Internet Appendix for “A Unified Model of Firm Dynamics with Limited Commitment and Assortative Matching”

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A The First-Best Case

Below we provide details of the solution to the value function in the first-best case and the associated power law of firm size. These derivations provide proofs for Proposition 1 and Proposition 2.

Proof of Proposition 1: Homogeneity implies that the value function of the optimization problem (16) is of the form $\bar{v}Z$. In this case, the HJB equation that describes the value function can be simplified to:

$$(r+\kappa)\,\bar{v} = \max_i \left\{A - h(i) + (i-\delta)\,\bar{v}\right\}. \tag{A.1}$$

Because $h(i) = i + \frac{1}{2}h_0 i^2$, the first-order condition for the above maximiation problem, $h'(i) = \bar{v}$ implies

$$\bar{v} = 1 + h_0 \hat{\imath}. \tag{A.2}$$

Together, (A.1) and (A.2) imply that $\hat{\imath}$ must satisfy

$$\frac{1}{2}h_0\hat{\imath}^2 - h_0\hat{r}\hat{\imath} + (A-\hat{r}) = 0,$$

where we denote $\hat{r} = r+\kappa+\delta$ as in Proposition 1. The relevant solution is

$$\hat{\imath} = \hat{r} - \sqrt{\hat{r}^2 - \frac{2}{h_0}(A-\hat{r})}.$$

Note that under Assumption 2, $\hat{\imath}$ defined above satisfies

$$\hat{\imath} = \arg\max_{i<\hat{r}} \frac{A-h(i)}{\hat{r}-i},$$

as needed.

A lemma for power law: To prove proposition 2, we first state a lemma that characterizes the stationary distribution of a Brownian motion with drift. A unit measure of particles enters the real line at x_0 at each point in time. They evaporate at a Poisson rate κ per unit of time. Conditioning on survival, each particle follows a Brownian motion with drift after entrance:

$$dx_t = \mu dt + \sigma dB_t.$$

Denote the density of the stationary distribution of the particles as $m\left(x|\, x_0\right)$, we have:

Lemma A.1. *Let $\theta_1 > 0$ and $\theta_2 < 0$ denote the two roots of the following quadratic equation:*

$$\kappa + \mu\theta - \frac{1}{2}\sigma^2\theta^2 = 0. \tag{A.3}$$

1. *The stationary distribution is given by:*

$$m\left(x|\,x_0\right) = \begin{cases} \frac{1}{\sqrt{\mu^2+2\kappa\sigma^2}} e^{\theta_2(x-x_0)} & x \geq x_0 \\ \frac{1}{\sqrt{\mu^2+2\kappa\sigma^2}} e^{\theta_1(x-x_0)} & x < x_0 \end{cases}$$

2. *Assume $\kappa > \mu + \frac{1}{2}\sigma^2$, then $\theta_2 < -1$, and*

$$\int_{-\infty}^{\infty} e^x m\left(x|\,x_0\right) dy = \frac{e^{x_0}}{\kappa - \left(\mu + \frac{1}{2}\sigma^2\right)} < \infty$$

3. *The density of $X = e^x$, denoted $M\left(X|\,x_0\right)$ is given by:*

$$M\left(X|\,x_0\right) = \begin{cases} \frac{1}{\sqrt{\mu^2+2\kappa\sigma^2}} e^{-\theta_2 x_0} X^{\theta_2 - 1} & X \geq e^{x_0} \\ \frac{1}{\sqrt{\mu^2+2\kappa\sigma^2}} e^{-\theta_1 x_0} X^{\theta_1 - 1} & X < e^{x_0} \end{cases}$$

   In particular, the right tail of X obeys power law with slope θ_2.

Proof. See Luttmer (2007). □

Proof of proposition 2: By proposition 1, $\ln Z$ is a Brownian motion:

$$d\ln Z = \left(\hat{\imath} - \delta - \frac{1}{2}\sigma^2\right) dt + \sigma dB_t,$$

with initial condition $\ln \bar{Z}$. Proposition 2 can be proved by applying Lemma A.1 directly.

B Optimal Contracting under limited commitment with CRS Matching Technology

In this section, we provide a characterization of the optimal contract under two-sided limited commitment and a proof for Proposition 3 of the paper.

B.1 Law of Motion of Continuation Utility

In the main text, we defined continuation utility as in (6). Since it is more convenient to work with the additively separable (expected utility) representation of the same preference, here we define

$$\hat{U}_t = \frac{1}{1-\gamma} U_t^{1-\gamma} = E_t \left[\int_t^{\tau_S \wedge \tau_D} (r+\kappa)\, e^{-r(s-t)} \frac{1}{1-\gamma} C_s^{1-\gamma} ds + \mathbb{1}_{\{\tau_S < \tau_D\}} e^{-r(\tau_S - t)} \hat{U}^S \left(\lambda Z_{\tau_S}\right) \right]$$

with $\hat{U}^S(X) = \frac{1}{1-\gamma}\left(U^S(X)\right)^{1-\gamma}$. Note that for $t < \tau_D \wedge \tau_S$

$$\int_0^t e^{-rs}(r+\kappa)\frac{C_s^{1-\gamma}}{1-\gamma}ds + e^{-rt}\hat{U}_t = E_t\begin{bmatrix}\int_0^{\tau_S\wedge\tau_D}(r+\kappa)e^{-rs}\frac{C_s^{1-\gamma}}{1-\gamma}ds \\ +\mathbb{1}_{\{\tau_S<\tau_D\}}e^{-r\tau_S}\hat{U}^S(\lambda Z_{\tau_S})\end{bmatrix} \tag{B.1}$$

is a martingale. Therefore, by the *Martingale Representation Theorem*, (B.1) implies

$$\begin{aligned}\int_0^t e^{-rs}(r+\kappa)\frac{C_s^{1-\gamma}}{1-\gamma}ds + e^{-rt}\hat{U}_t &= \int_0^t e^{-rs}G_B\left(Z_s,\hat{U}_s\right)dB_s \\ &\quad + \int_0^t e^{-rs}G_D\left(Z_s,\hat{U}_s\right)\left(dN_s^D - \kappa_D ds\right) \\ &\quad + \int_0^t e^{-rs}G_S\left(Z_s,\hat{U}_s\right)\left(dN_s^S - \kappa_S ds\right)\end{aligned} \tag{B.2}$$

with $\left\{G_B\left(Z_t,\hat{U}_t\right)\right\}$, $\left\{G_D\left(Z_t,\hat{U}_t\right)\right\}$, and $\left\{G_S\left(Z_t,\hat{U}_t\right)\right\}$ being three predictable and square-integrable processes. In Equation (B.2), $\left\{N_t^D\right\}$ and $\left\{N_t^S\right\}$ represent the counting processes of the death shock and the separating shock respectively. According to setup of the model, $G_D(Z_t,\hat{U}_t) = -\hat{U}_t$ and $G_S(Z_t,\hat{U}_t) = \hat{U}^S(\lambda Z_t) - \hat{U}_t$ for all t. Therefore, (B.2) implies:

$$\begin{aligned}d\hat{U}_t &= \left[(r+\kappa)\hat{U}_t - (r+\kappa)\frac{C_t^{1-\gamma}}{1-\gamma} - \kappa_S\hat{U}^S(\lambda Z_t)\right]dt + G_B\left(Z_t,\hat{U}_t\right)\sigma dB_t \\ &\quad -\hat{U}_t dN_t^D - \left(\hat{U}_t - \hat{U}^S(\lambda Z_t)\right)dN_t^S.\end{aligned}$$

Since $U_t = \left((1-\gamma)\hat{U}_t\right)^{\frac{1}{1-\gamma}}$, if we define $G(Z,U) = U^\gamma G_B\left(Z,\hat{U}\right)$ Itô's lemma implies (14). Furthermore, we have the law of motion for u, Equation (26), with

$$\begin{aligned}\mu_u(u) &= \frac{r+\kappa}{1-\gamma}\left(1-\left(\frac{c}{u}\right)^{1-\gamma}\right) - (i-\delta) + \left(\frac{1}{2}\gamma g^2 - g + 1\right)\sigma^2 - \frac{1}{2}(g-1)^2\sigma^2 \\ &\quad + \frac{\kappa_S}{1-\gamma}\left(1-\left(\frac{\lambda u_{MIN}}{u}\right)^{1-\gamma}\right), \\ \sigma_u(u) &= (g-1)\sigma.\end{aligned}$$

B.2 The HJB differential equation for $V(Z,U)$

For any pair of state variables (Z,U) such that $u_{MIN}(Z) < \frac{U}{Z} < u_{MAX}(Z)$, the limited commitment constraints do not bind and the value function $V(Z,U)$ must satisfy the following HJB equation in the interior.

$$(r+\kappa) V(Z,U) = \max_{C,i,g} \left\{ \begin{array}{c} AZ - h(i) Z - C + V_Z(Z,U) Z(i-\delta) + \kappa_S V^S(\lambda Z) \\ +V_U(Z,U) U \left[\begin{array}{c} \frac{r+\kappa}{1-\gamma}\left(1-\left(\frac{C}{U}\right)^{1-\gamma}\right) + \frac{1}{2}\gamma g^2\sigma^2 \\ +\frac{\kappa_S}{1-\gamma}\left(1-\left(\frac{U^S(\lambda Z)}{U}\right)^{1-\gamma}\right) \end{array} \right] \\ +\frac{1}{2}V_{UU}(Z,U) U^2 g^2 \sigma^2 + V_{Z,U}(Z,U) ZUg\sigma^2 \\ +\frac{1}{2}V_{ZZ}(Z,U) Z^2\sigma^2 \end{array} \right\}. \tag{B.3}$$

According to our normalization, $V(Z,U) = Zv(u) = Zv\left(\frac{U}{Z}\right)$. Then we have: $V_Z(Z,U) = v(u) - uv'(u)$, $V_U(Z,U) = v'(u)$, $V_{ZZ} = \frac{1}{Z}u^2 v''(u)$, $V_{UU}(Z,U) = \frac{1}{Z}v''(u)$, and $V_{ZU}(Z,U) = -\frac{1}{Z}uv''(u)$. Moreover, as we discussed in the main text, $U^S(\lambda Z) = \lambda u_{MIN} Z$ and $V^S(\lambda Z) = \lambda v(u_{MAX}) Z$. Therefore, we have the HJB equation (24).

