Internet Appendix for "The Collateralizability Premium"

A Proof of propositions

A.1 Proof of Proposition ??

It is convenient to derive the optimality conditions for firms' profit maximization using the dynamic programming formulation. Define $V_t(N_t)$ as firms' value function at time t. We have, for $t = 0, 1$,

$$V_t(N_t) = \max\{D_t + E[M_{t+1}N_{t+1}]\} \tag{A1}$$

$$D_t + q_t K_{t+1} = N_t + B_t \tag{A2}$$

$$N_{t+1} = A_{t+1}K_{t+1} + (1-\delta)\, q_{t+1}K_{t+1} - r_t B_t \tag{A3}$$

$$B_t \leq \zeta q_t K_{t+1} \tag{A4}$$

$$D_t \geq 0. \tag{A5}$$

We first derive a set of optimality conditions that characterize the equilibrium. Taking first order conditions of (??) w.r.t. K_{t+1} and B_t, we have:

$$\begin{aligned}
\bar{\mu}_t q_t &= E\left[M_{t+1}\mu_{t+1}\{A_{t+1} + q_{t+1}(1-\delta)\}\right] + \zeta\eta_t, \\
\bar{\mu}_t &= E\left[M_{t+1}\mu_{t+1}r_t\right] + \frac{\eta_t}{q_t}.
\end{aligned}$$

The envelope condition implies $\mu_t = \bar{\mu}_t$, which we can use to simplify the above equations to write:

$$q_t = E\left[M_{t+1}\frac{\mu_{t+1}}{\mu_t}\{A_{t+1} + q_{t+1}(1-\delta)\}\right] + \zeta\frac{\eta_t}{\mu_t}, \tag{A6}$$

$$1 = E\left[M_{t+1}\frac{\mu_{t+1}}{\mu_t}r_t\right] + \frac{\eta_t}{\mu_t}\frac{1}{q_t}. \tag{A7}$$

Also, note that whenever the collateral constraint is binding, equations (A2) and (A4) can be combined to write:

$$(1-\zeta)\, q_t\left[(1-\delta)\, K_t + I_t\right] = N_t.$$

Using the capital producer's optimality condition, and the functional form of the adjustment cost, we have $q_t = 1 + \tau (i_t - \delta)$. The above equation can be written as:

$$n_t = (1-\zeta) [1 + \tau (i_t - \delta)] [1 + i_t - \delta] . \tag{A8}$$

Note that equation (A8) implicitly define i as a function of n, which we denote as $i(n)$. Given the definition of $i(n)$, we can write Tobin's q as $q_t = 1 + \tau [i(n_t) - \delta]$, and normalized consumption as $c_t = c(A_t, n_t)$, where

$$c(A, n) \equiv A - i(n) - \frac{1}{2} \tau [i(n) - \delta]^2 . \tag{A9}$$

Using the above results, we can solve for the prices and quantities in period 1. In period 2, all of firms' cash flow are paid back to household as consumption goods. Therefore $\mu_2 = 1$. In addition, capital is valueless at the end of period 2 and $q_2 = 0$. Therefore equations (A6) can be written as $\mu_1 q_1 = E[M_2 A_2] + \zeta \eta_1$, and equation (A7) can be written as $\mu_1 = E[M_2] r_1 + \frac{\eta_1}{q_1}$. Under the assumption of log preference, $M_2 = \frac{C_1}{C_2} = \frac{A_1 K_1 - H(I_1, K_1)}{A_2 K_2} = \frac{c(A_1, n_1)}{A_2[(1-\delta)+i(n_1)]}$, and therefore, $M_2 A_2 = \frac{c(A_1, n_1)}{(1-\delta)+i(n_1)}$. Also, the household's intertemporal Euler equation implies $E[M_2 r_1] = 1$. Equations (A6) and (A7) can be further simplified as:

$$q_1 = \frac{1}{\mu_1} \frac{c(A_1, n_1)}{(1-\delta) + i(n_1)} + \zeta \frac{\eta_1}{\mu_1}, \tag{A10}$$

$$\mu_1 = 1 + \frac{\eta_1}{q_1}. \tag{A11}$$

Combining equations (A10) and (A11), and using the fact that $q_1 = 1 + \tau [i(n_1) - \delta]$, we can determine η_1 and μ_1 as functions of (n_1, A_1):

$$\eta_1 (A_1, n_1) = \frac{1}{1-\zeta} \left\{ \frac{A_1 - i(n_1) - \frac{1}{2}\tau [i(n_1) - \delta]^2}{1 - \delta + i(n_1)} - [1 + \tau (i(n_1) - \delta)] \right\}, \tag{A12}$$

and

$$\mu_1 (A, n) = 1 + \frac{\eta_1 (A, n_1)}{1 + \tau (i(n_1) - \delta)}. \tag{A13}$$

Equation (**??**) then follows directly from (A10), (A12), and (A13).

To derive the law of motion of n_1, note that the binding collateral constraint in period 0 implies $B_0 = \zeta q_0 K_1$. Equation $N_1 = A_1 K_1 + p_1 K_1 - r_0 B_0$ therefore implies

$$n_1 = A_1 - (1-\delta) q_1 - r_0 \zeta q_0. \tag{A14}$$

Using the households' consumption Euler equation, we express the interest r_0 as a function of consumption:

$$r_0 = \frac{1}{\beta E\left[\frac{C_0}{C_1}\right]} = \frac{1}{\beta c\left(A_0, n_0\right)} \frac{1}{E\left[\frac{1}{c(A_1,n_1)}\right]}. \tag{A15}$$

Equation (**??**) then follows from (A14) and (A15) by noting $q_t = 1 + \tau\left[i\left(n_t\right) - \delta\right]$.

A.2 Proof of Proposition ??

We prove Proposition 2 in two steps. First, we construct an equilibrium and show that under the assumptions of parameter values, the collateral constraint (A4) for both period 0 and period 1 binds. Second, we explicitly solve for the expression of the Lagrangian multipliers η_1 and μ_1 to verify the counter-cyclicality of $\frac{\eta_1}{\mu_1}$, i.e., inequality (**??**).

Proposed equilibrium prices and quantities Note that under the assumption of $\beta = \tau = \delta = 1$, the $i\left(n\right)$ function in (A8) and $c\left(A, n\right)$ function in (A9) take simple forms:

$$i\left(n\right) = \sqrt{\frac{n}{1-\zeta}},\ c\left(A, n\right) = A - \frac{1}{2} - \frac{1}{2}\frac{n}{1-\zeta}. \tag{A16}$$

We propose the following equilibrium prices and quantities and verify that they indeed satisfy the above listed equilibrium conditions:[1]

$$c_t = c\left(A_t, n_t\right);\ i_t = i\left(n_t\right);\quad q_t = 1 + \tau\left[i\left(n_t\right) - \delta\right],\quad t = 0, 1 \tag{A17}$$

$$\eta_1 = \eta_1\left(A, n\right), \mu_1 = \mu_1\left(A, n\right), \tag{A18}$$

where

$$\eta_1\left(A, n\right) = \frac{1}{\sqrt{1-\zeta}\sqrt{n}}\left\{A - \frac{1}{2} - \frac{3}{2}\frac{n}{1-\zeta}\right\}, \tag{A19}$$

$$\mu_1\left(A, n\right) = 1 + \frac{1}{n}\left\{A - \frac{1}{2} - \frac{3}{2}\frac{n}{1-\zeta}\right\}. \tag{A20}$$

and the first period net worth is given by:

$$n_1 = n\left(A_1 \middle| n_0\right) = A_1 - x\left(n_0\right), \tag{A21}$$

[1] Because all optimization problems are convexity programming problems, the first order conditions are both necessary and sufficient.

where $x(n_0)$ is given by equation (A23) below.

It is straight forward to show that the proposed prices and quantities satisfy the first order conditions (A6) and (A7). Below we verify that under our assumptions, the constructed Lagrangian multipliers are strictly positive, and therefore, the proposed allocation is indeed an equilibrium in which the collateral constraints are binding in both periods.

Verifying equilibrium conditions We verify that the collateral constraint must be binding under the proposed prices and quantities through a sequence of lemmas.

Lemma 1. *(Law of motion of net worth)*

The law of motion of net worth can be written as

$$n(A|\,n_0) = A - x(n_0).$$

Given $A_1 > \frac{1-\zeta}{1-2\zeta}$, $x(n_0)$ is strictly increasing with $x(0)=0$ and $\lim_{n_0 \to 2(1-\zeta)^2\left(A_0-\frac{1}{2}\right)} x(n_0) = \infty$.

Proof. Because $\beta = 1$, $\delta = 1$, we can write equation (**??**) as $n(A|\,n_0) = A - \zeta \frac{i^2(n_0)}{c(A_0,n_0)} \frac{1}{E\left[\frac{1}{c(A,n(A|n_0))}\right]}$. Using the definition of $i(n)$ and $c(A,n)$, $n(A|\,n_0) = A - x$, where x is implicitly defined as $x = \frac{\zeta}{1-\zeta} \frac{n_0}{A_0 - \frac{1}{2} - \frac{n_0}{2(1-\zeta)}} \frac{1}{E\left[\frac{1}{c(A,n(A|n_0))}\right]}$. Note that by the definition of $c(A,n)$ (equation (A16)), with $n_1 = A_1 - x$, we have

$$\begin{aligned} c(A_1,n_1) &= A_1 - \frac{1}{2} - \frac{1}{2}\frac{A_1 - x}{1-\zeta} \\ &= \frac{x}{2(1-\zeta)} + \left[1 - \frac{1}{2(1-\zeta)}\right] A_1 - \frac{1}{2}. \end{aligned} \tag{A22}$$

Therefore, $x(n_0)$ as a function of n_0 is defined by the solution to the following equation:

$$E\left[\frac{x}{\frac{x}{2(1-\zeta)} + \left[\frac{1-2\zeta}{2(1-\zeta)}A_1 - \frac{1}{2}\right]}\right] = \frac{\zeta}{1-\zeta} \frac{n_0}{A_0 - \frac{1}{2} - \frac{n_0}{2(1-\zeta)}}. \tag{A23}$$

Under the condition that $A_1 > \frac{1-\zeta}{1-2\zeta}$, the left-hand side is an increasing function of x, and as x increases from 0 to ∞, $E\left[\frac{x}{\frac{x}{2(1-\zeta)} + \left[\frac{1-2\zeta}{2(1-\zeta)}A_1 - \frac{1}{2}\right]}\right]$ increases from 0 to $2(1-\zeta)$. In addition, the right-hand side of equation (A23) is a strictly increasing function of n_0, and as n_0 increases from 0 to $2(1-\zeta)^2\left(A_0 - \frac{1}{2}\right)$, the right-hand side increases from 0 to $2(1-\zeta)$. As a result, equation

(A23) defines $x(n_0)$ as a strictly increasing function that maps $n_0 \in \left(0, 2(1-\zeta)^2\left(A_0 - \frac{1}{2}\right)\right)$ to $x \in (0, \infty)$. $\square$

The next lemma provide conditions under which the collateral constraint must be binding in period 1.

Lemma 2. *(Binding constraint for period 1)*

Assume

$$x(n_0) > \frac{2}{3}A_1, \tag{A24}$$

then the collateral constraint in period 1 is binding for all realizations of A_1, that is, $\eta_1(A_1, n_1) > 0$.

