. \tag{9}$$

Shareholders choose a contract, $\{D_t, C_t, i_t\}_{t=0}^{\infty}$ that maximizes the present value of firms' cash flows subject to the budget constraint in Equation (4), the incentive compatibility constraint, (7), and the limited commitment constraints in Equations (8) and (9).

2.3 Recursive Formulation

Following the standard approach in the dynamic contracting literature, we construct the solution to the optimal contracting problem recursively by using promised utility as a state variable. In our case, policy functions depend on two state variables (K, U), where K is the size of the firm and U is the continuation utility promised to the manager. We can think of the state variables, (K, U), as a summary of the firm's type. As in Atkeson and Lucas (1992), we construct the equilibrium allocation recursively. First, for firms of each type (K, U), we specify the flow rate of dividend payout, managerial compensation and investment-to-capital ratio using the policy functions $D(K, U)$, $C(K, U)$, $i(K, U)$. Next, we specify the law of motion of the state variables using:

$$\frac{dK}{K} = \left[i(K, U) - \delta\right] dt + \sigma_O dB_O + \sigma_U dB_U, \tag{10}$$

and

$$\begin{aligned} \frac{dU}{U} &= \left[-\frac{\beta+\kappa}{1-\gamma}\left(\left(\frac{C(K,U)}{U}\right)^{1-\gamma} - 1\right) + \frac{1}{2}\gamma\left(g_O^2\sigma_O^2 + g_U^2\sigma_U^2\right)\right] dt \\ &\quad + g_O\sigma_O dB_O + g_U\sigma_U dB_U. \end{aligned} \tag{11}$$

Equation (11) follows the formulation in Sannikov (2008) except that we use a stochastic differential utility representation of the preference so that utility is measured in consumption units (see Equation (6)). Here, g_O is the elasticity of continuation utility with respect to observable shocks, and g_U is the elasticity of U with respect to unobservable shocks. Intuitively, the policy functions g_U and g_O describe the rules of assigning continuation utilities based on the realizations of the Brownian shocks. At time t, for a given level of promised utility U_t, the principal allocates the manager's continuation utility over time and states by choosing an instantaneous consumption flow, $C(K_t, U_t)$, an elasticity of continuation utility with respect to unobservable shocks, $g_{U,t}$, and an elasticity with respect to observable shocks, $g_{O,t}$.

Because the production technology is constant return to scale, and utility functions are homogeneous, the optimal contracting problem is homogeneous in the state variable K. As a result, the value function, denoted by $V(K, U)$, satisfies

$$V(K, U) = v\left(\frac{U}{K}\right) K\,, \tag{12}$$

for some function v. We define $u = \frac{U}{K}$ as the normalized continuation utility and use homogeneity to write normalized consumption, investment, and dividend as functions of u:

$$c(u) = \frac{C(K,U)}{K};\ i(u) = \frac{I(K,U)}{K};\ d(u) = \frac{D(K,U)}{K}. \tag{13}$$

As shown in the appendix, the elasticities g_O and g_U are also functions of u.

Note that because K is firm size and U is the value of the manager's future compensation package, we can intuitively interpret u as the manager's share in the firm. In what follows, we use the terminology of the normalized utility and the manager's share in the firm interchangeably.

2.4 Entry, Exit, and Aggregation

A unit measure of managers arrives in the economy per unit of time with an outside option that provides them with a life-time utility U_0. Upon entry, a manager enters into a contract with shareholders and starts operating a firm of the initial size of K_0, which we normalize to one. Profit maximization implies that the initial normalized utility of the manager is $u_0 = \frac{U_0}{K_0}$. New managers arrive continuously and new firms are created upon their arrivals and, at the same time, existing firms continuously exit the economy. We focus on the stationary equilibrium where entry equals exit, and the total measure of firms in the economy is constant.

3 Optimal Contracting

3.1 Characterization of the Optimal Contract

As we show in Lemma 1 in Appendix B, an investment policy is incentive compatible if and only if the normalized policy function g_U satisfies

$$g_U(u) = (\beta + \kappa) c(u)^{-\gamma} u^{\gamma - 1} \cdot h'(i(u)) \tag{14}$$

for all u. The above condition is intuitive. The term $(\beta + \kappa) c(u)^{-\gamma} u^{\gamma-1}$ is the marginal utility of the manager, and $h'(i(u))$ is the marginal cost of investment measured in units of consumption goods. Thus, the right-hand side of Equation (14) is the marginal cost of investment measured in manager's utility units. Because shareholders do not observe managers' consumption and investment decisions, they assign continuation utilities according to the realized K. Therefore, from the manager's perspective, $g_U(u)$ is the increase in continuation utility for an additional unit of capital. Incentive compatibility requires that the investment policy specified by the contract is optimal from the manager's perspective. In the context of our model, it requires the marginal benefit of an additional unit of investment, $g_U(u)$, be equal to the marginal cost of investment for the manager, $(\beta + \kappa) c(u)^{-\gamma} u^{\gamma-1} \cdot h'(i(u))$.

Condition (14) reduces the requirement of incentive compatibility to restrictions on the policy functions for consumption and investment, and the sensitivity of the continuation utility to unobservable shocks. This allows us to characterize the value function as the solution to a HJB equation, which we describe in the following proposition.

Proposition 1. *The normalized value function, $v(u)$, satisfies the following HJB differential equation*

$$0 = \max_{\substack{c,i,g_O,\\ g_U = (\beta+\gamma)c^{-\gamma}u^{\gamma-1}h'(i)}} \left\{ \begin{array}{c} A - c - h(i) + v(u)(i - r - \kappa - \delta) \\ +uv'(u)\left[\frac{\beta+\kappa}{1-\gamma}\left(1 - \left(\frac{c}{u}\right)^{1-\gamma}\right) - (i-\delta) + \frac{1}{2}\gamma\left(g_U^2\sigma_U^2 + g_O^2\sigma_O^2\right)\right] \\ +\frac{1}{2}u^2 v''(u)\left[(g_U - 1)^2\sigma_U^2 + (g_O - 1)^2\sigma_O^2\right] \end{array} \right\} \tag{15}$$

on the domain $[u_{MIN}, u_{MAX}]$, with the following boundary conditions:

$$\lim_{u \to u_{MIN}} v''(u) = \lim_{u \to u_{MAX}} v''(u) = \infty, \text{ and } v(u_{MAX}) = 0.$$

Proof. See Appendix B.2. □

In Figure 1, we plot the normalized value function, $v(u)$, constructed using the estimates of the model parameters that we present in Section 4 below. Under the optimal contract, the normalized continuation utility of the agent, $u = \frac{U}{K}$, stays in the bounded interval, $[u_{MIN}, u_{MAX}]$. The limited commitment constraint in Equation (8) requires $u_t \geq u_{MIN}$ because any feasible contract must provide the manager a continuation utility at least as high as his outside option, $u_{MIN}K$. As u increases, the value function, $v(u)$, monotonically declines because a higher fraction of future cash flows is promised to the manager. Limited commitment on the shareholder side that requires the NPV of the firm to be non-negative at all times imposes an upper bound on u: u_{MAX} such that $v(u_{MAX}) = 0$ and $u_t \leq u_{MAX}$ for all t. As Figure 1 shows, the value function is concave on its domain.

The key tradeoff in moral-hazard models between risk sharing and incentive provision depends on the relative importance of unobservable versus observable shocks, $\frac{\sigma_U^2}{\sigma_O^2}$. Below, we show how the dynamics of continuation utility and firms' investment policies depend on $\frac{\sigma_U^2}{\sigma_O^2}$. These theoretical results allow us to identify the relative importance of observable and unobservable shocks by exploiting the cross-sectional distribution of CEO compensation and firm growth that we carry out in Section 4.

3.2 Dynamics of Continuation Utility

To understand the dynamics of continuation utility under the optimal contract, consider the elasticity of promised utility U with respect to observable and unobservable shocks, g_O and g_U, respectively. Figure 2 plots g_O (solid line) and g_U (dashed line) as functions of the key state variable, the normalized utility u. Note that the elasticity with respect to observable shocks is close to zero in most of its domain and converges to one at the boundaries. Due to optimal risk sharing, managers' continuation utility responds less to observable shocks than firms' output does, hence, $g_O(u) \leq 1$ for all u. As the normalized utility approaches its left boundary: $u \to u_{MIN}$, managers' continuation utility approaches its outside option: $U \to u_{MIN}K$. Because the elasticity of managers' outside option with respect to shocks in capital is one, the limited commitment constraint on the manager side, $U \geq u_{MIN}K$, requires that $g_O(u) = 1$ as $u \to u_{MIN}$. Similarly, $g_O(u)$ approaches one as the limited commitment constraint on the shareholder side starts to bind, because this constraint is equivalent to $U_t \leq u_{MAX}K_t$ and the elasticity of $u_{MAX}K_t$ with respect to shocks is one.

