

A comment on “Announcement risk premium reconsidered”

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Abstract A recent paper by Laarits [2] reconsiders the theorem of generalized risk sensitivity in Ai and Bansal [1]. We demonstrate that the key mistake in Laarits [2] is the confusion of basic equilibrium concepts in competitive equilibrium analysis. The concept of competitive equilibrium in exchange economies allows individual agents to optimally choose their consumption given prices and all available information, even though in the aggregate, total endowment is exogenous and cannot be modified. This confusion leads Laarits [2] to question the validity of individual investors’ optimality condition, i.e., the envelope theorem in Ai and Bansal [1].

1 Introduction

A recent paper by Laarits [2] reconsiders the theorem of generalized risk sensitivity in Ai and Bansal [1] (Hereafter AB). The key result in Laarits [2] is that “*Specifically, I show that the proof in Ai and Bansal (2018) contains a misapplication of the Envelope Theorem.*” The purpose of this note is to provide a more detailed explanation for the validity of the “envelope theorem”.

The key mistake in Laarits [2] is the confusion of basic equilibrium concepts in competitive equilibrium analysis. In competitive equilibriums in exchange economies, such as Lucas [4], aggregate endowment (or income) is exogenously given and is not affected by agents in the economy. However, this does not mean that individual investors cannot optimally choose their individual consumption. In fact, consumption-based asset pricing models are based on the concept that individual investors optimally choose their consumption given prices, which must, in equilibrium clear the market.

To clearly illustrate the above equilibrium concepts in the AB economy, this note provides a detailed discussion for the two-period economy in AB, which is less notation intensive compared to the fully dynamic model. The rest of the note is organized as follows. In Section 2, we describe the setup of the economy. Section 3 discusses the Arrow-Debreu market setup, Section 4 focuses on the sequential market setup. Section 5 briefly describes the extension to the infinite horizon setting. Section 6 comments on the example in Laarits [2] and Section 7 concludes.

2 Setup of the model

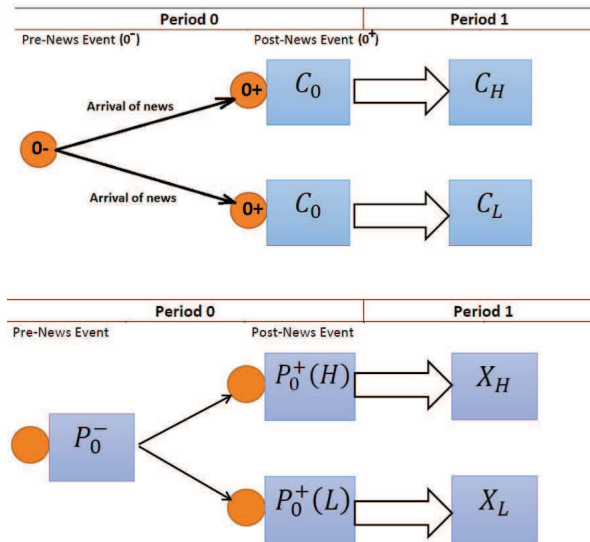
We consider the same two-period model in AB. This is a standard Lucas [4] endowment economy. The notations we use here are all consistent with those in the Supplemental Material of AB. To emphasize the consistence with the AB paper, we use typewriter font whenever we directly quote from their paper.

We consider a representative-agent economy with two periods, 0 and 1. Period 0 has no uncertainty and the aggregate endowment is a known constant, \bar{C}_0 . The aggregate endowment in period 1, denoted by \bar{C}_1 , is a random variable. We assume a finite number of states: $n = 1, 2, \dots, N$ and denote the possible realizations of \bar{C}_1 as $\{\bar{C}_1(n)\}_{n=1,2,\dots,N}$ and the possible realizations of asset payoff as $\{X(n)\}_{n=1,2,\dots,N}$. The probability of each state is $\pi(n) = \frac{1}{N}$ for $n = 1, 2, \dots, N$.