Proof. By equation (A19), the borrowing constraint binds, that is, $\eta_1(A_1, n_1) > 0$ if and only if

$$A_1 - \frac{1}{2} > \frac{3}{2}\frac{n_1}{1-\zeta}. \tag{A25}$$

Using $n_1 = A_1 - x(n_0)$, the above condition can be written as

$$\frac{3}{2(1-\zeta)}x(n_0) > \left(\frac{3}{2(1-\zeta)} - 1\right)A_1 + \frac{1}{2}. \tag{A26}$$

Note that under condition $A_1 > \frac{1-\zeta}{1-2\zeta}$, $\frac{1-2\zeta}{2(1-\zeta)}A_1 > \frac{1}{2}$. Therefore, a sufficient condition for (A25) is

$$\frac{3}{2(1-\zeta)}x(n_0) > \left(\frac{3}{2(1-\zeta)} - 1 + \frac{1-2\zeta}{2(1-\zeta)}\right)A_1 = \frac{2}{2(1-\zeta)}A_1,$$

which is equivalent to (A19). $\square$

Our next lemma provides conditions under which the collateral constraint is binding in period 0.

Lemma 3. *Suppose*

$$n_0 < \frac{1}{2}(1-\zeta)\left(A_0 - \frac{1}{2}\right) \tag{A27}$$

and

$$x(n_0) < \frac{1}{2+\zeta}\left[(1+2\zeta)A_1 + \frac{1}{2}(1-\zeta)\right], \tag{A28}$$

then the collateral constraint in period 0 must be binding, that is, $\eta_0 > 0$.

Proof. Combining equations (A6) and (A7), $\eta_0 > 0$ if and only if

$$E\left[M_1\mu_1\left(A_1+(1-\delta)\,q_1\right)\right] > E\left[M_1\mu_1\right] r_0 q_0.$$

Using the fact that $M_1 = \frac{c(A_0,n_0)}{c(A_1,n_1)i(n_0)}$ and $r_0 = \frac{1}{E[M_1]}$, the above condition can be written as:

$$\frac{E\left[\frac{\mu_1 A_1}{c(A_1,n_1)}\right] E\left[\frac{1}{c(A_1,n_1)}\right]}{E\left[\frac{\mu_1}{c(A_1,n_1)}\right]}\left(A_0-\frac{1}{2}-\frac{1}{2\,(1-\zeta)}n_0\right) > \frac{n_0}{1-\zeta}. \tag{A29}$$

We first show that

$$\frac{E\left[\frac{\mu_1 A_1}{c(A_1,n_1)}\right] E\left[\frac{1}{c(A_1,n_1)}\right]}{E\left[\frac{\mu_1}{c(A_1,n_1)}\right]} > \frac{2\,(2+\zeta)}{2\zeta+3}. \tag{A30}$$

To see this, using (A22), we have

$$\begin{aligned}\frac{c\,(A_1,n_1)}{A_1} &= \frac{\frac{x}{2(1-\zeta)}+\left[1-\frac{1}{2(1-\zeta)}\right]A_1-\frac{1}{2}}{A_1}\\ &< \frac{\frac{x}{2(1-\zeta)}+\left[1-\frac{1}{2(1-\zeta)}\right]A_1}{A_1}\end{aligned}$$

Under assumption (A28), $x\,(n_0) < \frac{1+2\zeta}{2+\zeta}A_1$ and therefore,

$$\frac{c\,(A_1,n_1)}{A_1} < \frac{1}{2\,(1-\zeta)}\frac{1+2\zeta}{2+\zeta}+\left[1-\frac{1}{2\,(1-\zeta)}\right] = \frac{2\zeta+3}{2\,(2+\zeta)}. \tag{A31}$$

As a result,

$$\begin{aligned}E\left[\frac{\mu_1 A_1}{c\,(A_1,n_1)}\right]E\left[\frac{1}{c\,(A_1,n_1)}\right] &= E\left[\frac{\mu_1 A_1}{c\,(A_1,n_1)}\right]E\left[\frac{A_1}{c\,(A_1,n_1)}\frac{1}{A_1}\right]\\ &> \frac{2\,(2+\zeta)}{2\zeta+3}E\left[\frac{\mu_1 A_1}{c\,(A_1,n_1)}\right]E\left[\frac{1}{A_1}\right]\\ &> \frac{2\,(2+\zeta)}{2\zeta+3}E\left[\frac{\mu_1}{c\,(A_1,n_1)}\right],\end{aligned}$$

where the first inequality uses (A31) and the second inequality above uses the fact that $\frac{\mu_1 A_1}{c(A_1,n_1)}$ is increasing in A_1 and therefore negatively correlated with $\frac{1}{A_1}$. This establishes (A30).

Given (A30), a sufficient condition for (A29) is

$$\frac{2\zeta+3}{2\,(2+\zeta)}\left(A_0-\frac{1}{2}-\frac{1}{2\,(1-\zeta)}n_0\right) > \frac{n_0}{1-\zeta}. \tag{A32}$$

To see the above inequality holds, note that assumption (A27) implies

$$A_0 - \frac{1}{2} > \frac{2}{1-\zeta} n_0.$$

Therefore,

$$A_0 - \frac{1}{2} - \frac{1}{2(1-\zeta)} n_0 > \frac{3}{2(1-\zeta)} n_0 > \frac{2(2+\zeta)}{2\zeta+3} n_0,$$

which proves (A31). □

To summarize the above results, we define n^* and $\hat{n}$ as follows. We denote x^{-1} to be the inverse function of $x(n_0)$ defined in (A23), which is a strictly increasing function due to our previous discussion. We denote $A_{\min}$ to be the lowest possible realization of A_1 and $A_{\max}$ to be the highest possible realization of A_1. We set

$$n^* = \min\left\{ x^{-1}\left(\frac{2}{3}A_{\max}\right), \frac{1}{2}(1-\zeta)\left(A_0 - \frac{1}{2}\right)\right\}; \tag{A33}$$

$$\hat{n} = x^{-1}\left(\frac{1}{2+\zeta}\left[(1+2\zeta)A_{\min} + \frac{1}{2}(1-\zeta)\right]\right). \tag{A34}$$

By the above lemmas, if $n_0 \in (n^*, \hat{n})$, the collateral constraints in both periods are binding.

Monotonicity of the Lagrangian multiplier In this section, we prove that under the conditions outlined in the previous section, (**??**) holds by establishing that $\frac{\eta(n_1,A_1)}{\mu(n_1,A_1)}$ is a strictly decreasing function of A_1.

Lemma 4. *(Monotonicity of the Lagrangian multiplier)*

Under the assumption of $n_0 \in (n^*, \hat{n})$,

$$\frac{d}{dA_1}\left[\frac{\eta(n_1,A_1)}{\mu(n_1,A_1)}\right] < 0,$$

*that is, the Lagrangian multiplier component of asset price (**??**) is counter-cyclical.*

Proof. Using equations (A19) and (A20), we have

$$\frac{\eta(n_1,A_1)}{\mu(n_1,A_1)} = \sqrt{\frac{n_1}{1-\zeta}} \frac{A_1 - \frac{1}{2} - \frac{3}{2}\frac{n_1}{1-\zeta}}{A_1 - \frac{1}{2} - \left(\frac{3}{2}\frac{1}{1-\zeta} - 1\right) n_1}.$$

Using the law of motion of net worth, $n_1 = A_1 - x$, we have:

$$\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)} = \sqrt{\frac{A_1 - x}{1-\zeta}} \frac{\frac{3}{2(1-\zeta)}x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 1\right) A_1}{\left(\frac{3}{2(1-\zeta)} - 1\right) x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 2\right) A_1}.$$

Therefore,

$$\frac{d}{dA_1} \ln \left[\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}\right] = \frac{1}{2}\frac{1}{A_1 - x} - \frac{\frac{3}{2(1-\zeta)} - 1}{\frac{3}{2(1-\zeta)}x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 1\right) A_1} + \frac{\frac{3}{2(1-\zeta)} - 2}{\left(\frac{3}{2(1-\zeta)} - 1\right) x - \frac{1}{2} - \left(\frac{3}{2(1-\zeta)} - 2\right) A_1}$$

To save notation, we denote $a = \frac{3}{2(1-\zeta)} - 1$. We have:

$$\begin{aligned} \frac{d}{dA_1} \ln \left[\frac{\eta(n_1, A_1)}{\mu(n_1, A_1)}\right] &= \frac{1}{2}\frac{1}{n_1} - \frac{a}{x - \frac{1}{2} - an_1} + \frac{a-1}{x - \frac{1}{2} - (a-1)\, n_1} \\ &= \frac{\left(x - \frac{1}{2}\right)\left[x - \frac{1}{2} - 2an_1\right] - n_1\left[x - \frac{1}{2} - a(a-1)\, n_1\right]}{2n_1\left[x - \frac{1}{2} - an_1\right]\left[x - \frac{1}{2} - (a-1)\, n_1\right]}. \end{aligned}$$

It is straightforward to show that condition (A25) implies $x - \frac{1}{2} - (a-1)\, n_1 > x - \frac{1}{2} - an_1 > 0$. We only need to show that the denominator is negative. Since $\zeta < \frac{1}{2}$, $(a-1) < 1$ and

$$\begin{aligned} \left(x - \frac{1}{2}\right)\left[x - \frac{1}{2} - 2an_1\right] - n_1\left[x - \frac{1}{2} - a(a-1)\, n_1\right] &< \left(x - \frac{1}{2}\right)\left[x - \frac{1}{2} - 2an_1\right] - n_1\left[x - \frac{1}{2} - 2an_1\right] \\ &= \left(x - \frac{1}{2} - n_1\right)\left[x - \frac{1}{2} - 2an_1\right]. \end{aligned}$$

Also, $\zeta > \frac{1}{4}$ implies $a > 1$. Therefore, $x - \frac{1}{2} - an_1 > 0$ implies $x - \frac{1}{2} > n_1$. It remains to show $x - \frac{1}{2} - 2an_1 < 0$. Using the definition $a = \frac{3}{2(1-\zeta)} - 1$, under assumption (A28),

$$\begin{aligned} x - \frac{1}{2} - 2an_1 &= x - \frac{1}{2} - 2a(A_1 - x) \\ &= \left(\frac{3}{(1-\zeta)} - 1\right) x - \frac{1}{2} - \left(\frac{3}{(1-\zeta)} - 2\right) A_1 \\ &< 0, \end{aligned}$$

which completes the proof. □

A.3 Proof of Proposition ??

We prove Proposition **??** in two steps: first, given prices, the quantities satisfy the household's and the entrepreneurs' optimality conditions; second, the quantities satisfy the market clearing conditions.

To verify the optimality conditions, note that the optimization problems of households and firms are all standard convex programming problems; therefore, we only need to verify first order conditions. Equation (**??**) is the household's first-order condition. Equation (**??**) is a normalized version of resource constraint (**??**). Both of them are satisfied as listed in Proposition **??**.