As Figure 2 further shows, the sensitivity of continuation utility with respect to unobservable shocks, g_U (dashed line), is positive and is substantially larger than g_O. The fact that $g_U > 0$ is the requirement of incentive provision. The optimal contract rewards high output and punishes low output. Because high output is more likely under high investment, this condition is necessary to deter managers from "stealing" firms' cash flows. Similar to observable shocks, $g_U(u)$ must converge to one at the boundaries where limited commitment constraints bind.

Two important observations follow. First, $g_O(u) < 1$ in the interior of its domain and g_U is much larger than g_O (in fact, $g_U(u) \geq 1$ in most of its domain). Note that under perfect risk sharing, the elasticity of continuation utility with respect to shocks is zero. In case of observable shocks, limited commitment is the only friction that prevents perfect risk sharing. Because the elasticity of outside options with respect to shocks is one, risk sharing requires that $g_O(u) < 1$ unless the constraints are binding. Further, $g_U(u)$ is significantly larger than $g_O(u)$ and is typically larger than one because incentive provision makes the continuation utility highly sensitive to innovations in output.

Second, $g_U(u)$ is a decreasing function over most of its domain. Recall that the normalized utility, u, proxies for the manager's share in the firm. When u is low, the manager is promised only a small fraction of the firm's output and, therefore, has a strong incentive to steal. To prevent stealing, the optimal contract is designed to provide a sufficiently high reward for good performance and, therefore, features high sensitivity to output shocks. As the manager's share in the firm increases, he owns a larger fraction of the firm's cash flows, which makes the incentive problem less severe and continuation utility less sensitive. The declining pattern in $g_U(u)$ illustrated in Figure 2 suggests that pay-performance sensitivity is decreasing in the normalized utility.

3.3 Investment Policy

As shown in Figure 3, our model implies that firm investment policy, $i(u_t) = \frac{I_t}{K_t}$, is an increasing function of the manager's normalized utility. Limited commitment and moral hazard both contribute to the robust positive relationship between the investment rate and the manager's share. When u approaches u_{MAX}, the limited commitment constraint on the shareholder side is likely to bind. A binding limited commitment constraint is associated with poor risk sharing. To grow out of the constraint and improve risk sharing, firms optimally increase their investments as u increases. Further, as the normalized utility rises, more of the firm's cash flows is promised to the manager. Hence, managers' incentives become more aligned with shareholders' interests, which motivates them to invest more. In contrast, when the manager's share in the firm declines, it is harder to incentivize the manager to invest. As a result, the cost of incentive provision increases and the rate of investment declines. In fact, for u close to its lower boundary, investment becomes negative.

It is well known that empirically, small firms invest at a higher rate and grow faster than large firms. In the next section, we show how the empirical relationship between investment and size, and the dependence of $i(u)$ on u implied by the model can be used to identify the relative magnitude of observable and unobservable shocks.

3.4 Implications for Identification

As discussed above, under the optimal contract, investment and growth rates increase with the normalized utility u, while pay-performance sensitivity decreases with u. These implications could potentially help identify the structural model parameters. However, the continuation utility is not directly observable. In this section, we show that we can identify the relative amount of private information and estimate the model using the readily available data on firm size by exploiting the relationship between firm size and continuation utility, which is endogenously determined by the optimal contract. In particular, we show that moral hazard determines the equilibrium correlation between firm size and the unobservable continuation utility. If the magnitude of unobservable shocks is relatively small, firm size and continuation utility are negatively correlated, whereas large values of $\frac{\sigma_U^2}{\sigma_O^2}$ imply a zero or even positive correlation between firm size and u. Because our model imposes monotone relationships between investment rate and u, and PPS and u, we can exploit the joint empirical distribution of growth rates, PPS and firm size to identify the model parameters.

Using Equations (10) and (11), we can write the law of motion of u as

$$\frac{du}{u} = \mu(u)dt + [g_O(u) - 1]\,\sigma_O dB_O + [g_U(u) - 1]\,\sigma_U dB_U, \tag{16}$$

where the function $\mu(u)$ is given in Equation (21) in the appendix. Note that $g_O - 1$ is the elasticity of u with respect to observable shocks, and $g_U - 1$ is the elasticity of u with respect unobservable shocks. As shown in Section 3.2, $g_O \leq 1$, g_U is significantly higher than g_O, and typically $g_U(u) \geq 1$. Consider first the case in which most of the shocks are observable, i.e., $\frac{\sigma_U^2}{\sigma_O^2}$ is close to 0. In this scenario, the relationship between firm size and the normalized continuation utility is negative because it is mostly driven by observable shocks and $g_O \leq 1$. Intuitively, while a positive shock increases both firm size K and continuation utility U, risk sharing requires U to increase at a lower rate than K. Hence, the manager's share u falls, and K and u are negatively correlated.

If the contribution of unobservable shocks increases, that is, if $\frac{\sigma_U^2}{\sigma_O^2}$ becomes large, the negative correlation between firm size and manager's share weakens because $g_U > g_O$. In fact, because $g_U \geq 1$ in most of its domain, the correlation between K and u might be even positive. As moral hazard becomes severe, the cost of incentive provision increases; therefore, as firms grow and become larger, they have to provide a higher fraction of their value to managers. We illustrate the relationship between firm size and the normalized utility in Figure 4. We consider several cases for the break-down between observable and unobservable shocks: $\frac{\sigma_U^2}{\sigma_O^2} = 0$ (dotted line), $\frac{\sigma_U^2}{\sigma_O^2} = 0.1$ (dashed line), $\frac{\sigma_U^2}{\sigma_O^2} = 1$ (dash-dotted line), and $\frac{\sigma_U^2}{\sigma_O^2} = 10$ (solid line). As the figure shows, when most shocks are observable, u is monotonically decreasing in firm size. As the relative magnitude of unobservable shocks increases, the negative relationship between u and K becomes considerably weaker and eventually changes its sign.

Figure 5 shows how the equilibrium relationship between firm size and the normalized utility translates into the relationship between size and investment. For each of the four cases, we plot the investment rate $i(u)$ as a function of firm size. For low levels of $\frac{\sigma_U^2}{\sigma_O^2}$ (0 and 0.1), our model features a strong inverse relationship between investment rate and firm size, which is due to a strong negative correlation between the normalized utility and size. As the magnitude of unobservable shocks increases, the negative relationship between investment and size disappears and ultimately reverses to positive.

Figure 6 shows the cross-sectional distribution of the average elasticity of continuation utility to productivity shocks. We compute average elasticity, ξ, as a weighted average of elasticities with respect to observable and unobservable shocks, $\xi = \sqrt{\frac{\sigma_U^2}{\sigma^2} g_U^2 + \frac{\sigma_O^2}{\sigma^2} g_O^2}$, where $\sigma^2 = \sigma_U^2 + \sigma_O^2$ is the total variance. Although the average elasticity of continuation utility with respect to shocks, ξ, is not observable, it determines pay-performance sensitivity (i.e., the elasticity of CEO pay to firm performance), which we can measure using the available data.[4] Figure 6 illustrates two important implications that help identify the relative magnitude of observable and unobservable shocks. First, as the amount of unobservable shocks increases, the overall elasticity of managerial pay to firm performance rises due to the higher cost of incentive provision. Second, without moral hazard, that is, when $\frac{\sigma_U^2}{\sigma_O^2} = 0$, ξ is a U-shaped function of size. A relatively modest amount of unobservable shocks (eg., $\frac{\sigma_U^2}{\sigma_O^2} = 0.1$) implies an increasing (almost monotone) relationship between ξ and firm size. As the contribution of unobservable shocks gets larger, the relationship between the elasticity of managerial pay and firm size virtually disappears. To summarize, Figure 6 shows that both the overall level and the cross-sectional variation of PPS provide important information about the relative magnitude of observable versus unobservable shocks.

Building on these theoretical insights, in the next section, we estimate the model by exploiting moment conditions of the joint distribution of firm size, investment and managerial

[4]The mapping between the elasticity of utility with respect to shocks, ξ, and the elasticity of managerial compensation with respect to firm performance (that is, the log-log based measure of PPS) is monotone, but nonlinear. For example, for $\frac{\sigma_U^2}{\sigma_O^2} = 0$, the log-log PPS is zero in the interior of (u_{MIN}, u_{MAX}) due to risk sharing. However, $\xi > 0$ because continuation utility accounts for a possibility of a binding constraint in the future, which is associated with a positive response of managerial pay with respect to shocks. Estimating the magnitude of $\frac{\sigma_U^2}{\sigma_O^2}$ from the level of PPS is thus a quantitative issue, which we address formally in the next section.

compensation.