Period 0 is further divided into two subperiods. In period 0^- , before any information about \bar{C}_1 is revealed, the pre-announcement market opens and asset prices at this point are called pre-announcement prices and are denoted by P^- . P^- cannot depend on the realization of \bar{C}_1 , which is unknown at this point. In period 0^+ , the agent receives an announcement s that carries information about \bar{C}_1 . Immediately after the announcement, the post-announcement asset market opens. The post-announcement asset prices depend on s and are denoted by $P^+(s)$. In period 0^+ , prices are denominated in current date-and-state-contingent consumption units, and the agent makes both optimal consumption and investment decisions given prices. In period 0^- , there is only investment decisions but no consumption decision. We denominate asset prices at 0^- in units of consumption goods delivered non-contingently in period 0^+ .

For simplicity, we assume that announcements fully reveal the true state, that is, $s \in \{1, 2, \dots, N\}$, although this assumption is not necessary as demonstrated in AB. In Figure 1, we illustrate the timing of information and consumption (top panel) and that of asset prices (bottom panel), assuming $N = 2$.

Figure 1: **consumption and asset prices in the two-period model**



Using the setup in AB, we assume that investors' preference is represented by a pair of functions $\{u, \mathcal{I}\}$, where u transforms consumption into utility and \mathcal{I} aggregates across different states. In our

setup, given a state-contingent consumption plan, $\{C_0(s), C_1(s)\}_{s=1}^N$, the utility is defined as:

$$\mathcal{I}[u(C_0(s)) + \beta u(C_1(s))]. \quad (2.1)$$

As explained in AB, although the aggregate endowment \bar{C}_0 does not depend on s , we allow agents' preference to be able to evaluate a larger class of consumption plans. In particular, we allow $C_0(s)$ to depend on the content of announcement. This is because in our competitive equilibrium setup below, investors are allowed to choose consumption in period 0^+ as a function of s .

The distinction between *aggregate* endowment, $\{\bar{C}_0, \bar{C}_1(n)\}_{n=1,2,\dots,N}$, where \bar{C}_0 does not depend on announcement s , and *individuals'* choices of consumption, $\{C_0(s), C_1(s)\}_{s=1}^N$, where $C_0(s)$ is determined given equilibrium prices and can potentially depend on s , is the key source of confusion in Laarits [2]. As in the standard Lucas [4] model, we consider an exchange economy, where aggregate endowment is exogenously specified. In particular, the aggregate endowment for period 0 does not depend on the content of announcement about the next period endowment, s , even though such announcements are made at time 0^+ . On the other hand, as explained clearly in AB, the model allows agents to make their consumption and investment choices to adjust their period 0 consumption, $C_0(s)$ based on the content of announcement. In the equilibrium, of course, market clearing implies that $C_0(s) = \bar{C}_0$. It is the price that has to adjust so that this condition is met. For pedagogical purpose, the notations we set up here allow us to clearly distinguish aggregate endowment and individual choices of consumption.¹

Let $\{X(s)\}_{s=1,2,\dots,N}$ be a vector of announcement payoff, that is, it is a contingent payoff realized at time 0^+ . Let $P^-(X)$ denote the price of the contingent payoff at time 0^- . The expected return of this asset is calculated as $\frac{E[X(s)]}{P^-(X)}$. In the special case where $X(s)$ is a constant, this is a risk-free announcement return, which we denote as R_f . A risky asset X requires an announcement premium if $\frac{E[X(s)]}{P^-(X)} \geq R_f$. Clearly, the choice of consumption numeraire in period 0^- will not affect the comparison between risky and risk-free returns from 0^- to 0^+ , as it affects the denomination of all returns proportionally.

3 The Arrow-Debreu market

Definition of equilibrium Here we consider a complete market setting in which all assets are traded at time 0^- . The complete market asset pricing setup can be found in standard textbooks, for example, chapter 8 of Ljungqvist and Sargent [3]. We use $\{\bar{C}_0, \{\bar{C}_1(s)\}_{s=1}^N\}$ to denote aggregate endowment in our two-period model and use $\{C_0(s), C_1(s)\}_{s=1}^N$ for the consumption choice of the agent. From an individual agent's perspective, the decision for C_0 is made after the announcement, and therefore is allowed to depend on s . At the aggregate level, \bar{C}_0 does not depend on s .

¹The Supplemental Material of AB is very clear about this distinction. The main text of the paper follows the convention in the consumption-based asset pricing literature not to draw the distinction to save notation.