To verify that the entrepreneur i's allocations $\{N_{i,t}, B_{i,t}, K_{i,t}, H_{i,t}, L_{i,t}\}$ as constructed in Proposition **??** satisfy the first order conditions for the optimization problem (**??**), note that the first order condition with respect to $B_{i,t}$ implies

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}\right] R_t^f + \frac{\eta_t^i}{q_{K,t}}. \tag{A35}$$

Similarly, the first order condition for $K_{i,t+1}$ is

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}}\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)\, q_{K,t+1}}{q_{K,t}}\right] + \zeta\frac{\eta_t^i}{q_{K,t}}. \tag{A36}$$

Finally, optimality with respect to the choice of type-H capital implies

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i \frac{\frac{\partial}{\partial H_{i,t+1}}\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)\, q_{H,t+1}}{q_{H,t}}\right]. \tag{A37}$$

Next, the law of motion of the endogenous state variable n can be constructed from equation (**??**):[2]

$$\begin{aligned} n' \quad &= \quad (1-\lambda)\left[\alpha\nu A' + \phi(1-\delta)\, q_K\left(A', n'\right) + (1-\phi)(1-\delta)\, q_H\left(A', n'\right) - \zeta\phi q_K(A,n)\, R_f(A,n)\right] \\ &\quad\quad + \lambda\chi\frac{n}{\Gamma(A,n)}. \end{aligned} \tag{A38}$$

With the law of motion of the state variables, we can construct the normalized utility of the

[2]We make use of the property that the ratio of K over H is always equal to $\phi/(1-\phi)$, as implied by the law of motion of the capital stock in (**??**).

household as the fixed point of

$$u(A,n)=\left\{(1-\beta)c(A,n)^{1-\frac{1}{\psi}}+\beta\Gamma(A,n)^{1-\frac{1}{\psi}}\left(E[u\left(A',n'\right)^{1-\gamma}]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The stochastic discount factors must be consistent with household utility maximization:

$$M' = \beta\left[\frac{c(A',n')\Gamma(A,n)}{c(A,n)}\right]^{-\frac{1}{\psi}}\left[\frac{u(A',n')}{E\left[u(A',n')^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} \tag{A39}$$

$$\tilde{M}' = M'[(1-\lambda)\mu\left(A',n'\right)+\lambda]. \tag{A40}$$

In our setup, thanks to the assumptions that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}$ and $H_{i,t+1}$ are made, we can construct an equilibrium in which μ_t^i and η_t^i are equalized across all the firms because $\frac{\partial}{\partial H_{i,t+1}}\pi\left(\bar{A}_{t+1},z_{i,t+1},K_{i,t+1},H_{i,t+1}\right)=\frac{\partial}{\partial K_{i,t+1}}\pi\left(\bar{A}_{t+1},z_{i,t+1},K_{i,t+1},H_{i,t+1}\right)$ are the same for all i.

Our next step is to verify the market clearing conditions. Given the initial conditions (initial net worth N_0, $\frac{K_1}{H_1}=\frac{\phi}{1-\phi}$, $N_{i,0}=z_{i,1}N_0$) and the net worth injection rule for the new entrant firms ($N_{t+1}^{entrant}=\chi N_t$ for all t), we establish the market clearing conditions through the following lemma. For simplicity, we assume the collateral constraint to be binding. The case in which this constraint is not binding can be dealt with in a similar way.

Lemma 5. *The optimal allocations $\{N_{i,t},B_{i,t},K_{i,t+1},H_{i,t+1}\}$ constructed as in Proposition* **??** *satisfy the market clearing conditions, i.e.,*

$$K_{t+1}=\int K_{i,t+1}\,di,\quad H_{t+1}=\int H_{i,t+1}\,di,\quad N_t=\int N_{i,t}\,di \tag{A41}$$

for all $t\geq 0$.

First, in each period t, given prices and $N_{i,t}$, the individual entrepreneur i's capital decisions $\{K_{i,t+1},H_{i,t+1}\}$ must satisfy the condition

$$N_{i,t}=(1-\zeta)\,q_{K,t}K_{i,t+1}+q_{H,t}H_{i,t+1} \tag{A42}$$

and the optimal decision rule (**??**). Equation (A42) is obtained by combining the entrepreneur's budget constraint (**??**) with a binding collateral constraint (**??**).

Next, we show by induction, that, given the initial conditions, market clearing conditions (A41) hold for all $t \geq 0$. In period 0, we start from the initial conditions. First, $N_{i,0} = z_{i,1} N_0$, where $z_{i,1}$ is chosen from the stationary distribution of z. Then, given $z_{i,1}$ for each firm i, we use equations (A42) and (**??**) to solve for $K_{i,1}$ and $H_{i,1}$. Clearly, $K_{i,1} = z_{i,1} K_1$ and $H_{i,1} = z_{i,1} H_1$. Therefore, the market clearing conditions (A41) hold for $t = 0$, i.e.,

$$\int K_{i,1}\, di = K_1, \quad \int H_{i,1}\, di = H_1, \quad \int N_{i,0}\, di = N_0. \tag{A43}$$

To complete the induction argument, we need to show that if market clearing holds for $t+1$, it must hold for $t+2$ for all t, which is the following claim:

Claim 1. *Suppose* $\int K_{i,t+1}\, di = K_{t+1}$, $\int H_{i,t+1}\, di = H_{t+1}$, $\int N_{i,t}\, di = N_t$, *and* $N_{t+1}^{entrant} = \chi N_t$, *then*

$$\int K_{i,t+2} di = K_{t+2} \quad \int H_{i,t+2} di = H_{t+2} \quad \int N_{i,t+1} di = N_{t+1} \tag{A44}$$

for all $t \geq 0$.

1. Using the law of motion for the net worth of existing firms, one can show that the total net worth of all surviving firms can be rewritten as follows:

$$\begin{aligned}
&(1-\lambda) \int N_{i,t+1} di \\
&= \quad (1-\lambda) \int \left[A_{t+1}\left(K_{i,t+1} + H_{i,t+1}\right) + (1-\delta)\, q_{K,t+1} K_{i,t+1} + (1-\delta)\, q_{H,t+1} H_{i,t+1} - R_{f,t} B_{i,t}\right] di, \\
&= \quad (1-\lambda)\left[A_{t+1}\left(K_{t+1} + H_{t+1}\right) + (1-\delta)\, q_{K,t} K_{t+1} + (1-\delta)\, q_{H,t} H_{t+1} - R_{f,t} B_t\right],
\end{aligned}$$

   since by assumption $\int K_{i,t+1}\, di = K_{t+1}$, $\int H_{i,t+1}\, di = H_{t+1}$, and $\int B_{i,t}\, di = B_t = \zeta q_{K,t} K_{t+1}$. Using the assignment rule for the net worth of new entrants, $N_{t+1}^{entrant} = \chi N_t$, we can show that the total net worth at the end of period $t+1$ across survivors and new entrants together satisfies $\int N_{i,t+1}\, di = N_{t+1}$, where aggregate net worth N_{t+1} is given by equation (**??**).

2. At the end of period $t+1$, we have a pool of firms consisting of old ones with net worth given by (**??**) and new entrants. All of them will observe $z_{i,t+2}$ (for the new entrants $z_{i,t+2} = \bar{z}$) and produce at the beginning of the period $t+1$.

   We compute the capital holdings for period $t+2$ for each firm i using (A42) and (**??**). At this point, the capital holdings and the net worth of all existing firms will not be proportional to $z_{i,t+2}$ due to heterogeneity in the shocks. However, we know that $\int N_{i,t+1}\, di = N_{t+1}$, and

$\int z_{i,t+2}\, di = 1$. Integrating (A42) and (??) across all i yields the two equations

$$(1-\zeta)\, q_{K,t+1} \int K_{i,t+2}\, di + q_{H,t+1} \int H_{i,t+2}\, di \quad = \quad N_{t+1} \tag{A45}$$

$$\int K_{i,t+2}\, di + \int H_{i,t+2}\, di \quad = \quad K_{t+2} + H_{t+2}, \tag{A46}$$

where we have used $\int N_{i,t+1}\, di = N_{t+1}$ and $\int z_{i,t+2}\, di = 1$. Given that the constraints of all entrepreneurs are binding, the budget constraint (A42) also holds at the aggregate level, i.e.,

$$N_{t+1} = (1-\zeta)\, q_{K,t+1} K_{t+2} + q_{H,t+1} H_{t+2}.$$

Together with the above system, this implies $\int K_{i,t+2} di = K_{t+2}$ and $\int H_{i,t+2} di = H_{t+2}$. Therefore, the claim is proved.

In summary, we have proved that the equilibrium prices and quantities constructed in Proposition **??** satisfy the household's and entrepreneur's optimality conditions, and that the quantities satisfy market clearing conditions.

Finally, we provide a recursive relationship that can be used to solve for $\theta(A, n)$ given the equilibrium constructed in Proposition **??**. The recursion (**??**) implies

$$\begin{aligned}\mu_t N_{i,t} + \theta_t z_{i,t+1} (K_t + H_t) =& E_t M_{t+1} \left[(1-\lambda) \left(\mu_{t+1} N_{i,t+1} + \theta_{t+1} (K_{t+1} + H_{t+1})\, z_{i,t+2}\right) + \lambda N_{i,t+1}\right] \\ =& E_t M_{t+1} \left[\left\{(1-\lambda)\, \mu_{t+1} + \lambda\right\} N_{i,t+1}\right] + (1-\lambda)\, z_{i,t+1} E_t \left[M_{t+1} \theta_{t+1} (K_{t+1} + H_{t+1})\right].\end{aligned} \tag{A47}$$

Below, we first focus on simplifying the term $E_t M_{t+1} \left[\left\{(1-\lambda)\, \mu_{t+1} + \lambda\right\} N_{i,t+1}\right]$. Note that a binding collateral constraint together with the entrepreneur's budget constraint (**??**) implies

$$(1-\zeta)\, q_{K,t} K_{i,t+1} + q_{H,t} H_{i,t+1} = N_{i,t}. \tag{A48}$$

Equation (A48) together with the optimality condition (**??**) determine $K_{i,t+1}$ and $H_{i,t+1}$ as functions of $N_{i,t}$ and $z_{i,t+1}$:

$$K_{i,t+1} = \frac{q_{H,t} z_{i,t+1} (K_{t+1} + H_{t+1}) - N_{i,t}}{q_{H,t} - (1-\zeta)\, q_{K,t}}; \quad H_{i,t+1} = \frac{N_{i,t} - (1-\zeta)\, q_{K,t} z_{i,t+1} (K_{t+1} + H_{t+1})}{q_{H,t} - (1-\zeta)\, q_{K,t}}. \tag{A49}$$

Using Equation (A49) and the law of motion of net worth (**??**), we can represent $N_{i,t+1}$ as a linear function of $N_{i,t}$ and $z_{i,t+1}$:

$$\begin{aligned} N_{i,t+1} &= z_{i,t+1}\alpha A_{t+1}\left(K_{t+1}+H_{t+1}\right)+(1-\delta)\,q_{K,t+1}\frac{q_{H,t}z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)-N_{i,t}}{q_{H,t}-(1-\zeta)\,q_{K,t}} \\ &\quad +(1-\delta)\,q_{H,t+1}\frac{N_{i,t}-(1-\zeta)\,q_{K,t}z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)}{q_{H,t}-(1-\zeta)\,q_{K,t}}-R_{f,t}\zeta q_{K,t}\frac{q_{H,t}z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)-N_{i,t}}{q_{H,t}-(1-\zeta)\,q_{K,t}}. \end{aligned}$$

Because we are only interested in the coefficients on $z_{i,t+1}$, collecting the terms that involves $z_{i,t+1}$ on both sides of (A47), we have:

$$\theta_t z_{i,t+1}\left(K_t+H_t\right)=z_{i,t+1}\left(K_{t+1}+H_{t+1}\right)\times Term,$$

where

$$Term=E_t\left[\tilde{M}_{t+1}\left\{\begin{array}{c}\alpha A_{t+1}+(1-\delta)\,q_{K,t+1}\frac{q_{H,t}}{q_{H,t}-(1-\zeta)q_{K,t}} \\ -(1-\delta)\,q_{H,t+1}\frac{(1-\zeta)q_{K,t}}{q_{H,t}-(1-\zeta)q_{K,t}}-R_{f,t}\zeta q_{K,t}\frac{q_{H,t}}{q_{H,t}-(1-\zeta)q_{K,t}}\end{array}\right\}\right]+(1-\lambda)\,E_t\left[M_{t+1}\theta_{t+1}\right].$$

We can simplify the first term using the first order conditions (**??**)-(**??**) to get

$$E_t\left[\tilde{M}_{t+1}\left\{\alpha\left(1-\nu\right)A_{t+1}\right\}\right].$$

Therefore, we have the following recursive relationship for $\theta\left(A,n\right)$:

$$\theta\left(A,n\right)=\left[1-\delta+i\left(A,n\right)\right]\left\{\alpha\left(1-\nu\right)E\left[M'\left\{\lambda+(1-\lambda)\,\mu\left(A',n'\right)\right\}A'\right]+(1-\lambda)\,E\left[M'\theta\left(A',n'\right)\right]\right\}. \tag{A50}$$

The term $\alpha\left(1-\nu\right)A'$ is the profit for the firm due to decreasing return to scale. Clearly, $\theta\left(A,n\right)$ has the interpretation of the present value of profit. In the case of constant returns to scale, $\theta\left(A,n\right)=0$.