4 Quantitative Evidence

4.1 Data

We use the standard panel of data that consist of US non-financial firms and come from the Center for Research in Securities Prices (CRSP) and Compustat. For each firm in our sample, we collect its size, investment, and managerial compensation. We measure firm size by the gross value of property, plant and equipment, and firm investment by capital expenditure. Executive compensation comprises salary, bonuses, the value of restricted stock and options granted, and long-term incentive payouts. All nominal quantities are converted to real using the consumer price index provided by the Bureau of Labor Statistics. The data are sampled on the annual frequency and cover the period from 1993 till 2013.

4.2 Structural Estimation

We estimate the model parameters using the simulated method of moments (McFadden (1989), and Pakes and Pollard (1989)). Our primary focus is on volatility parameters that govern the magnitude of observable and unobservable shocks, σ_O and σ_U, respectively. Following the discussion in Section 3, our identification strategy is to exploit the dynamics of firm growth and executive pay-performance sensitivity. Because firms' growth and CEO compensation are determined jointly by moral hazard and other model parameters, we also estimate the productivity parameter (A), the capital depreciation rate (δ), the adjustment cost parameters (ϕ and i^*), and parameters that determine the outside option of managers and the initial normalized utility (u_{MIN} and u_0, respectively).[5] Thus, we estimate a subset of structural parameters that together with volatility parameters have a first-order effect on the joint distribution of firm growth and managerial compensation. The remaining model parameters are calibrated consistent with the standards of the macroeconomics literature to match a different set of moments that we discuss below. Table 1 presents the list of the estimated and calibrated model parameters.

Let $\Theta = \{\sigma_O, \sigma_U, A, \delta, \phi, i^*, \bar{u}_{MIN}, \bar{u}_0\}$ denote the vector of parameters to estimate, and let $\mathcal{M}_D$ and $\mathcal{M}_M(\Theta)$ denote the vectors of data-based and model-implied moments,

[5] It is more convenient to estimate the equilibrium marginal product of capital directly. We can always use the equilibrium relationship, $A = z\mathbf{K}^{\alpha-1}$ to back out the primitive productivity parameter, z (see Appendix A).

respectively.[6] We estimate the model parameters by minimizing the following objective function:

$$\widehat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \big[\mathcal{M}_D - \mathcal{M}_M(\Theta)\big]' W \big[\mathcal{M}_D - \mathcal{M}_M(\Theta)\big], \tag{17}$$

where W is the weighting matrix. The model-based moments are computed via simulations. Specifically, we draw a panel of shocks, and for a given parameter configuration we solve the model numerically, discretize it and simulate a cross-section of firms. The simulated panel consists of 20,000 firms per year and the length of time series is set at 150 years. We discard the first 100 years of data and use the remaining 50 years of simulated data to calculate the vector of moments $\mathcal{M}_M(\Theta)$. Because our panel is fairly large, the simulated moments represent the population moments sufficiently well. We confirm that increasing either the length of the simulated sample or the size of the cross-section has virtually no effect on the model-implied moments and the parameter estimates. We estimate the model parameters using the optimal weight matrix. Hence, the estimation is carried out in two stages: in the first stage, we obtain the initial estimates by weighting each moment condition by the inverse of the variance of the sampling distribution of the corresponding statistic, and in the second stage we use the inverse of the variance-covariance matrix of the moment conditions evaluated at the first-stage estimates. We use the Newey and West (1987) estimator of the spectral density matrix at frequency zero with a truncation lag of two.

Guided by the model's implications discussed in Section 3, we select moments that are informative about the degree of moral hazard and help identify the structural parameters that we seek to estimate. The first set of moments characterizes aggregate and the cross-sectional dynamics of firms' growth. It consists of the average aggregate growth rate, the time-series average of the cross-sectional standard deviation of firms' growth rates, the time-series mean of median Tobin's Q, and average growth and investment rates of size-sorted portfolios. The second set of moments comprises CEO pay-performance sensitivity of firms with different size characteristics. The last moment is the power law in firm size. To calculate moments of the joint distribution of firm size, growth and managerial compensation, we construct five size-sorted portfolios using breakpoints that are equally-spaced in log size. Portfolios are re-balanced at the annual frequency, and for consistency, in the data and in the model, firm size is measured by capital stock.[7] The full list of moments that we exploit in estimation is presented in Table 2. In all, we use 15 moments to estimate eight model parameters.

[6] For computational convenience, in estimation we use the following parameterization: $\bar{u}_{MIN} \equiv u_{MIN}/u_{MAX}$, and $\bar{u}_0 \equiv u_0/u_{MAX}$.

[7] Because in estimation we exploit the cross-sectional differences in the dynamics of CEO compensation, in constructing portfolios, we only use firms with available executive compensation data. Also, to ensure that the variance-covariance matrix of the moment conditions is well behaved, in estimation, we exploit growth and investment rate moments for only three out of five size-sorted portfolios; yet we present the model implications for the entire cross section.

To keep the estimation tractable and to achieve identification, we fix several model parameters that either do not play a critical role in determining the cross-sectional distribution of firms' characteristics or cannot be identified separately. The values of the calibrated parameters are reported in Panel B of Table 1. Following Kydland and Prescott (1982), King and Rebelo (1999) and Rouwenhorst (1995), we set the degree of risk aversion at 2, the time-discount rate (hence, the annual interest rate) at 0.04 per year, and the capital share to 0.33. We calibrate the death rate to be 0.05 to match the average firms' exit rate in the data.

4.3 Parameter Estimates and Implications

Panel A of Table 1 presents the SMM estimates of the model parameters. First, notice that the set of moment conditions that we exploit in estimation allows us to identify the structural parameters sufficiently well; with only one exception, all the estimates have relatively small standard errors. Second, our estimates reveal a significant difference in the magnitude of observable and unobservable shocks. The estimates of volatility of observable and unobservable shocks are 0.28 (SE=0.014) and 0.09 (SE=0.014), respectively. That is, observable shocks account for about 90% of the overall variation in productivity while the unobservable shocks contribute a much modest 10%. The difference in volatilities is strongly statistically significant with a robust t-statistic of 7.6.

To evaluate the fit of the model, in Table 2 we report sample moments alongside moments implied by the model estimates. For completeness, in addition to the moments exploited in estimation we report four additional moments that characterize growth and investment rates of firms in the second and fourth size-sorted quintiles (those are marked with asterisks). In the last column, we present t-statistics for the difference between the data and the model-implied moments. Note first that our estimated model matches well the dynamics at the aggregate level. The model implies an average growth rate of firms' capital stock of about 4.7% and a median Tobin's Q of about 1.4. Both of them are statistically similar to the corresponding sample statistics. Below we focus on the cross-sectional moments that allow us to identify the relative magnitude of observable and unobservable shocks.

Investment, Growth Rates, and Firm Size As Table 2 shows, our model is able to account for the cross-sectional variation in firms' investment and growth observed in the data. As is well known, small firms, on average, invest more and grow at a much higher rate relative to large firms. Consistent with the data, our model generates a significant amount of dispersion in growth rates and investment rates across firms. In the model, the average investment rate declines monotonically from 19.1% for the bottom size-quintile to about 9.3%

for the top size-sorted quintile. Similarly, the average growth rates vary from about 12.4% for the small-size cohort to 3.8% for large firms. The cross-sectional standard deviation of growth rates in the model is about 26.7%, which is statistically similar to 23% in the data. Although the model does not fully account for the very large investment and growth rates of the smallest size-quintile observed in the data, the difference between the model and the data is not statistically significant.

Limited commitment allows our model to account for the robustly negative relationship between firm size and firm growth observed in the data. Equally important is the finding that the amount of unobservable shocks is modest compared to the size of observable shocks. As we explained in Section 3.4, as long as the amount of unobservable shocks is relatively small, risk sharing dominates incentive provision, which makes managerial compensation less sensitive to shocks than productivity. As a result, the manager's share in the firm, u, is negatively correlated with firm size. Because managers in small firms have a claim to a larger share of firm's cash flows, they have stronger incentives to invest than those in large firms. Hence, the negative relationship between size and u translates into a negative relationship between size and investment rate.