Taking first-order condition of the objective function on the right-hand-side of (B.3) with respect to C, we have

$$(r+\kappa) U^\gamma V_U(Z,U) = \frac{1}{C^{-\gamma}}.$$

To prove that compensation must be constant, it is enough to establish

$$d\left[U^\gamma V_U(Z,U)\right] = 0,$$

which follows from the proof of Lemma 5 in Ai and Li (2015). This proves the rest of Proposition 3.

C Power law of CEO Pay

In this section, we provide the details of the proof of Proposition 4.

C.1 Managers' Outside Options

Because once separated, managers and firms immediately find an opportunity to match, $V^S(Y) = \bar{V}(Y)$. Because the contracts signed on the directed matching market is with full commitment, we have

$$\bar{V}(Y) = \max_X V^{FB}\left(Y^{\psi_Y} X^{\psi_X}, \bar{U}(X)\right). \tag{C.1}$$

Note that $V^{FB}(Z,U) = \bar{v}Z - \frac{1}{r+\kappa_D}U$ by Proposition 1, where the constant $\bar{v}$ is defined in the same proposition. We conjecture and verify that the equilibrium outside option of the manager takes the form of

$$\bar{U}(X) = \bar{\bar{u}} X^\psi,$$

then the optimality condition for (C.1) implies that

$$\psi_X \bar{v} X^{\psi_X - 1} Y^{\psi_Y} = \frac{1}{r + \kappa_D} \bar{\bar{u}} \psi X^{\psi - 1}.$$

Together with the symmetric matching rule: $X(Y) = Y$, we determine the equilibrium outside option of the managers must satisfy:

$$\bar{\bar{u}} = \frac{\psi_X}{\psi} (r + \kappa_D) \bar{v}. \tag{C.2}$$

The outside option $\bar{\bar{u}} X^{\psi}$ is delivered by paying a constant consumption $\{\bar{\bar{u}} Z_t\}_{t=\tau_M}^{\tau_D}$ until the termination of the contract.

C.2 Optimality of Compensation and Investment Policies

A candidate optimal policy We use a guess-and-verify approach by first proposing a candidate optimal investment and consumption policy, which allows us to compute the candidate value function in closed form. We then verify that the constructed value function and the policy functions satisfy the HJB equation and the associated optimality conditions. Our construction follows the following procedure. First, given the proposed investment policy $\frac{I_t}{Z_t} = \iota$, the law of motion of Z_t is completely determined. Second, given the law of motion of Z_t, an initial level of consumption, C_0, and an initial guess of $\hat{c}$, we define

$$\Lambda_t = \ln C_0 - \psi \ln Z_t + l_t,$$

where $\{l_t\}_{t=0}^{\infty}$ is the regulator that keeps Λ_t above $\ln \hat{c}$ for all t (that is, $\{l_t\}_{t=0}^{\infty}$ is the minimum increasing process such that $l_0 = 0$ and $\Lambda_t \geq \ln \hat{c}$ for all t). Define the managerial compensation policy as

$$C_t = \exp(\Lambda_t) Z_t^{\psi}. \tag{C.3}$$

Third, given an initial guess of $\hat{c}$, the manager's expected utility can be evaluated as

$$\hat{U}(\exp(\Lambda), Z) = E\left[\int_0^{\tau_D \wedge \tau_S} (r + \kappa) e^{-rt} \frac{\left(\exp(\Lambda_t) Z_t^{\psi}\right)^{1-\gamma}}{1-\gamma} dt + \mathbb{1}_{\{\tau_S < \tau_D\}} e^{-r\tau_S} \bar{\bar{u}} X_{\tau_S}^{\psi} \,\middle|\, \Lambda_0 = \Lambda, Z_0 = Z\right]. \tag{C.4}$$

Our candidate compensation policy is the one associated with the $\hat{c}$ that is uniquely determined by:

$$\left[(1-\gamma) \hat{U}(\hat{c}, Z)\right]^{\frac{1}{1-\gamma}} = \bar{\bar{u}} Z^{\psi}. \tag{C.5}$$

The following lemma provides the functional form of the value function associated with above proposed policy functions.

Lemma A.2. *Define the normalized utility of the manager (with homogeneity of degree ψ w.r.t Z) $\omega = \frac{U}{Z^{\psi}}$. Under the investment policy $\frac{I_t}{Z_t} = \iota$ and the compensation policy* (C.3)*, for any $\omega \in [\bar{\bar{u}}, \infty)$, the normalized compensation policy, $c(\omega)$, is defined by the unique solution of the following equation on $[\hat{c}, \infty)$*

$$\omega^{1-\gamma} = c(\omega)^{1-\gamma} + \frac{1-\gamma}{\varsigma_1 - (1-\gamma)} \hat{c}^{\varsigma_1} c(\omega)^{1-\gamma-\varsigma_1} \tag{C.6}$$

with ς_1 given by (C.10) and

$$\hat{c} = c\left(\bar{\bar{u}}\right) = \bar{\bar{u}} \left(\frac{\varsigma_1 - (1-\gamma)}{1-\gamma} \right)^{\frac{1}{1-\gamma}} \tag{C.7}$$

being the minimum normalized compensation. The value function is given by

$$V\left(Z, U\right) = \frac{A-\iota}{r+\kappa+\delta-\iota} Z - \frac{1}{r+\kappa} \left[c\left(\frac{U}{Z^{\psi}}\right) + \frac{1}{\varsigma_1 - 1} \hat{c}^{\varsigma_1} c\left(\frac{U}{Z^{\psi}}\right)^{1-\varsigma_1} \right] Z^{\psi}. \tag{C.8}$$

Proof. The proof is similar to that of Lemma 8 in Ai and Li (2015). According to (C.3), the managerial compensation depends on Λ_t and Z_t^{ψ} and then we define the expected present value of the total compensation under the investment policy $\frac{I_t}{Z_t} = \iota$ and compensation policy (C.3)

$$G_C\left(\Lambda\right) Z^{\psi} = E\left[\int_0^{\infty} e^{-(r+\kappa)t} C_t | \Lambda_0 = \Lambda, Z_0 = Z \right].$$

with some function $G_C\left(\Lambda\right)$ for $\Lambda \geq \ln \hat{c}$. Since the law of motion of $\{Z_t\}$ implies

$$d \ln Z^{\psi} = \psi \left(\iota - \delta - \frac{1}{2}\sigma^2 \right) dt + \psi \sigma dB.$$

By (C.3), $G_C\left(\Lambda\right)$ satisfies the following ordinary differential equation.

$$\begin{aligned} 0 &= \exp\left(\Lambda\right) + \left[\psi \left(\iota - \delta + \frac{1}{2}\left(\psi - 1\right)\sigma^2 \right) - \left(r+\kappa\right) \right] G_C\left(\Lambda\right) \\ &\quad - \left[\psi\left(\iota - \delta\right) - \frac{1}{2}\psi\sigma^2 + \psi^2\sigma^2 \right] G_C'\left(\Lambda\right) + \frac{1}{2}\psi^2\sigma^2 G_C''\left(\Lambda\right). \end{aligned} \tag{C.9}$$

Therefore

$$G_C\left(\Lambda\right) = \frac{1}{r+\kappa} e^{\Lambda} + A_1 e^{(1-\varsigma_1)\Lambda} + A_2 e^{(1-\varsigma_2)\Lambda}$$

with A_1, A_2 being two unknown coefficients that are solved below, and $1-\varsigma_1$, $1-\varsigma_2$ being the roots of the following quadratic equation about α

$$\frac{1}{2}\psi^2\sigma^2\left(1-\alpha\right)^2 - \left[\psi\left(\iota-\delta\right) - \frac{1}{2}\psi\sigma^2 + \psi^2\sigma^2 \right]\left(1-\alpha\right) + \psi\left(\iota - \delta + \frac{1}{2}\left(\psi-1\right)\sigma^2 \right) - \left(r+\kappa\right) = 0.$$

By Assumption 2, we have

$$\varsigma_1 = \left[\sqrt{\left(\frac{\iota}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(r+\kappa)}{\sigma^2}}-\frac{1}{\psi}\left(\frac{\iota-\delta}{\sigma^2}-\frac{1}{2}\right)\right]\frac{1}{\psi}>\frac{1}{\psi}>1 \tag{C.10}$$

$$\varsigma_2 = \left[-\sqrt{\left(\frac{\iota}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(r+\kappa)}{\sigma^2}}-\frac{1}{\psi}\left(\frac{\iota-\delta}{\sigma^2}-\frac{1}{2}\right)\right]\frac{1}{\psi}<0. \tag{C.11}$$