Trading on the Arrow-Debreu market happens in period 0^- . Let $q_0(s)$ be the period 0^- price of an Arrow-Debreu security that delivers one unit of consumption good in period 0^+ and state s , for $s = 1, 2, \dots, N$. Similarly, let $q_1(s)$ be the Arrow-Debreu price of one unit of consumption good in period one and state s . Because markets are complete, the utility maximization problem of the representative agent can be written as:

$$\begin{aligned} & \max \mathcal{I}[u(C_0(s)) + \beta u(C_1(s))] \\ \text{subject to} & : \sum_{s=1}^N [q_0(s) C_0(s) + q_1(s) C_1(s)] \leq \sum_{s=1}^N [q_0(s) \bar{C}_0 + q_1(s) \bar{C}_1(s)] \end{aligned} \quad (3.1)$$

As in AB, $\mathcal{I}[\cdot]$ is the certainty equivalent functional. Because s is finite dimensional, we can think of \mathcal{I} as a mapping from R^N to R . Our definition of competitive equilibrium is standard, for example, Section 8.5 in Ljungqvist and Sargent [3]. A competitive equilibrium consists of prices, $\{q_0(s), q_1(s)\}_{s=1}^N$, and allocations, $\{\hat{C}_0(s), \hat{C}_1(s)\}_{s=1}^N$, such that:

1. Given asset prices, $\{q_0(s), q_1(s)\}_{s=1}^N$, the allocation solves the utility maximization problem (3.1).
2. The allocation satisfies the following market clearing conditions:

$$\hat{C}_0(s) = \bar{C}_0, \quad \hat{C}_1(s) = \bar{C}_1(s), \quad \text{all } s = 1, 2, \dots, N. \quad (3.2)$$

In the above setup, because the announcement is made at time 0^+ , from the agent's perspective, consumption at time 0^+ is allowed to depend on s , which we write as $C_0(s)$. Of course, the market clearing condition (3.2) implies that in the equilibrium $\hat{C}_0(s) = \bar{C}_0$ do not depend on s . However, this does not mean that the investors in the economy cannot choose $C_0(s)$ freely. As in the standard Lucas [4] economy, although the individual optimization problem (3.1) does not restrict that $C_0(s)$ cannot depend on s . It is the equilibrium prices, in particular, $q_0(s)$, that has to adjust so that the equilibrium utility maximizing choice of $\hat{C}_0(s)$ will equal to the aggregate endowment \bar{C}_0 for all s .

Equilibrium asset prices In our setup, given prices, $\{q_0(s), q_1(s)\}_{s=1}^N$, investors choose state-contingent consumption plan, $\{C_0(s), C_1(s)\}_{s=1}^N$, optimally to maximize utility. To save notation, as in the paper, we denote $V_s = u(\hat{C}_0(s)) + \beta u(\hat{C}_1(s))$. Optimality implies that,

$$\lambda q_0(s) = \frac{\partial \mathcal{I}[V]}{\partial V_s} u'(\hat{C}_0(s)), \quad \lambda q_1(s) = \frac{\partial \mathcal{I}[V]}{\partial V_s} \beta u'(\hat{C}_1(s)), \quad (3.3)$$

where λ is the Lagrangian multiplier of the budget constraint. In equilibrium, market clearing implies that $\hat{C}_0(s) = \bar{C}_0$ for all s .

Therefore, using the market clearing conditions to replace $\{\hat{C}_0(s), \hat{C}_1(s)\}_{s=1}^N$ in (3.3), equilibrium prices must satisfy

$$\lambda q_0(s) = \frac{\partial \mathcal{I}[V]}{\partial V_s} u'(\bar{C}_0), \quad \lambda q_1(s) = \frac{\partial \mathcal{I}[V]}{\partial V_s} \beta u'(\bar{C}_1(s)). \quad (3.4)$$

Note in particular, in equilibrium, $\hat{C}_0(s) = \bar{C}_0$ and cannot depend on s . Here, the interpretation is the same as the classical Lucas [4] model. Individual investors are free to choose $C_0(s)$ as a function of s . However, in equilibrium, asset prices must adjust so that market clears, and $\hat{C}_0(s) = \bar{C}_0$ cannot depend on s .

Announcement premium As is standard in competitive equilibrium asset pricing models, in equation (3.4), prices are determined up to a multiplicative constant λ . The determination of λ amounts to choosing a unit of denomination in period 0^- . Note however, the AB theorem is only concerned with the announcement premium, which is the difference in the returns on a risky asset and a risk-free asset. As a result, the choice of λ in period 0^- is irrelevant for the comparison of the two expected returns. Not surprisingly, the Theorem of Generalized Sensitivity in AB does not depend on the choice of numeraire in period 0^- .