Pay-Performance Sensitivity The level and the cross-sectional variation of pay-performance sensitivity also play an important role in identifying the amount of private information. In the data and in the model, we measure PPS by regressing changes in log compensation on log returns controlling for firm fixed effects. As Table 2 shows, in the data, pay-performance sensitivity varies substantially with size — small firms feature relatively low PPS while large firms are characterized by relatively high size sensitivity of managerial compensation. The empirical estimate of PPS almost doubles from about 0.22 (SE=0.01) for the bottom quintile to 0.44 (SE=0.05) for the top size-sorted portfolio. The difference in PPS between the largest and smallest size cohorts of about 0.22 is statistically significant with a t-statistics of 2.24. Our model implies similar magnitudes of PPS and a similar monotonically increasing pattern in pay-performance sensitivity across size-sorted portfolios. The model-implied spread between the top and the bottom quintile is 0.22, which replicates the observed dispersion.

The ability of the model to account for both the level and the cross-sectional variation in PPS relies crucially on the presence yet a relatively small magnitude of unobservable shocks. At the point estimates, unobservable shocks account for about 10% of the total firm-level volatility. As explained in Section 3.4, such a relatively low magnitude of unobservable shocks implies a negative relationship between firm size and manager's normalized utility, u. Recall that the sensitivity of manager's utility to productivity shocks is decreasing in u. Taken together, the two implications lead to a positive relationship between pay-performance sensitivity and firm size and allow the model to simultaneously match the level and the cross-sectional pattern in PPS observed in the data.

As Table 2 shows, overall, the model accounts quite well for the joint distribution of firm size, growth and managerial compensation. None of 15 moment conditions exploited in estimation are statistically significant at the conventional five-percent level, and the model is not rejected by the over-identifying conditions. The p-value of the chi-square test of over-identifying restrictions is 0.41.

4.4 The Role of Moral Hazard

In order to better understand why the data implies a relatively small magnitude of unobservable shocks, we consider a restricted version of the model specification that assigns a larger role to moral hazard. In particular, we impose the constraint that the magnitudes of observable and unobservable shocks are equal, i.e., $\sigma_U^2 = \sigma_O^2 = 0.5\sigma^2$. Instead of evaluating the restriction directly (without re-estimating other model parameters), we give the constrained specification a fair chance to match the data and estimate the rest of its parameters by exploiting the same set of moment conditions. Table 3 presents the estimates of the restricted specification, and Table 4 reports its implications.

First notice that the constrained specification has significant difficulties in accounting for aggregate growth. As shown in Table 4, it implies a 3.1% aggregate growth, on average, which is much lower than in the data. But a more pronounced deterioration in the fit is in the cross-sectional dimension. Imposing the constraint significantly limits the model's ability to generate a sizable cross-sectional variation in growth rates and pay-performance sensitivity.

Our benchmark unrestricted model generates a spread of 8.9% in average growth rates of firms in the bottom and the top size quintiles, and a 9.8% dispersion in their investment rates. Under the constraint, the difference in average growth rates between the two cohorts of firms shrinks to 4.5%, and the spread in investments rates declines to 5.3%. These implications are quite intuitive. Keeping everything else constant, a larger magnitude of unobservable shocks and a higher degree of moral hazard reduce the overall investment and growth in the economy because a significant share of capital is allocated to incentive provision. Because small firms have limited resources, this happens to hurt small firms more relative to large firms. Therefore, investment and growth rates of small firms are substantially reduced and so is the variation in average growth rates across size-sorted portfolios. The constrained specification is able to generate only a 7.6% growth rate and a 15.3% investment rate of small firms, which are significantly lower than the corresponding statistics in the data. While other model parameters (such as productivity and the parameters of the adjustment cost function) try to adjust and compensate the negative impact of moral hazard on growth, their adjustment is bound by the discipline imposed by other moment conditions.

Further notice the impact of moral hazard on pay-performance sensitivity. Consistent with the data, in the unrestricted specification, PPS almost doubles from 0.20 for the small quintile to 0.42 for the large size quintile. Under the constraint, the model-implied dispersion shrinks significantly. Once again, deviations in PPS implied by the restricted model specification and the data are more pronounced in the left tail of the size distribution — the restricted specification significantly overstates size elasticity of CEO compensation for small firms. In particular, it implies a 0.28 PPS of the smallest portfolios, which is significantly larger the corresponding empirical estimate. Recall that in order to stay away from the shareholder-side limited commitment constraint and to improve risk sharing, small firm try to allocate as much resources as possible to invest and grow. Thus, in the absence of moral hazard, CEO compensation in small firms is sensitive to negative shocks but is inelastic to positive productivity shocks. Introducing moral hazard makes CEO compensation also respond to positive innovations and, therefore, magnifies pay-performance sensitivity of small firms. This is why the restricted specification fails to match a relatively low PPS of small firms observed in the data.

4.5 Counter-factual Exercises

In this section, we use our estimated model to quantify the impact of moral hazard on CEO compensation and aggregate output. We consider a counter-factual exercise of eliminating all moral hazard in the economy (for example, by implementing more transparent accounting rules and a more stringent legal system). To carry out our analysis, we keep all preference and technology parameters, including the total volatility of shocks, fixed at their estimated values but assume that all shocks are observable, that is, we set $\sigma_U = 0$. We compare the steady-state of the economy without moral hazard ($\sigma_U = 0$) with our benchmark economy with moral hazard.

Impact of moral hazard on CEO compensation Moral hazard is often considered a key determinant of the level and the dynamics of CEO compensation. Our counter-factual exercise allows us to quantify the total fraction of managerial compensation that is attributed to incentive pay. To emphasize the dependence of policy functions on parameters, we denote the CEO compensation policy by $C\left(K,U|\,\Theta\right)$ and the steady-state distribution of the state variables by $\Phi\left(K,U|\,\Theta\right)$, where Θ represents the vector of parameter values of the model. We define λ_{CEOPAY} as the faction of incentive pay in total CEO compensation. Formally, we calculate the total amount of CEO compensation in the economy without moral hazard as a fraction of the total CEO pay in the economy with moral hazard and compute λ_{CEOPAY} as:

$$1 - \lambda_{CEOPAY} = \frac{\int C\left(K, U \middle| \Theta_0\right) \Phi\left(dK, dU \middle| \Theta_0\right)}{\int C\left(K, U \middle| \hat{\Theta}\right) \Phi\left(dK, dU \middle| \hat{\Theta}\right)}, \tag{18}$$

where $\hat{\Theta} = \{\hat{\sigma}_O, \hat{\sigma}_U, \hat{A}, \hat{\delta}, \hat{\phi}, \hat{\imath}^*, \hat{\bar{u}}_{MIN}, \hat{\bar{u}}_0\}$ is the estimated parameter vector, and Θ_0 is obtained from $\hat{\Theta}$ by setting $\sigma_U = 0$ and keeping all other parameters, including the total volatility of shocks, $\sqrt{\hat{\sigma}_U^2 + \hat{\sigma}_O^2}$, the same.

Our estimates imply that $\lambda_{CEOPAY} = 52.4\%$. That is, moral hazard accounts for about half of the overall CEO compensation. In other words, eliminating all moral hazard allows firms to save about 52% of CEO compensation while keeping managers' utility unchanged. In our model, eliminating moral hazard makes managerial compensation contract more efficient for two reasons. First, in the presence of moral hazard, incentive provision requires CEO compensation to respond to unobservable idiosyncratic shocks. This arrangement reduces welfare because managers are risk averse. In our model with moral hazard, shocks to firm output induce variation in CEO compensation of about 20% per year, whereas perfect risk sharing implies that CEO pay is constant over time. Note that the fact that moral hazard distorts the allocation of consumption across states of the world and limits risk sharing is true in both static and dynamic models.

Second, unique to our dynamic model, moral hazard also distorts the intertemporal allocation of managerial compensation. Because managers and shareholders have the same discount rate, efficiency requires CEO compensation to be constant over time. However, consistent with the previous literature (eg., Sannikov (2008), DeMarzo and Sannikov (2006)), the presence of moral hazard typically implies a back-loaded CEO compensation package. Intuitively, delayed compensation allows shareholders to condition future compensation on realized output to provide proper incentives to invest. In our model with moral hazard, the average growth rate of CEO compensation relative to the aggregate economy is about 1.8% per year, whereas it is zero in the model without moral hazard.