B Empirical Analysis

In this section, we provide empirical evidence on the relation between collateralizability and the cross-section of stock returns. First and most importantly, we show that high asset collateralizability firms have lower cash flow betas with respect two alternative proxies for financial shocks. Second, we conduct standard **?** two-pass regression and show the proxies of financial shocks are significantly negatively priced. High collateralizability firms are less negatively exposed to these shocks. These two pieces of evidence taken together strongly corroborate the model mechanism that collateralizable assets provide an insurance against aggregate shocks. We then perform other standard multi-factor asset pricing tests, and investigate the joint link between collateralizability and other firm characteristics on one hand and future stock returns on the other using multivariate regressions.

B.1 Cash flow risks of collateralizability-sorted portfolios

Our theory suggests that the collateralizability premium comes from the countercyclicality of the marginal value of collateralizable capital. In our model, firms, rather than households, directly trade physical assets directly, because they are more efficient than households in deploying these assets. Since firms are constrained, type-K and type-H capital, whose prices contain a Lagrangian multiplier component, can have different prices and expected returns even though they generate identical cash flows from the firm's perspective (measured in net worth units). The counter-cyclical nature of the Lagrangian multiplier provides a hedge against aggregate shocks and makes the price of collateralizable capital less sensitive to aggregate shocks and less cyclical. However, it is important to note, in our model, households are not constrained and free to trade the firms' equity and debt, so that differences in expected returns on the firms' equity must be due to differences in the cash flows accruing to equity holders (measured in consumption units). Put differently, the Lagrangian multiplier component of asset prices affects the risk exposure of cash flows to the equity holders, i.e., to households. We measure the cash flow to equity holders and show empirically at the portfolio level that the equity cash flows of firms with high asset collateralizability exhibit a lower, i.e., less negative, sensitivity respect to financial shocks, consistent with the model simulation.

We consider two alternative proxies for financial shocks: the change in the general cost of external finance (debt and equity) as suggested by **?** (ΔEM), and the log change in the cross-sectional dispersion of firm-level cash flow growth ($\Delta\sigma_{CS}$), similar in spirit to **?**.

When we measure the cash flow accruing to equity holders at the portfolio level, we follow **?** and first aggregate cash flow (represented by EBIT) across the firms in a given portfolio and then normalize this sum by the total lagged sales (SALE) of that portfolio. We then compute the sensitivity, i.e., the beta, of the portfolio cash flow growth with respect to the two proxies of financial shocks. The results are reported in Table B.1.

We make several observations. First and importantly, one can see from Panel A, the cash flow betas with respective the equity finance cost shock (ΔEM) display a monotonically increasing pattern from low to high collateralizability portfolios, and cash flow beta of low collateralizability portfolio is statistically significantly more negative that that of high collateralizability portfolio. In particular, the high collateralizability quintiles 4 and 5 exhibit insignificantly negative betas. This again highlights the main economic mechanism of our model that collateralizable assets provide an insurance against aggregate shocks. We also find this increasing pattern of cash flow betas across collateralizabilty portfolios with respect to $\Delta\sigma_{CS}$, although the cash flow beta difference is less significant.

Finally, to precisely connect the empirical evidence to our model, we run the same test based on data from a simulation of our model. As we show in Panel B of Table B.1, our model produces the same increasing pattern of cash flow betas with respective to two alternative simulated proxies of financial shocks across collateralizability-sorted portfolios. First, we directly use the innovation to the entrepreneurs' exit probability, ϵ_x.

Second, we construct the log change in the cross-sectional dispersion of cash flow growth $(\Delta\sigma_{CS})$ in a similar procedure as in the data. In our simulated model, this shock is positively correlated with innovations in λ, because higher values of λ in our model lower firm net worth and increase leverage. These effects make firm cash flow more sensitive to primitive productivity shocks including idiosyncratic productivity shocks. As a result, in our model, the cross section dispersion of cash flow growth is strongly positively correlated with shocks to λ. The **?** measure therefore backs out the structural shocks to λ and can explain the cross-section of collateralizability-sorted portfolios. In Table B.1, we report that cash flow betas of collateralizability-sorted portfolios with respect to this proxy display the same increasing pattern as the ones with structural shocks to λ. The results strongly confirm our key model mechanism and are consistent with the data.

B.2 Collateralizability spreads and financial shocks

In this section, we provide empirical evidence for the link between the collateralizability spread and financial shocks consistent with our model interpretation.

Empirically, we consider a two-factor asset pricing model with the market (Mkt) and one of

Table B.1: Cash Flow Exposure to the Financial Shock

This table shows the sensitivity of cash flows of collateralizability-sorted portfolios to the financial shock. Panel A and B report exposure coefficients from empirical data and model simulated data, respectively. The portfolio-level normalized cash flow is constructed by aggregating cash flow (EBIT) within each quintile portfolio, and then dividing it by the lagged sales of the same portfolio. In Panel A (data), we report the regression coefficients from regressing portfolio-level normalized cash flow on two alternative empirical proxies of the financial shocks: ΔEM and $\Delta\sigma_{CS}$. ΔEM is the first difference of average external finance cost from **?**. $\Delta\sigma_{CS}$ is the log change of the dispersion of firm-level cash flow growth. In the model (Panel B), we regress cash flow growth of each portfolios on two alternative proxies of financial shocks: the innovation to the liquidation probability ε_x and the log change of the dispersion of portfolio-level cash flow growth $\Delta\sigma_{CS}$. For dispersion measure $\Delta\sigma_{CS}$, every year t we sort firms in to 20 portfolios based on their collateralizability in year $t-1$, then we aggregate cash flow growth across all firms within each portfolio. Then we compute the cross-sectional standard deviation of 20 portfolios every year as the dispersion measure. We winsorize firm-level variables at the top and bottom 1%, respectively. All shocks are normalized to have zero mean and unit standard deviation. All regressions are conducted at the annual frequency. The t-statistics (in parentheses) are adjusted following **?**. All regression coefficients are multiplied by 100.

Panel A: Data

Financial Shocks	1	2	3	4	5	1-5
ΔEM	-1.94	-1.95	-1.41	-0.34	-0.14	-2.13
	(-1.83)	(-1.36)	(-1.81)	(-0.67)	(-0.63)	(-2.08)
$\Delta\sigma_{CS}$	-0.40	0.08	0.28	0.31	0.17	-0.55
	(-0.88)	(0.30)	(1.51)	(1.47)	(1.12)	(-1.10)

Panel B: Model

Financial Shocks	1	2	3	4	5	1-5
ε_x	-3.97	-3.67	-2.46	-0.93	-1.06	-2.90
	(-14.18)	(-8.75)	(-3.06)	(-0.74)	(-1.12)	(-3.11)
$\Delta\sigma_{CS}$	-2.24	-1.87	-1.62	0.58	2.26	-4.50
	(-5.07)	(-3.64)	(-2.61)	(0.48)	(2.58)	(-4.56)

the two financial shock proxies (ΔEM or $\Delta\sigma_{CS}$) as factors. Following the standard approach developed by **?**, we first estimate the exposures (betas) of excess returns of five collateralizability-sorted portfolios with respect to the market and the financial shock factor using the whole sample. Next, we run period-by-period cross-sectional regressions of realized portfolio returns on betas to estimate the market prices of risks, which are calculated as the average slopes from the period-by-period cross-sectional regressions.

We also conduct the **?** two-pass regression based on data from a simulation of our model. The only difference is that, rather than running a two-factor model, we run a one-factor regression with just the financial shock ϵ_x, since our model, by design, features a one-factor structure due to the perfect correlation between TFP and financial shocks.

The results are presented in Table B.2, respectively, where Panel A and Panel B present the exposures of the five portfolios to factors, while the estimated market prices of risk are shown in Panel C.

We make several observations. First, the betas with respect to ΔEM display a monotonically increasing pattern from low to high collateralizability portfolios. In particular, the high collateralizability quintile exhibits a (marginally) significantly less negative beta than the low collateralizability quintile. This pattern is confirmed in an even stronger fashion when we use $\Delta\sigma_{CS}$ as a proxy for financial shocks. The betas with respect to this factor also display a monotonically increasing pattern from low to high collateralizability portfolios. It is worth noting that, with respect to this proxy of financial shocks, the difference in return beta between portfolio 1 and 5 is statistically significant. When we run the test using simulated data from the model, the return betas of collateralizability-sorted portfolios display a pattern consistent with the data.

Furthermore, in the second stage cross-sectional regressions, we use five collateralizability-sorted portfolios as the test assets, we compare the two-factor model ($Mkt + \Delta EM$ or $Mkt + \Delta\sigma_{CS}$) with the standard CAPM with only the market factor. We observe that the CAPM fails. When we add the financial shock factor, the estimated market price of risk for this new factor is negative and significant. The average pricing error (i.e., the intercept) becomes smaller and even statistically insignificant. The second stage empirical results are again confirmed by the model simulation.

Table B.2: Betas and Price of Risks of the Financial Shock

This table presents the risk price estimates for the financial shock. The factors considered in the empirical data are the market return and one of the two alternative empirical proxies for the financial shock, that is, external finance shock (ΔEM) and cross-sectional dispersion shock of firm-level cash flow growth ($\Delta\sigma_{CS}$). In the data, we construct the external finance shock by taking the first difference of the average costs of external finance from **?**. Dispersion shock $\Delta\sigma_{CS}$ is the log change of the dispersion of firm-level cash flow growth. In the model, we use innovation to the probability of liquidity ε_x and log change of dispersion of portfolio-level cash flow growth $\Delta\sigma_{CS}$ as our proxies for the financial shock. The dispersion measure $\Delta\sigma_{CS}$ is constructed in the same procedure as in Table B.1. Panels A and B present the first-stage estimates of factor exposures of collateralizability-sorted portfolios in the data and in the model, respectively. Panel C reports the risk prices (λ_{Fin}) of the financial shock estimated from the second-stage regressions. The risk prices reported in Panel C are the mean slopes of period-by-period cross-sectional regressions of portfolio excess returns on risk exposures (betas). All shocks are normalized to have zero mean and unit standard deviation. The regressions are conducted at the annual frequency. R^2 is calculated as the mean across R^2 of the period-by-period regressions. The mean absolute pricing errors (MAE) across the test assets in Panel C are expressed in percentage terms. The t-statistics (in parentheses) are adjusted following **?**.

