The Collateralizability Premium

Hengjie Ai, Jun Li, Kai Li and Christian Schlag*

April 25, 2018

Abstract

This paper studies the implications of credit market frictions for the cross-section of expected stock returns. A common prediction of macroeconomic theories of credit market frictions is that the tightness of financial constraints is countercyclical. As a result, capital that can be used as collateral to relax such constraints provides insurance against aggregate shocks and should command a lower risk compensation compared to non-collateralizable assets. Based on a novel measure of asset collateralizability, we provide empirical evidence supporting this prediction. A long-short portfolio constructed from firms with low and high asset collateralizability generates an average excess return of around 8% per year. We develop a general equilibrium model with heterogeneous firms and financial constraints to quantitatively account for the effect of collateralizability on the cross-section of expected returns.

JEL Codes: E2, E3, G12

Keywords: Cross-section of Returns, Financial Frictions, Collateral Constraint

First Draft: January 31, 2017

* Hengjie Ai (hengjie.ai@umn.edu) is at the Carlson School of Management of University of Minnesota; Jun Li (jun.li@hof.uni-frankfurt.de) is at Goethe University Frankfurt and SAFE; Kai Li (kaili@ust.hk) is at Hong Kong University of Science and Technology; and Christian Schlag (schlag@finance.uni-frankfurt.de) is at Goethe University Frankfurt and SAFE. This paper was previously circulated under the title "Asset Collateralizabiltiy and the Cross-Section of Expected Returns". The authors thank Frederico Belo, Adrian Buss (EFA discussant), Zhanhui Chen (ABFER and Finance Down Under discussant), Nicola Fuchs-Schündeln, Bob Goldstein, Jun Li (UT Dallas), Xiaoji Lin (AFA discussant), Dimitris Pananikolaou, Julien Penasse, Vincenzo Quadrini, Adriano Rampini (NBER SI discussant), Amir Yaron, Harold Zhang, Lei Zhang (CICF discussant) as well as the participants at ABFER annual meeting, SED, NBER Summer Institute (Capital Market and the Economy), EFA, CQAsia Annual Conference in Hong Kong, Finance Down Under, University of Minnesota (Carlson), Goethe University Frankfurt, Hong Kong University of Science and Technology, UT Dallas, City University of Hong Kong for their helpful comments. The usual disclaimer applies.

1 Introduction

A large literature in economics and finance emphasizes the importance of credit market frictions in affecting macroeconomic fluctuations.[1] Although models differ in details, a common prediction is that financial constraints exacerbate economic downturns because they are more binding in recessions. As a result, theories of financial frictions predict that assets relaxing financial constraints should provide insurance against aggregate shocks. We evaluate the implication of this mechanism for the cross-section of equity returns.

From an asset pricing perspective, when financial constraints are binding, the value of collateralizable capital includes not only the dividends it generates, but also the present value of the Lagrangian multipliers of the collateral constraints it relaxes. If financial constraints are tighter in recessions, then a firm holding more collateralizable capital should require a lower expected return in equilibrium, since the collateralizability of its assets provides a hedge against the risk of being financially constrained in recessions, making the firm less risky.

To examine the relationship between asset collateralizability and expected returns, we first construct a measure of firms' asset collateralizability. Guided by the corporate finance theory linking firms' capital structure decisions to collateral constraints (e.g., Rampini and Viswanathan (2013)), we measure asset collateralizability as the value-weighted average of the collateralizability of the different types of assets owned by the firm. Our measure can be interpreted as the fraction of firm value that can be attributed to the collateralizability of its assets.

We sort stocks into portfolios according to this collateralizability measure and document that the spread between the low collateralizability portfolio and the high-collateralizability portfolio is on average close to 8% per year within the subset of financially constrained firms. The difference in returns remains significant after controlling for conventional factors such as the market, size, value, momentum, and profitability.

To quantify the effect of asset collateralizability on the cross-section of expected returns, we develop a general equilibrium model with heterogeneous firms and financial constraints. In our model, firms are operated by entrepreneurs who experience idiosyncratic productivity shocks. As in Kiyotaki and Moore (1997, 2012), lending contracts can not be fully enforced and therefore require collateral. Firms with high productivity and low net worth have higher financing needs and in equilibrium, so that they acquire more collateralizable assets in order to borrow. In the constrained efficient allocation in our model, heterogeneity in productivity and net worth translate into heterogeneity in the collateralizability of firm assets. In this

[1] Quadrini (2011) and Brunnermeier et al. (2012) provide comprehensive reviews of this literature.

setup, we show that, at the aggregate level, collateralizable capital requires lower expected returns in equilibrium, and in the cross-section, firms with high asset collateralizability earn low risk premiums.

Our calibrated model quantitatively matches the conventional macroeconomic quantity dynamics and asset pricing moments, and, more importantly, it is able to quantitatively account for the empirical relationship between asset collateralizability, leverage, and expected returns.

Related Literature This paper builds on the large macroeconomics literature studying the role of credit market frictions in generating fluctuations across the business cycle (see Quadrini (2011) and Brunnermeier et al. (2012) for recent reviews). The papers that are most related to ours are those emphasizing the importance of borrowing constraints and contract enforcements, such as Kiyotaki and Moore (1997, 2012), Gertler and Kiyotaki (2010), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Elenev, Landvoigt, and Van Nieuwerburgh (2017). Gomes, Yamarthy, and Yaron (2015) study the asset pricing implications of credit market frictions in a production economy. A common prediction of the papers in this literature is that the tightness of borrowing constraints is counter-cyclical. We study the implications of this prediction on the cross-section of expected returns.

Our paper is also related to the corporate finance literature that emphasize the importance of asset collateralizability for the capital structure decisions of firms. Albuquerque and Hopenhayn (2004) study dynamic financing with limited commitment, Rampini and Viswanathan (2010, 2013) develop a joint theory of capital structure and risk management based on asset collateralizability, and Schmid (2008) considers the quantitative implications of dynamic financing with collateral constraints. Falato, Kadyrzhanova, and Sim (2013) provide empirical evidence for the link between asset collateralizability and leverage in aggregate time series and in the cross section.

Our paper further belongs to the literature on production-based asset pricing, for which Kogan and Papanikolaou (2012) provide an excellent survey. From the methodological point of view, our general equilibrium model allows for a cross section of firms with heterogeneous productivity and is related to previous work including Gomes et al. (2003), Gârleanu, Kogan, and Panageas (2012), Ai and Kiku (2013), and Kogan, Papanikolaou, and Stoffman (2017). Compared to these papers, our model incorporates financial frictions. In addition, our aggregation result is novel in the sense that despite heterogeneity in productivity and the presence of aggregate shocks, the equilibrium in our model can be solved for without having to use any distribution as a state variable.

Our paper is also connected to the broader literature linking investment to the cross-section of expected returns. Zhang (2005) provides an investment-based explanation for the value premium. Li (2011) and Lin (2012) focus on the relationship between R&D investment and expected stock returns. Eisfeldt and Papanikolaou (2013) develop a model of organizational capital and expected returns. Belo, Lin, and Yang (2017) study implications of equity financing frictions on the cross-section of stock returns.

The rest of the paper is organized as follows. We summarize our empirical results on the relationship between asset collateralizability in Section 2. We describe a general equilibrium model with collateral constraints in Section 3 and analyze its asset pricing implications in Section 4. In Section 5, we provide a quantitative analysis of our model. Section 6 concludes.

2 Empirical Facts

2.1 Measuring collateralizability

To empirically examine the link between asset collateralizability and expected returns, we first construct a measure of asset collateralizability at the firm level. Models with financial frictions typically feature a collateral constraint that takes the following general form:

$$B_{i,t} \leq \sum_{j=1}^{J} \zeta_j q_{j,t} K_{i,j,t+1}, \tag{1}$$

where $B_{i,t}$ denotes the total amount of borrowing by firm i at time t, $q_{j,t}$ is the price of type-j capital at time t, and $K_{i,j,t+1}$ is the associated amount of capital used by firm i at time $t+1$, which is determined at time t. This means we assume a one period time to build like in standard real business cycle models.

The different types of capital differ with respect to their collateralizability. The parameter $\zeta_j \in [0,1]$ measures the degree to which type-j capital is collateralizable. $\zeta_j = 1$ implies that type-j capital can be fully collateralized, while $\zeta_j = 0$ means that this type of capital cannot be collateralized at all. Equation (1) thus says that total borrowing by the firm is constrained by the total collateral it can provide.

Our collateralizability measure is a value-weighted average of collateralizabilities of different types of firm assets. Specifically, the overall collateralizability of firm i's assets at time

t, $\bar{\zeta}_{i,t}$, is defined as:

$$\bar{\zeta}_{i,t} \equiv \sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}, \qquad (2)$$

where $V_{i,t}$ denotes the total value of firm i's assets. In models with collateral constraints, the value of the collateralizable capital typically includes the present value of both the cash flows it generates and of the Lagrangian multipliers of the collateral constraint. These represent the marginal value of relaxing the constraint through the use of collateralizable capital. In Section 5.4 below we show that, in our model, the firm-level collateralizability measure $\bar{\zeta}_{i,t}$ can be intuitively interpreted as the relative weight of present value of the Lagrangian multipliers in the total value of the firm's assets.[2] As a result, it summarizes the heterogeneity in firms' risk exposure due to asset collateralizability.

To empirically construct the collateralizability measure $\bar{\zeta}_{i,t}$ for each firm, we follow a two-step procedure. First, we use a regression-based approach to estimate the callateralizability parameters ζ_j for each type of capital. Motivated by previous work (e.g., Rampini and Viswanathan (2013, 2017)), we broadly classify assets into three categories based on their collateralizability: structure, equipment, and intangible capital. Focusing on the subset of financially constrained firms for which the constraint (1) holds with equality, we divide both sides of the equation by the total value of firm assets at time t, $V_{i,t}$, and obtain

$$\frac{B_{i,t}}{V_{i,t}} = \sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}.$$

The above equation links firm i's leverage ratio, $\frac{B_{i,t}}{V_{i,t}}$ to its value-weighted collateralizability measure. Empirically, we run a panel regression of firm leverage, $\frac{B_{i,t}}{V_{i,t}}$, on the value weights of the different types of capital, $\frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}$, to estimate the collateralizability parameter ζ_j for structure and equipment, respectively.[3]

Second, the firm specific "collateralizability score" at time t, denoted as $\bar{\zeta}_{i,t}$, is constructed as a weighted average of the collateralizability of individual assets via

$$\bar{\zeta}_{i,t} = \sum_{j=1}^{J} \widehat{\zeta}_j \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}},$$

[2]See equation (31) below.

[3]We impose the restriction that $\zeta_j = 0$ for intangible capital both because previous work typically argues that intangible capital cannot be used as collateral, and because its empirical estimate is slightly negative in unrestricted regressions.

where $\widehat{\zeta}_j$ denotes the coefficient estimates from the panel regression described above. We provide further details concerning the construction of the collateralizability measure in Appendix C.2.

2.2 Collateralizability and expected returns

Equipped with the time series of the collateralizability measure for each firm, we follow the standard procedure and construct collateralizability-sorted portfolios. Consistent with our theory, we focus on the subset of financially constrained firms, whose asset valuations contain a non-zero Lagrangian multiplier component.

Table 1 reports average annualized excess returns, t-statistics, and Sharpe ratios of the five collateralizability-sorted portfolios. We consider three alternative measures for the degree to which a firm is financially constrained: the WW index (Whited and Wu (2006), Hennessy and Whited (2007)), the SA index (Hadlock and Pierce (2010)), and an indicator of whether the firm has paid dividends over the past year. We classify a firm as being financially constrained if it has a WW index higher than the median (top panel), or an SA index higher than the median (middle panel), or if it has not paid dividends during the previous year (bottom panel).

The top panel shows that, based on the WW index, the the average equity return for firms with low collateralizability (Quintile 1) is around 8% higher on an annualized basis than that of a typical high collateralizability firm (Quintile 5). We call this return spread the (negative) collateralizability premium. The return difference is statistically significant with a t-value of 2.76, and its Sharpe ratio is 0.45. The premium is robust with respect to the way we measure if a firm is financially constrained, as can be seen from the middle and bottom panels of Table 1.

In sum, the evidence on the collateralizability spread among financially constrained firms strongly supports our theoretical prediction that the collateralizable assets are less risky and therefore are expected to earn a lower return. In the next section, we develop a general equilibrium model with heterogenous firms and financial constraints to formalize the above intuition and to quantitatively account for the negative collateralizability premium.

Table 1: Portfolios Sorted on Collateralizability

This table reports average value-weighted monthly excess returns (in percent and annualized) for portfolios sorted on collateralizability. The sample period is from July 1979 to December 2016. At the end of June of each year t, we sort the constrained firms into five quintiles based on their collateralizability measures at the end of year $t-1$, where Quintile 1 (Quintile 5) contains the firms with the lowest (highest) share of collateralizable assets. A firm is classified as constrained at the end of year $t-1$, if its WW or its SA index are higher than the corresponding cross-sectional median in year $t-1$, or if the firm has not paid dividends in year $t-1$. The WW and SA indices are constructed according to Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. Standard errors are estimated using Newey-West estimator allowing for one lag. The table reports average excess returns $E[R]-R_f$, as well as the associated t-statistics, and Sharpe ratios (SR).

	1	2	3	4	5	1-5
	Financially constrained firms - WW index					
$E[R]-R_f(\%)$	13.33	11.59	9.43	9.37	5.36	7.96
t	(2.82)	(2.71)	(2.32)	(2.33)	(1.44)	(2.76)
SR	0.46	0.44	0.38	0.38	0.24	0.45
	Financially constrained firms - SA index					
$E[R]-R_f(\%)$	10.42	11.40	11.42	8.47	4.47	5.95
t	(2.16)	(2.55)	(2.61)	(2.14)	(1.12)	(2.11)
SR	0.35	0.42	0.43	0.35	0.18	0.34
	Financially constrained firms - Non-Dividend					
$E[R]-R_f(\%)$	14.98	9.91	12.10	6.34	7.97	7.00
t	(3.30)	(2.33)	(2.78)	(1.48)	(2.08)	(2.50)
SR	0.54	0.38	0.45	0.24	0.34	0.41

3 A General Equilibrium Model

This section describes the ingredients of our quantitative theory of the collateralizability spread. The aggregate aspect of the model is intended to follow standard macro models with collateral constraints such as Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010). We allow for heterogeneity in the collateralizability of assets as in Rampini and Viswanathan (2013). The key additional elements in our theory are idiosyncratic productivity shocks and firm entry and exit. These features allow us to generate quantitatively plausible firm dynamics in order to study the implications of financial constraints for the cross-section of equity returns.