It is customary in consumption-based asset pricing models to use current-period consumption as the numeraire to denominate asset prices. However, in our simple model, there is no consumption at time 0^- . Here, we follow the treatment in AB to use one unit of state-non-contingent consumption goods delivered in period 0^+ as the numeraire in the denomination of prices. If we normalize the price of one unit state-non-contingent consumption at time 0^+ to be one, that is, $\sum_{s=1}^N q_0(s) = 1$; then, for all s ,

$$q_0(s) = \frac{\frac{\partial \mathcal{I}[V]}{\partial V_s}}{\sum_{s=1}^N \frac{\partial \mathcal{I}[V]}{\partial V_s}}, \quad (3.5)$$

and $\frac{q_1(s)}{q_0(s)} = \beta \frac{u'(\bar{C}_1(s))}{u'(\bar{C}_0)}$. That is, we can simply use ratios of marginal utilities to compute Arrow-Debreu prices. Clearly, (3.5) implies the expression of the A-SDF in equation (12) of AB.

4 The sequential market setup

It is well known that Arrow-Debreu setups as described in the last section yield identical equilibrium outcomes as sequential market setups, see for example, Chapter 8 Ljungqvist and Sargent [3]. Not surprisingly, investors optimality conditions in a sequential market setup should give the same A-

SDF as in (3.5). As in standard dynamic equilibrium models, the first order optimality conditions in an Arrow-Debreu market setup implies an envelope condition for investors' value functions in the sequential market setup.

Because the main argument in Laarits [2] focuses on the application of the envelope theorem in AB in the sequential market setup, we describe the sequential market setup below and provide a derivation using the envelope condition, which is of course equivalent to the Arrow-Debreu setting above.

Definition of equilibrium In the sequential market setup, we think of the representative investor of the economy as starting with the initial endowment of financial wealth W , which can be interpreted as the present value of a "Lucas tree" that yields a stream of consumption goods $\{\bar{C}_0, \{\bar{C}_1(s)\}_{s=1}^N\}$. Investors trade a vector of assets sequentially. Instead of describing the payoffs and prices of the traded assets, we normalize the payoffs so that all assets have unit prices. This allows us to work with returns and save some notation. At time 0^- , a vector of returns are available for trading: $\{R_{A,j}(s)\}_{j=1,2,\dots,J}$. These are assets that require one unit of input at time 0^- , and provide a state-contingent return $R_{A,j}(s)$ upon announcement at time 0^+ . We allow the realizations of $R_{A,j}(s)$ to depend on the announcement s .

At time 0^+ , after the realization of the announcement, asset markets open again. We call the return on an asset purchased after announcements at time 0^+ post-announcement returns. Note that because announcements fully reveal the true state of the economy, there is no uncertainty at 0^+ . As a result, returns on all assets from period 0^+ to period 1 must be equal, and we can drop the j subscript to save notation. We use $R_{P,s}$ to denote the post-announcement return in state s . Investors' utility maximization problem on the post-announcement market can be described as:

$$\begin{aligned} V_s(W) &= \max_{C_0(s), C_1(s)} u(C_0(s)) + \beta u(C_1(s)) \\ \text{subject to} & : C_1(s) = (W - C_0(s)) R_{P,s}. \end{aligned} \tag{4.1}$$

The above is equation (S.1.3) in the Supplemental Material of AB. Here, we define investors' value function $V_s(W)$ as the maximum level of utility that can be achieved for a given level of W in state s . Note that asset prices depend on s , and therefore so does the value function. Again, as in the standard Lucas [4] asset pricing framework, from individual investors' perspective, $C_0(s)$ is a choice variable and can potentially depend on s , as it is chosen after the announcement is made. Optimality conditions with respect to $C_0(s)$ will have to hold as long as investors maximize their utilities.