Panel A: Portfolio Factor Exposures - Data

	1	2	3	4	5	1-5
ΔEM	-0.142	-0.128	-0.113	-0.091	-0.084	-0.058
	(-2.580)	(-2.804)	(-2.135)	(-2.126)	(-1.723)	(-1.598)
$\Delta\sigma_{CS}$	-0.029	-0.007	-0.003	0.007	0.016	-0.045
	(-0.990)	(-0.251)	(-0.113)	(0.260)	(0.592)	(-2.074)

Panel B: Portfolio Factor Exposures - Model

	1	2	3	4	5	1-5
ε_x	-0.047	-0.042	-0.035	-0.019	-0.004	-0.043
	(-15.665)	(-12.723)	(-12.757)	(-3.327)	(-1.291)	(-10.291)
$\Delta\sigma_{CS}$	-0.022	-0.020	-0.016	-0.008	-0.003	-0.019
	(-4.274)	(-3.690)	(-3.990)	(-2.627)	(-1.698)	(-4.459)

Panel C: Price of Risks

	Data					Model	
	CAPM	ΔEM	Mkt+ΔEM	$\Delta\sigma_{CS}$	Mkt+$\Delta\sigma_{CS}$	ε_x	$\Delta\sigma_{CS}$
λ_{Mkt}	0.700		-0.005		0.072		
(t)	(2.227)		(-0.011)		(0.338)		
λ_{Fin}		-1.230	-1.094	-1.435	-1.272	-0.854	-2.118
(t)		(-2.478)	(-1.729)	(-2.442)	(-1.876)	(-6.879)	(-6.955)
Intercept	-0.441	-0.045	0.103	0.091	0.009	-0.004	-0.004
(t)	(-1.888)	(-0.777)	(0.214)	(2.386)	(0.041)	(-1.808)	(-1.616)
MAE	4.612	3.947	3.266	4.033	3.149	0.561	0.554
R^2	0.387	0.445	0.624	0.451	0.633	0.739	0.737

B.3 Asset pricing tests

We now perform a number of standard asset pricing tests to show that the collateralizability premium cannot be explained by standard risk factors, as represented by the ? four factor model, the ? five factor model, or the organizational capital factor proposed by ?. We also investigate the incremental predictive power of current asset collateralizability for future stock returns at the firm-level.

First, we investigate to what extent the variation in the returns of the collateralizability-sorted portfolios can be explained by standard risk factors suggested by ? and ?. In particular, we run monthly time-series regressions of the (annualized) excess returns of each portfolio on a constant and the risk factors included in the above models. Table B.3 reports the intercepts (i.e., alphas) and exposures (i.e., betas). The intercepts can be interpreted as pricing errors (abnormal returns), which remain unexplained by the given set of factors.

We make two key observations. First, the pricing errors of the collateralizablity-sorted portfolios with respect to the given sets of factors are large and statistically significant. The estimated alphas of the low-minus-high portfolio are 9.34% for the ? model and 5.80% for the ? five-factor model, respectively, with associated t-statistics of around 3.5 and 2.1.

Second, in order to distinguish our collateralizability measure from organizational capital, we also control for this factor constructed by ?,[3] together with the three Fama-French factors.

The results are shown in Panel C of Table B.3. The pricing error of the low-minus-high portfolio is still significant in the presence of the organizational capital factor (OMK) and amounts almost 9% per year with a t-statistic of greater than 2.6. In particular, the five portfolios sorted on collateralizability are not strongly exposed to this factor, indicated by economically small and statistically insignificant coefficients, except for Quintile 5.

Taken together, the cross-sectional return spread across collateralizability sorted portfolios cannot be explained by either the ? four-factor model, the ? five-factor model, or the organizational capital factor proposed by ?.

Second, we extend the previous analysis to the investigation of the link between collateralizability and future stock returns using firm-level multivariate regressions that include firm's collat-

[3] We would like to thank Dimitris Papanikolaou for sharing this time series of the organizational factor.

eralizability and other controls as return predictors. In particular, we perform standard firm-level cross-sectional regressions (?) to predict future stock returns:

$$R_{i,t+1} = \alpha_i + \beta \cdot \mathit{Collateralizability}_{i,t} + \gamma \cdot \mathit{Controls}_{i,t} + \varepsilon_{i,t+1},$$

where $R_{i,t+1}$ is stock i's cumulative (raw) return over the respective next year, i.e., from July of year t to June of each year $t+1$. The control variables include current collateralizability, size, book-to-market (BM), profitability (ROA), and book leverage. To avoid using future information, all the balance sheet variables are based on the values available before the end of year t. Table B.4 reports the results. The regressions exhibit a significantly negative slope coefficient for collateralizability across all specifications, which supports our theory, since a higher current degree of collateralizability implies lower overall risk exposure, so that expected future returns should indeed be smaller with higher collateralizability.

In our empirical measure, only structure and equipment capital contribute to firms' collateralizability, but not intangible capital. Therefore, by construction, our collateralizability measure weakly negatively correlates with measures of intangible capital. In order to empirically distinguish our theoretical channel from the ones focusing on organizational capital (?) and R&D capital (?, ?), we also control for OG/AT, the ratio of organizational capital to total assets, and XRD/AT, the ratio of R&D expenses to total assets, as suggested in the literature. The results in Table B.4 show that the negative slope coefficients for collateralizability remain significant, although they become smaller in magnitude, after controlling for these two firm characteristics. Instead of using the ratio of R&D expenditure to total assets, we also used the ratio of R&D capital to total assets as a control. The results remain very similar.

B.4 Additional empirical evidence

In this section, we provide additional empirical evidence regarding the collateralizability premium. First, we demonstrate the robustness of our findings by forming collateralizability portfolios within industries to make sure that our baseline result is not driven by industry-specific effects, and by performing a rolling-window estimation of the collateralizability parameters. Second, we present correlations between collateralizability and firm characteristics. Finally, we perform double sorts with respect to collateralizability and financial leverage.

Table B.3: Asset Pricing Tests of Collateralizability-sorted Portfolios

This table shows the coefficients of regressions of excess returns of collateralizability-sorted portfolios on the factors from the **?** four-factor model (Panel A), the **?** five-factor model (Panel B), and a model featuring the Fama-Franch three-factor model augmented by the organizational capital factor from **?** (Panel C). The t-statistics are computed based on **?** adjusted standard errors. The analysis is performed for financially constrained firms. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The sample period is from July 1979 to December 2016, with the exception of Panel C, where the sample ends in December 2008 due to the length of the organizational capital factor. We annualize returns by multiplying by 12.

Panel A: Carhart Four-Factor Model

	1	2	3	4	5	1-5
α	5.43	2.94	2.04	-1.87	-3.91	9.34
(t)	2.80	1.76	1.35	-1.45	-2.44	3.47
β_{MKT}	1.07	1.07	1.07	1.12	1.10	-0.03
(t)	25.25	27.91	32.58	35.68	26.93	-0.48
β_{HML}	-0.62	-0.49	-0.21	-0.12	0.01	-0.63
(t)	-9.72	-8.60	-3.87	-2.31	0.16	-6.03
β_{SMB}	1.34	1.11	1.06	0.97	0.84	0.50
(t)	15.66	15.77	22.71	15.28	8.72	3.27
β_{MOM}	-0.04	-0.06	-0.05	-0.02	-0.07	0.03
(t)	-0.73	-1.74	-1.27	-0.56	-1.31	0.33
R^2	0.85	0.87	0.88	0.90	0.84	0.27

Panel B: Fama-French Five-Factor Model

	1	2	3	4	5	1-5
α	13.02	12.45	12.87	9.22	7.22	5.80
(t)	2.84	2.75	3.07	2.16	1.67	2.06
β_{MKT}	0.49	0.07	0.08	0.20	0.07	0.42
(t)	0.75	0.13	0.15	0.37	0.12	1.01
β_{SMB}	2.03	1.24	1.17	1.28	1.38	0.65
(t)	2.00	1.55	1.43	1.62	1.79	1.08
β_{HML}	-3.84	-4.34	-3.67	-3.15	-2.49	-1.35
(t)	-2.55	-3.21	-2.99	-2.65	-1.92	-1.12
β_{RMW}	-2.77	-3.12	-2.32	-1.90	-1.34	-1.43
(t)	-1.47	-2.11	-1.48	-1.33	-0.97	-1.11
β_{CMA}	2.10	1.00	1.74	0.92	0.94	1.17
(t)	0.83	0.46	1.02	0.53	0.62	0.58
R^2	0.09	0.10	0.08	0.07	0.06	0.04

Panel C: Control for Organizational Capital Factor

	1	2	3	4	5	1-5
α	5.06	3.42	1.56	-0.95	-3.90	8.96
(t)	2.30	1.67	0.85	-0.61	-1.74	2.66
β_{MKT}	1.10	1.07	1.10	1.11	1.08	0.03
(t)	19.82	21.75	24.43	30.89	23.11	0.35
β_{HML}	-0.56	-0.50	-0.19	-0.14	-0.00	-0.55
(t)	-7.23	-7.46	-2.71	-1.85	-0.04	-3.68
β_{SMB}	1.40	1.12	1.05	0.97	0.81	0.59
(t)	14.95	16.91	18.30	14.51	6.09	3.07
β_{OMK}	-0.02	0.01	0.01	-0.04	-0.14	0.13
(t)	-0.31	0.23	0.14	-0.99	-2.33	1.29
R^2	0.86	0.87	0.88	0.89	0.83	0.29

Table B.4: Fama Macbeth Regressions

This table reports the results for Fama-MacBeth regressions of annual cumulative firm-level excess stock returns on lagged firm characteristics. The coefficients reported in the table are the time-series averages of the slope coefficients from year-by-year cross-sectional regressions. The reported R^2 is the time-series average of the cross-sectional R^2. The columns labeled “SA,” “WW” and “Non-Dividend” refer to the financially constrained samples. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. ROA is Compustat item IB divided by book assets. OG/AT is organizational capital over total book assets, XRD/AT is R&D expenditure over total book assets. The t-statistics (in parentheses) are adjusted following **?**. The sample period is from 1979 to 2016.

	(1) SA	(2) WW	(3) No-Dividend	(4) SA	(5) WW	(6) No-Dividend	(7) SA	(8) WW	(9) No-Dividend
Collateralizability	-0.114***	-0.118***	-0.089***	-0.083**	-0.103***	-0.079***	-0.057**	-0.056**	-0.042**
	(-3.782)	(-4.040)	(-3.523)	(-2.388)	(-3.119)	(-3.309)	(-2.600)	(-2.371)	(-2.444)
log(ME)	-0.147***	-0.142***	-0.073***	-0.143***	-0.138***	-0.070***	-0.154***	-0.150***	-0.078***
	(-4.536)	(-5.329)	(-3.786)	(-4.458)	(-5.332)	(-3.685)	(-4.668)	(-5.480)	(-3.889)
BM	0.044**	0.032**	0.053***	0.051***	0.035**	0.057***	0.051***	0.041***	0.059***
	(2.505)	(2.164)	(4.582)	(2.731)	(2.243)	(4.440)	(3.029)	(2.860)	(5.003)
Lagged return	0.012	0.003	0.018	0.010	0.002	0.016	0.012	0.004	0.019
	(0.756)	(0.194)	(1.014)	(0.649)	(0.105)	(0.910)	(0.758)	(0.261)	(1.005)
ROA	0.068	0.053	0.089	0.082	0.067	0.102	0.178***	0.186***	0.208***
	(0.994)	(0.906)	(1.438)	(1.143)	(1.094)	(1.564)	(2.769)	(3.146)	(3.700)
Book Leverage	-0.090	-0.031	-0.013	-0.070	-0.018	0.001	-0.044	0.020	0.042
	(-1.639)	(-0.640)	(-0.236)	(-1.280)	(-0.369)	(0.015)	(-0.894)	(0.470)	(0.791)
OG/AT				0.109***	0.065**	0.072**			
				(3.873)	(2.206)	(2.296)			
XRD/AT							0.642***	0.774***	0.584***
							(2.795)	(3.053)	(2.833)
Constant	-3.574***	-3.634***	-3.783***	-3.655***	-3.685***	-3.839***	-3.620***	-3.678***	-3.833***
	(-4.275)	(-4.212)	(-4.435)	(-4.323)	(-4.229)	(-4.472)	(-4.319)	(-4.266)	(-4.503)
Observations	32864	37090	43078	32864	37090	43078	32864	37090	43078
R^2	0.087	0.085	0.065	0.091	0.088	0.069	0.093	0.091	0.072

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

B.4.1 Alternative portfolio sorts

To implement the first robustness check, we consider the Fama-French industry classification with 17 sectors. We sort firms into collateralizability quintiles according to their collateralizability score within their respective industry. Portfolio 1 will thus contain all firms which are in the lowest quintile relative to their industry peers, and so on for portfolios 2 to 5. By doing so, we essentially control for industry fixed effects. Table B.5 reports the results of this exercise, and one can see that the results are very close to the findings of our benchmark analysis presented in Table **??**.