3.1 Households

Time is infinite and discrete. The representative household consists of a continuum of workers and a continuum of entrepreneurs. Workers (entrepreneurs) receive their labor (capital) incomes every period and submit them to the planner of the household, who makes decisions for consumption for all members of the household. Entrepreneurs and workers make their financial decisions separately.[4]

The household ranks the utility of consumption plans according to the following recursive preferences as in Epstein and Zin (1989):

$$U_t = \left\{ (1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(E_t [U_{t+1}^{1-\gamma}] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

where β is the time discount rate, ψ is the intertemporal elasticity of substitution, and γ is the relative risk aversion. As we will show later in the paper, together with the endogenous equilibrium long run risk, the recursive preferences in our model generate a volatile pricing kernel and a significant equity premium as in Bansal and Yaron (2004).

In every period t, the household purchases the amount $B_{i,t}$ of risk-free bonds from entrepreneur i, from which it will receive $B_{i,t} R_{t+1}^f$ next period, where R_{t+1}^f denotes the gross risk-free interest rate from period t to $t+1$. In addition, it receives capital income $\Pi_{i,t}$ from entrepreneur i and labor income $W_t L_t$ from all members who are workers. Without loss of generality, we assume that all workers are endowed with the same number L_t of hours per

[4]Like Gertler and Kiyotaki (2010), we make the assumption that household members make joint decisions on their consumption to avoid the need to keep the distribution of entrepreneur income as the state variable.

period. The household budget constraint at time t can therefore be written as

$$C_t + \int B_{i,t} di = W_t L_t + R_t^f \int B_{i,t-1} di + \int \Pi_{i,t} di.$$

Let M_{t+1} denote the the stochastic discount factor implied by household optimization. Under recursive utility, $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$, and the optimality of the intertemporal saving decisions implies that the risk-free interest rate must satisfy

$$E_t[M_{t+1}] R_{t+1}^f = 1. \tag{3}$$

3.2 Entrepreneurs

Entrepreneurs are agents operating productive ideas. An entrepreneur who starts at time 0 draws an idea with initial productivity $\bar{z}$ and begins the operation with initial net worth N_0. Under our convention, N_0 is also the total net worth of all entrepreneurs at time 0 because the total measure of all entrepreneurs is normalized to one.

Let $N_{i,t}$ denote entrepreneur i's net worth at time t, and let $B_{i,t}$ denote the total amount of risk-free bonds the entrepreneur issues to the household at time t. Then the time-t budget constraint for the entrepreneur is given as

$$q_{K,t} K_{i,t+1} + q_{H,t} H_{i,t+1} = N_{i,t} + B_{i,t}. \tag{4}$$

In (4) we assume that there are two types of capital, K and H, that differ in their collateralizability, and we use $q_{K,t}$ and $q_{H,t}$ to denote their prices at time t. $K_{i,t+1}$ and $H_{i,t+1}$ are the amount of capital that entrepreneur i purchases at time t, which can be used for production over the period from t to $t+1$. We assume that the entrepreneur has access to only risk-free borrowing contracts, i.e., we do not allow for state-contingent debt. At time t, the entrepreneur is assumed to have an opportunity to default on his contract and abscond with all of the type-H capital and a fraction of $1-\zeta$ of the type-K capital. Because lenders can retrieve a ζ fraction of the type-K capital upon default, borrowing is limited by

$$B_{i,t} \leq \zeta q_{K,t} K_{i,t+1}. \tag{5}$$

Type-K capital can therefore be interpreted as collateralizable, while type-H capital cannot be used as collateral.

From time t to $t+1$, the productivity of entrepreneur i evolves according to the law of motion

$$z_{i,t+1} = z_{i,t} e^{\varepsilon_{i,t+1}}, \tag{6}$$

where $\varepsilon_{i,t+1}$ is a Gaussian shock with mean μ_ε and variance σ_ε^2, assumed to be i.i.d. across agents i and over time. We use $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ to denote entrepreneur i's equilibrium profit at time $t+1$, where $\bar{A}_{t+1}$ is aggregate productivity in period $t+1$, and $z_{i,t+1}$ denotes entrepreneur i's idiosyncratic productivity. The specification of the aggregate productivity processes will be provided below in Section 5.1.

In each period, after production, the entrepreneur experiences a liquidation shock with probability λ, upon which he loses his idea and needs to liquidate his net worth to return it back to the household.[5] If the liquidation shock happens, the entrepreneur restarts with a draw of a new idea with initial productivity $\bar{z}$ and an initial net worth χN_t in period $t+1$, where N_t is the total (average) net worth of the economy in period t, and $\chi \in (0,1)$ is a parameter that determines the ratio of the initial net worth of entrepreneurs relative to that of the economy-wide average. Conditional on no liquidation shock, the net worth $N_{i,t+1}$ of entrepreneur i at time $t+1$ is determined as

$$\begin{aligned} N_{i,t+1} &= \pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)\, q_{K,t+1} K_{i,t+1} \\ &\quad + (1-\delta)\, q_{H,t+1} H_{i,t+1} - R_{f,t+1} B_{i,t}. \end{aligned} \tag{7}$$

The interpretation is that the entrepreneur receives the profit $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ from production. His capital holdings depreciate at rate δ, and he needs to pay back the debt borrowed from last period plus interest, amounting to $R_{f,t+1} B_{i,t}$

Because of the fact that whenever a liquidity shock occurs, entrepreneurs submit their net worth to the household who chooses consumption collectively for all members, entrepreneurs value their net worth using the same pricing kernel as the household. Let $V_t^i\left(N_{i,t}\right)$ denote the value function of entrepreneur i. It must satisfy the following Bellman equation:

$$V_t^i\left(N_{i,t}\right) = \max_{\{K_{i,t+1}, H_{i,t+1}, N_{i,t+1}, B_{i,t}\}} E_t\left[M_{t+1}\{\lambda N_{i,t+1} + (1-\lambda)\, V_{t+1}^i\left(N_{i,t+1}\right)\}\right], \tag{8}$$

subject to the budget constraint (4), the collateral constraint (5), and the law of motion of $N_{i,t+1}$ given by (7).

We use variables without an i subscript to denote economy-wide aggregate quantities.

[5] This assumption effectively makes entrepreneurs less patient than the household and prevents them from saving their way out of the financial constraint.

The aggregate net worth in the entrepreneurial sector satisfies

$$N_{t+1} = (1-\lambda)\left[\begin{array}{c} \pi\left(\bar{A}_{t+1}, K_{t+1}, H_{t+1}\right) + (1-\delta)\, q_{K,t+1}K_{t+1} \\ +(1-\delta)\, q_{H,t+1}H_{t+1} - R_{f,t+1}B_t \end{array}\right] + \lambda\chi N_t, \tag{9}$$

where $\pi\left(\bar{A}_{t+1}, K_{t+1}, H_{t+1}\right)$ denotes the aggregate profit of all entrepreneurs.

3.3 Production

Final output With $z_{i,t}$ denoting the idiosyncratic productivity for firm i at time t, output $y_{i,t}$ of firm i at time t is assumed to be generated through the following production technology:

$$y_{i,t} = \bar{A}_t\left[z_{i,t}^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}\right]^{\alpha} L_{i,t}^{1-\alpha} \tag{10}$$

In our formulation, α is capital share, and ν is the span of control parameter as in Atkeson and Kehoe (2005). Note that collateralizable and non-collateralizable capital are perfect substitutes in production. This assumption is made for tractability.

Firm i's profit at time t, $\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right)$ is given as

$$\begin{aligned} \pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) &= \max_{L_{i,t}} y_{i,t} - W_t L_{i,t} \\ &= \max_{L_{i,t}} \bar{A}_t\left[z_{i,t}^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}\right]^{\alpha} L_{i,t}^{1-\alpha} - W_t L_{i,t}, \end{aligned} \tag{11}$$

where W_t is the equilibrium wage rate, and $L_{i,t}$ is the amount of labor hired by entrepreneur i at time t.

It is convenient to write the profit function explicitly by maximizing out labor in equation (11) and using the labor market clearing condition $\int L_{i,t}di = 1$ to get

$$L_{i,t} = \frac{z_{i,t}^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}}{\int z_{i,t}^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu} di}, \tag{12}$$

so that entrepreneur i's profit function becomes

$$\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha\bar{A}_t z_{i,t}^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}\left[\int z_{i,t}^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu} di\right]^{\alpha-1}. \tag{13}$$

Given the output of entrepreneur i, $y_{i,t}$, from equation (10), the total output of the economy is given as

$$\begin{aligned} Y_t &= \int y_{i,t} di, \\ &= \bar{A}_t \left[\int z_{i,t}^{1-\nu} \left(K_{i,t} + H_{i,t} \right)^{\nu} di \right]^{\alpha}. \end{aligned} \tag{14}$$

Capital goods We assume that capital goods are produced from a constant-return-to-scale and convex adjustment cost function $G(I, K + H)$, that is, one unit of the investment good costs $G(I, K + H)$ units of consumption goods. Therefore, the aggregate resource constraint is

$$C_t + I_t + G(I_t, K_t + H_t) = Y_t. \tag{15}$$

Without loss of generality, we assume that $G(I_t, K_t + H_t) = g\left(\frac{I_t}{K_t+H_t}\right)(K_t + H_t)$ for some convex function g.

We further assume that the fractions ϕ and $1 - \phi$ of the new investment goods can be used for type-K and type-H capital, respectively. This is another simplifying assumption. It implies that, at the aggregate level, the ratio of type-K to type-H capital is always equal to $\phi/(1 - \phi)$, and thus the total capital stock of the economy can be summarized by a single state variable. The aggregate stocks of type-H and type-K capital satisfy

$$\begin{aligned} H_{t+1} &= (1 - \delta) H_t + (1 - \phi) I_t \\ K_{t+1} &= (1 - \delta) K_t + \phi I_t. \end{aligned} \tag{16}$$

4 Equilibrium Asset Pricing

4.1 Aggregation

Our economy is one with both aggregate and idiosyncratic productivity shocks. In general, we would have to use the joint distribution of capital and net worth as an infinite-dimensional state variable in order to characterize the equilibrium recursively. In this section, we present a novel aggregation result and show that the aggregate quantities and prices of our model can be characterized without any reference to distributions. Given aggregate quantities and prices, quantities and shadow prices at the individual firm level can be computed using equilibrium conditions.

Distribution of idiosyncratic productivity In our model, the law of motion of idiosyncratic productivity shocks, $z_{i,t+1} = z_{i,t}e^{\varepsilon_{i,t+1}}$, is time invariant, implying that the cross-sectional distribution of the $z_{i,t}$ will eventually converge to a stationary distribution.[6] At the macro level, the heterogeneity of idiosyncratic productivity can be conveniently summarized by a simple statistic: $Z_t = \int z_{i,t}di$. It is useful to compute this integral explicitly.

Given the law of motion of $z_{i,t}$ from equation (6) and the fact that entrepreneurs receive a liquidation shock with probability λ, we have:

$$Z_{t+1} = (1-\lambda)\int z_{i,t}e^{\varepsilon_{i,t+1}}di + \lambda\bar{z}.$$

The interpretation is that only a fraction $(1-\lambda)$ of entrepreneurs will survive until the next period, while the rest will restart with a productivity of $\bar{z}$. Note that based on the assumption that $\varepsilon_{i,t+1}$ is independent of $z_{i,t}$, we can integrate out $\varepsilon_{i,t+1}$ and rewrite the above equation as [7]

$$\begin{aligned} Z_{t+1} &= (1-\lambda)\int z_{i,t}E\left[e^{\varepsilon_{i,t+1}}\right]di \ + \lambda\bar{z}, \\ &= (1-\lambda)\,Z_t e^{\mu_\varepsilon+\frac{1}{2}\sigma_\varepsilon^2} + \lambda\bar{z}, \end{aligned}$$

where the last equality follows from the fact that $\varepsilon_{i,t+1}$ is normally distributed. It is straightforward to see that if we choose the normalization $\bar{z} = \frac{1}{\lambda}\left[1-(1-\lambda)\,e^{\mu_\varepsilon+\frac{1}{2}\sigma_\varepsilon^2}\right]$ and initialize the economy by setting $Z_0 = 1$, then $Z_t = 1$ for all t. This will be the assumption we maintain for the rest of the paper.

Firm profits We assume that $\varepsilon_{i,t+1}$ is observed at the end of period t when the entrepreneurs plan next period's capital. As we show in Appendix A, this implies that entrepreneur will choose $K_{i,t+t} + H_{i,t+1}$ to be proportional to $z_{i,t+1}$. Additionally, because $\int z_{i,t+1}di = 1$, we must have

$$K_{i,t+1} + H_{i,t+1} = z_{i,t+1}\left(K_{t+1} + H_{t+1}\right), \tag{17}$$

where K_{t+1} and H_{t+1} are the aggregate quantities of type-K and type-H capital, respectively.

[6] In fact, the stationary distribution of $z_{i,t}$ is a double-sided Pareto distribution. Our model is therefore consistent with the empirical evidence regarding the power law distribution of firm size.

[7] The first line requires us to define the set of firms and the notion of integration in a mathematically careful way. Rather than going to the technical details, we refer the readers to Feldman and Gilles (1985) and Judd (1985). Constantinides and Duffie (1996) use a similar construction in the context of heterogenous consumers.