In period 0^- , there is no consumption decision and the agent chooses

investment in a vector of announcement returns to maximize:

$$\begin{aligned} & \max_{\{\xi_j\}_{j=1}^J} \mathcal{I} [V (W')] \\ & \text{subject to} : W'_s = W - \sum_{j=1}^J \xi_j + \sum_{j=1}^J \xi_j R_{A,j} (s), \text{ all } s, \end{aligned} \quad (4.2)$$

where $W' = \{W'_s\}_{s=1}^N$ is the vector of realizations of wealth in the next period, and $V (W') = \{V_s (W'_s)\}_{s=1}^N$ is a vector of value functions. For each s , the value function $V_s (W)$ is defined by the optimal portfolio choice problem on the post-announcement market.

The equilibrium consists of an initial wealth W , a vector of returns, $\{\{R_{A,j} (s)\}_j, R_{P,s}\}_{s=1}^N$ and the equilibrium choice of consumption, $\{\hat{C}_0 (s), \hat{C}_1 (s)\}_{s=1}^N$, and portfolio holdings, $\{\xi_j\}_j$, such that

1. Given W and $\{\{R_{A,j} (s)\}_j, R_{P,s}\}_{s=1}^N$, the consumption and portfolio choices solve the optimization problems in (4.1) and (4.2).
2. The markets for consumption goods and assets clear.

Equilibrium asset prices The first order condition for (4.2) with respect to ξ_j implies that for any announcement returns $R_{A,j}$,

$$\sum_{s=1}^N \frac{\partial}{\partial V_s} \mathcal{I} [V (W')] \frac{\partial V_s (W'_s)}{\partial W'_s} [R_{A,j} (s) - 1] = 0, \quad (4.3)$$

where W'_s denote the equilibrium wealth of the agent in period 0^+ after announcement s . The envelope condition for (4.1) implies that $\frac{\partial V_s (W'_s)}{\partial W'_s} = u' (\hat{C}_0 (s)) = u' (\bar{C}_0)$, where the second equality uses the market clearing condition. Here $\frac{\partial V_s (W'_s)}{\partial W'_s} = u' (\hat{C}_0 (s))$ is an optimality condition: because $\hat{C}_0 (s)$ is the optimal choice of investors, it has to satisfy the envelope condition. Here we do not discuss the technical assumptions of the differentiability of utility functions under which the envelope condition holds. In the finite dimensional setting, this can be found in standard textbooks, such as Simon and Blume [5]. AB provide a careful analysis on this issue in their infinite dimensional setup. The second part of the above equality uses the market clearing condition to replace the optimal choice $\hat{C}_0 (s)$ by \bar{C}_0 because in equilibrium they have to be equal.

Laarits [2]'s main objection to AB is the applicability of the envelope theorem, $\frac{\partial V_s (W'_s)}{\partial W'_s} = u' (\hat{C}_0 (s)) = u' (\bar{C}_0)$. Again, we emphasize here that the first equality holds because from an individual investor's perspective, $\hat{C}_0 (s)$ is chosen after announcements are made and is allowed to

depend on s . The second equality uses the fact that equilibrium prices have to adjust so that the equilibrium market clearing conditions are met.

As $u' > 0$, equation (4.3) implies

$$\sum_{s=1}^N \frac{\frac{\partial}{\partial V_s} \mathcal{I}[V(W')]}{\sum_{s=1}^N \frac{\partial}{\partial V_s} \mathcal{I}[V(W')]} R_{A,j}(s) = 1,$$

as in equation (11) of the paper.

5 The fully dynamic model

The derivation in the fully dynamic model in AB is completely analogous to the above, and there is little need to repeat. Here we simply reiterate a few places in AB where notations are carefully chosen to reflect the difference between the aggregate endowment, which is given exogenously in the model, and individual consumption, which is chosen optimally given prices.

1. In the second paragraph of Section 4.1 on page 1395, the individual consumption choice, $\{C_t\}_{t=1}^T$, is adapted to $\{\mathcal{F}_t^+\}_{t=1}^T$, which reflect the fact that consumption is chosen after announcements are made. On page 1395 of the paper, AB wrote, A consumption plan is an $\{\mathcal{F}_t^+\}_{t=1}^T$ -adapted process $\{C_t\}_{t=1}^T$, such that C_t is a Y -valued square-integrable random variables for all t .
2. In the subsequent paragraph, AB remarks, The aggregate endowment of the economy, denote as $\bar{C} \in C$ is required to be $\{\mathcal{F}_t^-\}_{t=1}^T$ -adapted. As in the two-period model, individual consumption choices are allowed to be made contingent on the announcements, $\{s_{t-1}^+\}_{t=1}^T$. However, announcements carry information about future endowments but do not affect current-period endowments. That is, $\forall t$, the aggregate consumption \bar{C}_t , must be \mathcal{F}_t^- measurable. The above setup allows us to model announcements as revelations of public information associated with realizations of $\{s_{t-1}^+\}_{t=1}^T$, separately from the realizations of consumption.
3. Laarits [2] cited and agreed with the optimality condition (AB51):