To decompose the overall impact of moral hazard into its effect on risk sharing and its impact on the intertemporal allocation of consumption, let $\bar{C}(K, U)$ denote CEO compensation policy implied by the optimal contract in the moral hazard model that has no idiosyncratic risk (that is, by setting the diffusion coefficient of $C\left(K, U \middle| \hat{\Theta}\right)$ to zero but keeping the drift). Also, let $\bar{\Phi}(K, U)$ be the stationary distribution of CEO compensation under the policy $\bar{C}(K, U)$. Then, the total impact of moral hazard can be decomposed as follows:

$$[1-\lambda_{CEOPAY}] = \frac{\int \bar{C}\left(K,U\right)\bar{\Phi}\left(dK,dU\right)}{\int C\left(K,U\middle|\,\hat{\Theta}\right)\Phi\left(dK,dU\middle|\,\hat{\Theta}\right)} \times \frac{\int C\left(K,U\middle|\,\Theta_0\right)\Phi\left(dK,dU\middle|\,\Theta_0\right)}{\int \bar{C}\left(K,U\right)\bar{\Phi}\left(dK,dU\right)}. \quad (19)$$

The first term of the product on the right-hand side measures the efficiency loss due to limiting risk sharing, and the second component measures the efficiency loss due to distortions in the intertemporal allocation of CEO compensation. We find that $1-\frac{\int \bar{C}(K,U)\bar{\Phi}(dK,dU)}{\int C\left(K,U|\hat{\Theta}\right)\Phi\left(dK,dU|\hat{\Theta}\right)} = 39\%$ and $1-\frac{\int C(K,U|\Theta_0)\Phi(dK,dU|\Theta_0)}{\int \bar{C}(K,U)\bar{\Phi}(dK,dU)} = 22\%$. Thus, our estimates imply that 52.4% of CEO compensation is the result of incentive provision. About two thirds of the incentive pay is compensation for the additional risk managers have to bear and one third of it is compensation for distortions in the intertemporal allocation of their compensation packages. Our decomposition highlights the importance of a dynamic setting in estimating efficiency losses — static models would miss distortions in the intertemporal allocation, which we find to be a qualitatively signifying part of incentive compensation.

Impact of moral hazard on aggregate output Our general equilibrium framework also allows us to quantify the impact of moral hazard on aggregate output. We compute it as a percentage increase in output that can be achieved by eliminating all moral hazard in the economy:

$$\lambda_{OUTPUT} = \frac{\int zK\left(K,U\right)^{\alpha} N\left(K,U\right)^{1-\alpha} d\Phi\left(dK,dU\middle|\,\Theta_0\right)}{\int zK\left(K,U\right)^{\alpha} N\left(K,U\right)^{1-\alpha} \Phi\left(dK,dU\middle|\,\hat{\Theta}\right)} - 1.$$

Under the estimated parameter values of the model, we find that $\lambda_{OUTPUT} = 3.44\%$. That is, eliminating all moral hazard results in a permanent increase in aggregate output of the economy by about 3.4%.

Note the importance of general equilibrium for the welfare analysis. Taking as given the equilibrium level of marginal product of capital, A, a reduction in moral hazard improves the efficiency of production at a firm level. This could potentially lead to an infinitely large increase in firm value because revenue is linear in A. However, a lower degree of moral hazard implies that firms invest more and capital accumulates faster. Hence, the equilibrium level of marginal product of capital must fall as the economy accumulates more capital stock. The return to scale of the technology with respect to capital at the aggregate level is key determinant of the increase in output due to a reduction in moral hazard. Our general equilibrium model allows us to quantify the effect of moral hazard by taking into account the equilibrium relationship between the marginal product of capital and the total amount of capital in the economy.

5 Conclusion

We quantify the impact of moral hazard using a structural estimation of a dynamic general equilibrium model with agency frictions. The degree of moral hazard is defined by the relative magnitude of unobservable versus observable productivity shocks. We show that moral hazard has important implications for the cross-sectional relationships between firm size and investment, and firm size and pay-performance sensitivity. We exploit the predictions of our model and identify the amount of observable and unobservable shocks by exploiting moment conditions of the joint empirical distribution of firm size, growth and PPS. We find that the magnitude of unobservable shocks is relatively small and accounts for about 10% of the total variation. Our estimates imply that moral-hazard induced incentives are quantitatively significant and explain 52% of managerial compensation. Our welfare analysis suggests that eliminating moral hazard results in about 3.4% increase in aggregate output.

Appendix

A Details of General Equilibrium

Here we provide the details of aggregation and general equilibrium in our model.

A.1 The product market

The optimality condition for the choice of labor for (1) implies

$$(1-\alpha) z \left(\frac{K_j}{N_j}\right)^{\alpha} = W, \tag{20}$$

The above implies that capital-to-labor ratio is constant across all firms, which also equals the economy-wide capital-to-labor ratio. Because we normalize the total supply of labor to 1, $\frac{K_j}{N_j} = \mathbf{K}$, where $\mathbf{K}$ is the aggregate capital stock of the economy. Therefore, the above equation implies $W = (1-\alpha) z\mathbf{K}^{\alpha}$ is the equilibrium wage. We can therefore compute the profit function in (1) as

$$\pi(K_j) = z\mathbf{K}^{\alpha-1}K_j$$

In our notion, this means $A = z\mathbf{K}^{\alpha-1}$ is the equilibrium marginal product of capital.

A.2 Aggregation

The value and policy functions of the optimal contracting problem satisfy the homogeneity property (12) and (13). This implies that aggregate quantities can be computed by using the following "summary measure":

$$m(u) = \int K\Phi(u, K)\, dK,$$

where $\Phi(u, K)$ is the joint distribution of normalized utility and firm size. For example total managerial compensation can be calculated as

$$\int C(uK, K)\,\Phi(u, K)\, dK = \int Kc(u)\,\Phi(u, K)\, dK = \int c(u)\,\phi(u)\, du.$$

Also, because output at the firm level satisfies $y_j = zK_j^{\alpha} N_j^{1-\alpha} = \frac{1}{\alpha} AK_j$ by the result from the last section, total output of the economy can be calculated as

$$\int \frac{1}{\alpha} AK_j \Phi(u, K) \, dK = \frac{1}{\alpha} A \int \phi(u) \, du.$$

Notice that the law of motion of the normalized continuation utility satisfies

$$du = u\mu(u)dt + u \left[g_O(u) - 1 \right] \sigma_O dB_O + u \left[g_U(u) - 1 \right] \sigma_U dB_U$$

with

$$\mu(u) = \begin{bmatrix} -\frac{\beta+\kappa}{1-\gamma} \left(\left(\frac{c(u)}{u} \right) - 1 \right) + \frac{1}{2}\gamma \left(g_O^2(u)\sigma_O^2 + g_U^2(u)\sigma_U^2 \right) \\ + \left(g_O(u) - 1 \right) \sigma_O^2 + \left(g_U(u) - 1 \right) \sigma_U^2 \end{bmatrix}. \tag{21}$$

Therefore, numerically, we can compute the summary measure $m(u)$ by solving the following forward equation:

$$\begin{aligned} 0 \;=\; & \bar{m}(u) - \left(\kappa - (i(u) - \delta) \right) m(u) - \frac{d}{du} \left[m(u)\mu_u(u) + u \left(\left(g_O(u) - 1 \right) \sigma_O^2 + \left(g_U(u) - 1 \right) \sigma_U^2 \right) \right] \\ & + \frac{1}{2} \frac{d^2}{du^2} \left(m(u) u^2 \left(\left(g_O(u) - 1 \right)^2 \sigma_O^2 + \left(g_U(u) - 1 \right)^2 \sigma_U^2 \right) \right). \end{aligned}$$

Here $\bar{m}(u) = \int K \bar{\Phi}(u, K) \, dK$ with $\bar{\Phi}(u, K)$ being the entrance measure of new firms.

B Optimal Contracting

B.1 Incentive Compatibility

Lemma 1. [Incentive compatibility]*A contract constructed form the allocation rule, $C(Z,U)$, $I(Z,U)$, $D(Z,U)$, $N(Z,U)$, $g_U(Z,U)$, $g_O(Z,U)$ satisfies the obedience constraint* (7) *if and only if for all $t \in [0, \infty)$*

$$\frac{g_U(Z_t, U_t)}{U_t^{\gamma-1} Z_t} = (\beta + \kappa) \, C(Z_t, U_t)^{-\gamma} H_I \left(I(Z_t, U_t), Z_t \right) \tag{22}$$

or in normalized terms

$$g_U(u_t) = (\beta + \kappa) \, c_t^{-\gamma} u_t^{\gamma-1} h'(i_t). \tag{23}$$

Lemma 2. *We show that the obedience constraint* (7) *is satisfied if and only if for all*

$t \in [0, \infty)$

$$\left(C\left(Z_t, U_t\right), I\left(Z_t, U_t\right)\right) \in \arg \max_{\substack{C,I \ s.t \\ C+H(I,Z_t)=AZ_t-D(Z_t,U_t)}} \frac{\beta+\kappa}{1-\gamma} C^{1-\gamma} + \frac{1}{U_t^{\gamma-1} Z_t} g_U\left(Z_t, U_t\right) I \tag{24}$$

which, along with concavity of $H(I, Z)$, implies (22) *and* (23).