The assumption that capital is chosen after $z_{i,t+1}$ is observed implies that total output does not depend on the joint distribution of idiosyncratic productivity and capital. This is because given idiosyncratic shocks, all entrepreneurs choose the optimal level of capital such that the marginal productivity of capital is the same across all entrepreneurs. This fact allows us to write $Y_t = \bar{A}_t \left(K_{t+1} + H_{t+1}\right)^{\alpha\nu} \int z_{i,t} di = \bar{A}_t \left(K_{t+1} + H_{t+1}\right)^{\alpha\nu}$. It also implies that the profit at the firm level is proportional to aggregate productivity, i.e.,

$$\pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha \bar{A}_t z_{i,t} \left(K_t + H_t\right)^{\alpha\nu},$$

and the marginal products of capital are equalized across firms for the two types of capital:

$$\frac{\partial}{\partial K_{i,t}} \pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \frac{\partial}{\partial H_{i,t}} \pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha\nu \bar{A}_t \left(K_t + H_t\right)^{\alpha\nu - 1}. \tag{18}$$

To prove (18), we take derivatives of entrepreneur i's profit function (13) with respect to $K_{i,t}$ and $H_{i,t}$, and then impose the optimality condition (17),

Intertemporal optimality Having simplified the profit functions, we can derive the optimality conditions for the entrepreneur's maximization problem (8). Note that given equilibrium prices, the objective function and the constraints are linear in net worth. Therefore, the value function V_t^i must be linear as well. We write $V_t^i\left(N_{i,t}\right) = \mu_t^i N_{i,t}$, where μ_t^i can be interpreted as the marginal value of net worth for entrepreneur i. Furthermore, let η_t^i be the Lagrangian multiplier associated with the collateral constraint (5). The first order condition with respect to $B_{i,t}$ implies

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i\right] R_{t+1}^f + \eta_t^i, \tag{19}$$

where we use the definition

$$\widetilde{M}_{t+1}^i \equiv M_{t+1}[\left(1-\lambda\right)\mu_{t+1}^i + \lambda]. \tag{20}$$

The interpretation is that one unit of net worth allows the entrepreneur to reduce one unit of borrowing, the present value of which is $E_t\left[\widetilde{M}_{t+1}^i\right] R_{t+1}^f$, and relaxes the collateral constraint, the benefit of which is measured by η_t^i.

Similarly, the first order condition for $K_{i,t+1}$ is

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i \frac{\frac{\partial}{\partial K_{i,t+1}} \pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + \left(1-\delta\right) q_{K,t+1}}{q_{K,t}}\right] + \zeta \eta_t^i. \tag{21}$$

An additional unit of type-K capital allows the entrepreneur to purchase $\frac{1}{q_{K,t}}$ units of capital, which pays a profit of $\frac{\partial \pi}{\partial K}\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ over the next period before it depreciates at rate δ. In addition, a fraction ζ of type-K capital can be used as collateral to relax the borrowing constraint.

Finally, optimality with respect to the choice of type-H capital implies

$$\mu_t^i = E_t\left[\widetilde{M}_{t+1}^i \frac{\frac{\partial}{\partial H_{i,t+1}}\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)\, q_{H,t+1}}{q_{H,t}}\right]. \tag{22}$$

Recursive construction of the equilibrium Note that in our model, firms differ in their net worth. First, the net worth depends on the entire history of idiosyncratic productivity shocks, as can be seen from equation (7), since, due to (6), $z_{i,t+1}$ depends on $z_{i,t}$, which in turn depends on $z_{i,t-1}$ etc. Furthermore, the net worth also depends on the need for capital which relies on the realization of next period's productivity shock. Therefore, in general, the marginal benefit of net worth, μ_t^i, and the tightness of the collateral constraint, η_t^i, depend on the individual firm's entire history. Below we show that despite the heterogeneity in net worth and capital holdings across firms, our model allows an equilibrium in which μ_t^i and η_t^i are equalized across firms, and aggregate quantities can be determined independently of the distribution of net worth and capital.

The assumptions that type-K and type-H capital are perfect substitutes in production and that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}$ and $H_{i,t+1}$ are made imply that the marginal product of both types of capital are equalized within and across firms, as shown in equation (18). As a result, equations (19) to (22) permit solutions where μ_t^i and η_t^i are not firm-specific. Intuitively, because the marginal product of capital depends only on the sum of $K_{i,t+1}$ and $H_{i,t+1}$, but not on the individual summands, entrepreneurs will choose the total amount of capital to equalize its marginal product across firms. This is also because $z_{i,t+1}$ is observed at the end of period t. Depending on his borrowing need, an entrepreneur can then determine $K_{i,t+1}$ to satisfy the collateral constraint. Because capital can be purchased on a competitive market, entrepreneurs will choose $K_{i,t+1}$ to equalize its price to its marginal benefit, which includes the marginal product of capital and the Lagrangian multiplier η_t^i. Because both the prices and the marginal product of capital are equalized across firms, so is the tightness of the collateral constraint.

We formalize the above observation by constructing a recursive equilibrium in two steps. First, we show that the aggregate quantities and prices can be characterized by a set of equilibrium functionals. Second, we further construct individual firm's quantities from the aggregate quantities and prices. We make one final assumption, namely that the aggregate productivity is given by $\bar{A}_t = A_t (K_t + H_t)^{1-\nu\alpha}$, where $\{A_t\}_{t=0}^{\infty}$ is an exogenous Markov productivity process. On the one hand, this assumption follows Frankel (1962) and Romer (1986) and is a parsimonious way to generate endogenous growth. On the other hand, combined with recursive preferences, this assumption increases the volatility of the pricing kernel, as in the stream of long-run risk model (see, e.g., Bansal and Yaron (2004) and Kung and Schmid (2015)). From a technical point of view, thanks to this assumption, equilibrium quantities are homogenous of degree one in the total capital stock, $K + H$, and equilibrium prices do not depend on $K+H$. It is therefore convenient to work with normalized quantities.

Let lower case variables denote aggregate quantities normalized by the current capital stock, so that, for instance, n_t denotes aggregate net worth N_t normalized by the total capital stock $K_t + H_t$. The equilibrium objects are consumption, $c(A,n)$, investment, $i(A,n)$, the marginal value of net worth, $\mu(A,n)$, the Lagrangian multiplier on the collateral constraint, $\eta(A,n)$, the price of type-K capital, $q_K(A,n)$, the price of type-H capital, $q_H(A,n)$, and the risk-free interest rate, $R_f(A,n)$ as functions of the state variables A and n.

To introduce the recursive formulation, we denote a generic variable in period t as X and in period $t+1$ as X'. Given the above equilibrium functionals, we can define

$$\Gamma(A,n) \equiv \frac{K' + H'}{K + H} = (1-\delta) + i(A,n)$$

as the growth rate of the capital stock and construct the law of motion of the endogenous state variable n from equation (9):[8]

$$\begin{aligned} n' \;=\;& (1-\lambda)\left[\alpha A' + \phi(1-\delta) q_K(A',n') + (1-\phi)(1-\delta) q_H(A',n') - \zeta\phi q_K(A,n) R_f(A,n)\right] \\ & + \lambda\chi \frac{n}{\Gamma(A,n)}. \end{aligned} \tag{23}$$

With the law of motion of the state variables, we can construct the normalized utility of the household as the fixed point of

$$u(A,n) = \left\{(1-\beta)c(A,n)^{1-\frac{1}{\psi}} + \beta\Gamma(A,n)^{1-\frac{1}{\psi}} \left(E[u(A',n')^{1-\gamma}]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

[8]We make use of the property that the ratio of K over H is always equal to $\phi/(1-\phi)$, as implied by the law of motion of the capital stock in (17).

The stochastic discount factors can then be written as

$$M' = \beta \left[\frac{c(A', n')\,\Gamma(A, n)}{c(A, n)} \right]^{-\frac{1}{\psi}} \left[\frac{u(A', n')}{E\left[u(A', n')^{1-\gamma}\right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi} - \gamma} \tag{24}$$

$$\widetilde{M}' = M'[(1-\lambda)\,\mu(A', n') + \lambda]. \tag{25}$$

Formally, an equilibrium in our model consists of a set of aggregate quantities, $\{C_t, B_t, \Pi_t, K_t, H_t, I_t, N_t\}$, individual entrepreneur choices, $\{K_{i,t}, H_{i,t}, L_{i,t}, B_{i,t}, N_{i,t}\}$, and prices $\left\{M_t, \widetilde{M}_t, W_t, q_{K,t}, q_{H,t}, \mu_t, \eta_t, R_{f,t}\right\}$ such that, given prices, quantities satisfy the household's and the entrepreneurs' optimality conditions, the market clearing conditions, and the relevant resource constraints. Below, we present a procedure to construct a Markov equilibrium where all prices and quantities are functions of the state variables (A, n). For simplicity, we assume that the initial idiosyncratic productivity across all firms satisfies $\int z_{i,1} di = 1$, the initial aggregate net worth is N_0, aggregate capital holdings start with $\frac{K_1}{H_1} = \frac{\phi}{1-\phi}$, and firm's initial net worth satisfies $n_{i,0} = z_{i,1} N_0$ for all i.

Again we use, x and X to denote a generic normalized and non-normalized quantity, respectively. For example, c denotes normalized aggregate consumption, while C is the original value.

Proposition 1. *(Markov equilibrium)*

Suppose there exists a set of equilibrium functionals $\{c(A,n), i(A,n), \mu(A,n), \eta(A,n), q_K(A,n), q_H(A,n), R_f(A,n)\}$ *satisfying the following set of functional equations:*

$$E[M' | A]\, R_f(A, n) = 1,$$

$$\mu(A, n) = E\left[\widetilde{M}' \middle| A\right] R_f(A, n) + \eta(A, n),$$

$$\mu(A, n) = E\left[\widetilde{M}' \frac{\alpha A' + (1-\delta)\, q_K(A', n')}{q_K(A, n)} \middle| A\right] + \zeta \eta(A, n),$$

$$\mu(A, n) = E\left[\widetilde{M}' \frac{\alpha A' + (1-\delta)\, q_H(A', n')}{q_H(A, n)} \middle| A\right],$$

$$\frac{n}{\Gamma(A, n)} = (1-\zeta)\,\phi q_K(A, n) + (1-\phi)\, q_H(A, n),$$

$$G'(i(A, n)) = \phi q_K(A, n) + (1-\phi)\, q_H(A, n),$$

$$c(A, n) + i(A, n) + g(i(A, n)) = A,$$

where the law of motion of n is given by (23), and the stochastic discount factors M' and $\widetilde{M}'$

are defined in (24) and (25). Then the equilibrium prices and quantities can be constructed as follows and they constitute a Markov equilibrium:

1. *Given the sequence of exogenous shocks $\{A_t\}$, the sequence of n_t can be constructed using the law of motion in (23), the normalized policy functions are constructed as:*

$$x_t = x(A_t, n_t), \; for \; x = c, i, \mu, \eta, q_K, q_H, R_f.$$

2. *Given the sequence of normalized quantities, aggregate quantities are constructed as:*

$$\begin{aligned} H_{t+1} &= H_t[1-\delta+i_t], \quad K_{t+1} = K_t[1-\delta+i_t] \\ X_t &= x_t[H_t + K_t] \end{aligned}$$

for $x = c, i, b, n$, $X = C, I, B, N$, and all t.

3. *Given the aggregate quantities, the individual entrepreneurs' net worth follows from (7). Given the sequences $\{N_{i,t}\}$, the quantities $B_{i,t}, K_{i,t}$ and $H_{i,t}$ are jointly determined by equations (4), (5), and (17). Finally, $L_{i,t} = z_{i,t}$ for all i, t.*

Proof. See Appendix A. □

The above proposition says that we can solve for aggregate quantities first, and then use the firm-level budget constraint and the law of motion of idiosyncratic productivity in to construct the cross-section of net worth and capital holdings.

4.2 The collateralizability spread

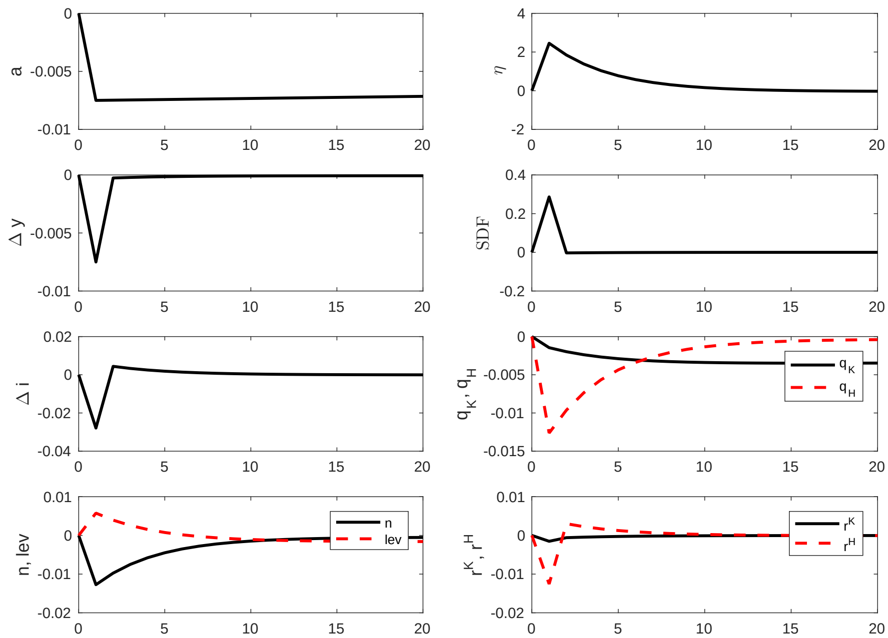
Our model allows for two types of capital, where type-K capital is collateralizable, while type-H capital is not. Note that one unit of type j capital costs $q_{j,t}$ in period t and it pays off $\Pi_{j,t+1} + (1-\delta)\, q_{j,t+1}$ in the next period, for $j \in \{K, H\}$. Therefore, the un-levered returns on the claims to the two types of capital are given by:

$$R_{j,t+1} = \frac{\alpha A_{t+1} + (1-\delta)\, q_{j,t+1}}{q_{j,t}} \quad (j = K, H). \tag{26}$$

Risk premiums are determined by the covariances of the payoffs with respect to the stochastic discount factor. Given that the components representing the marginal products of capital are identical for the two types of capital, the key to understanding the collateralizability premium, as shown formally in equation (29), is the cyclical properties of the prices of the two types of capital, $q_{j,t+1}$.