Notice that $G_C(\Lambda)$ satisfies the boundary conditions: $G'_C(\Lambda_{MIN})=0$ where $\Lambda_{MIN}=\ln c(\bar{\bar{u}})$, and $\lim_{\Lambda\to\infty} G_C(\Lambda)=\frac{1}{r+\kappa}e^{\Lambda}$. Then

$$G_C(\Lambda)=\frac{1}{r+\kappa}\left[e^{\Lambda}+\frac{1}{\varsigma_1-1}e^{\varsigma_1\Lambda_{MIN}+(1-\varsigma_1)\Lambda}\right].$$

Similarly, the manager's expected utility is

$$\hat{U}(\Lambda,Z)=E_t\left[\int_t^{\tau_S\wedge\tau_D} e^{-(r+\kappa)(s-t)}(r+\kappa)\frac{C_s^{1-\gamma}}{1-\gamma}ds+\mathbb{1}_{\{\tau_S<\tau_D\}}e^{-r(\tau_S-t)}\bar{\bar{u}}X_{\tau_S}^{\psi}|\Lambda_t=\Lambda,Z_t=Z\right].$$

and, because of homogeneity, we define function $G_U(\Lambda)$ for $\Lambda\geq\ln\hat{c}$ such that

$$G_U(\Lambda)Z^{\psi(1-\gamma)}=\hat{U}(\Lambda,Z).$$

So $G_U(\Lambda)$ satisfies the following ordinary differential equation

$$\begin{aligned}0 &= (r+\kappa)e^{(1-\kappa)\Lambda}-\left[r+\kappa+\frac{1}{2}\gamma(1-\gamma)\psi^2\sigma^2-\psi\left(\iota-\delta+\frac{1}{2}(\psi-1)\sigma^2\right)(1-\gamma)\right]G_U(\Lambda)\\ &\quad+\left[-\psi\left(\iota-\delta-\frac{1}{2}\sigma^2\right)-(1-\gamma)\psi^2\sigma^2\right]G'_U(\Lambda)+\frac{1}{2}\psi^2\sigma^2G''_U(\Lambda)\end{aligned} \tag{C.12}$$

and boundary conditions: $G'_U(\Lambda_{MIN})=0$ and $\lim_{\Lambda\to\infty}G_U(\Lambda)=e^{(1-\gamma)\Lambda}$. Therefore

$$G_U(\Lambda)=e^{(1-\gamma)\Lambda}+\frac{1-\gamma}{\varsigma_1-(1-\gamma)}e^{\varsigma_1\Lambda_{MIN}+(1-\gamma-\varsigma_1)\Lambda}$$

with ς_1 being defined by equation (C.10). Notice that $e^{\Lambda_{MIN}}=\hat{c}$ and $U^{1-\gamma}=\hat{U}(\Lambda,Z)$, hence we have (C.6) and (C.7).

On the other hand, since the expected present value of the operating profit of the firm with a capital stock Z under the policies is $Z(A-\iota)/(r+\kappa+\delta-\iota)$, we have

$$V(Z,U)=\frac{A-\iota}{r+\kappa+\delta-\iota}Z-G_C\left(\ln c\left(\frac{U}{Z^{\psi}}\right)\right)Z^{\psi} \tag{C.13}$$

which implies (C.8). □

Verification of optimality It is straightforward to verify that the proposed compensation policy satisfies the first-order conditions in the HJB equation and the limited commitment constraint (by the construction of $\hat{c}$ in (C.4)). The fact that the regulated Brownian motion construction is equivalent to the running maximum characterization of the compensation policy in equation (29) follows from Harrison (1985). To verify the optimality of investment policy, it is enough to show

$$V_Z(Z,U) \geq 1$$

for all (Z,U) under Assumptions 2 and 3. To show this inequality, we prove the following lemma.

Lemma A.3.

$$V_Z(U,Z) = \frac{A-\iota}{r+\kappa+\delta-\iota} - \psi Z^{\psi-1}\left[G_C(\Lambda) - \frac{1}{r+\kappa}e^{\gamma\Lambda}G_U(\Lambda)\right].$$

Here $\Lambda = \ln c\left(\frac{U}{Z^\psi}\right)$ *which increases with* $\frac{U}{Z^\psi}$*.*

Proof. With a little abusing of notation, we define function $\Lambda(\omega)$ for $\omega \in [\bar{\bar{u}}, \infty)$. Equation (C.13) implies

$$V_Z(Z,U) = \frac{A-\iota}{r+\kappa+\delta-\iota} - \psi Z^{\psi-1}G_C\left(\Lambda\left(\frac{U}{Z^\psi}\right)\right) - Z^\psi G_C'\left(\Lambda\left(\frac{U}{Z^\psi}\right)\right)\Lambda'\left(\frac{U}{Z^\psi}\right)\left(-\psi Z^{-\psi-1}U\right) \tag{C.14}$$

Notice that for any $\Lambda \in [\Lambda_{MIN}, \infty)$

$$\begin{aligned} G_C'(\Lambda) &= \frac{1}{r+\kappa}\left(e^\Lambda + e^{\varsigma_1\Lambda_{MIN}+(1-\varsigma_1)\Lambda}\right) \\ G_U'(\Lambda) &= (1-\gamma)(r+\kappa)e^{-\gamma\Lambda}\frac{1}{r+\kappa}\left(e^\Lambda + e^{\varsigma_1\Lambda_{MIN}+(1-\varsigma_1)\Lambda}\right) \end{aligned}$$

and then we have

$$\frac{G_C'(\Lambda)}{G_U'(\Lambda)} = \frac{1}{(1-\gamma)(r+\kappa)}e^{\gamma\Lambda}. \tag{C.15}$$

On the other hand, according to the definition of $G_U(\Lambda)$, for any $u \in [\bar{\bar{u}}, \infty)$,

$$G_U(u)^{\frac{1}{1-\gamma}} = u$$

and then we have

$$\Lambda'(u) = G_U(\Lambda(u))^{\frac{\gamma}{\gamma-1}}\frac{1}{G_U'(\Lambda(u))}(1-\gamma). \tag{C.16}$$

In addition

$$U = G_U\left(\Lambda\left(\frac{U}{Z^\psi}\right)\right)^{\frac{1}{1-\gamma}}Z^\psi. \tag{C.17}$$

By plugging (C.15), (C.16) and (C.17) into (C.14) we have the expression of $V_Z(Z,U)$ in the lemma and (C.16) implies that $\Lambda'(u) > 0$. $\square$

Lemma A.3 implies

$$\begin{aligned}
V_Z(U,Z) &= \frac{A-\iota}{r+\kappa+\delta-\iota}-\psi Z^{\psi-1}\frac{1}{r+\kappa}\frac{\varsigma_1\gamma}{(\varsigma_1-1)(\varsigma_1-(1-\gamma))}e^{\Lambda-\varsigma_1(\Lambda-\Lambda_{MIN})}\\
&= \frac{A-\iota}{r+\kappa+\delta-\iota}-\psi\left(\frac{C}{e^{\Lambda}}\right)^{1-\frac{1}{\psi}}\frac{1}{r+\kappa}\frac{\varsigma_1\gamma}{(\varsigma_1-1)(\varsigma_1-(1-\gamma))}e^{\Lambda-\varsigma_1(\Lambda-\Lambda_{MIN})}\\
&\geq \frac{A-\iota}{r+\kappa+\delta-\iota}-\psi C_0^{1-\frac{1}{\psi}}\frac{1}{r+\kappa}\frac{\varsigma_1\gamma}{(\varsigma_1-1)(\varsigma_1-(1-\gamma))}e^{\left(\frac{1}{\psi}-\varsigma_1\right)\Lambda-\varsigma_1\Lambda_{MIN}}.
\end{aligned}$$

The last inequality is due to (C.3) which implies that the fact that $C_0 \leq C_t$ for all $t \geq 0$. Notice that (C.10) implies $\varsigma_1 > \frac{1}{\psi}$ and that $\Lambda \geq \Lambda_{MIN}$, therefore

$$V_Z(U,Z) \geq \frac{A-\iota}{r+\kappa+\delta-\iota}-\psi C_0^{1-\frac{1}{\psi}}\frac{1}{r+\kappa}\frac{\varsigma_1\gamma}{(\varsigma_1-1)(\varsigma_1-(1-\gamma))}e^{\frac{1}{\psi}\Lambda_{MIN}}. \tag{C.18}$$

To characterize the starting compensation C_0, notice that the starting continuation utility is $\bar{U}$ and

$$\bar{U} = G_U\left(\Lambda\left(\frac{\bar{U}}{Z_0^{\psi}}\right)\right)^{\frac{1}{1-\gamma}} Z_0^{\psi}$$

with Z_0 being the initial organizational capital level. Let $\Lambda_0 = \Lambda\left(\frac{\bar{U}}{Z_0^{\psi}}\right)$. Then

$$\begin{aligned}
C_0 \equiv \hat{C}(\Lambda_0) &= Z_0^{\psi}e^{\Lambda_0}\\
&= \bar{U}G_U(\Lambda_0)^{\frac{1}{\gamma-1}}e^{\Lambda_0}\\
&= \bar{U}\left[1+\frac{1-\gamma}{\varsigma_1-(1-\gamma)}e^{\varsigma_1(\Lambda_{MIN}-\Lambda_0)}\right]^{\frac{1}{\gamma-1}}
\end{aligned}$$

Obviously $\hat{C}(\Lambda)$ is an increasing function over $[\Lambda_{MIN},\infty)$. Therefore

$$C_0 \geq \hat{C}(\Lambda_{MIN}) = \bar{U}\left[1+\frac{1-\gamma}{\varsigma_1-(1-\gamma)}\right]^{\frac{1}{\gamma-1}}.$$

Therefore (C.18) implies

$$V_Z(U,Z) \geq \frac{A-\iota}{r+\kappa+\delta-\iota}-\psi\bar{U}^{1-\frac{1}{\psi}}\left[1+\frac{1-\gamma}{\varsigma_1-(1-\gamma)}\right]^{\frac{\psi-1}{\psi(\gamma-1)}}\frac{1}{r+\kappa}\frac{\varsigma_1\gamma}{(\varsigma_1-1)(\varsigma_1-(1-\gamma))}e^{\frac{1}{\psi}\Lambda_{MIN}}.$$

Notice that

$$e^{\Lambda_{MIN}} = \left(\frac{1}{\varsigma_1-(1-\gamma)}\right)^{\frac{1}{\gamma-1}}\bar{\bar{u}}.$$

Hence condition (28) in Assumption 2 implies that $V_Z(U,Z) \geq 1$ and the investment policy is optimal.