In our benchmark analysis, we estimate the collateralizability coefficients for structure and equipment capital, ζ_S and ζ_E, using the whole sample. One might argue that this introduces a look-ahead bias, since the estimation is based on data not observable at the time when decisions are made. To see whether a potential look-ahead bias indeed has an effect on our results, we now perform the portfolio sort in year t exclusively on information up to $t-1$. In more detail, we use estimates denoted by $\widehat{\zeta}_{S,t-1}$ and $\widehat{\zeta}_{E,t-1}$ derived from expanding window regressions using data available up to the end of year $t-1$. The first window consists of data for the period from 1975 to 1980.[4].

Table B.6 presents the results in a fashion analogous to Table **??**. For all three measures for financial constraints, the collateralizability spread is positive, large, and significant. This shows that our baseline results do not suffer from a look-ahead bias with respect to the estimation of the collateralizability coefficients.

In order to capture the fact that structure capital is more collateralizable than equipment capital (**?**, **?**), we employ a constrained version of the leverage regression in Table **??** by estimating the equation

$$\frac{B_{i,t}}{AT_{i,t}} = (\zeta_E + e^{\Delta})StructShare_{l,t} + \zeta_E EquipShare_{l,t} + \gamma X_{i,t} + \varepsilon_{i,t},$$

i.e., we impose the restriction $\zeta_S = \zeta_E + e^{\Delta} > \zeta_E$. Then we perform a maximum likelihood estimation of above equation to obtain the time series of the estimates of ζ_E and Δ. In our sample, the estimated e^{Δ} across expanding windows is of mean 0.15 with standard error of 0.02.

[4]The regressor, marginal tax rate, is only available after 1980, therefore we drop this regressor. All other regressors are available from 1975 onwards. The results are similar if we start our sample in 1980 with marginal tax rate.

Table B.5: Portfolios Sorted on Collateralizability within Industries

This table reports annualized average monthly value-weighted excess returns ($E[R] - R^f$) for collateralizability-sorted portfolios, and their alphas with respect to different factor models. The sample period is from July 1979 to December 2016. $\alpha^{FF3+MOM}$ and α^{FF5} are the alphas with respect to the **?** four-factor model and the **?** five-factor model, respectively. At the end of June each year t, we consider each of the 17 Fama-French industries and sort the constrained firms in a given industry into quintiles based on their collateralizability scores at the end of year $t-1$. We hold the portfolios for a year, from July of year t until the June of year $t+1$. Portfolios are rebalanced in July every year. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The WW and the SA index are constructed according to **?** and **?**, respectively. Additionally, we consider a subsample where the firms are classified as constrained by all three measures jointly. We annualize returns by multiplying by 12. The t-statistics are estimated following **?**.

	1	2	3	4	5	1-5
	Financially constrained firms - All measures					
$E[R]-R_f(\%)$	13.14	10.46	11.67	7.97	6.86	6.28
(t)	2.63	2.22	2.41	1.79	1.46	2.60
$\alpha^{FF3+MOM}$	4.59	2.06	2.64	-1.31	-2.74	7.33
(t)	2.48	0.98	1.35	-0.68	-1.47	3.21
α^{FF5}	13.87	10.94	13.01	10.97	7.04	6.83
(t)	2.75	2.31	2.52	2.36	1.38	2.85
	Financially constrained firms - WW index					
$E[R]-R_f(\%)$	12.53	11.77	9.83	8.36	6.02	6.51
(t)	2.68	2.71	2.22	1.99	1.41	3.07
$\alpha^{FF3+MOM}$	4.23	2.83	1.62	-0.76	-3.05	7.28
(t)	2.51	1.78	0.97	-0.50	-2.37	3.57
α^{FF5}	14.25	12.26	11.76	10.28	6.37	7.88
(t)	2.97	2.75	2.56	2.35	1.33	3.58
	Financially constrained firms, SA index					
$E[R]-R_f(\%)$	11.28	11.51	8.08	8.32	6.02	5.26
(t)	2.35	2.41	1.77	1.94	1.35	2.54
$\alpha^{FF3+MOM}$	3.58	4.65	-0.62	-0.49	-2.50	6.08
(t)	1.99	2.25	-0.41	-0.33	-1.50	2.99
α^{FF5}	12.20	13.36	10.07	9.54	7.37	4.83
(t)	2.60	2.57	2.08	2.09	1.58	2.22
	Financially constrained firms, Non-Dividend					
$E[R]-R_f(\%)$	14.99	12.98	6.99	7.92	9.69	5.30
(t)	3.50	2.83	1.70	1.98	2.12	2.27
$\alpha^{FF3+MOM}$	7.39	5.23	0.39	-0.12	2.24	5.15
(t)	3.90	2.59	0.23	-0.08	1.22	2.18
α^{FF5}	14.90	14.95	8.16	8.88	11.45	3.44
(t)	3.59	3.59	2.16	2.16	2.61	1.47

As a further note, one advantage of our approach to sort stocks into portfolios does not rely on absolute precision in the estimation of ζ_E and ζ_S (which could potentially be subject to various sources of biases, e.g., due to endogeneity of capital structure choices, measurement errors in capital etc.). The outcome of the portfolio sort only depends on the ranking of the collateralizability measure for a given firm, not on its exact magnitude. In our empirical construction of the collateralizability measure, we consider three types of capital according to BEA, structure, equipment, and intellectual capital. As long as $\zeta_S > \zeta_E$ and intellectual capital does not contribute to collateralizability, the rank of a firm with respect to asset collateralizability will depend only on the composition of its capital, not on the numerical values of the estimated ζ-coefficients.

Table B.6: Portfolios Sorting based on Expanding Window Estimated Collateralizability

This table reports average value-weighted monthly excess returns (in percent and annualized) for portfolios sorted on collateralizability. The sample period is from July 1981 to December 2016. At the end of June of each year t, we sort the constrained firms into five quintiles based on their collateralizability measures (estimated using expanding window) at the end of year $t-1$, where quintile 1 (quintile 5) contains the firms with the lowest (highest) share of collateralizable assets. We hold the portfolios for a year, from July of year t until the June of year $t+1$. Firms are classified as constrained in year t, if their year end WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The WW and SA indices are constructed according to **?** and **?**, respectively. Standard errors are estimated using Newey-West estimator. The table reports average excess returns $E[R]-R_f$, as well as the associated t-statistics, and Sharpe ratios (SR). We annualize returns by multiplying by 12.

	1	2	3	4	5	1-5
	Financially constrained firms - WW index					
$E[R]-R_f(\%)$	11.76	10.68	9.80	7.18	5.20	6.55
(t)	2.33	2.31	2.24	1.71	1.30	2.18
SR	0.41	0.41	0.40	0.30	0.24	0.38
	Financially constrained firms - SA index					
$E[R]-R_f(\%)$	9.61	10.74	9.36	7.82	3.64	5.97
(t)	1.84	2.21	2.11	1.74	0.88	2.07
SR	0.32	0.40	0.38	0.31	0.16	0.35
	Financially constrained firms - Non-Dividend					
$E[R]-R_f(\%)$	14.32	9.18	6.93	7.13	6.75	7.57
(t)	3.11	2.07	1.59	1.59	1.63	2.83
SR	0.54	0.36	0.28	0.28	0.29	0.49

B.4.2 Collateralizability and additional firm characteristics

As indicated by the results in Table **??**, our model can quantitatively replicate the patterns of leverage, asset growth and the investment rate. In Table B.7, we now present additional characteristics of the firms in our collateralizability-sorted portfolios.

Cash flow and size are relatively flat across the five portfolios, low collateralizability firms on average hold more cash. Although cash is not modeled in our paper, this empirical finding is still consistent with our model intuition. Firms with less collaterizable assets hold more cash to compensate for the fact that they can hardly obtain collateralized loans, and even less so in recessions. The probability of debt issuance is increasing with asset collateralizability, while the probability of equity financing shows the opposite tendency. This reflects the substitution effect between the two types of external financing. Additionally, firms with more collateralizable assets on average have more short-term and long-term debt.

In Table B.8, we report the correlations of the collateralizability measure with other firm characteristics which have been shown in the past literature to predict the cross-section of stock returns, including the book-to-market ratio (BM) , the $R\&D$-to-asset ratio (XRD/AT), the organizational capital-to-asset ratio (OG/AT), (log) size (log(ME)), the investment rate, i.e., the ratio of investment to capital (I/K), and the return on assets (ROA). Notably, the collateralizability measure and these firm characteristics are only weakly correlated, with the correlation coefficients ranging between -33% to 16%.

B.4.3 Double sorting on collateralizability and leverage

As discussed in the main text, firms with higher asset collateralizablility have higher debt capacity and thus tend to have higher financial leverage. When a firm is highly levered, its equity is more exposed to aggregate risks. The effects of collateralizability and leverage can thus offset each other in determining the overall riskiness of the firm and consequently its expected equity return.

In order to disentangle these two effects, we conduct an independent double sort on collateralizability and financial leverage. The average returns for the resulting portfolios are reported in Table B.9. First, within each quintile sorted on book leverage, the collateralizability spread is always significantly positive. Second, the average returns of the high-minus-low leverage portfolios within each collateralizability quintile are not statistically significant.

Table B.7: Firm Characteristics

This table reports the median of firm characteristics across portfolios of firms sorted on collateralizability. The sample starts in 1979 and ends in 2016. Collateralizability is defined as in Section D.2. Book leverage is lease adjusted following **?**. BM is the book-to-market ratio. $\frac{I}{K+H}$ is the sum of physical investments (CAPX), R&D and organizational capital investments over the sum of PPEGT and intangible capital. More details on the definition of R&D and organizational capital investments can be found Appendix D.3. $\log(ME)$ is the nature log of the market capitalization. Cash flow is defined as OIBDP to total asset ratio. Gross profitability is defined as revenue minus cost of goods denominated by total assets. ROE is the return on equity, which is the OIBDP divided by book equity. Asset growth is the growth rate of total assets. Type-K asset growth is the growth rate of PPEGT. Age is defined as the years a firm being recorded in COMPUSTAT. WW and SA index are following **?** and **?**, respectively. Dividend is calculated as the mean of the dividend dummy within each portfolio, which represents the probability of a firm paying dividend of that portfolio. Cash/AT is defined as cash and cash equivalents over total asset ratio. The probability of equity (debt) issuance is defined as the mean of a dummy variable within that quintile, which takes value of one if the flow to equity (debt) is negative. Flow to equity is defined as purchases of common stock plus dividends less sale of common stock. Flow to debt is defined as debt reduction plus changes in current debt plus interest paid, less debt issuance. Probability of external financing is defined as the mean of a dummy variable, which takes value of one when the sum of flow to debt and equity are negative.