We can iterate equations (21) and (22) forward to obtain an expression for $q_{K,t}$ and $q_{H,t}$ as the present value of all future cash flows. Clearly, $q_{K,t}$ contains the Lagrangian multipliers $\left\{\eta_{t+s}^i\right\}_{s=0}^{\infty}$, while $q_{H,t}$ does not. Because the Lagrangian multipliers are counter-cyclical and act as a hedge, $q_{K,t}$ will be less sensitive to aggregate shocks and less cyclical. These asset pricing implications of our model are best illustrated with impulse response functions.

Figure 1: Impulse responses to a negative aggregate productivity shock



The graphs in this figure represent log-deviations from the steady state for quantities (left column) and prices (right column) induced by a one-standard deviation negative shock to aggregate productivity. The parameters are shown in Table 2. The horizontal axis represents time in months.

Based on the graphs in Figure 1 we make two observations. First, a negative productivity shock lowers output and investment (second and third graph in the left column) as in standard macro models. In addition, as shown in the bottom graph on the left, entrepreneur net worth drops sharply (third graph in the right column) and leverage goes up immediately. Second, upon a negative productivity shock, because entrepreneur net worth drops sharply, the price of type-H capital also decreases sharply. The decrease in the price of the collateralizable capital, on the other hand, is much smaller. This is because the Lagrangian multiplier η (first graph in the right column) on the collateral constraint increases upon impact and offsets the effect of a negative productivity shock on the price of type-K capital. As a result, the return of type-K capital responds much less to negative productivity shocks than that of type-H capital (bottom graph in the right column), implying that collateralizable capital is indeed less risky than non-collateralizable capital in our model.

5 Quantitative model predictions

In this section, we calibrate our model at the monthly frequency and evaluate its ability to replicate key moments of both macroeconomic quantities and asset prices at the aggregate level. More importantly, we investigate its performance in terms of quantitatively accounting for key features of firm characteristics and producing a collateralizability premium in the cross-section. For macroeconomic quantities, we focus on a long sample of U.S. annual data from 1930 to 2016. All macroeconomic variables are real and per capita. Consumption, output and physical investment data are from the Bureau of Economic Analysis (BEA). In order to obtain the time series of total amount of tangible and intangible asset, we firstly aggregate the total amount of intangible or tangible capital across all U.S. compustat firms for each year. The aggregate intangible to tangible asset ratio is the time series of the aggregate intangible capital divided by tangible capital. For the purpose of cross-sectional analyses we make use of several data sources at the micro-level, including (1) firm level balance sheet data in the CRSP/Compustat Merged Fundamentals Annual Files, (2) monthly stock returns from CRSP, and (3) industry level non-residential capital stock data from the BEA table "Fixed Assets by Industry". Appendix C provides more details on our data sources at the firm and industry level.

5.1 Specification of aggregate shocks

We first formalize the specification of the exogenous aggregate shocks in this economy. First, log aggregate productivity $a \equiv \log(A)$ follows

$$a_{t+1} = a_{ss}\left(1 - \rho_A\right) + \rho_A a_t + \sigma_A \varepsilon_{A,t+1}, \tag{27}$$

where a_{ss} denotes the steady-state value of a. Second, we also introduce the shocks to entrepreneurs' liquidation probability λ. As is well known in the literature of macroeconomic models with financial frictions, the aggregate productivity shock alone does not create quantitatively enough volatility in capital prices and entrepreneurs' net worth. Additional source of shocks, for instance, capital quality shocks as in Gertler and Kiyotaki (2010) and Elenev, Landvoigt, and Van Nieuwerburgh (2017), is needed to generate a higher volatility in net worth. In our model, because a shock to λ affects the entrepreneurs' discount rate and therefore their net worth, without directly affecting the real production, we interpret it as a financial shock, in a spirit similar to Jermann and Quadrini (2012). Importantly, our general model intuition that collateralizable assets provide a hedge against aggregate shocks holds for both productivity and financial shocks.

To technically maintain $\lambda \in (0, 1)$ in a parsimonious way, we set

$$\lambda_t = \frac{\exp(x_t)}{\exp(x_t) + \exp(-x_t)},$$

where x_t follows the autocorrelated process,

$$x_{t+1} = x_{ss}(1 - \rho_x) + \rho_x x_t + \sigma_x \varepsilon_{x,t+1},$$

with x_{ss} denoting the steady-state value. We assume the innovations to a and x have the following structure:

$$\begin{bmatrix} \varepsilon_{A,t+1} \\ \varepsilon_{x,t+1} \end{bmatrix} \sim Normal\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{A,x} \\ \rho_{A,x} & 1 \end{bmatrix} \right),$$

in which the parameter $\rho_{A,x}$ captures the correlation between the two shocks. In the benchmark calibration, we assume $\rho_{A,x} = -1$. First, a negative correlation indicates that a negative productivity shock is associated with a positive discount rate shock. This assumption is necessary to quantitatively generate a positive correlation between consumption and investment growth consistent with the data. If only the financial shock, $\varepsilon_{x,t+1}$, is present, it will affect contemporaneous consumption and investment but not output. In this case the resource constraint in equation (15) implies a counterfactually negative correlation between consumption and investment growth. Second, the assumption of a perfectly negative correlation is for parsimony, and it effectively implies there is only one aggregate shock in this economy.

5.2 Calibration

We calibrate our model at the monthly frequency and present the parameters in Table 2. The first group of parameters are those which can be determined based on the literature. In particular, we set the relative risk aversion γ to 10 and the intertemporal elasticity of substitution ψ to 1.25. These parameter values are in line with the long-run risks literature, such as Bansal and Yaron (2004). The capital share parameter α is set to 0.33, as in the standard real business cycles literature. The span of control parameter ν is set to 0.85, consistent with Atkeson and Kehoe (2005).

Table 2: **Calibrated Parameter Values**

Parameter	Symbol	Value
Relative risk aversion	γ	10
IES	ψ	1.25
Capital share in production	α	0.33
Span of contral parameter	ν	0.85
Mean productivity growth rate	$e^{a_{ss}}$	0.042
Time discount rate	β	0.999
Share of type-K investment	ϕ	0.667
Capital depreciation rate	δ	0.007
Average exit rate of entrepreneurs	$\bar{\lambda}$	0.010
Collateralizability parameter	ζ	0.702
Transfer to entering entrepreneurs	χ	0.915
Persistence of TFP shocks	ρ_A	0.988
Vol. of TFP shock	σ_A	0.007
Persistence of financial shocks	ρ_x	0.988
Vol. of financial shock	σ_x	0.053
Corr. between TFP and financial shocks	$\rho_{A,x}$	-1
Invest. adj. cost paramter	τ	30
Mean idiosync. productivity growth	μ_ε	0.002
Vol. of idiosync. productivity growth	σ_ε	0.029

The parameters in the second group are determined by matching a set of first moments of quantities and prices. We set the long-term average economy-wide productivity growth rate a_{ss} to match a value for the U.S. economy of 2% per year. The time discount factor β is set to match the average real risk free rate of 1% per year. The share of type-K capital investment ϕ is set to 0.67 to match an average intangible-to-tangible-asset ratio of 57% for the average U.S. Compustat firm.[9] The capital depreciation rate is set to be 8% per year. For parsimony, we assume the same depreciation rate for both types of capital. The parameter x_{ss} is set to

[9]The construction of intangible capital is explained in detail in Appendix C.3.

match an average exit probability $\overline{\lambda}$ of 0.01, targeting an average corporate duration of 10 years of US Compustat firms. We calibrate the remaining two parameters related to financial frictions, the collateralizability parameter ζ and the transfer to entering entrepreneurs χ, to generate an average non-financial corporate sector leverage ratio equal to 0.5 and an average consumption-to-investment ratio of 4.5. These values are broadly in line with the data, where leverage is measured by the median lease capital adjusted leverage ratio of U.S. non-financial firms in Compustat.

The parameters in the third group are determined by second moments in the data. The persistence parameters ρ_A and ρ_x are set to 0.988 each to roughly match the autocorrelations of consumption and output growth. As discussed above, we impose a perfectly negative correlation between productivity and financial shocks, i.e., we set $\rho_{x,A} = -1$. The standard deviations of the shock to the exit probability λ, σ_x, and to productivity, σ_A, are jointly calibrated to match the volatilities of consumption growth and the correlation between consumption and investment growth. For the capital adjustment cost function we choose a standard quadratic form, i.e.,

$$g\left(\frac{I_t}{K_t + H_t}\right) = \frac{I_t}{K_t + H_t} + \frac{\tau}{2}\left(\frac{I_t}{K_t + H_t} - \frac{I_{ss}}{K_{ss} + H_{ss}}\right)^2,$$

where X_{ss} denotes the steady state values for $X = I, K, H$. The elasticity parameter of the adjustment cost function, τ, is set to allow our model to achieve a sufficiently high volatility of investment, broadly in line with the data.

The last group contains the parameters related to the idiosyncratic productivity shocks, μ_ε and σ_ε. We calibrate them to match the mean (2.5%) and the volatility (10%) of the idiosyncratic productivity growth of the cross-section of U.S. non-financial firms in the Compustat database.

5.3 Aggregate moments

We now turn to the quantitative performance of the model at the aggregate level. We solve and simulate our model at the monthly frequency and aggregate the model-generated data to compute annual moments.[10] We show that our model is broadly consistent with the key empirical features of macroeconomic quantities and asset prices. More importantly, it

[10]Because the limited commitment constraint is binding in the steady-state, we solve the model using a second-order local approximation around the steady state using the `Dynare` package. We have also solved version solved versions of our model using the global method developed in Ai, Li, and Yang (2016) and verified the accuracy of the local approximation.

produces a sizable negative collateralizability spread at the aggregate level.

Table 3 reports the key moments of macroeconomic quantities (top panel) and those of asset returns (bottom panel) respectively, and compares them to their counterparts in the data where available.

In terms of aggregate moments on macro quantities (top panel), our calibration features a low volatility of consumption growth (2.62%) and a relatively high volatility of investment (8.48%). Thanks to the negative correlation between the productivity and financial shocks, our model can reproduce a positive consumption-investment correlation (33%), consistent with the data. The model also generates a persistence of output growth (65%) in line with aggregate data and an average intangible-to-tangible-capital ratio of 50%, a value broadly consistent with the average ratio across U.S. Compustat firms. In summary, our model inherits the success of real business cycles models on the quantity side of the economy.

Table 3: Model Simulations and Aggregate Moments

This table presents the annualized moments from the model simulation. We simulate the economy at monthly frequency based on the monthly calibration as in Table 2, then aggregate the monthly observations to annual frequency. The model moments are obtained from repetitions of small simulation samples. Data counterparts refer to the US and span the sample period 1930-2016. The market return R_M corresponds to the return on entrepreneurs' net worth at the aggregate level and embodies the endogenous financial leverage. R_K^{Lev} and R_H denote the levered return on type-K capital and the unlevered return on type-H capital, respectively. GMM standard errors (in parentheses) are adjusted following Newey and West (1987).

Moments	Data	Benchmark
$\sigma(\Delta c)$	2.53 (0.56)	2.62
$\sigma(\Delta i)$	10.30 (2.36)	8.48
$corr(\Delta c, \Delta i)$	0.40 (0.28)	0.33
$AC1(\Delta y)$	0.49 (0.15)	0.65
$E[H/K]$	0.57 (0.02)	0.50
$E[R_M - R_f]$	6.51 (2.25)	8.21
$E[R_f]$	1.10 (0.16)	1.24
$E[R_H - R_f]$		12.28
$E[R_K - R_f]$		0.84
$E[R_K^{Lev} - R_H]$		-9.45

Turning the attention to the asset pricing moments (bottom panel), our model produces a low risk free rate (1.24%) and a high equity premium (8.21%), comparable to key empirical moments for aggregate stock market. Moreover, in our model the risk premium on type-K capital of 0.84% is much lower than that on type-H capital 12.28%.

Quantitatively, there is an offsetting effect for the negative colllateralizability premium via the financial leverage channel. Type-K capital is collateralizable, and allows the firm to borrow more, so that leverage increases, which in turn increases the expected return on equity. If we assume a binding borrowing constraint and replace $B_{i,t}$ by $\zeta q_{j,t} K_{j,t+1}$, one can see that buying type-K capital effectively delivers a levered return, since

$$R_{K,t+1}^{Lev} = \frac{\alpha A_{t+1} + (1-\delta) q_{K,t+1} - R_{f,t+1} \zeta q_{K,t}}{q_{K,t}(1-\zeta)}, = \frac{1}{1-\zeta}(R_{K,t+1} - R_{f,t+1}) + R_{f,t+1}. \tag{28}$$

In the first line, the denominator $q_{K,t}(1-\zeta)$ represents the amount of internal net worth required to buy one unit of type-K capital, and it can be interpreted as the minimum down payment per unit of capital. The numerator $\alpha A_{t+1} + (1-\delta) q_{K,t+1} - R_{f,t+1}\zeta q_{K,t}$ is tomorrow's payoff per unit of capital, after subtracting the debt repayment. Because type-H capital is non-collateralizable and has to be purchased 100% with equity, it cannot be levered up. In sum, the (negative) collateralizability premium at the aggregate level can be interpreted as the difference between the average return of a levered claim on the type-K capital and an un-levered claim on type-H capital.