To simply notation, we focus on the case $\sigma_O = 0$, that is, all shocks are unobservable. We also omit the arguments Z and U in the policy functions in the statement of the lemma and define

$$W_t = \frac{1}{1-\gamma} U_t^{1-\gamma} = E_t\left[\int_t^\tau e^{-\beta(s-t)} \left(\beta+\kappa\right) C_s^{1-\gamma} ds\right], \tag{25}$$

which is the expected-utility representation of the agent's continuation utility. The Martingale representation theorem implies

$$dW_t = \left(\beta+\kappa\right)\left[W_t - \frac{C_t^{1-\gamma}}{1-\gamma}\right] dt + G_U\left(Z_t, U_t\right) \sigma_U dB_{U,t} \tag{26}$$

with

$$G_U\left(Z_t, U_t\right) = \frac{g_U\left(Z_t, U_t\right)}{U_t^{\gamma-1}} \tag{27}$$

and $g_U\left(Z_t, U_t\right)$ being the sensitivity term in Equation (11). Suppose that $\left\{\tilde{C}_t, \tilde{I}_t\right\}_{t=0}^\infty$ is an alternative consumption and investment plan other than $\left\{C_t, I_t\right\}_{t=0}^\infty$, such that

$$\tilde{C}_t + H\left(\tilde{I}_t, Z_t\right) = C_t + H\left(I_t, Z_t\right) = AZ_t - D_t$$

We define

$$\mathcal{G}_t^{\tilde{C},\tilde{I}} = \int_0^t e^{-(\beta+\kappa)s} \left(\beta+\kappa\right) \frac{1}{1-\gamma} \tilde{C}_s^{1-\gamma} ds + e^{-(\beta+\kappa)t} W_t.$$

So $\mathcal{G}_t^{\tilde{C},\tilde{I}}$ is the time-t conditional expected utility of the agent's life-time utility if he follows plan $\left\{\tilde{C}_t, \tilde{I}_t\right\}_{t=0}^\infty$ over $[0, t]$ and switches to $\left\{C_t, I_t\right\}_{t=0}^\infty$ at t. Obviously, $\mathcal{G}_0^{\tilde{C},\tilde{I}} = W_0$ and

$$e^{(\beta+\kappa)t} d\mathcal{G}_t^{\tilde{C},\tilde{I}} = \left(\beta+\kappa\right) \frac{1}{1-\gamma} \tilde{C}_t^{1-\gamma} dt - \left(\beta+\kappa\right) W_t dt + dW_t. \tag{28}$$

Let $\left\{B_{U,t}^{C,I}\right\}_{t=0}^\infty$ and $\left\{B_{U,t}^{\tilde{C},\tilde{I}}\right\}_{t=0}^\infty$ be the Itô's processes which are standard Brownian motion under the probability measures induced by $\left\{C_t, I_t\right\}_{t=0}^\infty$ and $\left\{\tilde{C}_t, \tilde{I}_t\right\}_{t=0}^\infty$ respectively according to Girsanov theorem. Then (10) implies

$$\sigma_U dB_{U,t}^{C,I} = \sigma_U dB_{U,t}^{\tilde{C},\tilde{I}} + \frac{1}{Z_t}\left(\tilde{I}_t - I_t\right) dt. \tag{29}$$

Then (28) and (26) implies

$$
\begin{aligned}
e^{(\beta+\kappa)t} d\mathcal{G}_t^{\tilde{C},\tilde{I}} &= \frac{\beta+\kappa}{1-\gamma}\left(\tilde{C}_t^{1-\gamma} - C_t^{1-\gamma}\right) dt + G_U\left(Z_t, U_t\right)\sigma_U dB_{U,t}^{C,I} \\
&= \left[\frac{\beta+\kappa}{1-\gamma}\left(\tilde{C}_t^{1-\gamma} - C_t^{1-\gamma}\right) + G_U\left(Z_t, U_t\right)\frac{1}{Z_t}\left(\tilde{I}_t - I_t\right)\right] dt \\
&\quad + G_U\left(Z_t, U_t\right) dB_{U,t}^{\tilde{C},\tilde{I}}. \qquad (30)
\end{aligned}
$$

The last equality above is due to (29). Suppose that (24) is not satisfied and, according to (27),

$$
\frac{\beta+\kappa}{1-\gamma} C_t^{1-\gamma} + \frac{1}{Z_t} G_U\left(Z_t, U_t\right) I_t < \frac{\beta+\kappa}{1-\gamma}\tilde{C}_t^{1-\gamma} + \frac{1}{Z_t} G_U\left(Z_t, U_t\right)\tilde{I}_t
$$

over a time interval with a positive measure. Then there exists a $\bar{t}$ such that $\left\{\mathcal{G}_t^{\tilde{C},\tilde{I}}\right\}$ is a sub-martingale over $[0, \bar{t}]$ and

$$
W_0 = \mathcal{G}_0^{\tilde{C},\tilde{I}} < E_0\left[\mathcal{G}_{\bar{t}}^{\tilde{C},\tilde{I}}\right]
$$

so that $\{C_t, I_t\}_{t=0}^{\infty}$ is dominated by following $\{C_t, I_t\}_{t=0}^{\infty}$ from the beginning and switching to $\{C_t, I_t\}_{t=0}^{\infty}$ at $\bar{t}$. Conversely, if (24) is satisfied for all t, there is no such profitable deviation for all $t \in [0, \infty)$. Proof of (22) is straightforward.

B.2 Proof of Proposition 1

According to the definition of $V(Z, U)$, the laws of motion of Z and U, (10) and (11), when neither limited commitment constraint is binding, $V(Z, U)$ satisfies the following HJB differential equation.

$$
(\mathbf{r}+\kappa) V(Z, U) = \max_{c, i, g_U, g_O}\left\{\begin{array}{c}
AZ - h(i) Z - cZ + V_Z(Z, U) Z[i-\delta] \\
+\frac{1}{2} V_{ZZ}(Z, U) Z^2\left(\sigma_U^2 + \sigma_O^2\right) \\
+V_U(Z, U) U\left[\frac{r+\kappa}{1-\gamma}\left(1 - \left(\frac{C}{U}\right)^{1-\gamma}\right) + \frac{1}{2}\gamma\left(g_U^2\sigma_U^2 + g_O^2\sigma_O^2\right)\right] \\
+\frac{1}{2} V_{UU}(Z, U) U^2\left(g_U^2\sigma_U^2 + g_O^2\sigma_O^2\right) \\
+V_{Z,U}(Z, U) ZU\left(g_U\sigma_U^2 + g_O\sigma_O^2\right)
\end{array}\right\}. \qquad (31)
$$

Notice that the incentive constraint (23) implies the following restriction on the maximization problem on the right hand side of (31).

$$
g_U = (\beta+\gamma) c^{-\gamma} u^{\gamma-1} h'(i). \qquad (32)
$$

Furthermore, according to the normalization, we have $V_Z(Z, U) = v(u) - uv'(u)$, $V_U(Z, U) = v'(u)$, $V_{ZZ}(Z, U) = \frac{1}{Z} u^2 v''(u)$, $V_{UU}(Z, U) = \frac{1}{Zv''(u)}$, and $V_{ZU}(Z, U) = -\frac{1}{Z} uv''(u)$. Therefore we have (15).

The boundary conditions can be shown by following the argument in the proof of Lemma 1 in Ai and Li (2015).

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Table 1

Model Parameters

Panel A: Estimated Parameters

Parameter	Estimate	
σ_O	0.2753	(0.0139)
σ_U	0.0909	(0.0135)
A	0.1691	(0.0021)
δ	0.0570	(0.0022)
ϕ	0.9684	(0.0943)
i^*	0.0019	(0.0001)
$\bar{u}_{MIN}$	0.0232	(0.0029)
$\bar{u}_0$	0.2257	(0.3225)

Panel B: Calibrated Parameters

Parameter	Value
γ	2
β	0.04
κ	0.05
α	0.33

Table 1 presents the estimates of the model parameters and their standard errors in parentheses (Panel A) and the values of the calibrated parameters (Panel B). The estimated parameters include volatility of observable and unobservable shocks (σ_O and σ_U, respectively), the total factor productivity (A), the capital depreciation rate (δ), the adjustment cost parameters (ϕ and i^*), and parameters that determine the outside option of managers and the initial normalized utility ($\bar{u}_{MIN}$ and $\bar{u}_0$, respectively). The set of calibrated parameters consists of risk aversion (γ), the discount rate (β), the death rate of managers (κ), and the capital share, α.