	1	2	3	4	5
Collateralizability	0.081	0.168	0.260	0.377	0.619
Book leverage	0.104	0.163	0.228	0.343	0.460
BM	0.441	0.576	0.611	0.673	0.670
$\frac{I}{K+H}$	0.174	0.169	0.162	0.165	0.191
$\log(ME)$	3.822	3.988	4.000	4.153	4.178
Cash flow	0.037	0.094	0.110	0.113	0.098
Gross profitability	0.478	0.423	0.375	0.339	0.276
ROE	0.060	0.164	0.204	0.231	0.223
Asset growth	0.003	0.048	0.068	0.079	0.116
Type-K asset growth	0.075	0.092	0.100	0.108	0.129
Age	7.000	9.000	9.000	8.000	8.000
WW	-0.159	-0.183	-0.189	-0.194	-0.191
SA	-2.284	-2.506	-2.540	-2.576	-2.580
Prob(Dividend)	0.136	0.146	0.178	0.172	0.162
Cash/AT	0.246	0.142	0.114	0.087	0.104
Prob(Equity issuance)	0.665	0.594	0.523	0.501	0.496
Prob(Debt issuance)	0.097	0.118	0.114	0.122	0.143
Prob(External finance)	0.240	0.215	0.191	0.190	0.208
Short-term debt/AT	0.007	0.011	0.012	0.015	0.017
Long-term debt/AT	0.006	0.011	0.014	0.019	0.012

Table B.8: Correlations Among Firm Characteristics

This table reports the correlation between collateralizability and other firm characteristics. The sample period is from 1978 to 2016, it focuses on constrained firms identified using **?** index. Log(ME) is the log of market capitalization deflated by CPI. BM is the book-to-market ratio. XRD/AT is R&D expenditure over total book assets. OG/AT is organizational capital over total book assets. I/K is the investment rate, it is calculated as the Compustat item CAPX divided by PPENT. ROA is Compustat item IB divided by book assets.

Variables	Collateralizability	BM	XRD/AT	OG/AT	log(ME)	I/K	ROA
Collateralizability	1.000						
BM	0.105	1.000					
XRD/AT	-0.333	-0.180	1.000				
OG/AT	-0.233	-0.065	0.117	1.000			
log(ME)	-0.013	-0.159	-0.006	-0.207	1.000		
I/K	0.011	-0.021	0.008	-0.001	0.004	1.000	
ROA	0.161	-0.041	-0.456	-0.154	0.126	-0.015	1.000

Table B.9: Independent Double Sort on Collateralizability and Leverage

This table reports annualized average value-weighted monthly excess returns for portfolios double-sorted independently on collateralizability and leverage. The sample starts in July 1979 and ends in December 2016. At the end of June in each year t, we independently sort financially constrained firms into quintiles based on collateralizability (horizontal direction) and into quintiles based on book financial leverage (vertical direction), then we compute the value-weighted returns of each portfolio. The book financial leverage is defined as financial debt over total asset ratio. A firm is considered financially constrained in year t, if its WW index (**?**) is above the respective median at the end of year $t-1$. The t-statistics are estimated following **?**. All returns are annualized by multiplying with 12.

	L Col	2	3	4	H Col	L-H	t-stat
L Lev	11.96	7.58	10.51	10.14	5.48	6.48	1.81
2	13.84	11.38	11.19	5.31	5.98	7.85	1.96
3	13.07	14.16	11.05	9.70	4.50	8.57	2.06
4	15.48	10.10	11.73	5.39	5.04	10.43	2.51
H Lev	16.94	10.82	10.74	8.39	7.25	9.69	2.09
H-L	4.98	3.24	0.23	-1.75	1.76		
t-stat	1.17	0.81	0.06	-0.55	0.60		

C Sensitivity analysis

In this section, we discuss the sensitivity of our quantitative results to several important parameters. To save space, we only discuss the moments which are sensitive to the respective each parameter. The results are reported in Table C.10.

Collateralizability parameter (ζ) The parameter ζ determines the collateralizability of type-K capital. We vary this parameter by $\pm 10\%$ around the benchmark value of 0.513 from Table **??** and make the following observations.

First, since we assume the collateral constraint is binding, higher collateralizability mechanically increases the average leverage ratio. Second, higher collateralizability leads to a lower risk premium for type-K capital, but to a higher risk premium for type-H capital, which overall implies a higher collateralizability premium. This is consistent with our model mechanism. Note that the price of type-K capital contains not only the present value of future cash flows, but also the present value of Lagrangian multipliers. According to equation (**??**), an increase in ζ makes the second component more important, which in turn makes the hedging channel more important and type-K capital less risky. On the other hand, a higher leverage ratio makes the entrepreneur's net worth more volatile, and therefore increases the risk premium of type-H capital.

Type-K and type-H capital ratio (ϕ) We vary this parameter by $\pm 10\%$. A higher ϕ implies a larger proportion of collateralizable assets in the economy, and as a result, it mechanically increases the leverage ratio and the overall asset collateralizability. A higher leverage ratio in turn leads to a more volatile entrepreneur's net worth, and therefore, increases the risk premia for both types of capital. On the other hand, higher ϕ implies more type-K capital, which can be used to hedge against aggregate risk. Therefore, higher ϕ may also reduce the overall riskiness of the aggregate economy and lower down the risk premium. As shown in Panel B, hedging effect dominates, thus the overall risk premium is lower and return spread is also lower.

Shock correlation ($\rho_{A,x}$) As explained in Section **??**, we assume a negative correlation between the aggregate productivity shock and the financial shock in order for the model to generate a positive correlation between consumption and investment growth, consistent with the data. For parsimony, we had imposed a perfectly negative correlation in our benchmark calibration. We vary this parameter and consider the cases $\rho_{A,x} = -0.8$ and -0.9.

In terms of results, the correlation between consumption and investment growth becomes less positive, confirming our model intuition presented in Section ??. Furthermore, varying this correlation parameter does not qualitatively change the collateralizability spread and has limited effects on various risk premia as well.

Table C.10: Sensitivity Analysis

The table shows the results of sensitivity analyses, where key parameters of the model are varied around the values from the benchmark calibration shown in Table ??. A star superscript denotes the parameter value from the benchmark calibration.

Panel A: the role of collateralizability parameter ζ

	Data	Benchmark	0.9ζ	1.1ζ
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.50	3.49
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.34	0.33	0.38
$E[R^M - R^f]$	5.71 (2.25)	4.16	3.26	4.36
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	12.00	12.27
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-6.22	-7.78

Panel B: the role of capital composition ϕ: $\pm 10\%$

	Data	Benchmark	0.9ϕ	1.1ϕ
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.57	3.49
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.34	0.31	0.35
$E[R^M - R^f]$	5.71 (2.25)	4.16	4.26	3.69
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	12.18	12.11
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-7.44	-6.81

Panel C: the role of shock correlations

	Data	Benchmark	$corr(\varepsilon_A, \varepsilon_x) = -0.8$	$corr(\varepsilon_A, \varepsilon_x) = -0.9$
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.42	3.55
$corr(\Delta c, \Delta i)$	0.40 (0.28)	0.42	0.31	0.37
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.34	0.32	0.32
$E[R^M - R^f]$	5.71 (2.25)	4.16	3.73	3.82
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	11.05	11.69
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-6.09	-6.69

Persistence parameters of exogenous shocks (ρ_x and ρ_A) We vary persistence parameters of exogenous shocks (ρ_x and ρ_A) one at a time. The parameter variations we consider change

Table C.10: Sensitivity Analysis (Continued)

Panel D: the role of ρ_x: $\pm$ half life

	Data	Benchmark	80% Half life	120% Half life
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.41	3.62
$AC1(\Delta y)$	0.49 (0.15)	0.53	0.50	0.56
$AC1(\frac{B_t}{K_t+H_t})$	0.86 (0.33)	0.85	0.86	0.85
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.34	0.34	0.35
$E[R^M - R^f]$	5.71 (2.25)	4.16	3.98	4.44
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	11.66	12.49
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-6.73	-7.80

Panel E: the role of ρ_A: $\pm$ half life

	Data	Benchmark	80% Half life	120% Half life
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.51	3.52
$AC1(\Delta y)$	0.49 (0.15)	0.53	0.53	0.53
$AC1(\frac{B_t}{K_t+H_t})$	0.86 (0.33)	0.85	0.85	0.86
$E[\frac{B_t}{K_t+H_t}]$	0.32(0.01)	0.34	0.34	0.35
$E[R^M - R^f]$	5.71 (2.25)	4.16	3.63	4.60
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	12.10	12.31
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-6.79	-7.70

Panel F: the role of σ_x: $\pm 10\%$

	Data	Benchmark	$0.9\sigma_x$	$1.1\sigma_x$
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.41	3.62
$\sigma(\Delta i)$	10.30 (2.36)	8.67	8.00	9.16
$E[R^M - R^f]$	5.71 (2.25)	4.16	3.98	4.44
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	11.66	12.49
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-6.73	-7.80

Panel G: the role of σ_A: $\pm 10\%$

	Data	Benchmark	$0.9\sigma_A$	$1.1\sigma_A$
$\sigma(\Delta y)$	3.05 (0.60)	3.52	3.25	3.78
$\sigma(\Delta i)$	10.30 (2.36)	8.67	8.48	8.78
$E[R^M - R^f]$	5.71 (2.25)	4.16	3.20	5.29
$\sigma(R^M - R^f)$	20.89 (2.21)	12.14	11.37	12.88
$E[\bar{R}^{K,Lev} - \bar{R}^H]$		-7.35	-6.76	-7.84

the half-life of a shock to x or a by $\pm 20\%$.

First, an increase in ρ_x has opposite effects on the risk premia of type-K and type-H capital. On the one hand, a more persistent financial shock makes type-K capital an even better hedging device, which reduces the equilibrium risk premium. On the other hand, entrepreneurs' net worth becomes more volatile, and as a result, the risk premium of type-H capital increases. Put together, this leads to a higher risk premium for the aggregate market and to a larger collateralizability spread. Second, an increase in ρ_A generates a stronger long-run risk channel in cash flows, and as a result, we observe higher risk premia for both type-K and type-H capital. The effects of lower ρ_x and ρ_A are exactly opposite to those generated by higher values for these parameters.

Shock volatilities (σ_x and σ_A) We vary the shock volatilities σ_x and σ_A, one at a time, by $\pm 10\%$. We observe that the effect caused by increasing the two shock volatilities are very similar. A higher σ_x or σ_A leads to an increase in both the market risk premium and the collateralizability spread, which is intuitively clear, since the economy in general becomes riskier.

D Data and measurement

We now provide details on the data sources, the construction of our empirical collateralizability measure, and on the measurement of intangible capital.

D.1 Data sources

Our major sources of data are (1) firm level balance sheet data from the CRSP/Compustat Merged Fundamentals Annual Files, (2) monthly stock returns from CRSP, and (3) industry level non-residential capital stock data from the BEA table.[5] We adopt the standard screening process for the CRSP/Compustat Merged Database. We exclude utilities and financial firms (SIC codes between 4900 and 4999 and between 6000 and 6999, respectively). Additionally, we only keep common stocks that are traded on NYSE, AMEX and NASDAQ. The accounting treatment of R&D expenses was standardized in 1975, and we allow three years for firms to adjust to the new accounting rules, so that our sample starts in 1978. Following **?**, we exclude firm-year observations for which the value

[5]The BEA table is from "private fixed asset by industry", Table 3.1ESI.

of total assets or sales is less than \$1 million. We focus on the impact of asset collateralizability on debt capacity of firms, therefore we drop small firms, which do not have much debt in the first place. In practice, we drop firm-year observations with market capitalization below \$8 million, which roughly corresponds to the bottom 5% of firms. All firm characteristics are winsorized at the 1% level. The potential delisting bias of stock returns is corrected following **?** and **?**.