Combining the two Euler equations, (19) and (21), and eliminating η_t, we obtain

$$E_t\left[\widetilde{M}_{t+1} R_{K,t+1}^{Lev}\right] = \mu_t,$$

and a rearrangement of equation (22) gives

$$E_t\left[\widetilde{M}_{t+1} R_{H,t+1}\right] = \mu_t.$$

Therefore, the expected return spread is equal to

$$E_t\left(R_{K,t+1}^{Lev} - R_{H,t+1}\right) = -\frac{1}{E_t\left(\widetilde{M}_{t+1}\right)}\left(Cov_t\left[\widetilde{M}_{t+1}, R_{K,t+1}^{Lev}\right] - Cov_t\left[\widetilde{M}_{t+1}, R_{H,t+1}\right]\right), = -\frac{1}{E_t\left(\widetilde{M}_{t+1}\right)}\left(\frac{1}{1-\zeta} Cov_t\left[\widetilde{M}_{t+1}, R_{K,t+1}\right] - Cov_t\left[\widetilde{M}_{t+1}, R_{H,t+1}\right]\right). \tag{29}$$

On the right-hand side of equation (29), we can see the two offsetting effects at work. On one hand, the counter-cyclical tightness of the collateral constraint makes $R_{K,t+1}$ covary less with the stochastic discount factor $\widetilde{M}_{t+1}$. However, the leverage multiplier $\frac{1}{1-\zeta}$ may offset this effect by amplifying the cyclical fluctuations of a levered claim on type-K capital. The relative riskiness of the type-K versus type-H capital thus depends on the relative contributions of the Lagrangian multiplier effect and the offsetting leverage effect. In the last row of Table

3, we report a sizable negative average return spread of -9.45% between a levered claim on type-K capital and non-collateralizable capital, $(E[R_K^{Lev} - R_H])$. This means, in our calibrated model, the first effect clearly dominates, and there is a negative collateralizability premium.

5.4 The cross section of collateralizability and equity returns

In this section, we study the collateralizability spread at the cross-sectional level. In particular, we simulate firms from the model, measure the collateralizability of firm assets, and conduct the same collateralizability-based portfolio sorting procedure as we do in the data.

Equity claims to firms in our model can be freely traded among entrepreneurs. The return on an entrepreneur's net worth is $\frac{N_{i,t+1}}{N_{i,t}}$. Using equations (4) and (7), we obtain

$$\begin{aligned}\frac{N_{i,t+1}}{N_{i,t}} &= \frac{\alpha A_{t+1}\left(K_{i,t+1}+H_{i,t+1}\right)+(1-\delta)\,q_{K,t+1}K_{i,t+1}+(1-\delta)\,q_{H,t+1}H_{i,t+1}-R_{f,t+1}B_{i,t}}{q_{K,t}K_{i,t+1}+q_{H,t}H_{i,t+1}-B_{i,t}}, \\ &= \frac{(1-\zeta)q_{K,t}K_{i,t+1}}{N_{i,t}}R_{K,t+1}^{Lev}+\frac{q_{H,t}H_{i,t+1}}{N_{i,t}}R_{H,t+1},\end{aligned}$$

where $R_{K,t+1}^{Lev}$ is a levered return on the type-K capital, as defined in equation (28). The above expression has an intuitive interpretation. The return on equity is the weighted average of the levered return on the type-K capital and the un-levered return on the type-H capital. The weights $\frac{(1-\zeta)q_{K,t}K_{i,t+1}}{N_{i,t}}$ and $\frac{q_{H,t}H_{i,t+1}}{N_{i,t}}$ are the relative shares of the entrepreneur's net worth represented by type-K and type-H capital, respectively. In the case of a binding collateral constraint, these weights sum up to one. Since, in our model, $R_{K,t+1}^{Lev}$ and $R_{H,t+1}$ are the same across all firms, firm level expected returns differ only because of the way total capital is composed of type H and type K. This composition can be equivalently summarized by the collateralizability measure for the firm's assets.

To see this, note that μ_t^i and η_t^i are identical across firms, so that equations (21) and (22) can be summarized as

$$\mu_t q_{j,t}K_{j,t+1} = E_t\left[\widetilde{M}_{t+1}^i\left\{\Pi_{j,t+1}+(1-\delta)\,q_{j,t+1}\right\}K_{j,t+1}\right]+\zeta_j\eta_t q_{j,t}K_{j,t+1}. \tag{30}$$

Dividing the above equation by the total value of the firm's assets V_t and summing over all types of capital j, we obtain:

$$\mu_t = \frac{\sum_{j=1}^{J}E_t\left[\widetilde{M}_{t+1}^i\left\{\Pi_{j,t+1}+(1-\delta)\,q_{j,t+1}\right\}K_{j,t+1}\right]}{V_t}+\eta_t\sum_{j=1}^{J}\zeta_j\frac{q_{j,t}K_{j,t+1}}{V_t}. \tag{31}$$

μ_t is the shadow value of entrepreneur's net worth. Equation (31) decomposes μ_t into two parts. Since the term $E_t\left[\tilde{M}_{t+1}^i\left\{\Pi_{j,t+1}+(1-\delta)\,q_{j,t+1}\right\}K_{j,t+1}\right]$ can be interpreted as the present value of the cash flows generated by type-j capital, the first component is the fraction of firm value that comes from cash flows. The second component is the relative contribution of the Lagrangian multiplier for the collateral constraint, multiplied by our measure of asset collateralizability.

In our model, μ_t and η_t are common across all firms. All types of capital generate the same marginal product in all periods. As a result, expected returns differ only because of the effect coming from the second component which is associated with the Lagrangian multiplier. Different compositions of asset collateralizability lead to different sensitivity to the valuation of Lagrangian multiplier. This is completely summarized by the asset collateralizability measure, $\sum_{j=1}^{J}\zeta_j\frac{q_{j,t}K_{j,t+1}}{V_t}$. As we show next in this section, this parallel between our model and our empirical procedure allows our model to match very well the quantitative features of the collateralizability spread in the data.

In Table 4, we report our model's implications for the cross-section of asset collateralizability, leverage ratio, and expected returns and compare them with the data. In the data, we focus on financially constrained firms, which are defined according to the WW index, and report our results in the upper panel in Table 4. As we show in Section 2, other measures of financial constraints yields quantitatively similar results on the collateralizability premium. We follow the same procedure with the simulated data in our model and sort stocks into five portfolios based on the collateralizability measure. The corresponding moments are reported in the bottom panel of Table 4.

We make three observations. First, the collateralibility scores in our model are similar to those in the data across the quintile portfolios. Despite its simplicity, our model endogenously generates a plausible distribution of asset collateralizability in the cross-section.

Second, as in the data, leverage is by and large increasing in asset collateralizability. This implication of our model is consistent with the corporate finance literature emphasizing the importance of collateral in firms' capital structure decisions (e.g., Rampini and Viswanathan (2013)). The dispersion in leverage in our model is somewhat higher than that in the data. This is not surprising, as in our model, asset collateralizability is the only factor determining leverage, while in the data there are many other determinants of the capital structure.

Lastly and most importantly, firms with high asset collateralizability, despite their high leverage, have a significantly lower expected return than those with low asset collateralizability. Quantitatively, our model produces a sizable collateralizability spread (5.30%), comparable to that (7.96%) in the data.

Table 4: Firm Characteristics and Expected Returns

This table shows model-simulated moments and their counterparts in the data at the portfolio level. The sample period is from July 1979 to December 2016. At the end of June of each year t, we sort the constrained firms into five quintiles based on collateralizability measure at the end of year $t-1$. The table shows the mean of the collateralizability measure across firms, the mean of book leverage (lease adjusted), and the average value-weighted excess returns $E[R]-R^f(\%)$ (annualized), for quintile portfolios sorted on collateralizability. Panel A reports the statistics computed from the sample of financially constrained firms (as measured by the WW index, see Whited and Wu (2006)). In each year, a firm is classified as financially constrained if its WW index is higher than the cross-sectional median in that year. Panel B reports the statistics computed from simulated data. In particular, we simulate the firm level characteristics and returns at the monthly level, and then perform the same portfolio sorts as in the data.

Panel A: Data

	1	2	3	4	5	5-1
Collateralizability	0.05	0.10	0.14	0.22	0.79	
Book Leverage	0.49	0.41	0.45	0.59	0.53	
$E[R]-R^f(\%)$	13.33	11.59	9.43	9.37	5.36	7.96

Panel B: Model

	1	2	3	4	5	5-1
Collateralizability	0.28	0.51	0.59	0.64	0.68	
Book Leverage	0.23	0.50	0.64	0.73	0.83	
$E[R]-R^f(\%)$	11.68	9.59	8.18	7.24	6.37	5.30

As discussed above, an increase in the holdings of type-K capital raises the firm's asset collateralizability and has two effects on the expected return of its equity. On one hand, because collateralizable capital has a lower expected return than non-collateralizable capital, higher asset collateralizability tends to lower the expected return on the firm's equity. On the other hand, because higher asset collateralizability allows the firm to borrow more, it increases leverage, which in turn tends to increase the expected return on equity. Our quantitative analysis shows that the first effect dominates the second, leading to a negative collateralizablity premium.

6 Conclusion

In this paper, we present a general equilibrium asset pricing model with heterogenous firms and collateral constraints. Our model predicts that the collateralizable asset provides insurance against aggregate shocks and should therefore earn a lower expected return, since it relaxes the collateral constraint, which is more binding in recessions than in booms.

We develop an empirical collateralizability measure for firms' assets, and document empirical evidence consistent with the predictions of our model. In particular, we find in the data that the difference in average equity returns between firms with a low and a high degree of asset collateralizability amounts to almost 8% per year. When we calibrate our model to the dynamics of macroeconomic quantities, we show that the credit market friction channel is a quantitatively important determinant for the cross-section of asset returns.

References

Ai, H. and D. Kiku (2013): "Growth to value: Option exercise and the cross section of equity returns," *Journal of Financial Economics*, 107, 325–349.

Albuquerque, R. and H. Hopenhayn (2004): "Optimal Lending Contracts and Firm Dynamics," *Review of Economic Studies*, 71, 285–315.

Atkeson, A. and P. J. Kehoe (2005): "Modeling and Measuring Organization Capital," *Journal of Political Economy*, 113, 1026–53.

Bansal, R. and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution," *Journal of Finance*, 59, 1481–1509.

Belo, F., X. Lin, and F. Yang (2017): "External Equity Financing Shocks, Financial Flows, and Asset Prices," Working Paper.

Brunnermeier, M. K., T. Eisenbach, and Y. Sannikov (2012): "Macroeconomics with Financial Frictions: A Survey," Working Paper.

Brunnermeier, M. K. and Y. Sannikov (2014): "A macroeconomic model with a financial sector," *American Economic Review*, 104, 379–421.

Campello, M. and E. Giambona (2013): "Real assets and capital structure," *Journal of Financial and Quantitative Analysis*, 48, 1333–1370.

Carhart, M. M. (1997): "On persistence in mutual fund performance," *Journal of Finance*, 52, 57–82.

Chan, L. K. C., J. Lakonishok, and T. Sougiannis (2001): "The Stock Market Valuation of Research," *Journal of Finance*, 56, 2431 – 2456.

Constantinides, G. M. and D. Duffie (1996): "Asset pricing with heterogeneous consumers," *Journal of Political economy*, 104, 219–240.

Croce, M. M., T. T. Nguyen, S. Raymond, and L. Schmid (2017): "Government Debt and the Returns to Innovation," forthcoming *Journal of Financial Economics*.

Eisfeldt, A. L. and D. Papanikolaou (2013): "Organization capital and the cross-section of expected returns," *Journal of Finance*, 68, 1365–1406.

——— (2014): "The value and ownership of intangible capital," *American Economic Review*, 104, 189–194.

Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2017): "A Macroeconomic Model with Financially Constrained Producers and Intermediaries," Tech. rep., Working Paper.

Epstein, L. G. and S. E. Zin (1989): "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework," *Econometrica*, 57, 937–969.

Falato, A., D. Kadyrzhanova, and J. W. Sim (2013): "Rising Intangible Capital, Shrinking Debt Capacity, and the US Corporate Savings Glut," Working Paper.

Fama, E. F. and K. R. French (2015): "A five-factor asset pricing model," *Journal of Financial Economics*, 116, 1–22.

Fama, E. F. and J. D. MacBeth (1973): "Risk, return, and equilibrium: Empirical tests," *Journal of Political Economy*, 81, 607–636.

Feldman, M. and C. Gilles (1985): "An expository note on individual risk without aggregate uncertainty," *Journal of Economic Theory*, 35, 26–32.

Frankel, M. (1962): "The Production Function in Allocation and Growth: A Synthesis," *American Economic Review*, 52, 996–1022.

Gârleanu, N., L. Kogan, and S. Panageas (2012): "Displacement risk and asset returns," *Journal of Financial Economics*, 105, 491–510.

Gertler, M. and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, 3, 547–599.

Gomes, J., L. Kogan, and L. Zhang (2003): "Equilibrium Cross Section of Returns," *Journal of Political Economy*, 111, 693–732.

Gomes, J., R. Yamarthy, and A. Yaron (2015): "Carlstrom and Fuerst meets Epstein and Zin: The Asset Pricing Implications of Contracting Frictions," Working Paper.

Graham, J. R. (2000): "How big are the tax benefits of debt?" *Journal of Finance*, 55, 1901–1941.

Hadlock, C. J. and J. R. Pierce (2010): "New evidence on measuring financial constraints: Moving beyond the KZ index," *Review of Financial Studies*, 23, 1909–1940.

He, Z. and A. Krishnamurthy (2013): "Intermediary Asset Pricing," *American Economic Review*, 103, 732–70.

Hennessy, C. a. and T. M. Whited (2007): "How costly is external financing? Evidence from a structural estimation," *Journal of Finance*, 62, 1705–1745.

Hulten, C. R. and X. Hao (2008): "What is a company really worth? Intangible capital and the "market-to-book value" puzzle," Working Paper.

Jermann, U. and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102, 238–271.

Judd, K. L. (1985): "The law of large numbers with a continuum of iid random variables," *Journal of Economic theory*, 35, 19–25.