Table 2

Sample and Model-Implied Moments

Moments		Data	Model	t-stat(Diff)
Growth:	Aggregate	0.048	0.047	0.10
	P1 (Small)	0.194	0.124	1.70
	P2*	0.108	0.082	1.26
	P3	0.071	0.059	1.30
	P4*	0.053	0.044	1.35
	P5 (Large)	0.032	0.038	−1.02
CS-Std of Growth Rates		0.231	0.267	−1.55
Tobin's Q		1.446	1.401	0.84
I/K:	P1 (Small)	0.251	0.191	1.64
	P2*	0.160	0.144	1.10
	P3	0.120	0.120	0.01
	P4*	0.101	0.102	−0.24
	P5 (Large)	0.092	0.093	−0.11
PPS:	P1 (Small)	0.215	0.202	0.58
	P2	0.234	0.193	1.48
	P3	0.240	0.279	−0.86
	P4	0.309	0.376	−1.84
	P5 (Large)	0.443	0.422	0.22
Power Law		1.180	1.198	−0.69

Table 2 shows the set of moments exploited in estimation. We report moments in the data and in the model, and t-statistics for the difference between sample- and model-implied moments. The set of moments consists of average growth rates (Growth) in the aggregate economy and across five size-sorted portfolios (P1–P5), the cross-sectional standard deviation of firms' growth rates, median Tobin's Q, average investment rates (I/K), and executive pay-performance sensitivity (PPS) for the cross section of size-sorted portfolios, and the power law in firm size. Growth rates are measured in logs, PPS is measured in a panel regression of log change in CEO compensation on log firm return, controlling for firm fixed effects, and the power law is estimated using firms in the top size-sorted portfolio. Moments with asterisks are not exploited in estimation. The data are annual, measured in real terms and cover the period from 1993 to 2013.

Table 3
Estimates of the Restricted Model Specification

Parameter	Estimate
σ	0.3846 (0.0088)
A	0.2042 (0.0022)
δ	0.0701 (0.0015)
ϕ	2.5611 (0.1422)
i^*	0.0004 (0.0001)
$\bar{u}_{MIN}$	0.0176 (0.0043)
$\bar{u}_0$	0.5002 (0.1144)
σ_O^2	$0.5\sigma^2$
σ_U^2	$0.5\sigma^2$

Table 3 presents the estimates of the restricted model specification and their standard errors in parentheses. The estimated parameters include total volatility (σ), the total factor productivity (A), the capital depreciation rate (δ), the adjustment cost parameters (ϕ and i^*), and parameters that determine the outside option of managers and the initial normalized utility ($\bar{u}_{MIN}$ and $\bar{u}_0$, respectively). The bottom rows show the constrains imposed on volatilities of observable and unobservable shocks (σ_O and σ_U, respectively).

Table 4

Model-Implied Moments of the Restricted Specification

Moments		Data	Model	t-stat(Diff)
Growth:	Aggregate	0.048	0.031	2.26
	P1 (Small)	0.194	0.076	2.16
	P2*	0.108	0.080	1.31
	P3	0.071	0.044	2.09
	P4*	0.053	0.025	2.30
	P5 (Large)	0.032	0.032	0.14
CS-Std of Growth Rates		0.231	0.350	−2.44
Tobin's Q		1.446	1.512	−1.15
I/K:	P1 (Small)	0.251	0.153	2.09
	P2*	0.160	0.154	0.48
	P3	0.120	0.117	0.33
	P4*	0.101	0.095	1.21
	P5 (Large)	0.092	0.100	−1.73
PPS:	P1 (Small)	0.215	0.278	−2.05
	P2	0.234	0.284	−1.62
	P3	0.240	0.275	−0.80
	P4	0.309	0.309	−0.01
	P5 (Large)	0.443	0.374	0.71
Power Law		1.180	1.169	0.41

Table 4 shows the implications of the restricted model specification detailed in Table 3. We report moments in the data and in the model, and t-statistics for the difference between sample- and model-implied moments. The set of moments consists of average growth rates (Growth) in the aggregate economy and across five size-sorted portfolios (P1–P5), the cross-sectional standard deviation of firms' growth rates, median Tobin's Q, average investment rates (I/K), and executive pay-performance sensitivity (PPS) for the cross section of size-sorted portfolios, and the power law in firm size. Growth rates are measured in logs, PPS is measured in a panel regression of log change in CEO compensation on log firm return, controlling for firm fixed effects, and the power law is estimated using firms in the top size-sorted portfolio. Moments with asterisks are not exploited in estimation. The data are annual, measured in real terms and cover the period from 1993 to 2013.

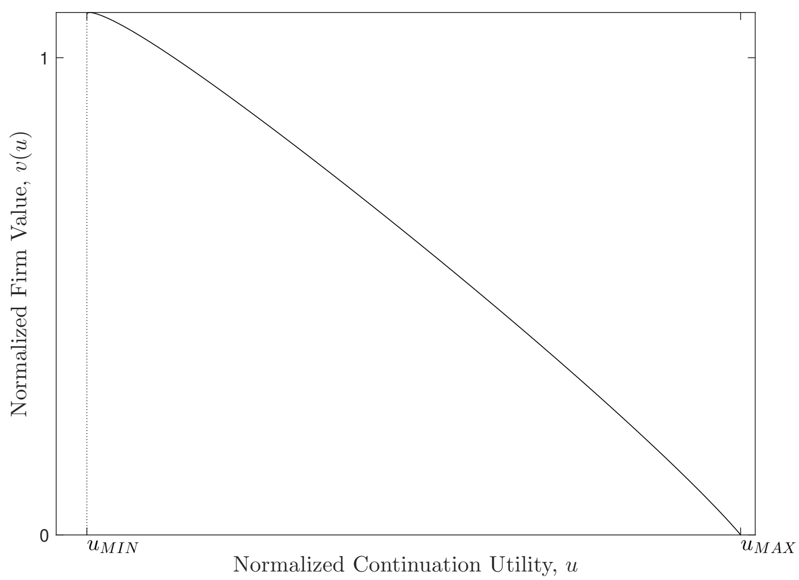

Figure 1. The Normalized Value Function

Figure 1 plots the normalized value function. The horizontal axis represents the normalized continuation utility of the manager, u, and the vertical axis represents the normalized firm value. u_{MIN} and u_{MAX} are the lower and upper bounds of u respectively.

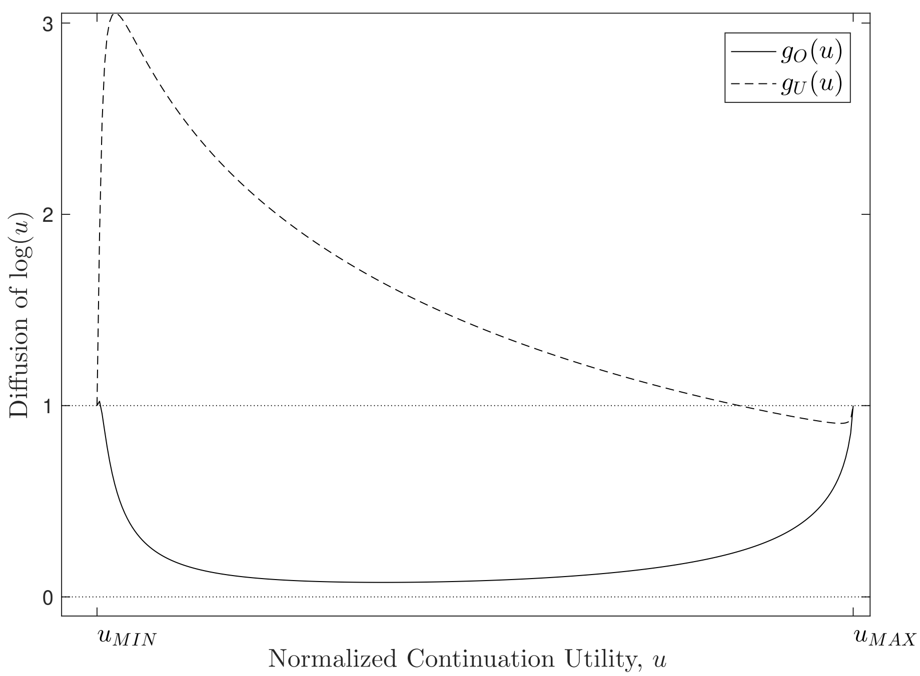

Figure 2. Elasticity of Continuation Utility

Figure 2 plots the sensitivity of continuation utility with respect to observable shocks, g_O (solid line), and with respect to unobservable shocks, g_U (dashed line), under the optimal contract.