In order to obtain a long sample with broader coverage,[6] we use the narrowly defined industry level non-residential fixed asset (structure, equipment and intellectual) from the BEA tables to back out industry level structure and equipment capital shares.

In Table D.11, we provide the definitions of the variables used in our empirical analyses.

D.2 Measurement of collateralizability

This section provides details on the construction of the firm specific collateralizability measure, complementing the description of the methodology provided in Section **??**.

We first construct proxies for the share of the two types of capital, denoted by *StructShare* and *EquipShare*. Then we run the leverage regression (**??**), which allows us to later calculate the firm-specific collateralizability score.

The BEA classification features 63 industries. We match the BEA data to Compustat firm level data using NAICS codes, assuming that, for a given year, firms in the same industry have the same structure and equipment capital shares. We construct measures of structure and equipment shares for industry l in year t as

$$StructShare_{l,t} = \frac{\text{Structure}_{l,t}^{BEA}}{\text{Fixed Asset}_{l,t}^{BEA}} \frac{\text{Fixed Asset}_{l,t}^{\text{Compustat}}}{\text{PPEGT}_{l,t}^{\text{Compustat}} + \text{Intangible}_{l,t}^{\text{Compustat}}}$$

and

$$EquipShare_{l,t} = \frac{\text{Equipment}_{l,t}^{BEA}}{\text{Fixed Asset}_{l,t}^{BEA}} \frac{\text{Fixed Asset}_{l,t}^{\text{Compustat}}}{\text{PPEGT}_{l,t}^{\text{Compustat}} + \text{Intangible}_{l,t}^{\text{Compustat}}},$$

where $AT_{l,t}$ are total assets in industry l in year t, i.e., the sum of assets across all firms in our sample belonging to industry l in year t. The first component on the right hand side refers to the

[6] COMPUSTAT shows the components of physical capital (PPEGT) only for the period from 1969 to 1997. However, even for the years between 1969 and 1997, only 40% of the observations have non-missing entries for the components of PPEGT, which are buildings (PPENB), machinery and equipment (PPENME), land and improvements (PPENLI).

structure (equipment) share from BEA data, which is given as the ratio of structure (equipment) to fixed assets at the industry level. The second component refers to the industry level fixed asset to total asset ratio in Compustat. We use PPEGT in Compustat as the equivalent for fixed assets in the BEA data. By doing so, we map the BEA industry level measure of structure (equipment) to fixed asset ratio to corresponding measures in the Compustat, at the industry level. Since we distinguish assets by their collateralizability, we normalize fixed assets by the total value of physical and intangible capital.

We interpret the weighted sum, $\zeta_S StructShare_{l,t} + \zeta_E EquipShare_{l,t}$, as the contribution of structure and equipment capital to financial leverage. The product of this sum and the book value of assets, $(\zeta_S StructShare_{l,t} + \zeta_E EquipShare_{l,t}) \cdot AT_{i,t}$, then represents the total collateralizable capital of firm i in year t.[7] Given this, the collateralizability score for firm i in year t is computed as

$$\zeta_{i,t} = \frac{(\zeta_S \cdot StructShare_{l,t} + \zeta_E \cdot EquipShare_{l,t}) \cdot AT_{i,t}}{PPEGT_{i,t} + Intangible_{i,t}}, \tag{D51}$$

where $PPEGT_{i,t}$ and $Intangible_{i,t}$ are the physical capital and intangible capital of firm i in year t, respectively. The importance of taking intangible capital into account has been emphasized in the recent literature, e.g., by **?** and **?**. The asset-specific collateralizablity parameters ζ_S and ζ_E we adopt in our empirical analyses are the ones shown in the last column of Table **??**, where firms are classified as constrained based jointly on all three measures (SA index, WW index, and non-dividend paying).

In the above collateralizability measure, we implicitly assume the collateralizability parameter for intangible capital to be equal to zero. We do this based on empirical evidence that intangible capital can hardly be used as collateral, since only 3% of the total value of loans to companies are actually collateralized by intangibles like patents or brands (**?**). Our results remain qualitatively very similar when we exclude intangible capital from the denominator of the collateralizability measure in (D51) and only exploit the differences in collateralizability between structure and equipment capital.

[7] Alternatively, we also used the market value of assets to compute total collateralizable capital. The empirical collteralizability spread based on this sorting measure is even stronger than that obtained in our benchmark analysis.

D.3 Measuring intangible capital

In this section, we provide details regarding the construction of firm-specific intangible capital. The total amount of intangible capital of a firm is given by the sum of externally acquired and internally created intangible capital, where the latter consists of R&D capital and organizational capital.

Externally acquired intangible capital is given by item *INTAN* in Compustat. Firms typically capitalize this type of asset on the balance sheet as part of intangible assets. For the average firm in our sample, *INTAN* amounts to about 19% of total intangible capital with a median of 3%, consistent with **?**. We set externally acquired intangibel capital to zero, whenever the entry for INTAN is missing.

Concerning internally created intangible capital, R&D capital does not appear on the firm's balance sheet, but it can be estimated by accumulating past expenditures. Following **?** and **?**, we capitalize past R&D expenditures (Compustat item *XRD*) using the so-called perpetual inventory method, i.e.,[8]

$$RD_{t+1} = (1 - \delta_{RD})RD_t + XRD_t,$$

where δ_{RD} is the depreciation rate of R&D capital. Following **?**, we set the depreciation rates for different industries following **?**. For unclassified industries, the depreciation rate is set to 15%.[9]

Finally, we also need the initial value RD_0. We use the first non-missing R&D expenditure, XRD_1, as the first R&D investment, and specify RD_0 as

$$RD_0 = \frac{XRD_1}{g_{RD} + \delta_{RD}}, \tag{D52}$$

where g_{RD} is the average annual growth rate of firm level R&D expenditure. In our sample, g_{RD} is around 29%.

Following **?** and **?**, our organizational capital is constructed by accumulating a fraction of Compustat item *XSGA*, "Selling, General and Administrative Expense", which indirectly reflects the reputation or human capital of a firm. However, as documented by **?**, *XSGA* also includes R&D expenses *XRD*, unless they are included in the cost of goods sold (Compustat item *COGS*). Additionally, *XSGA* sometimes also incorporates the in-process R&D expense (Compustat item *RDIP*).

[8] This method is also used by the BEA R&D satellite account.

[9] Our results are not sensitive to the choice of depreciation rates.

Hence, following **?**, we subtract XRD and $RDIP$ from $XSGA$.[10] Additionally, also following **?**, we add the filter that when XRD exceeds $XSGA$, but is less than $COGS$, or when $XSGA$ is missing, we keep XSGA with no further adjustment. Afterwards, we replace missing XSGA with zero. As in **?**, **?**, and **?**, we count only 30% of SGA expenses as investment in organizational capital, the rest is treated as operating costs.

Using a procedure analogous to the one described above for internally created R&D capital, organizational capital is constructed as

$$OG_{t+1} = (1 - \delta_{OG})OG_t + SGA_t,$$

where $SGA_t = 0.3(XSGA_t - XRD_t - RDIP_t)$ and the depreciation rate δ_{OG} is set to 20%, consistent with **?** and **?**. Again analogous to the case of R&D capital we set the initial level of organizational capital OG_0 according to

$$OG_0 = \frac{SGA_1}{g_{OG} + \delta_{OG}}.$$

The average annual growth rate of firm level $XSGA$, g_{OG}, is 18.9% in our sample.

[10] $RDIP$ (in-process R&D expense) is coded as negative in Compustat. Subtracting RDIP from XSGA means RDIP is added to XSGA. As discussed in **?**, XSGA does not include this component, so we add this component back to XSGA, then subtract the total amount of R&D expenditures.

Table D.11: Definition of Variables

Variables	Definition	Sources
Structure share	We first construct the structure shares from BEA industry capital stock data, defined as structure capital over total fixed asset ratio. Then we rescale the structure shares by the corresponding industry average of physical asset (PPEGT) to book asset ratio (AT).	BEA + Compustat
Equipment share	We first construct the equipment shares from BEA industry capital stock data, defined as equipment capital over total fixed asset ratio. Then we rescale the equipment shares by the corresponding industry average of physical asset (PPEGT) to book asset ratio (AT).	BEA + Compustat
Intangible capital	Intangible capital is defined following **?**. We capitalize R&D and SGA expenditures using the perpetual inventory method.	Compustat
Collateralizability	Collateralizable capital divided by PPEGT + Intangible. Collateralizable capital and intangible capital are defined in Section D.2	BEA + Compustat
BE	Book value of equity, computed as the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) as the book value of preferred stock.	Compustat
ME	Market value of equity is copmputed as price per share times the number of shares outstanding. The share price is taken from CRSP, the number of shares outstandings from Compustat or CRSP, depending on availability.	CRSP+Compustat
BM	Book to market value of equity ratio.	Compustat
Tangibility	Physical capital (PPEGT) to the sum of physical (PPEGT) and intangible capital ratio.	Compustat
Book size	Natural log of the sum of PPEGT and intangible capital.	Compustat
Gross profitability	Compustat item REVT minus COGS divided by AT.	Compustat
OG/AT	Organizational capital divided by total assets (AT).	Compustat
XRD/AT	R&D expenditure to book asset ratio.	Compustat
Book leverage	Lease adjusted book leverage is defined as lease adjusted debt over total asset ratio (AT). The lease adjusted debt is the financial debt (DLTT+DLC) plus the net present value of capital lease as in **?**.	Compustat
Dividend dummy	Dummy variable equal to 1, if the firm's dividend payment (DVT, DVC or DVP) over the year was positive.	Compustat
Sales growth volatility	Rolling window standard deviation of past 4 year's sales growth.	Compustat
Rating dummy	Dummy variable equal to 1, if the firm has either a bond rating or a commercial paper rating, and 0 otherwise.	Compustat
Marginal tax rate	Following **?**.	John Graham's website
WW index	Following **?**.	Compustat
SA index	Following **?**.	Compustat
Return on asset	Cash flow (EBITDA) divided by total assets (AT).	Compustat

Table D.11: Definition of Variables (Continued)

Variables	Definition	Sources
Cash	Compustat item CHE.	Compustat
Equity issuance	The negative of flow to debt. Compustat item -(PRSTKC+DV-SSTK).	Compustat
Debt issuance	The negative of flow to debt. Compustat item -(DLTR+DLCCH+XINT-DLTIS).	Compustat
External finance	The sum of equity and debt issuance.	Compustat
Short-term debt	Compustat item DLC.	Compustat
Long-term debt	Compustat item DLTR.	Compustat
Return on equity	Operating income before depreciation (OIBDP) divided by book equity.	Compustat
Financial leverage	Total financial debt (DLTT + DLC) over total book asset (AT) ratio.	Compustat
Cash flow	Compustat item EBITDA divided by total assets	Compustat
Asset growth	Growth rate of total assets	Compustat
Type-K asset growth	Growth rate of PPEGT	Compustat
Return on equity	Cash flow (EBITDA) divided by book equity	Compustat
Age	The current year minus the year where a firm has the first non-missing observation.	Compustat