Kiyotaki, N. and J. Moore (1997): "Credit cycles," *Journal of Political Economy*, 105, 211–248.

——— (2012): "Liquidity, business cycles, and monetary policy," Working Paper.

Kogan, L. and D. Papanikolaou (2012): "Economic Activity of Firms and Asset Prices," *Annual Review of Financial Economics*, 4, 1–24.

Kogan, L., D. Papanikolaou, and N. Stoffman (2017): "Winners and Losers: Creative Destruction and the Stock Market," Working Paper.

Kung, H. and L. Schmid (2015): "Innovation, growth, and asset prices," *Journal of Finance*, 70, 1001–1037.

Li, D. (2011): "Financial constraints, R&D investment, and stock returns," *Review of Financial Studies*, 24, 2974–3007.

Li, W. C. and H. B. Hall (2016): "Depreciation of Business R&D Capital," Working Paper.

Lin, X. (2012): "Endogenous technological progress and the cross-section of stock returns," *Journal of Financial Economics*, 103, 411–427.

Newey, W. K. and K. D. West (1987): "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–708.

Peters, R. H. and L. A. Taylor (2017): "Intangible Capital and the Investment-q Relation," *Journal of Financial Economics*, 123, 251–272.

Quadrini, V. (2011): "Financial Frictions in Macroeconomic Fluctuations," *FRB Richmond Economic Quarterly*, 97, 209–254.

Rampini, A. and S. Viswanathan (2010): "Collateral, risk management, and the distribution of debt capacity," *Journal of Finance*, 65, 2293–2322.

——— (2013): "Collateral and capital structure," *Journal of Financial Economics*, 109, 466–492.

——— (2017): "Financial Intermediary Capital," Working Paper.

Romer, P. M. (1986): "Increasing Returns and Long-run Growth," *Journal of Political Economy*, 94, 1002 – 1037.

Schmid, L. (2008): "A Quantitative Dynamic Agency Model of Financing Constraints," Working Paper.

Shumway, T. (1997): "The delisting bias in CRSP data," *Journal of Finance*, 52, 327–340.

Shumway, T. and V. A. Warther (1999): "The delisting bias in CRSP's Nasdaq data and its implications for the size effect," *Journal of Finance*, 54, 2361–2379.

Whited, T. M. and G. Wu (2006): "Financial constraints risk," *Review of Financial Studies*, 19, 531–559.

Zhang, L. (2005): "The value premium," *Journal of Finance*, 60, 67–103.

Appendix A: Proof of proposition 1

To prove Proposition 1, we need to prove the following: first, given prices, the quantities satisfy the household's and entrepreneur's optimality conditions; second, the quantities satisfy market clearing conditions.

First, the household's first-order condition (3) and the resource constraint (15) are satisfied by construction, since their normalized versions represent two of the functional equations listed in Proposition 1.

Second, we prove the entrepreneur i's allocations $\{N_{i,t}, B_{i,t}, K_{i,t}, H_{i,t}, L_{i,t}\}$ as constructed in Proposition 1 are indeed optimal solutions to his optimization problem (8). Note that the entrepreneur's optimization problem is a standard convex programming problem. Therefore, the first order conditions, i.e. equations (19) to (22), together with the constraints (4), (5) and (7), constitute both necessary and sufficient conditions for optimality. It is easy to show that, given prices, the equilibrium quantities $\{N_{i,t}, B_{i,t}, K_{i,t}, H_{i,t}, L_{i,t}\}$ as constructed in Proposition 1 satisfy the above conditions.

Lastly, we show the market clearing conditions hold. Given the initial conditions (initial net worth N_0, $\frac{K_1}{H_1} = \frac{\phi}{1-\phi}$, $N_{i,0} = z_{i,1}N_0$) and the net worth injection rule for the new entrant firms ($N_{t+1}^{entrant} = \chi N_t$ for all t), we can prove the following lemma:

Lemma 1. *The optimal allocations $\{N_{i,t}, B_{i,t}, K_{i,t+1}, H_{i,t+1}\}$ constructed as in Proposition 1 satisfy the market clearing conditions, i.e.,*

$$K_{t+1} = \int K_{i,t+1}di, \quad H_{t+1} = \int H_{i,t+1}di, \quad N_t = \int N_{i,t}di, \text{ for all } t \geq 0. \tag{A1}$$

First, at each period t, given prices and $N_{i,t}$, the individual entrepreneur i's capital decisions $\{K_{i,t+1}, H_{i,t+1}\}$ must satisfy

$$N_{i,t} = (1-\zeta)\, q_{K,t}K_{i,t+1} + q_{H,t}H_{i,t+1}, \tag{A2}$$

and the optimal decision rule (17). Equation (A2) is obtained by combining entrepreneur's budget constraint (4) with a binding borrowing constraint (5).

Next, we show, given the initial conditions, market clearing conditions (A1) hold for all $t \geq 0$.

In period 0, we start from the initial conditions. Given

$$N_{i,0} = z_{i,1}N_0,$$

where $z_{i,1}$ is chosen from the stationary distribution of z. Given $z_{i,1}$ for each firm i, we use equations (A2) and (17) to solve for $K_{i,1}$ and $H_{i,1}$. Clearly, $K_{i,1} = z_{i,1}K_1$ and $H_{i,1} = z_{i,1}H_1$. Therefore, the market clearing conditions (A1) hold for $t = 0$:

$$\int K_{i,1}di = K_1 \quad \int H_{i,1}di = H_1 \quad \int N_{i,0}di = N_0. \tag{A3}$$

We then show the market clearing conditions (A1) also hold for $t > 0$. In particular, we prove the following claim:

Claim 1. *Suppose* $\int K_{i,t+1}di = K_{t+1}$, $\int H_{i,t+1}di = H_{t+1}$, $\int N_{i,t}di = N_t$, *and* $N_{t+1}^{entrant} = \chi N_t$, *then*

$$\int K_{i,t+2}di = K_{t+2} \quad \int H_{i,t+2}di = H_{t+2} \quad \int N_{i,t+1}di = N_{t+1}, for\ all\ for\ t \geq 0. \tag{A4}$$

1. Using the law of motion of the net worth of existing firms, one can show that the total net worth of all surviving firms satisfies:

$$\begin{aligned} &(1-\lambda)\int N_{i,t+1}di \\ &= (1-\lambda)\int \left[A_{t+1}\left(K_{i,t+1}+H_{i,t+1}\right)+(1-\delta)\,q_{K,t+1}K_{i,t+1}+(1-\delta)\,q_{H,t+1}H_{i,t+1}-R_{f,t+1}B_{i,t}\right]di, \\ &= (1-\lambda)\left[A_{t+1}\left(K_{t+1}+H_{t+1}\right)+(1-\delta)\,q_{K,t}K_{t+1}+(1-\delta)\,q_{H,t}H_{t+1}-R_{f,t+1}B_t\right], \end{aligned}$$

   since $\int K_{i,t+1}di = K_{t+1}$, $\int H_{i,t+1}di = H_{t+1}$, and $\int B_{i,t}di = B_t = \zeta q_{K,t}K_{t+1}$. With the assignment rule for the net worth of new entrants, $N_{t+1}^{entrant} = \chi N_t$, we can show that the total net worth at the end of period $t+1$ across survivors and new entrants satisfies $\int N_{i,t+1}di = N_{t+1}$, in which the aggregate net worth N_{t+1} is given in equation (9).

2. At the end of period $t + 1$, we have a pool of firms that consists of old ones with net worth given by (7) and new entrants. All of them will observe $z_{i,t+2}$ (for the new entrants $z_{i,t+2} = \bar{z}$) and produce at the beginning of the period $t + 1$.

   We compute the capital holdings for period $t + 2$ for each firm i using (A2) and (17). At this point, the capital holdings and the net worth of all existing firms will not be proportional to $z_{i,t+2}$ due to heterogeneity in the shocks. However, we know that $\int N_{i,t+1}di = N_{t+1}$, and $\int z_{i,t+2}di = 1$. Integrating (A2) and (17) across all i, we obtain

$$(1-\zeta)\,q_{K,t+1}\int K_{i,t+2}di + q_{H,t+1}\int H_{i,t+2}di = \int N_{i,t+1}di = N_{t+1}, \tag{A5}$$

$$\int K_{i,t+2}di + \int H_{i,t+2}di = (K_{t+2}+H_{t+2})\int z_{i,t+2}di = K_{t+2}+H_{t+2}. \tag{A6}$$

The first equality in equation (A6) is obtained by imposing the optimal decision rule (17). Consider $\int K_{i,t+2}di$ and $\int H_{i,t+2}di$ are the unique solution to the linear equation system (A5) and (A6). Given that the constraints of all entrepreneurs are binding, the budget constraint (A2) also holds at the aggregate level, that is,

$$N_{t+1} = (1-\zeta)\, q_{K,t+1}K_{t+2} + q_{H,t+1}H_{t+2}.$$

It implies that the solution must be $\int K_{i,t+2}di = K_{t+2}$ and $\int H_{i,t+2}di = H_{t+2}$. Therefore, the claim is proved.

In summary, we have proved the equilibrium prices and quantities constructed in Proposition 1 satisfy the household's and entrepreneur's optimality conditions; and the quantities satisfy market clearing conditions. Therefore, the proof of Proposition 1 is completed.

Appendix B: Additional empirical evidence

In this section, we provide additional empirical evidence regarding the collateralizability premium, including the standard multi-factor asset pricing tests and cross-sectional regressions following Fama and MacBeth (1973). We also demonstrate the robustness of our basic finding by forming collateralizability portfolios within industries and by performing double sorts with respect to collateralizability and financial leverage.

B.1. Collateralizability versus standard risk factors

In this section, we investigate to what extent the variations in the average returns of the collateralizability-sorted portfolios can be explained by exposures to standard risk factors, as those features in the models proposed by Carhart (1997) and Fama and French (2015). In particular, we run monthly time-series regressions of the (annualized) excess returns of each portfolio on a constant and the risk factors included in the above models. Table B.1 reports the intercepts and exposures (i.e., betas). The intercepts can be interpreted as pricing errors (abnormal returns), which remain unexplained by the given set of factors.

We make two key observations. First, the pricing errors of the collateralizablity sorted portfolio with respect to the given set of factors remain large and significant, with 10 % for the Carhart (1997) model and 11.47% for the Fama and French (2015) five-factor model, both with highly significant t-statistics. Second, the pricing errors implied by both factor models are even larger than the collateralizability spread itself reported in Table 1), mostly

Table B.1: Alphas of Collateralizability Portfolios

This table shows the coefficients of regressions of the returns for quintile portfolios sorted by collateralizability on the factors from the Carhart (1997) four-factor model (Panel A), the Fama and French (2015) five factor model (Panel B), and a model featuring the tree Fama-Franch factors augmented by the organizational capital factor suggested by Eisfeldt and Papanikolaou (2013) (Panel C). The t-statistics are adjusted following Newey and West (1987). The analysis is performed for financially constrained firms. A firm is classified as constrained in year t, if its WW index according to Whited and Wu (2006) is greater than the sample median for the given year. The sample period is from July 1979 to December 2016, with the exception of Panel C, where the sample ends in December 2008.

Panel A: Carhart Four-Factor Model

	1	2	3	4	5	1-5
α	5.69	3.59	0.92	0.32	-4.35	10.04
t-stat	(2.88)	(2.30)	(0.59)	(0.23)	(-2.79)	(3.81)
β_{MKT}	1.08	1.08	1.06	1.09	1.13	-0.05
t-stat	(28.01)	(29.04)	(29.57)	(35.25)	(28.00)	(-0.91)
β_{HML}	-0.63	-0.46	-0.32	-0.14	-0.01	-0.63
t-stat	(-9.71)	(-8.80)	(-6.26)	(-3.01)	(-0.09)	(-6.60)
β_{SMB}	1.30	1.12	1.09	1.12	0.76	0.54
t-stat	(19.06)	(16.45)	(18.92)	(24.89)	(9.23)	(4.62)
β_{MOM}	-0.06	-0.06	-0.04	-0.08	-0.02	-0.04
t-stat	(-1.17)	(-1.72)	(-1.22)	(-2.43)	(-0.50)	(-0.48)
R^2	0.85	0.87	0.89	0.89	0.84	0.28

Panel B: Fama-French Five-Factor Model

	1	2	3	4	5	1-5
α	7.18	5.38	2.06	1.00	-4.29	11.47
t-stat	(4.83)	(4.65)	(1.77)	(0.86)	(-3.38)	(5.63)
β_{MKT}	1.02	1.01	1.03	1.07	1.13	-0.11
t-stat	(26.70)	(32.58)	(33.92)	(39.15)	(28.95)	(-2.11)
β_{SMB}	1.11	0.96	0.98	1.03	0.90	0.21
t-stat	(16.58)	(16.79)	(19.79)	(21.94)	(15.55)	(2.37)
β_{HML}	-0.77	-0.50	-0.49	-0.29	-0.05	-0.71
t-stat	(-8.83)	(-7.84)	(-7.47)	(-4.97)	(-0.73)	(-6.03)
β_{RMW}	-0.65	-0.56	-0.39	-0.33	0.22	-0.88
t-stat	(-6.37)	(-7.02)	(-5.94)	(-4.52)	(2.89)	(-6.75)
β_{CMA}	0.13	-0.05	0.19	0.15	-0.15	0.28
t-stat	(0.97)	(-0.48)	(2.16)	(1.58)	(-1.86)	(1.76)
R^2	0.88	0.89	0.90	0.90	0.86	0.40

Panel C: Control for Organizational Capital Factor

	1	2	3	4	5	1-5
α	6.07	3.82	0.89	0.97	-3.65	9.72
t-stat	(2.61)	(2.00)	(0.43)	(0.51)	(-1.67)	(2.99)
β_{MKT}	1.12	1.08	1.08	1.09	1.10	0.02
t-stat	(21.33)	(24.32)	(23.13)	(25.25)	(26.57)	(0.40)
β_{HML}	-0.57	-0.45	-0.35	-0.11	-0.04	-0.53
t-stat	(-7.06)	(-7.22)	(-5.59)	(-1.59)	(-0.34)	(-3.97)
β_{SMB}	1.36	1.13	1.08	1.14	0.73	0.63
t-stat	(17.77)	(17.17)	(19.59)	(27.50)	(6.59)	(4.56)
β_{OMK}	-0.04	0.00	0.04	-0.04	-0.16	0.12
t-stat	(-0.58)	(0.02)	(1.03)	(-0.84)	(-2.44)	(1.20)
R^2	0.86	0.87	0.89	0.88	0.83	0.31

due to negative exposures of the low-minus-high collateralizability portfolio to HML (in both Panel A and B) and to RMW (Panel B).