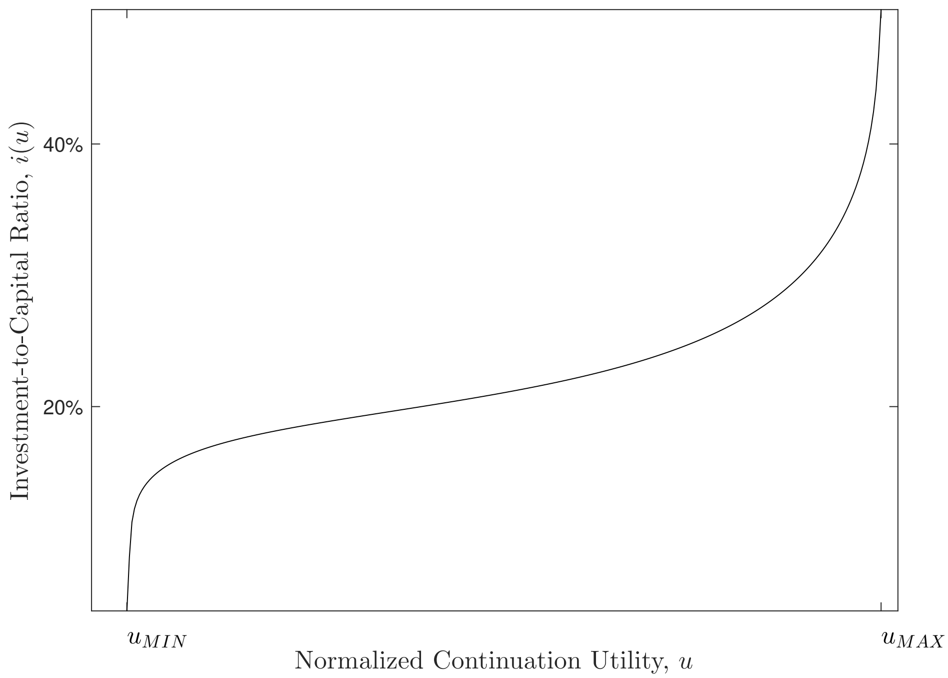

Figure 3. Investment Rate

Figure 3 plots the investment-to-capital ratio $i(u)$ under the optimal contract as a function of the normalized utility.

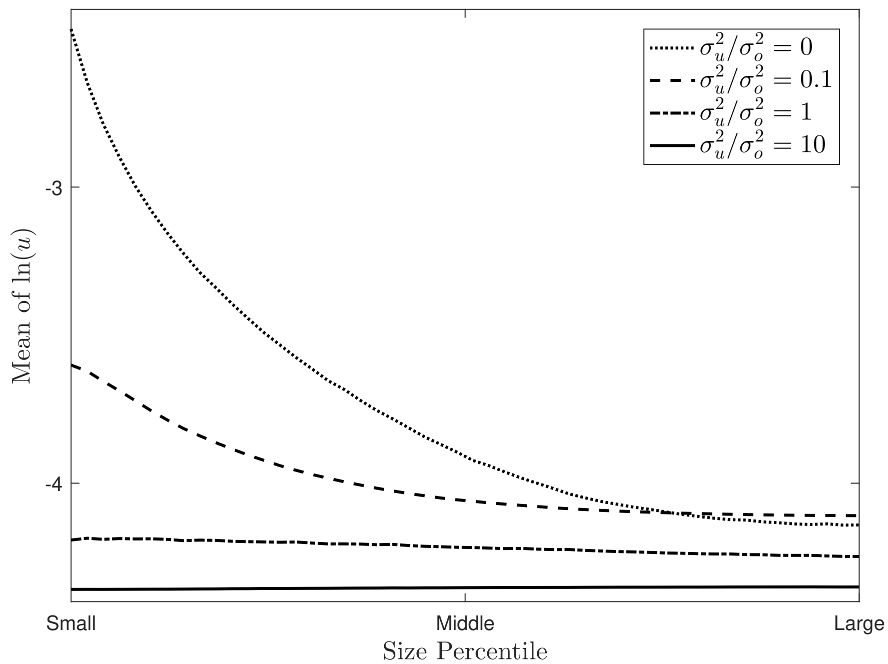

Figure 4. Equilibrium Correlation Between Manager's Share and Firm Size

Figure 4 plots the average manager's share (in logs) as a function of firm size in the equilibrium stationary distribution under different assumptions about the relative magnitude of observable and unobservable shocks.

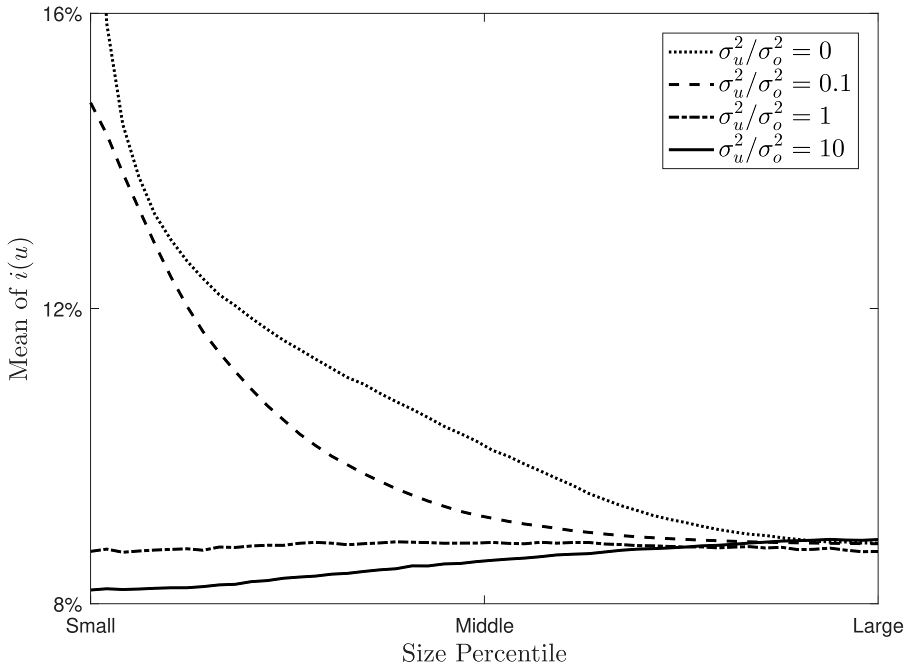

Figure 5. Investment and Firm Size

Figure 5 plots the average investment-to-capital ratio, $i(u)$, as a function of firm size in the equilibrium stationary distribution under different assumptions about the relative magnitude of observable and unobservable shocks.

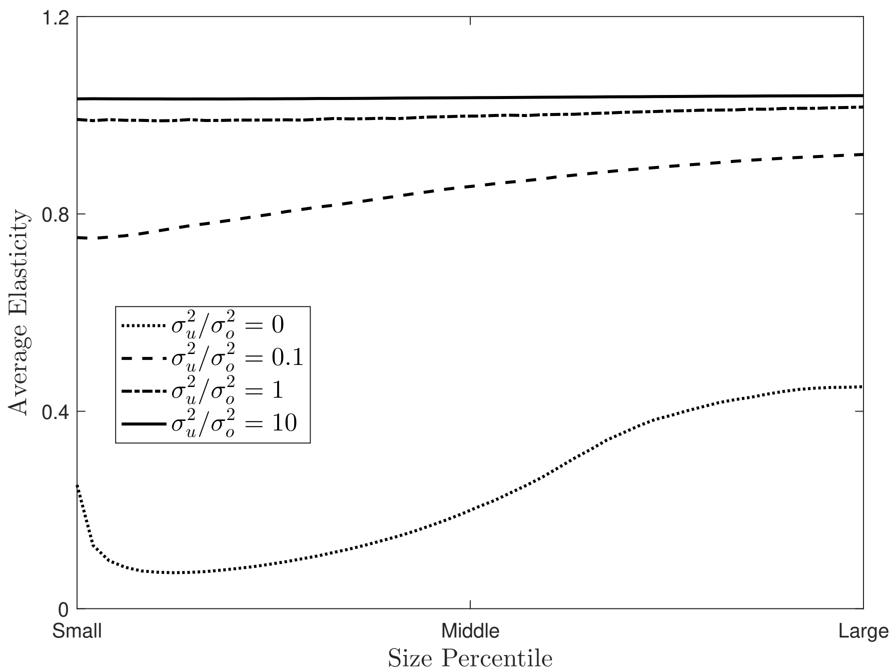

Figure 6. Pay-Performance Sensitivity and Firm Size

Figure 6 plots the average elasticity of continuation utility with respect to observable and unobservable shocks as a function of firm size in the equilibrium stationary distribution under different assumptions about the relative magnitude of observable and unobservable shocks.