C.3 Distribution of Running Maximums of Geometric Brown Motions

We first present a lemma that computes the integral of discounted normal density. The proof is standard and is omitted here.

Lemma A.4. *Assume $\kappa_D > 0$, $\kappa_S > 0$, $\psi > 0$ and $y \geq x_0$. Let θ_2 be the negative root of the quadratic equation (A.3) defined in Lemma A.1. Then*

$$\int_0^{\infty} e^{-\kappa t} \Phi_0 \left(\frac{y - \psi \mu t - x_0}{\psi \sigma \sqrt{t}} \right) dt = \frac{1}{\kappa} + \frac{\psi}{\theta_2 \sqrt{\mu^2 + 2\kappa\sigma^2}} e^{\frac{\theta_2}{\psi}(y - x_0)},$$

where $\Phi_0(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Next, we present a lemma that characterizes the distribution of the right tail of the running maximum of Brownian motions. Continue to consider the setup of Lemma A.1. For all j, let $\{x_{j,s}\}_{s=0}^{\infty}$ be a Brownian motion starts at x_0 as in Lemma A.1. Define the running maximum of the Brownian motions as

$$\hat{x}_{j,t} = \sup_{0<s<t} x_{j,s},$$

and let $Y_{j,t}$ be

$$Y_{j,t} = \max\{y_0, \psi_0 + \psi \hat{x}_{j,t}\},$$

where y_0 is a constant, and ψ_0 and ψ are two real valued parameters. We continue and use m to denote the stationary distribution of the particles $\{x_j\}$ which evaporate at Poisson rate κ. The following lemma characterizes the distribution of the right tail of Y.

Lemma A.5. *Assume $\mu > 0$. For y large enough,*

$$m(Y_j > y) \sim -\frac{\psi}{\theta_2 \sqrt{\psi^2\mu^2 + 2\kappa\psi^2\sigma^2}} e^{\frac{\theta_2}{\psi}[y - (\psi_0 + \psi x_0)]},$$

where θ_2 is the negative root of the quadratic equation (A.3) defined in Lemma A.1.

Proof. Note that for $y > y_0$, $Y_{j,t} = \max\{y_0, \psi_0 + \psi\hat{x}_{j,t}\} > y$ is equivalent to $\psi_0 + \psi\hat{x}_{j,t} > y$, or $\hat{x}_{j,t} > \frac{1}{\psi}(y - \psi_0)$. Using equation (9.4) on page 15 of Harrison (1985), for any x,

$$P(\hat{x}_{j,t} < x) = \Phi_0\left(\frac{x - x_0 - \mu t}{\sigma\sqrt{t}}\right) - e^{-\frac{2\mu(x - x_0)}{\sigma^2}} \Phi_0\left(\frac{-x + x_0 - \mu t}{\sigma\sqrt{t}}\right),$$

where Φ_0 is the cumulative distribution function of the standard normal distribution. Therefore,

$$P\left(\hat{x}_{j,t} > \frac{1}{\psi}(y - \psi_0)\right) = 1 - \Phi_0\left(\frac{\frac{1}{\psi}(y - \psi_0) - x_0 - \mu t}{\sigma\sqrt{t}}\right) + e^{-\frac{2\mu\left(\frac{1}{\psi}(y - \psi_0) - x_0\right)}{\sigma^2}} \Phi_0\left(\frac{-\frac{1}{\psi}(y - \psi_0) + x_0 - \mu t}{\sigma\sqrt{t}}\right), \tag{C.19}$$

Note that the second normal cdf can be written as

$$\begin{aligned}\Phi_0\left(\frac{-\frac{1}{\psi}(y-\psi_0)+x_0-\mu t}{\sigma\sqrt{t}}\right) &= \Phi_0\left(\frac{-y+\psi_0+\psi x_0-\psi\mu t}{\psi\sigma\sqrt{t}}\right) \\ &= 1-\Phi_0\left(\frac{y+\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right).\end{aligned}$$

As a result, (C.19) can be written as:

$$\begin{aligned}P\left(\hat{x}_{j,t}>\frac{1}{\psi}(y-\psi_0)\right) &= 1-\Phi_0\left(\frac{y-\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right) \\ &\quad +e^{-\frac{2\mu(y-\psi_0-\psi x_0)}{\psi\sigma^2}}\left[1-\Phi_0\left(\frac{y+\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right)\right]. \qquad \text{(C.20)}\end{aligned}$$

Because particles evaporate at rate κ, the law of large numbers implies that the total measure of particles that satisfies $\hat{x}_{j,t}>\frac{1}{\psi}(y-\psi_0)$ is given by:

$$m(Y_j>y)=\int_0^\infty e^{-\kappa t}P\left(\hat{x}_{j,t}>\frac{1}{\psi}(y-\psi_0)\right)dt.$$

Using (C.20),

$$\begin{aligned}m(Y_j>y) &= \frac{1}{\kappa}-\int_0^\infty e^{-\kappa t}\Phi_0\left(\frac{y-\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right)dt \\ &\quad +e^{-\frac{2\mu(y-\psi_0-\psi x_0)}{\psi\sigma^2}}\left[\frac{1}{\kappa}-\int_0^\infty e^{-\kappa t}\Phi_0\left(\frac{y+\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right)dt\right]. \qquad \text{(C.21)}\end{aligned}$$

Using Lemma A.4, for $y>(\psi_0+\psi x_0)$,

$$\int_0^\infty e^{-\kappa t}\Phi_0\left(\frac{y-\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right)dt=\frac{1}{\kappa}+\frac{\psi}{\theta_2\sqrt{\psi^2\mu^2+2\kappa\psi^2\sigma^2}}e^{\frac{\theta_2}{\psi}[y-(\psi_0+\psi x_0)]},$$

where θ_2 is the negative root of the quadratic equation (A.3). Similarly,

$$\int_0^\infty e^{-\kappa t}\Phi_0\left(\frac{y+\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right)dt=\frac{1}{\kappa}+\frac{\psi}{\tilde{\theta}_2\sqrt{\psi^2\mu^2+2\kappa\psi^2\sigma^2}}e^{\frac{\theta_2}{\psi}[y-(\psi_0+\psi x_0)]}, \qquad \text{(C.22)}$$

where $\tilde{\theta}_2$ is the negative root of the quadratic equation:

$$\kappa-\mu\tilde{\theta}-\frac{1}{2}\sigma^2\tilde{\theta}=0.$$

Note that $\tilde{\theta}_2=-\theta_1$, where θ_1 is the positive root of (A.3). Therefore, equation (C.22) can be written as:

$$\int_0^\infty e^{-\kappa t}\Phi_0\left(\frac{y+\psi\mu t-(\psi_0+\psi x_0)}{\psi\sigma\sqrt{t}}\right)dt=\frac{1}{\kappa}-\frac{\psi}{\theta_1\sqrt{\psi^2\mu^2+2\kappa\psi^2\sigma^2}}e^{-\frac{\theta_1}{\psi}[y-(\psi_0+\psi x_0)]}. \qquad \text{(C.23)}$$

Summarize (C.21), (C.22) and (C.23), we have:

$$\begin{aligned} m\left(Y_j > y\right) &= -\frac{\psi}{\theta_2\sqrt{\psi^2\mu^2 + 2\kappa\psi^2\sigma^2}} e^{\frac{\theta_2}{\psi}\left[y-(\psi_0+\psi x_0)\right]} \\ &\quad + \frac{\psi}{\theta_1\sqrt{\psi^2\mu^2 + 2\kappa\psi^2\sigma^2}} e^{-\frac{2\mu+\theta_1\sigma^2}{\psi\sigma^2}\left[y-(\psi_0+\psi x_0)\right]} \end{aligned}$$

It is straight forward to show that under the assumption $\mu > 0$, $\frac{\theta_2}{\psi} > -\frac{2\mu+\theta_1\sigma^2}{\psi\sigma^2}$. Therefore the first term dominates and determines the tail behavior of $m\left(Y_j > y\right)$ for large y. This proves the lemma. □

C.4 Proof of Proposition 4

The optimal compensation policy in equation (29) implies

$$\ln C_{j,.t} = \max\left\{\ln c\left(u_{MIN}\right) + \psi \ln Z_{j,t},\ \ln C_0\right\}.$$

Under the optimal investment policy, the law of motion of $Z_{j,t}$ is given by:

$$d\ln Z_{j,t} = (\iota - \delta)\,dt + \sigma dB_{j,t},$$

where the Brownian motion $B_{j,t}$ is independent across j. Under Assumption 2, $\iota - \delta > 0$. Using the result of Lemma A.5 above, for y large,

$$m\left(\ln C_{j,.t} > y\right) \sim -\frac{\psi}{\xi\sqrt{\psi^2\left(\iota-\delta\right)^2 + 2\kappa\psi^2\sigma^2}} e^{\frac{\xi}{\psi}\left[y-\left(\ln c(u_{MIN})+\psi\ln C_0\right)\right]}.$$

Clearly, the right tail of C obeys a power law with tail slope $\frac{\xi}{\psi}$.