Additionally, in order to distinguish our collateralizability measure from organizational capital, we also control for this factor originally suggested by Eisfeldt and Papanikolaou (2013),[11] together with the three Fama-French factors.

The results are shown in Panel C of Table B.1. The pricing errors are still significant in the presence of the organizational capital factor, with the magnitude of 9.7% per year and a t-statistic of 3. In particular, the five portfolios sorted on collateralizability are not strongly exposed to this factor, indicated by small and with the exception of portfolio 5) insignificant coefficients.

Taken together, the cross-sectional return spread across collateralizability sorted portfolios cannot be explained by either the Carhart (1997) four-factor model, the Fama and French (2015) five-factor model, or the organizational capital factor proposed by Eisfeldt and Papanikolaou (2013).

B.2. Firm-level return predictability regression

In this section, we extend the previous analysis to investigate the link between collateralizability and the future stock returns in the cross-section.

We perform standard firm-level cross-sectional regressions (Fama and MacBeth (1973)) to predict future stock returns:

$$R_{i,t+1} = \alpha^i + \beta \cdot \mathit{Collateralizability}_{i,t} + \gamma \cdot \mathit{Controls}_{i,t} + \varepsilon_{i,t+1},$$

where $R_{i,t+1}$ is stock i's cumulative (raw) return from July of year t to June of each year $t+1$. The control variables include the lagged firm collateralizability, size, book-to-market (BM), profitability (ROA) and book leverage. To avoid using future information, all the balance sheet variables are based on the values available at the end of year $t-1$. Table B.2 reports the results. The regressions exhibit a significantly negative slope coefficient for collateralizability across all specifications, supporting our theory.

In our empirical measure, only structure and equipment capital contribute to firms' collateralizability, but not intangible capital. Therefore, by construction, our collateralizability measure weakly negatively correlates with measures of intangible capital. In order to empirically distinguish our theoretical channel from the ones focusing on organizational capital

[11] We would like to thank Dimitris Papanikolaou for sharing the time series of the organizational factor.

(Eisfeldt and Papanikolaou (2013)) and R&D capital (Chan et al. (2001), Croce et al. (2017)), we also control for OG/AT, the ratio of organizational capital to total assets, and XRD/AT, the ratio of R&D expenses to total assets .

As shown in Table B.2 , the negative slope coefficients for collateralizability remain significant, although it becomes smaller in magnitude, after controlling for these two firm characteristics. Instead of using the ratio of R&D expenditure to total assets, we also used the ratio of R&D capital to total assets as a control. The results remain qualitatively very similar.

B.3. Alternative portfolio sorts

As a further robustness check, we consider each of the 17 Fama-French industries and sort firms into collateralizability quintile portfolios according to their collateralizability score within their respective industry. Portfolio 1 will thus contain all firms which are in the lowest quintile with respect to collateralizability relative to their industry peers, and so on for portfolios 2 to 5. By doing so, we essentially control for the industry fixed effect and compare firms with different collateralizability within each industry. Table B.3 reports the results of this exercise. The results are virtually unchanged when compared to the findings of our benchmark analysis shown in Table 1.

B.4. Double sorting on collateralizability and leverage

As discussed in the main text, firms with higher asset collateralizablility have higher debt capacity and thus tend to have higher financial leverage. If a firm is highly levered, then its equity is more exposed to aggregate risks. The effects of collateralizability and leverage can thus offset each other in determining the overall riskiness of the firm and consequently its average equity return.

In order to disentangle these two effects, we conduct a double sort on collateralizability and book leverage. The average returns for the resulting portfolios are reported in Table B.4. First, within each tercile sorted on book leverage, the collateralizability spread is always significantly positive. Second, the average returns of the high-minus-low leverage portfolios within each collateralizability quintile are not statistically significant.

Table B.2: Fama Macbeth Regressions

This table reports the results for Fama-MacBeth regressions of annual cumulative individual firm (raw) stock returns on lagged firm characteristics. The numbers reported in the table are the time-series averages of the slope coefficients from year-by-year cross-sectional regressions. The reported R^2 is the time-series average of the cross-sectional R^2. The columns labeled "SA", "WW", and "Non-Dividend" refer to the samples, where firms are classified as financially constrained using the SA index (Hadlock and Pierce (2010)), the WW index (Whited and Wu (2006)), or the fact that they did not pay dividends in the given year. ROA is Compustat item IB divided by book assets. OG/AT is organizational capital over total book assets, XRD/AT is R&D expenditure over total book assets. The t-statistics (in parentheses) are adjusted following Newey and West (1987). The sample period is from 1979 to 2016.

	(1) SA	(2) WW	(3) Non-Dividend	(4) SA	(5) WW	(6) Non-Dividend	(7) SA	(8) WW	(9) Non-Dividend
Collateralizability	-0.170***	-0.169***	-0.152***	-0.105*	-0.132**	-0.126***	-0.0892***	-0.0669**	-0.0694**
	(-3.68)	(-3.73)	(-3.84)	(-1.92)	(-2.49)	(-2.82)	(-2.75)	(-2.22)	(-2.25)
log(ME)	-0.108***	-0.108***	-0.0496***	-0.104***	-0.105***	-0.0466***	-0.113***	-0.116***	-0.0539***
	(-4.72)	(-6.23)	(-4.61)	(-4.58)	(-6.21)	(-4.53)	(-4.88)	(-6.46)	(-4.90)
BM	0.0626***	0.0429***	0.0538***	0.0663***	0.0442***	0.0555***	0.0679***	0.0510***	0.0598***
	(3.28)	(3.50)	(4.90)	(3.37)	(3.41)	(4.77)	(3.56)	(4.11)	(5.21)
Lagged return	0.0144	0.00527	0.0175	0.0123	0.00354	0.0156	0.0147	0.00694	0.0180
	(0.92)	(0.33)	(0.92)	(0.81)	(0.23)	(0.83)	(0.94)	(0.43)	(0.93)
ROA	0.0804	0.0729	0.0888*	0.0873	0.0814	0.0962*	0.202***	0.223***	0.225***
	(1.18)	(1.21)	(1.70)	(1.23)	(1.30)	(1.74)	(3.42)	(4.25)	(4.90)
Book Leverage	-0.0854	-0.0364	-0.00990	-0.0653	-0.0242	0.000523	-0.0378	0.0169	0.0475
	(-1.68)	(-0.78)	(-0.20)	(-1.29)	(-0.50)	(0.01)	(-0.85)	(0.44)	(1.07)
OG/AT				0.0662***	0.0392**	0.0374**			
				(3.73)	(2.26)	(2.05)			
XRD/AT							0.529***	0.690***	0.570***
							(3.27)	(3.87)	(3.21)
Constant	0.580***	0.593***	0.389***	0.513***	0.551***	0.352***	0.537***	0.542***	0.335***
	(5.72)	(7.88)	(5.94)	(5.28)	(7.64)	(5.45)	(5.42)	(7.81)	(5.11)
Observations	32258	37378	43907	32258	37378	43907	32258	37378	43907
R^2	0.0799	0.0753	0.0603	0.0836	0.0780	0.0637	0.0862	0.0816	0.0675

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.3: Portfolios Sorted on Collateralizability within Industries

This table reports reports annualized average monthly value-weighted excess returns ($E[R]-r^f$) for portfolios sorted on collateralizability, their α with respect to different factor models as well as the associated t-statistics. α^{FF+MOM} and α^{FF5} are the alphas with respect to the Carhart (1997) four-factor model and the Fama and French (2015) five-factor model, respectively. At the end of June each year t, we consider each of the 17 Fama-French industries and sort the constrained firms in a given industry into quintiles based on their collateralizability scores at the end of year $t-1$. Firms are classified as constrained at the end of year $t-1$, if their WW or SA index are higher than the corresponding median in year $t-1$, or if the firms do not pay dividends in year $t-1$. The WW and the SA index are constructed according to Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. Additionally, we consider a subsample where the firms are classified as constrained by all three measures jointly. The t-statistics are adjusted following Newey and West (1987). The sample period is from July 1979 to December 2016.

	1	2	3	4	5	1-5
	Financially constrained firms - All measures					
$E[R]-r^f(\%)$	12.56	12.26	12.21	8.22	5.32	7.24
t-stat	(2.63)	(2.60)	(2.89)	(1.92)	(1.24)	(3.19)
$\alpha^{FF3+MOM}$	4.03	3.50	3.34	-1.75	-3.47	7.50
t-stat	(2.05)	(1.75)	(1.69)	(-0.89)	(-1.90)	(3.13)
α^{FF5}	5.64	5.42	3.97	0.04	-1.78	7.42
t-stat	(3.51)	(3.54)	(2.56)	(0.02)	(-1.22)	(3.67)
	Financially constrained firms - WW index					
$E[R]-r^f(\%)$	12.20	13.17	10.05	8.56	5.75	6.44
t-stat	(2.77)	(2.99)	(2.49)	(2.18)	(1.43)	(3.46)
$\alpha^{FF3+MOM}$	3.30	5.10	1.92	-0.60	-3.25	6.56
t-stat	(2.05)	(2.97)	(1.30)	(-0.41)	(-2.23)	(3.31)
α^{FF5}	5.03	6.08	2.73	0.14	-1.34	6.37
t-stat	(4.06)	(4.71)	(2.28)	(0.13)	(-1.14)	(4.03)
	Financially constrained firms, SA index					
$E[R]-r^f(\%)$	10.94	10.99	9.70	9.07	6.18	4.76
t-stat	(2.38)	(2.42)	(2.28)	(2.26)	(1.45)	(2.25)
$\alpha^{FF3+MOM}$	3.00	3.88	2.65	0.46	-2.17	5.17
t-stat	(1.53)	(2.23)	(1.32)	(0.28)	(-1.20)	(2.40)
α^{FF5}	5.72	6.30	4.91	2.00	-0.57	6.30
t-stat	(4.38)	(4.48)	(2.54)	(1.31)	(-0.44)	(3.80)
	Financially constrained firms, Non-Dividend					
$E[R]-r^f(\%)$	12.42	13.83	8.58	7.75	7.42	5.00
t-stat	(2.92)	(3.11)	(2.09)	(1.93)	(1.76)	(2.08)
$\alpha^{FF3+MOM}$	4.66	6.61	0.76	0.53	0.08	4.58
t-stat	(2.26)	(3.12)	(0.42)	(0.31)	(0.04)	(1.86)
α^{FF5}	5.20	7.01	1.35	0.60	1.55	3.65
t-stat	(2.70)	(4.30)	(0.97)	(0.40)	(0.94)	(1.65)

Table B.4: Independent Double Sort on Collateralizability and Leverage

This table reports annualized average value-weighted monthly excess returns for portfolios double-sorted independently on collateralizability and leverage. The sample starts in July 1979 July and ends in December 2016. At the end of June in each year t, we independently sort financially constrained firms into five quintiles based on collateralizability (vertical direction) and into terciles based on book financial leverage (horizontal direction), then we compute the value-weighted returns of each portfolio. The book financial leverage is defined as financial debt over total asset ratio. A firm is considered financially constrained, if its WW index (Whited and Wu (2006)) is above the respective median. The t-statistics are adjusted following Newey and West (1987).

	L Lev	2	H Lev	H-L	t-stat
L Col	16.56	17.51	22.14	5.58	(1.67)
2	13.77	16.69	18.67	4.89	(1.70)
3	14.50	13.52	13.01	-1.49	(-0.53)
4	13.61	16.59	10.46	-3.15	(-1.14)
H Col	8.70	8.75	10.16	1.46	(0.52)
L-H	7.86	8.76	11.98	4.12	(1.15)
t-stat	(2.34)	(2.48)	(3.21)		

Appendix C: Data and measurement

We now provide details on the data sources, the construction of our empirical collateralizability measure, and on the measurement of intangible capital.

C.1. Data sources

Our major sources of data are (1) firm level balance sheet data from the CRSP/Compustat Merged Fundamentals Annual Files, (2) monthly stock returns from CRSP, and (3) industry level non-residential capital stock data from the BEA table "Fixed Assets by Industry". We adopt the standard screening process for the CRSP/Compustat Merged Database. We exclude utilities and financial firms (SIC codes between 4900 and 4999 and between 6000 and 6999, respectively). Additionally, we keep common stocks that are traded on NYSE, AMEX and NASDAQ. The accounting treatment of R&D expense reporting was standardized in 1975, and we allow three years for firms to adjust to the new accounting rules, so that our sample starts in 1978. Following Campello and Giambona (2013), we exclude firm-year observations for which the value of total assets or sales is less than $1 million. We focus on the impact of asset collateralizability on debt capacity of firms, therefore we drop small firms, which do not have much debt. In practice we drop firm-year observations with market value of equity below $8 million, which roughly corresponds to the bottom 5% of firms. All firm characteristics are winsorized at the 1% level. The potential delisting bias of stock returns is corrected following Shumway (1997) and Shumway and Warther (1999).

In order to obtain a long sample with broader coverage[12], we use the narrowly defined industry level non-residential fixed asset (structure, equipment and intellectual) from the BEA tables to back out industry level structure and equipment capital shares.

In Table C.6, we provide the definitions of the variables used in our empirical analyses.

C.2. Measurement of collateralizability

This section provides details on the construction of the firm specific collateralizability measure, complementing the description of the methodology provided in Section 2.