$$E \left[D\mathcal{I} \left[V_{z_t^+}(W') \right] \frac{d}{dW'} V_{z_t^+}(W') (R_j(z_t^+) - 1) \Big| z_t^- \right] = 0. \quad (\text{AB51})$$

In the dynamic model, AB claim that the value function $V_{z_t^+}(W')$ must satisfy (AB52):

$$V_{z_t^+}(W) = \max_{\xi} \left\{ u \left(W - \sum_{j=0}^J \xi_j \right) + \beta \mathcal{I} \left[V_{z_{t+1}^-} \left(\sum_{j=0}^J \xi_j R_j(z_{t-1}^-) \right) \Big| z_t^+ \right] \right\}, \quad (\text{AB52})$$

which implies the unnumbered equation under (AB52) in AB:

$$\frac{d}{dW} V_{z_t^+}(W) = u' \left(W - \sum_{j=0}^J \xi_j \right) = u'(C_t) = u'(\bar{C}_t). \quad (5.1)$$

Equation (5.1) is the key equation that Laarits [2] disputes. Again, as in the two period model, the first equality is the optimality condition, because C_t is chosen after announcements are made, while the last equality imposes market clearing, which implies that the equilibrium quantity of \bar{C}_t cannot depend on the content of announcements.

6 The Laarits example

Laarits [2]’s paper claims that it has an example with expected utility that generates an announcement premium. The example is given in Section 4.2 of the paper. This example is rather confusing, because unlike AB, Laarits [2] does not provide a precise definition of equilibrium and does not specify clearly the unit of denomination of prices. Below we point out the main questionable step in Laarits [2]’s derivation.

On page 15 of Laarits [2], the paper asks “What is the SDF from $t = 0^+$ to $t = 1$?” and concludes $m_{\pm}^1 = 1$. Presumably, m_{\pm}^1 calculates the present value, evaluated in units of consumption numeraire at 0^+ , for one unit of payoff realized in period 1. Standard consumption-based asset pricing models determine this SDF using investors marginal rate of substitution between consumption at 0^+ and consumption at 1. If Laarits [2] were to follow this modeling choice, the paper would have reached the same conclusion as AB.

Laarits [2] refuses to do so. The paper insists that C_0 is already determined and cannot change by individual consumers. Then it would not make sense to talk about the relative price of consumption goods paid at 0^+ versus that of consumption goods paid at time 1. If agents cannot affect their consumption by trading on asset markets, then the model is simply incapable of generating unique predictions on relative prices. If the author wants the model to say something about relative prices, he will have to allow agents to change their consumption by trading assets, which is a standard practice in consumption-based asset pricing. Note that allowing agents to change C_0 by trading assets does not necessarily mean that C_0 will have to be a one-to-one function of s . In fact, equilibrium market clearing would imply that C_0 cannot depend on s , which of course pins down a unique set of prices. In any case, $m_{\pm}^1 = 1$ does not follow logically from the paper.

We also note that throughout the paper, Laarits [2] does not distinguish the individual choice of consumption and equilibrium aggregate consumption. We emphasize that the distinction is important. A clear definition of equilibrium is necessarily the starting point for talking about equilibrium prices.

7 Conclusion

The basic premise of asset pricing theory in exchange economies is that individual consumers take prices as given in making optimal consumption and investment choices. As a result the optimality conditions for these consumption investment problems can be used to derive pricing relationships. The fact that aggregate consumption is exogenously determined and cannot be changed by individual consumer's decision does not affect the validity of individual consumer's optimality conditions. This point is articulated very clearly in the classical paper of Lucas [4] as well as in standard textbooks such as Ljungqvist and Sargent [3]. To make predictions on announcement risk premiums, AB allow investors in their model to choose current-period consumption after announcements are made. The optimality condition for consumption-saving choice in AB provides a basis for the announcement stochastic discount factor derived in AB.

References

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