D Solution to the general model

In this section, we describe the solution to the general model with a decreasing returns to scale matching technology and assortative matching. We also provide details of the numerical procedure that we use to solve the model.

D.1 The outside options of the firms and the managers

We start from a description of the equilibrium conditions that our model has to satisfy. Thanks to the symmetric matching rule, equilibrium quantities can be determined by jointly solve for the outside options of firms and managers, $V^S(Y)$ and $U^S(X)$, and the value function $V(Z,U)$ that satisfy the following conditions.

1. Given $U^S(X)$ and $V^S(Y)$, the domain of the value function, $V(Z,U)$, is

$$\left\{(Z,U) : Z \geq \hat{Z} \text{ and } U \in \left[U_{MIN}(Z), U_{MAX}(Z)\right]\right\}$$

with

$$\hat{Z} = \min\left\{Z : U_{MAX}(Z) \geq U_{MIN}(Z)\right\}. \tag{D.1}$$

Here, given $Z \geq \hat{Z}$, $U_{MIN}(Z)$ is the lower bound of the manager's continuation value which represents his outside option and then

$$U_{MIN}(Z) = U^S(\lambda Z).$$

Similarly, $U_{MAX}(Z)$ is the upper bound representing the firm-side limited commitment constraint. Obviously, higher promised value to the manager makes the firm value lower than that of its outside option. Namely, on the upper bound,

$$V(Z, U_{MAX}(Z)) = V^S(\lambda Z).$$

Because the matching technology exhibits decreasing return to scales, when the manager's human capital, X, and the firm's organization capital Y are too small, their outside options after separating are better than what they receive under a contract. In fact, when $Z < \hat{Z}$, the level of Z at which the upper- and lower-bound function U_{MAX} and U_{MIN} intersect, the limited commitment constraints on the two sides cannot be satisfied simultaneously so that the contract is not feasible. Hence, $V(Z,U)$ is the solution to the following optimal contracting problem:

$$\begin{aligned} V(Z,U) &= E\left[\int_0^{\tau_D \wedge \tau_S} e^{-rt}\left[AZ_t - C_t - h\left(\frac{I_t}{Z_t}\right)Z_t\right]dt + \mathbb{1}_{\{\tau_S < \tau_D\}} e^{-r\tau_S} V^S\left(\lambda Z_{\tau_S}\right)\right] \\ &\quad \text{subject to } Z_t \geq \hat{Z} \text{ and } U_t \in \left[U_{MIN}(Z_t), U_{MAX}(Z_t)\right] \end{aligned} \tag{D.2}$$

2. Given $V(Z,U)$, $\bar{U}(X)$ is consistent with firms' optimality under the symmetric matching rule:

$$V_Z\left(X^{\psi_Y+\psi_X}, \bar{U}(X)\right)\psi_X X^{\psi_Y+\psi_X-1} + V_U\left(X^{\psi_Y+\psi_X}, \bar{U}(X)\right)\frac{d}{dX}\bar{U}(X) = 0. \tag{D.3}$$

3. Given $V(Z,U)$, $\bar{V}(Y)$ is determined by the maximum profit firms can achieve on the matching market under the symmetric matching rule:

$$\bar{V}(Y) = V\left(Y^{\psi_Y+\psi_X}, \bar{U}(Y)\right).$$

4. Given $\bar{U}(X)$ and $\bar{V}(Y)$, the value functions upon separation, $U^S(X)$ and $V^S(X)$ are given by (7) and (9) respectively.

D.2 The normalized value function

Now, we introduce the normalization of the state variables and the value functions that we used in our computation. Since Z and U grows exponentially in equilibrium, it is convenient to use $\ln\left(\frac{U}{Z}\right)$ and $\ln Z$ as the state variables. To simplify notations, we denote

$$z = \ln Z \text{ and } \omega = \ln U - \ln Z,$$

and work with the normalized value function: $v(z,\omega) = e^{-z}V\left(e^{k}, e^{\omega+z}\right)$. Then we have

$$\begin{aligned}
V_Z(Z,U) &= v(z,\omega) + v_z(z,\omega) - v_\omega(z,\omega), \\
V_U(Z,U) &= \frac{Z}{U} v_\omega(z,\omega), \\
V_{ZZ}(Z,U) &= \frac{1}{Z}\left[v_z(z,\omega) - v_\omega(z,\omega) + v_{zz}(z,\omega) - 2v_{z\omega}(z,\omega) + v_{\omega\omega}(z,\omega),\right] \\
V_{UU}(Z,U) &= \frac{Z}{U^2}\left(-v_\omega(z,\omega) + v_{\omega\omega}(z,\omega)\right) \\
\text{and } V_{ZU}(Z,U) &= \frac{1}{U}\left(v_\omega(z,\omega) + v_{z\omega}(z,\omega) - v_{\omega\omega}(z,\omega)\right).
\end{aligned}$$

We also define the normalized lower and upper bounds of ω:

$$\begin{aligned}
\omega_{MIN}(z) &= \ln\left(\frac{U_{MIN}(\exp(z))}{\exp(z)}\right) = \ln\left(\frac{U_{MIN}(Z)}{Z}\right); \\
\omega_{MAX}(z) &= \ln\left(\frac{U_{MAX}(\exp(z))}{\exp(z)}\right) = \ln\left(\frac{U_{MAX}(Z)}{Z}\right).
\end{aligned}$$

Therefore, (B.3) becomes

$$0 = \max_{c,i,g}\left\{\begin{array}{c}
[A - c - h(i)] - v(z,\omega)(r+\kappa+\delta-i) + v_z(z,\omega)\left(i-\delta+\frac{1}{2}\sigma^2\right) \\
+v_\omega(z,\omega)\left[\begin{array}{c}\frac{r+\kappa}{1-\gamma}\left(1-\left(\frac{c}{\exp(\omega)}\right)^{1-\gamma}\right) + \frac{\kappa_S}{1-\gamma}\left(1-\left(\frac{\exp(\omega_{MIN}(z))}{\exp(\omega)}\right)^{1-\gamma}\right) \\ +\frac{1}{2}\sigma^2\left(\gamma g^2 - (g-1)^2\right) - (i-\delta)\end{array}\right] \\
+\frac{1}{2}v_{zz}(z,\omega)\sigma^2 + \frac{1}{2}v_{\omega\omega}(z,\omega)(g-1)^2\sigma^2 + v_{z\omega}(z,\omega)(g-1)\sigma^2 + \kappa_S v(z,\omega_{MAX}(z))
\end{array}\right\}, \tag{D.4}$$

The first-order conditions implies the following optimal choices of c, i and g (recall that $h(i) = i + \frac{1}{2}h_0 i^2$):

$$\begin{aligned}
c^* &= \exp(\omega)\left(-(r+\kappa)v_\omega(z,\omega)\right)^{\frac{1}{\gamma}}, \\
i^* &= \frac{1}{h_0}\left(v(z,\omega) + v_z(z,\omega) - v_\omega(z,\omega) - 1\right), \\
g^* &= -\frac{v_\omega(z,\omega) + v_{z\omega}(z,\omega) - v_{\omega\omega}(z,\omega)}{(\gamma-1)v_\omega(z,\omega) + v_{\omega\omega}(z,\omega)}.
\end{aligned} \tag{D.5}$$

Therefore, the HJB equation (D.6) becomes the following PDE.

$$0=\left\{\begin{array}{c} A-\exp(\omega)\left(-\left(r+\kappa\right)v_{\omega}\right)^{\frac{1}{\gamma}}-h\left(\frac{1}{h_0}\left(v+v_z-v_{\omega}-1\right)\right) \\ -v\left(r+\kappa+\delta-\frac{1}{h_0}\left(v+v_z-v_{\omega}-1\right)\right)+v_z\left[\frac{1}{h_0}\left(v+v_z-v_{\omega}-1\right)-\delta+\frac{1}{2}\sigma^2\right] \\ +v_{\omega}\left[\begin{array}{c}\frac{r+\kappa}{1-\gamma}\left(1-\left(-\left(r+\kappa\right)v_{\omega}\right)^{\frac{1-\gamma}{\gamma}}\right)+\frac{\kappa_S}{1-\gamma}\left(1-\left(\frac{\exp(\omega_{MIN}(z))}{\exp(\omega)}\right)^{1-\gamma}\right) \\ +\frac{1}{2}\sigma^2\left(\gamma g^2-\left(g-1\right)^2\right)-\left(\frac{1}{h_0}\left(v+v_z-v_{\omega}-1\right)-\delta\right)\end{array}\right] \\ +\frac{1}{2}v_{zz}\sigma^2+\frac{1}{2}v_{\omega\omega}\left(-\frac{\gamma v_{\omega}+v_{z\omega}}{(\gamma-1)v_{\omega}+v_{\omega\omega}}\right)^2\sigma^2+v_{z\omega}\left(-\frac{\gamma v_{\omega}+v_{z\omega}}{(\gamma-1)v_{\omega}+v_{\omega\omega}}\right)\sigma^2+\kappa_S v\left(z,\omega_{MAX}(z)\right)\end{array}\right\}, \tag{D.6}$$