We first construct proxies for the share of the two types of capital, denoted by *StructShare*

[12] COMPUSTAT shows the components of physical capital (PPEGT) only for the period from 1969 to 1997. However, even for the years between 1969 and 1997, only 40% of the observations have non-missing entries for the components of PPEGT, which are buildings (PPENB), machinery and equipment (PPENME), land and improvements (PPENLI).

and $EquipShare$. Then we run the leverage regression as in equation (2), which allows us to later calculate the firm-specific collateralizability score.

The BEA classification features 63 industries. We match the BEA data to Compustat firm level data using NAICS codes, assuming that, for a given year, firms in the same industry have the same structure and equipment capital shares. We construct measures of structure and equipment shares for industry j in year t as

$$\begin{aligned} StructShare_{j,t} &= \frac{\text{Structure}^{BEA}_{j,t}}{\text{Fixed Asset}^{BEA}_{j,t}} \frac{\text{Fixed Asset}^{\text{Compustat}}_{j,t}}{\text{AT}^{\text{Compustat}}_{j,t}}, \\ EquipShare_{j,t} &= \frac{\text{Equipment}^{BEA}_{j,t}}{\text{Fixed Asset}^{BEA}_{j,t}} \frac{\text{Fixed Asset}^{\text{Compustat}}_{j,t}}{\text{AT}^{\text{Compustat}}_{j,t}}, \end{aligned}$$

where $AT_{j,t}$ are total assets in industry j in year t, i.e., the sum across all firms in our sample belonging to industry j in year t. The first component on the right hand side refers to the structure (equipment) share from BEA data, which is structure (equipment) to fixed asset ratio at the industry level. The second component refers to the industry level fixed asset to total asset ratio in Compustat. We use PPEGT in Compustat as the equivalent for fixed asset in BEA data. By doing so, we map the BEA industry level measure of structure (equipment) to fixed asset ratio to structure (equipment) to total asset ratio, at the industry level. Keeping denominator as the total asset is important, since we motivate the collateralizability measure from a classical collateral constraint. As discussed in Section 2, both the collateralizable asset and financial debt should be denominated by total assets.

In order to construct an empirical collateralizability measure analogous to the theoretically motivated one from equation (2), we run the following regression:

$$\begin{aligned} \frac{B_{i,t}}{AT_{i,t}} &= c + \zeta_S StructShare_{j,t} + \zeta_E EquipShare_{j,t} \\ &\quad + \gamma X_{i,t} + \sum_t Year_t + \varepsilon_{i,t}, \end{aligned} \tag{C7}$$

where, for a given firm i, j denotes the industry which the firm belongs to in year t. $X_{i,t}$ represents a vector of controls typically used in capital structure regressions, including size, book-to-market ratio, profitability, marginal tax rate, earnings volatility, and bond ratings. $B_{i,t}$ is total debt, defined as long term debt (DLTT) plus short term debt (DLC). Additionally, in order to capture non-financial debt, we adjust debt by adding capitalized rental expenses following Rampini and Viswanathan (2013).

The results are shown in Table C.5. We run the leverage regression on firms classified as financially constrained (based on either its SA index, its WW index, or on it not paying dividends over the given year, or on all these indicators together), and for the full sample.

As we can see in all of the specifications, there is a significant difference between structure and equipment capital in terms of their respective collateralizability with structure capital supporting substantially more debt. This result is is in line with the findings in Campello and Giambona (2013).

We interpret the weighted sum, $\zeta_S StructShare_{j,t} + \zeta_E EquipShare_{j,t}$, as the contribution of structure and equipment capital to financial leverage, and the product of this sum and the book value of assets, $(\zeta_S StructShare_{j,t} + \zeta_S EquipShare_{j,t}) \cdot AT_{i,t}$, represents total collateralizable capital of firm i in year t.[13] The collateralizability score for firm i in year t is then computed as

$$\zeta_{i,t} = \frac{(\zeta_S \cdot StructShare_{j,t} + \zeta_S \cdot EquipShare_{j,t}) \cdot AT_{i,t}}{PPEGT_{i,t} + Intangible_{i,t}}, \tag{C8}$$

where $PPEGT_{i,t}$ and $Intangible_{i,t}$ are the physical capital and intangible capital of firm i in year t, respectively. The importance of taking intangible capital into account has been emphasized in the recent literature, e.g., by Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017). The asset-specific collateralizablity parameters ζ_S and ζ_E we adopt in our empirical analyses are the ones from the last column of Table C.5, where firms are classified as constrained based jointly on all three measures (SA index, WW index, and non-dividend paying).

In the above collateralizability measure, we implicitly assume the collateralizability parameter for intangible capital to be equal to zero. We do this based on empirical evidence that intangible capital can hardly be used as collateral, since only 3% of the total value of loans to companies are actually collateralized by intangibles like patents or brands (Falato et al. (2013)). Our results remain qualitatively very similar when we exclude intangible capital from the denominator of the collateralizability measure in (C8) and only exploit the asymmetric collateralizability between structure and equipment capital within tangible assets.

C.3. Measuring intangible capital

In this section, we provide details on the construction of firm specific intangible capital. The total amount of intangible capital of a firm is given by the sum of externally acquired and internally created intangible capital, where the latter consists of R&D capital and organizational capital.

[13] Alternatively, we also used the market value of assets to compute total collateralizable capital. The empirical collteralizability spread based on this sorting measure is even stronger.

Table C.5: Capital Structure Regressions

This table reports the results for regression (C7) using lease adjusted book leverage as the left-hand side variable. *StructShare* and *EquipShare* are constructed using BEA and Compustat data, as described in Section C.2. *Book Size* is the log of the sum of Compustat item *PPEGT* and intangible capital, *BM* is the book-to-market ratio. *Profitability* is given by Compustat item *OIBDP/AT*. *Marginal Tax Rate* data is downloaded from John Graham's website (Graham (2000)). *Sales Grth Volatility* is computed as the standard deviation of sales growth over rolling 4-year windows. *Rating Dummy* is a dummy variable that takes a value of 1 if the firm has either a bond rating or a commercial paper rating, and 0 otherwise. Standard errors are clustered at the firm-year level. The column labeled "Full" corresponds to the regression performed on all firms. The columns labeled "Non-Dividend", "SA", and "WW" show the result for regressions using the samples of firms classified as constrained based on them not having paid dividends over the year, or their SA or WW index being above the median in the given year, respectively. The column "All Cons." refers to the regression for the sample of firms which are classified as constrained using all three measures jointly.

	(1) Full	(2) Non-Dividend	(3) SA	(4) WW	(5) All Cons.
StructShare	0.434***	0.622***	0.559***	0.558***	0.626***
	(10.95)	(9.03)	(6.31)	(7.19)	(13.79)
EquipShare	0.00553	0.155**	0.172	0.0924	0.223***
	(0.13)	(2.16)	(1.53)	(1.16)	(3.06)
Book Size	-0.0113***	0.00829*	0.0467***	0.0546***	0.0639***
	(-3.96)	(1.71)	(5.78)	(7.93)	(13.04)
BM	0.0207***	0.0264***	-0.00688	0.00325	0.00657
	(3.59)	(3.31)	(-0.53)	(0.33)	(0.72)
Profitability	-0.0480	-0.0414	-0.0168	-0.0322	-0.00835
	(-1.54)	(-1.15)	(-0.45)	(-0.88)	(-0.26)
Marginal Tax Rate	-0.180***	-0.108***	-0.251***	-0.209***	-0.153***
	(-8.08)	(-3.09)	(-5.59)	(-5.69)	(-4.46)
Sales Grth Volatility	-0.00175**	-0.00218**	-0.00184**	-0.00196**	-0.00180***
	(-2.35)	(-2.18)	(-2.34)	(-2.11)	(-2.84)
Rating Dummy	0.0592***	0.0457**	-0.0139	0.0806***	0.0787**
	(4.78)	(2.14)	(-0.32)	(2.60)	(2.40)
Constant	0.429***	0.262***	0.198***	0.171***	0.0907**
	(20.93)	(7.26)	(4.14)	(4.44)	(2.38)
Observations	58903	27849	17496	23976	12709
R^2	0.0495	0.0727	0.0580	0.0773	0.0721

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Externally acquired intangible capital is given by item *INTAN* in Compustat. Firms typically capitalize this type of asset on the balance sheet as part of intangible assets. For the average firm, *INTAN* amounts to about 19% of total intangible capital with a median of 3%, consistent with Peters and Taylor (2017). We set externally acquired intangibel capital to zero, when the entry for INTAN is missing.

Concerning internally created intangible capital, R&D capital does not appear on the firm's balance sheet, but it can be estimated by accumulating past expenditures. Following Falato et al. (2013) and Peters and Taylor (2017), we capitalize past R&D expenditures (Compustat item *XRD*) using the so-called perpetual inventory method, i.e.,[14]

$$RD_{t+1} = (1 - \delta_{RD})RD_t + XRD_t,$$

where δ_{RD} is the depreciation rate of R&D capital. Like Peters and Taylor (2017), we set the depreciation rates for different industries following Li and Hall (2016). For unclassified industries, the depreciation rate is set to 15%.[15]

Finally, we also need the initial value RD_0. We use the first non-missing R&D expenditure, XRD_1, as the first R&D investment, and specify RD_0 as

$$RD_0 = \frac{XRD_1}{g_{RD} + \delta_{RD}}, \tag{C9}$$

where g_{RD} is the average annual growth rate of firm level R&D expenditure. In our sample, g_{RD} is around 29%.

Following Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017), our organizational capital is constructed by accumulating a fraction of Compustat item *XSGA*, "Selling, General and Administrative Expense", which indirectly reflects the reputation or human capital of a firm. However, as documented by Peters and Taylor (2017), *XSGA* also includes R&D expenses *XRD*, unless they are included in the cost of goods sold (Compustat item *COGS*). Additionally, *XSGA* sometimes also incorporate the in-process R&D expense (Compustat item *RDIP*). Hence, following Peters and Taylor (2017), we subtract *XRD* and *RDIP* from *XSGA*.[16] Additionally, also following Peters and Taylor (2017), we add the filter that when *XRD* exceeds *XSGA*, but is less than *COGS*, or when *XSGA* is missing, we keep XSGA with no further adjustment. Afterwards, we replace missing XSGA with

[14] This method is also used by the BEA R&D satellite account.

[15] Our results are not sensitive to the choice of depreciation rates.

[16] *RDIP* (in-process R&D expense) is coded as negative in Compustat. Subtracting RDIP from XSGA means RDIP is added to XSGA. As discussed in Peters and Taylor (2017), XSGA does not include this component, so we add this component back to XSGA, then subtract the total amount of R&D expenditures.

zero. As in Hulten and Hao (2008), Eisfeldt and Papanikolaou (2014), and Peters and Taylor (2017), we count only 30% of SGA expenses as investment in organizational capital, the rest is treated as operating costs.

Using a procedure analogous to the one described above for internally created R&D capital, organizational capital is constructed as

$$OG_{t+1} = (1 - \delta_{OG})OG_t + SGA_t,$$

where $SGA_t = 0.3(XSGA_t - XRD_t - RDIP_t)$ and the depreciation rate δ_{OG} is set to 20%, consistent with Falato, Kadyrzhanova, and Sim (2013) and Peters and Taylor (2017). Again anlagous to the case of R&D capital we set the initial level of organizational capital OG_0 according to

$$OG_0 = \frac{SGA_1}{g_{OG} + \delta_{OG}}.$$

The average annual growth rate of firm level $XSGA$, g_{OG}, is 18.9% in our sample.

Table C.6: Definition of variables

Variables	Definition	Sources
Structure share	Firstly we construct the structure shares from BEA industry capital stock data, defined as structure capital over total fixed asset ratio. Then we rescale the structure shares by the corresponding industry average of physical asset (PPEGT) to book asset ratio (AT).	BEA + Compustat
Equipment share	Firstly we construct the equipment shares from BEA industry capital stock data, defined as equipment capital over total fixed asset ratio. Then we rescale the equipment shares by the corresponding industry average of physical asset (PPEGT) to book asset ratio (AT).	BEA + Compustat
Intangible capital	Intangible capital is defined following Peters and Taylor (2017). We capitalize R&D and SGA expenditures using the perpetual inventory method.	Compustat
Collateralizability	Collateralizable capital divided by PPEGT + Intangible. Collateralizable capital and intangible capital are defined in Section C.2.	BEA + Compustat
BE	Book value of equity is the book value of stockholders equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) as the book value of preferred stock.	Compustat
ME	Market value of equity is copmputed as price per share times the number of shares outstanding. The share price is taken from CRSP, the number of shares outstandings from Compustat or CRSP, depending on availability.	CRSP+Compustat
log(ME)	The natural log of market value of equity.	CRSP+Compustat
BM	Book to market value of equity ratio.	Compustat
Tangibility	Physical capital (PPEGT) to the sum of physical (PPEGT) and intangible capital ratio.	Compustat
Book size	The natural log of the sum of PPEGT and intangible capital.	Compustat
Profitability	Compustat item OIBDP divided by AT.	Compustat
OG/AT	Organizational capital divided by total assets (AT).	Compustat
XRD/AT	R&D expenditure to book asset ratio.	Compustat
Book leverage	Lease adjusted book leverage is defined as financial debt (DLTT+DLC) plus XRENT*10, divided by AT.	Compustat
Dividend Dummy	A dummy variable takes value of one if the firm's dividend payment (DVT, DVC or DVP) over the year was positive.	Compustat
Sales Grth Volatility	Rolling window standard deviation of past 4 year's sales growth.	Compustat
Rating Dummy	A dummy variable taking the value of 1, if the firm has either a bond rating or a commercial paper rating, and 0 otherwise.	Compustat
Marginal Tax Rate	Following Graham (2000).	John Graham's website
WW index	Following Whited and Wu (2006).	Compustat
SA index	Following Hadlock and Pierce (2010).	Compustat
ROA	Income before extraordinary items (IB) divided by total assets (AT).	Compustat