With a little abuse of notations, we denote the policy functions characterizing the optimal contract by $c\left(z,\omega\right)$, $i\left(z,\omega\right)$ and $g\left(z,\omega\right)$. Then under the optimal contract and prior to the death and separation shocks,

$$\begin{aligned} dz &= \left(i\left(z,\omega\right)-\delta-\frac{1}{2}\sigma^2\right)dt+\sigma dB \\ d\omega &= \left[\begin{array}{c}\frac{r+\kappa}{1-\gamma}\left(1-\left(\frac{c(z,\omega)}{\exp(\omega)}\right)^{1-\gamma}\right)++\frac{\kappa_S}{1-\gamma}\left(1-\left(\frac{\exp(\omega_{MIN}(z))}{\exp(\omega)}\right)^{1-\gamma}\right) \\ -\left(i\left(z,\omega\right)-\delta\right)+\frac{1}{2}\left(\gamma g\left(z,\omega\right)^2-\left(g\left(z,\omega\right)-1\right)^2\right)\sigma^2\end{array}\right]dt \\ &\quad +\left(g\left(z,\omega\right)-1\right)\sigma dB. \end{aligned}$$

As in (D.1), define

$$\hat{z}=\min\left\{z:\omega_{MAX}(z)\geq\omega_{MIN}(z)\right\}.$$

Because of the two-sided limited commitment, the domain of the value function is

$$\left\{(z,\omega):\omega\in\left[\omega_{MIN}(z),\omega_{MAX}(z)\right]\text{ and }z\geq\hat{z}\right\}.$$

For any $z\geq\hat{z}$, on the upper bound, $\{(z,\omega):\omega=\omega_{MAX}(z)\}$, the expected drifts dz and $d\omega$ satisfy $dz+\omega'_{MAX}(z)d\omega\leq 0$ and the diffusion rates of satisfy

$$g(z,\omega)-1=\omega'_{MAX}(z)\text{ or }-\frac{\gamma v_{\omega}\left(z,\omega\right)+v_{z\omega}(z,\omega)}{\left(\gamma-1\right)v_{\omega}(z,\omega)+v_{\omega\omega}(z,\omega)}=\omega'_{MAX}(z). \tag{D.7}$$

On the lower bound, $\{(z,\omega):\omega=\omega_{MIN}(z)\}$, the drifts satisfy $dz+\omega'_{MIN}(z)d\omega\geq 0$. and the diffusion rates of z and ω satisfy

$$g(z,\omega)-1=\omega'_{MIN}(z)\text{ or }-\frac{\gamma v_{\omega}\left(z,\omega\right)+v_{z\omega}(z,\omega)}{\left(\gamma-1\right)v_{\omega}(z,\omega)+v_{\omega\omega}(z,\omega)}=\omega'_{MIN}(z). \tag{D.8}$$

Once z reaches $\hat{z}$, the the manager and the firm separate.

D.3 Computation procedure

In this subsection, we describe the computation procedure for the numerical solution of the value function $v(z, \omega)$ and its domain. Briefly speaking, we start from a simple initial guess and conduct a sequence of rounds of an outer iteration until it converges. In each round of the iteration we update (i) the value functions of an unemployed manager and an idle firm when they receive a matching opportunity; (ii) the value functions when they remain unmatched; and then (iii) their outside options and the value function of the firm under the optimal contract.

We set v_0, the initial guess of v, based on the first-best value function of the firm characterized in (18), and set the outside option of the manager to be extremely low. The reason for doing so is that, after each round of the iteration, the firm's value function is lowered over its entire domain and the region of the (z, ω)-space in which the two-sided limited commitment constraints are satisfied shrinks. This feature makes our computation procedure easier. Specifically, according to (18), the first-best value function under the normalization is

$$v_0(z, \omega) = e^z \bar{v} - \frac{1}{r + \kappa} \exp(\omega + z).$$

Then we choose upper and lower bounds for z and ω as the initial domain[1] which is a rectangle in the (z, ω)-space. We choose the lower bound of z to be -5 which is low enough such that when the iteration converges the minimal feasible level of z, $\hat{z}$, is interior. Since, in theory, there is no upper bound of z, but, for the computation purposes, we have to set an upper bound over which the value of v is not updated. Hence, we have to make sure that the upper bound is high enough such that further raising it would not affect our simulation results. Therefore, we choose the upper bound of z to be 12.

Given any $z \in [-5, 12]$, we set the upper bound of ω to

$$\omega^0_{MAX}(z) = \ln\left((r + \kappa)\, \bar{v}\right) \text{ for all } z,$$

which is the highest level of ω that a firm can promise to the manager under the first-best contract with a non-negative firm value. We set the lower bound of ω to be

$$\omega^0_{MIN}(z) = \underline{\omega}^0_{MIN} \text{ for all } z,$$

with $\underline{\omega}^0_{MIN}$ being the normalized value of the manager of permanent unemployment, namely, $\tau_M = 0$. Starting with this initial guess, we conduct an outer iteration. Each round of the outer iteration consists of the following three steps.

Step 1: Let the initial guess we input at the beginning of a round be $v_n(z, \omega)$, and the upper

[1] Notice that the domain of the value function should be endogenously determined by the outside options of the manager and the firm in equilibrium. This condition will be satisfied when the outer iteration converges.

and lower bound of ω for each z be

$$\omega^n_{MAX}(z) \text{ and } \omega^n_{MIN}(z)$$

respectively. Define

$$\hat{z}^n = \min\{z : \omega^n_{MAX}(z) \geq \omega^n_{MIN}(z)\} \text{ and } \hat{\omega}^n = \frac{1}{2}\left[\omega^n_{MAX}\left(\hat{z}^n\right) + \omega^n_{MIN}\left(\hat{z}^n\right)\right].^2$$

Given $v^n(z,\omega)$ and its domain, we characterize the normalized value function of an unemployed manager upon a match, $\bar{\omega}^n(x)$,[3] by solving the ODE (D.3), which is equivalent to

$$\frac{d\bar{\omega}^n(x)}{dx} = (\varphi_X - 1) - \varphi_X \frac{v_z\left(\bar{\omega}^n(x), (\varphi_X + \varphi_Y)\, x\right)}{v_\omega\left(\bar{\omega}^n(x), (\varphi_X + \varphi_Y)\, x\right)}$$

with a boundary condition $\bar{\omega}^n\left(\hat{z}^n\right) = \hat{\omega}^n$. Then, the normalized value function of an idle firm when it is matched with an manager is

$$\bar{v}^n(y) = v^n\left(\left(\varphi_X + \varphi_Y\right) y, \bar{\omega}^n(y)\right).$$

Step 2: We characterize the value of the firm and the manager of their outside options or upon a separation, $\omega^{S,n}(x)$ and $v^{S,n}(y)$, based on $\bar{\omega}^n(x)$ and $\bar{v}^n(y)$. The un-normalized value functions of an unemployed manager and an idle firm when a matching opportunity arrives can be written as

$$\bar{U}^n(X) = X \exp\left(\bar{\omega}^n\left(\ln(X)\right)\right) \text{ and } \bar{V}^n(Y) = Y \bar{v}^n\left(\ln(Y)\right)$$

respectively. By plugging in these two value functions into the right-hand sides of (7) and (9), we solve the value functions when they separate, $U^{S,n}(X)$ and $V^{S,n}(Y)$. According to our normalization,

$$\omega^{S,n}(x) = \ln\left(U^{S,n}\left(\exp(x)\right)\right) - x \text{ and } v^{S,n}(y) = \frac{1}{\exp(y)} V^{S,n}\left(\exp(y)\right).$$

Step 3: Given $\omega^{S,n}(x)$ and $v^{S,n}(y)$, we solve $v_{n+1}(z,\omega)$, the updated value function, and the domain using Markov-Chain approximation.[4] Since the manager's outside option has the normalized value $\omega^{S,n}(x)$, we update the lower bound of ω of the domain of the value function to

$$\omega^{n+1}_{MIN}(z) = \omega^{S,n}(\ln(\lambda) + z)$$

and keep the upper bound $\omega^n_{MAX}(z)$. We then plug $v_n(z,\omega)$ into the right-hand side of the HJB (D.6), set $\omega_{MIN}(z)$ to be $\omega^{n+1}_{MIN}(z)$, and set $\omega_{MAX}(z)$ to be $\omega^n_{MAX}(z)$. Then, we conduct an inner iteration process of Markov-Chain approximation. In each round of the iteration, the optimal

[2] Obviously, $\hat{\omega}^n = \omega^n_{MAX}\left(\hat{z}^n\right) = \omega^n_{MIN}\left(\hat{z}^n\right)$ if the upper and lower bound functions intersect in the interior region of our compuational domain.

[3] Here, we choose the normalization $x = \ln(X)$ and $y = \ln(Y)$.

[4] See Kushner and Dupuis (2001).

policies in the interior of the domain are given by (D.5), with the derivatives of the updated value function. However, on the upper bound of ω of the domain we set the sensitivity $g(z,\omega)$ according to (D.7) and on the lower bound we set it according to (D.8). We keep the inner iteration process until it converges. The limit value function is $v_{n+1}(z,\omega)$. Notice that the value function sinks as we keep updating it so that the limited commitment constraint on the firm's side would not be satisfied, because the firm value on the upper bound $\omega^n_{MAX}(z)$ would be strictly lower than the firm's outside option. Therefore, we the update the upper bound to

$$\omega^{n+1}_{MAX}(z) = \max\left\{\omega : v_{n+1}(z,\omega) \geq v^{S,n}\left(\ln(\lambda)+z\right)\right\}.$$

Intuitively, we define the updated domain of the value function as the region in the (z,w)-space where the firm-side limited commitment constraint is not violated. We take $v_{n+1}(z,\omega)$, $\omega^{n+1}_{MAX}(z)$, and $\omega^{n+1}_{MIN}(z)$ back to Step 1 of a new round of the outer iteration.

E Calibration details

E.1 Equilibrium relationships

In this section, we derive several equilibrium relationships that are used in solving and calibrating our model.

Marginal product of capital Note that in our model, the marginal product of organization capital of firm j is

$$A_j = (1-\psi)\, Z_j^{-\nu}\left(K_j^{\alpha} N_j^{1-\alpha}\right)^{\nu}.$$

Because physical capital and labor move free across firms, in equilibrium, $K_j = \frac{Z_j}{\mathbf{Z}}\mathbf{K}$, where $\mathbf{K}$ is the total stock of the physical capital in the economy, and $\mathbf{Z}$ denotes the total stock of organization capital. Also, $N_j = \frac{Z_j}{\mathbf{Z}}$ because total labor supply is normalized to one. Therefore, the marginal product of organization capital does not depend on Z_j:

$$A = (1-\nu)\,\mathbf{Z}^{-\nu}\mathbf{K}^{\alpha\nu}. \tag{E.1}$$

The total amount of organization capital $\mathbf{Z}$ in the economy in the first best case can be easily solved in closed form. Note that the organization capital of a firm starts at $\bar{Z}$, grows at rate of $\hat{\imath}-\delta$ and vanishes at the rate of κ. As a result, the expected value of a firm's organization capital is $\frac{\bar{Z}}{\kappa+\delta-\hat{\imath}}$. Because we normalize the entry rate so that the total measure of operating firms is one,

$$\mathbf{Z} = \frac{\bar{Z}}{\kappa+\delta-\hat{\imath}}; \quad A = (1-\nu)\left(\frac{\bar{Z}}{\kappa+\delta-\hat{\imath}}\right)^{-\nu}\mathbf{K}^{\alpha\nu}. \tag{E.2}$$

in the first best case. Equations (19) and (E.2) jointly determine the equilibrium quantities: A and

$\hat{\imath}$.

The marginal product of physical capital of firm j is

$$MPK_j = \alpha\nu K_j^{\alpha\nu-1} Z_j^{1-\nu} N_j^{\nu(1-\alpha)}.$$

Using $K_j = \frac{Z_j}{\mathbf{Z}}\mathbf{K}$ and $N_j = \frac{Z_j}{\mathbf{Z}}$, the marginal product of physical capital,

$$MPK = \alpha\nu \mathbf{K}^{\alpha\nu-1}\mathbf{Z}^{1-\nu} \tag{E.3}$$

does not depend on j. Comparing (E.3) with (E.1), we have:

$$\frac{A}{MPK} = \frac{1-\nu}{\alpha\nu}k, \tag{E.4}$$

where $k \equiv \frac{\mathbf{K}}{\mathbf{Z}}$ is the economy-wide physical capital to organization capital ratio.

Because we have a partial equilibrium model with the constant returns to scale production function, to determine the scale of the economy, we normalize total labor supply to 1. Given this normalization, the marginal product of physical capital and that of organization capital depends on the total stock of organization capital, $\mathbf{Z}$, and the ratio of physical capital to organization capital, k.

The total aggregate of organization capital, $\mathbf{Z}$, depends on the initial size of the firm, $\bar{Z}$ and the entry rate of firms, $\bar{e}$. We normal $\bar{e}$ so that the total measure of operating firms in this economy equals 1.[5] We then choose $\bar{Z}$ so that $A = 0.21$, which allows our model to match a median sales growth of 3.2% per year in the data. Without calibrating $\mathbf{Z}$ directly, this procedure pins down the equilibrium stock of organization capital. The relationship between firm growth rate and $\mathbf{Z}$ is clear from equation (E.2) in the first best case. In general, there is a one-to-one mapping between growth rate and $\mathbf{Z}$. Therefore, given other parameter values, there is a one-to-one mapping between the equilibrium marginal product of organization capital, A and average firm growth rate, and we use this relationship to calibration A.

In our setup, given A, the stock of physical capital does not affect the model's implications for firm size and CEO pay. However, it is relevant for the calculation of model-implied Tobin's Q, which we use to pin down the adjustment cost parameter h_0. We use the relationship (E.4) to calibrate k. We set $MPK = 0.14$ to match the marginal product of capital in standard RBC models. Given A and MPK, equation (E.4) determines the ratio of physical capital and organization capital, k. We now turn to the normalization procedure for firm entry rate, $\bar{e}$ and the calculation of Tobin's Q.

Steady state measure of firms Let m and s be the steady-state total measure of operating firms, i.e. matched manager-firm pairs, and that of idle firms, respectively. The rate of death of matched firms is $m\kappa_D$, and the rate of separation is $m\tilde{\kappa}_S$, where $\tilde{\kappa}_S$ is the equilibrium separation

[5]We describe the details of this normalization procedure in the next section.

rate, which includes both endogenous separation and exogenous separation.[6] The rate of death of unmatched firms is $s\kappa_D$ and the rate of newly formed match is $s\kappa_M$. In a steady-state equilibrium, the entry and exit into the pool of operating firms must be equal:

$$\bar{e} + s\kappa_M = m\left(\tilde{\kappa}_S + \kappa_D\right),$$

and the entry and exit into the pool of unmatched firm must also equal:

$$m\tilde{\kappa}_S = s\left(\kappa_M + \kappa_D\right).$$

The above two equations imply that if we choose

$$\bar{e} = \frac{\kappa_M + \kappa_D}{\kappa_M\kappa_D + \kappa_D\left(\kappa_S + \kappa_D\right)},$$

then the steady-state measure of operating firms is 1. Given $\bar{e}$, the steady-state total measure of matched and idle firms are given by:

$$m = 1; \quad s = \frac{\tilde{\kappa}_S}{\kappa_M + \kappa_D}.$$

Tobin's Q Tobin's Q is defined as the total value of firm's capital income divided by the firm's capital stock. Here capital income includes compensation to both physical capital and organization capital. The present value of compensation to organization capital is:

$$p\left(u\right)Z = E\left[\int_0^\infty e^{-rt}\left[AZ_t - h\left(\frac{I_t}{Z_t}\right)Z_t\right]dt\right].$$

Therefore the Tobin's Q of a firm is

$$\frac{p\left(u\right)Z + K}{K} = 1 + \frac{p\left(u\right)}{k}. \tag{E.5}$$

If we choose the marginal product of physical capital to be $MPK = 14\%$, Given the calibrated parameter values $\nu = 0.75$, $A = 0.21$, and $\alpha = 0.36$, we have $k = \frac{\nu\alpha}{1-\nu}\frac{A}{MPK} = 1.62$. The function $p\left(u\right)$ is implied by our model. Given $p\left(u\right)$ and k, equation (E.5) can be used to construct model implied Tobin's Q.

F Power-Law Estimates

As stated in the paper, the probability distribution function of a continuous power-law random variable x is given by:

$$f(x) = k\zeta x^{-(1+\zeta)}, \tag{F.1}$$

[6]Note that $\tilde{\kappa}_S = \kappa_S$ in the constant returns to scale model because there is no endogenous separation.

where $k = x_{min}^{\zeta}$, x_{min} is the lower bound of the power-law behavior, and ζ is the power-law exponent. It is common in empirical work to treat x_{min} as if it were known (typically by choosing a point beyond which the empirical distribution appears approximately linear on a log-log plot) and estimate the scaling parameter ζ by maximum likelihood. However, unless the right-tail cutoff is chosen at or close to the true value, the estimates of the exponent may be significantly biased. To address this issue, we estimate both parameters by minimizing the Kolmogorov-Smirnov (KS) distance. In particular, for each potential lower bound $\tilde{x}$, we estimate the power-law exponent using the data above $\tilde{x}$ as:

$$\tilde{\zeta} = N\left[\sum_{i=1}^{N} \ln \frac{x_i}{\tilde{x}}\right]^{-1}, \quad x_i \geq \tilde{x},\ i = 1, ..., N. \tag{F.2}$$

Our estimates of x_{min} and ζ is the pair that yields the power-law distribution that provides the best fit to the observed data according to the KS criteria, i.e.,

$$\{\hat{x}_{min}, \hat{\zeta}\} = \min_{\tilde{x},\tilde{\zeta}} \Big\{ KS\!-\!distance \Big\} \equiv \min_{\tilde{x},\tilde{\zeta}} \Big\{ \max_{x\geq\tilde{x}} \big| F(x;\tilde{x},\tilde{\zeta}) - \hat{F}(x) \big| \Big\}, \tag{F.3}$$

where $F(x;\tilde{x},\tilde{\zeta})$ is the candidate power-law cumulative distribution function and $\hat{F}(x)$ is the empirical distribution.

G Organization-Capital Based Evidence

In this section, we briefly discuss the robustness of our baseline evidence with respect to an alternative proxy for organization capital. We follow the procedure of Eisfeldt and Papanikolaou (2013, 2014) and construct the stock of organization capital by cumulating firms' selling, general, and administrative (SG&A) expenses using the perpetual inventory method.

We find that physical and organizational capital are highly correlated, and so are the corresponding investment rates. In particular, on average, the cross-sectional correlations between investment rates of physical and organizational capital are 0.95, 0.93, 0.95, 0.95 and 0.81 within Consumer, Manufacturing, HiTec, Health and Other industries, respectively.[7]

Table A.I further confirms a strong positive relationship between the two investment rates. It reports variations in physical and organization-capital investment rates (I/K and I°/K°, respectively) across portfolios sorted on either gross capital (K) or organization capital (K°). As the table shows, sorting on either measure of firm size generates a strong declining pattern in the two investment rates, i.e., both I/K and I°/K° are inversely related to both physical capital and organization capital. We also find that firms with low stock of organization capital are less likely to pay dividends compared with large firms, and that the managerial share declines in organization capital. This evidence is reported in Table A.II below. In all, we find that the cross-sectional distribution of organization-capital investment is consistent with the key model's predictions.

[7]We examine the within-industry relationship between investment rates to account for the variation in prevailing accounting practices regarding SG&A expenses across industries.

H Cross-Industry Analysis

Our model has a unique set of cross-sectoral implications. In particular, our model predicts that the impact of agency frictions increases with the returns to scale of the matching technology. Hence, industries with less decreasing returns to scale of the matching technology, feature high managerial compensation relative to firm size and a strong inverse relationship between firm investment and size, whereas industries with significantly decreasing returns to scale of the matching technology (i.e., $\psi \ll 1$) are characterized by a relatively low ratio of CEO pay to size and a relatively thin right tail of the distribution of CEO compensation.

In this section, we evaluate the cross-sectoral implications of the model using a set of five industry portfolios. We first sort all firms into five industries: "Consumer", "Manufacturing", "Hitech", "Health", and "Others", and compute their average CEO compensation.[8] As Table A.III shows, we find a sizable dispersion in the average CEO pay to size ratio across the five industries. In particular, keeping firm size equal, CEOs in "Hitech" and "Health" sectors typically earn three to four times more relative as their peers in other industries. We confirm below that the gap in managerial compensation between the two groups is highly statistically significant. According to our model, this evidence suggests that organization capital in "Hitech" and "Health" sectors is highly valuable in the sense that it can be productively employed in another match if managers decide to voluntarily separate with their current employers. That is, "Hitech" and "Health" industries are likely to have a relatively high returns to scale of the matching technology. In contrast, industries like "Consumer" and "Manufacturing" are likely to feature a significantly decreasing returns to scale of the matching technology.

Guided the model's intuition, we group the five industries into two categories: "High-ψ" group that consists of "Hitech" and "Health" sectors, and "Low-ψ" group that consists of the remaining industries.[9] We next examine if the two industry groups feature any discernible differences in their compensation and investment policies as implied by our model.

First, consistent with the model's predictions and our classification of industries, we find that in "High-ψ" group, the power-law coefficient of CEO pay is closer to that of firm size, whereas CEO compensation in "Low-ψ" group has a considerably thinner tail compared to the distribution of firm size. For each industry group, we estimate the power-law exponents for firm size measured by the number of employees and CEO compensation year by year over the 1992-2016 sample period, and in Table A.IV we report time-series averages on the estimated coefficients. Recall that the ratio of power law in firm size to power law in CEO pay measures the degree of returns to scale of the matching technology, ψ (see Proposition 4). As the table shows, "High-ψ" group indeed features a higher ψ compared with "Low-ψ" group. In particular, the implied returns to scale of the matching technology of "Low-ψ" and "High-ψ" industries are about 0.47 and 0.75, respectively.

[8] We use the Fama-French industry definition that is available on Kenneth French website.

[9] We combine the five industries into two groups to gain statistical power, which is particularly important in analyzing the tail behavior of the empirical distributions of firm size and CEO compensation.

We next examine CEO compensation and investment policies in the two industry groups and compare them with the implications of our model calibrated under two alternative assumptions for the returns to scale of the matching technology: $\psi = 0.47$ and $\psi = 0.75$ that correspond to our empirical estimates. To evaluate the cross-sectional variation in policy decisions, we sort firms in each industry group into three portfolios based on firm size.[10] To save space, we report average rates only for the bottom and top tercile portfolios, i.e., small and large firms.

Table A.V shows that in the data, CEO compensation to firm size ratio differs significantly in the two industry groups. The difference in the average CEO pay to size ratio between "High-ψ" and "Low-ψ" sectors is 3% with a t-statistic of 8.9. Further, consistent with the model's implications, the cross-sectional dispersion in managers' equity share is significantly larger in "High-ψ" group compared with "Low-ψ" group.

Table A.VI presents the average investment rates and their cross-sectional dispersion in the two sectors. First, notice that on average, firms in "High-ψ" industry group invest at a higher rate compared with "Low-ψ" industries. The difference in investment rates is about 2.4% per annum with a t-statistics of 2.3. Second, consistent with the model's predictions, "High-ψ" group features a significantly larger cross-sectional variation in investment rates relative to "Low-ψ" group. On average, compared with large firms, small firms invest by 6.3% and 9% more in "Low-ψ" and "High-ψ" sectors, respectively, and the difference in small-minus-large spreads between the two groups is strongly statistically significant. Intuitively, the higher the magnitude of the returns to scale of the matching technology, the more severe agency frictions and hence, the stronger the negative relationship between firm investment and size. Our calibrated model matches well the observed dispersion in investment rates, in particular, the large-minus-small spread in investment rates implied by the model increases in magnitude from -6.7% for "Low-ψ" sector to -8.8% for "High-ψ" sector.

[10] In Tables A.V-A.VI, we use gross capital as a measure of firm size; the evidence is similar if size is measured by the number of employees.

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Table A.I

Investment Rates

	Sorting on Physical Capital (K)		Sorting on Organization Capital (K°)	
	I/K	I°/K°	I/K	I°/K°
Small	0.182	0.246	0.166	0.270
2	0.140	0.223	0.124	0.245
3	0.125	0.216	0.114	0.228
4	0.107	0.204	0.103	0.214
Large	0.088	0.193	0.095	0.194
Large−Small	−0.094	−0.053	−0.071	−0.076
	(−5.65)	(−4.09)	(−5.86)	(−5.09)

Table A.I presents average investment rates of physical and organization capital, I/K and I°/K°, respectively, of quintile portfolios sorted on either physical (K) or organization (K°) capital. T-statistics for the difference between large and small firms based on the Newey and West (1987) estimator with four lags are reported in parentheses.

Table A.II

Sorting on Organization Capital (K°)

	Div Payers Fraction	CEOpay/K°
Small	0.120	0.041
2	0.269	0.026
3	0.393	0.020
4	0.520	0.013
Large	0.751	0.005
Large−Small	0.631	−0.037
	(27.11)	(−9.55)

Table A.II presents the average fraction of dividend-paying firms and the median ratio of CEO compensation to organization capital (CEOpay/K°) of quintile portfolios sorted on organization capital. T-statistics for the difference between large and small firms based on the Newey and West (1987) estimator with four lags are reported in parentheses.

Table A.III

Industry Sort

Industry	C/K	Group
Consumer	0.0136	Low-ψ
Manufacturing	0.0045	Low-ψ
HiTec	0.0380	High-ψ
Health	0.0420	High-ψ
Other	0.0171	Low-ψ

Table A.III presents the median ratio of CEO compensation to gross capital for five industry portfolios and the corresponding classification of the industry sectors into two groups, "Low-ψ" and "High-ψ".

Table A.IV

Industry Sort: Power Law

	Power Law ($\hat{\zeta}$)		Implied ψ
Industry Group	Employees	CEO Pay	
Low-ψ	1.12	2.36	0.47
High-ψ	1.64	2.17	0.75

Table A.IV presents the estimates of the exponent of the power-law distribution (ζ) for the number of firm employees and CEO compensation in "Low-ψ" and "High-ψ" industry groups. The table reports time-series averages of the parameters estimated year-by-year in the 1992-2016 sample. The right panel shows the implied degree of returns to scale of the matching technology (ψ).

Table A.V

Industry Sort: CEO Pay to Firm Size Ratio

	Firm Size				Model
Industry Group	All	Small	Large	Large−Small	Large−Small
Low-ψ	0.009	0.032	0.002	−0.030	−0.070
High-ψ	0.039	0.094	0.005	−0.089	−0.098
High−Low	0.030			−0.058	−0.028
	(8.92)			(−8.01)	

Table A.V presents the median ratio of CEO compensation to gross capital in "Low-ψ" and "High-ψ" industry groups. Small and large firms represent the bottom and top size-sorted tercile portfolios, respectively. Size in the data is measured by gross capital. T-statistics for the difference between "High-ψ" and "Low-ψ" industry groups based on the Newey and West (1987) estimator with four lags are reported in parentheses. The model-implied spreads are presented in "Model" column.

Table A.VI

Industry Sort: Investment Rates

	Firm Size				Model
Industry Group	All	Small	Large	Large−Small	Large−Small
Low-ψ	0.091	0.149	0.086	−0.063	−0.067
High-ψ	0.115	0.198	0.107	−0.090	−0.088
High−Low	0.024			−0.027	−0.021
	(2.26)			(−3.59)	

Table A.VI presents the average investment-to-capital ratio in "Low-ψ" and "High-ψ" industry groups. Small and large firms represent the bottom and top size-sorted tercile portfolios, respectively. Size in the data is measured by gross capital. T-statistics for the difference between "High-ψ" and "Low-ψ" industry groups based on the Newey and West (1987) estimator with four lags are reported in parentheses. The model-implied spreads are presented in "Model" column.