Misallocation and Risk Sharing

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ABSTRACT

This paper shows that factor misallocation is closely tied to the risk-sharing avenues available to firm owners. In contrast to the commonly studied bond-only economy with collateral constraints (for example Moll (2014)), we find that the degree of misallocation is *increasing* in the persistence of the idiosyncratic risk when firms have access to state-contingent contracts. The possibility to transfer wealth from high productivity states to low productivity states allows firm owners to trade off the efficient allocation of consumption against the efficient allocation of capital. We show that for reasonable values of risk aversion, insurance needs more than offset production efficiency concerns and thereby generates large capital misallocation.

1 Introduction

An extensive empirical literature (see Hsieh and Klenow (2009) and several others) documents dispersion in marginal products of capital across firms for several countries. These patterns are commonly interpreted as evidence of capital misallocation and being responsible for cross-country total factor productivity gaps. Several authors have proposed frictions that limit productive firms from borrowing and investing as a factor responsible for capital misallocation.[1] However, existing quantitative models that feature financial frictions have had at best only a modest success in accounting for the large observed dispersion in marginal products. In this paper, we show that the role of financial frictions in generating misallocation is substantially amplified when firm owners have access to state-contingent insurance.

In a seminal work, Moll (2014) showed that in an economy where firm owners exclusively borrow and save using a risk-free bond, the degree of misallocation is decreasing in the persistence of idiosyncratic shocks. Agents increase precautionary savings to smooth consumption against persistent shocks. However higher saving rates and larger wealth accumulation also means that borrowing constraints are less likely to bind. Since empirical firm-level estimates of productivity show that shocks are indeed quite persistent, this logic implies that financial frictions will not generate significant misallocation. Our thesis is that a mechanical relationship between self-insurance motives and efficient capital allocation is not a general result. In contrast to the bond-only economy, we find that for the same type of financial frictions, the degree of misallocation is *increasing* in the persistence of the idiosyncratic risk when firms have access to state-contingent contracts. Thus the empirically found large persistence of productivity shocks implies that financial market frictions can account for the observed level of capital misallocation.

To demonstrate our results, we study an environment that is similar to Moll (2014) except that there is a complete set of Arrow-Debreu securities additionally available to firm owners. The financial constraints are modeled

[1] For instance Banerjee and Moll (2010); Buera et al. (2011); Buera and Shin (2011, 2013); Buera et al. (2015); Midrigan and Xu (2014)

as a collateral constraint whereby the amount of capital that a firm can use is limited by the wealth of the entrepreneur. We characterize the stationary competitive equilibrium and study comparative statics with respect to the persistence of the idiosyncratic risk. We show that the degree of misallocation is increasing in the persistence of the productivity shocks for a large enough value of risk aversion.

The intuition for the result is as follows. In our setup with state-contingent payments, there is a trade-off between the allocation of wealth for insuring against adverse income states versus allocation of wealth for productive efficiency. Insurance requires entrepreneurs to borrow from states with high productivity and transfer wealth in states with low productivity. Efficiency requires that entrepreneurs have more wealth in states when it is a good time to invest, i.e., scale up production without hitting the borrowing constraint. These two distinct motives pull in opposite directions. For a given level of risk aversion, as shocks become more persistent, the insurance motives are stronger. Entrepreneurs choose to enter productive states with a low level of wealth; sacrificing productive efficiency in order to attain better consumption insurance. This makes misallocation higher.

After showing the main result, we quantitatively evaluate the magnitude of risk aversion needed for financial frictions to generate plausible levels for misallocation. We find that for even for moderate values of risk aversion, say in the range of 3 to 5, the degree of misallocation is increasing in persistence.

Finally, we discuss alternative implementations of the allocations in our economy. In our setup, allowing for state-contingent contracts is equivalent to issuing outside equity. Alternatively, insurance arrangements can arise out of implicit contracts such as defaultable debt, insurance within the family, or even fiscal policy that provides state-contingent payments. A rigorous empirical examination will require us to study consumption patterns of firm-owners. This is outside the scope of the current study.

The paper is organized as follows. In section 2, we describe the environment. In section 3, we specialize it to two-state economy and prove our main result. In section 4 we numerically analyze a more general economy. Section 5 concludes. The proofs and other details omitted from the main text are relegated to the Appendix.

2 Model

Preferences and Technology Time is continuous. The economy consists of a unit mass of entrepreneurs and a mass $\mathbf{L}$ of workers. Entrepreneurs are endowed with an idiosyncratic productivity process θ and a technology

$$y = f(\theta, K, L) = (\theta K)^{\alpha_K} L^{\alpha_L} \tag{1}$$

with $\alpha_K + \alpha_L \leq 1$, and where productivity θ follows a Levy process with

$$d\theta_t = \mu(\theta_t)dt + \sigma(\theta_t)dM_t,$$

where M_t is a (semi) martingale with appropriate restrictions such that a stationary distribution for θ_t exists and is unique.[2] Workers are endowed with one efficiency unit of labor.

Entrepreneurs and workers have common preferences given by

$$E_0 \left[\int_0^\infty e^{-\beta t} \frac{C_{i,t}^{1-\gamma}}{1-\gamma} dt \right]. \tag{2}$$

Aggregate capital $\mathbf{K}$ depreciates at the rate δ. The resource constraint is given by

$$\mathbf{L} C_L + \int C_{i,t} di + d\mathbf{K}_t + \delta \mathbf{K}_t = \int y_{i,t} di.$$

Markets and Frictions Labor markets are competitive. There is a perfectly competitive financial intermediary that receives deposits, invests and rents out physical capital as well as offers insurance to entrepreneurs using a full set of Arrow securities. Later we show that the insurance contracts can be implemented using bonds, outside and inside equity. Entrepreneurs face a collateral constraint which limits their ability to use capital. In particular, for

[2]For technical reasons we also assume that the ergodic distribution of θ has a compact support when $\alpha_K + \alpha_L = 1$.

an entrepreneur with wealth W_i, we impose

$$K_i \leq \lambda W_i, \ \lambda > 1. \tag{3}$$

Budgets Given the interest rate r and wage rate w, entreprenurs' profits are

$$\begin{aligned} \Pi\left(W, \theta; w, r\right) &= \max_{K,L} \left\{ (\theta K)^{\alpha_K} L^{\alpha_L} - wL - (r+\delta) K \right\} \\ 0 &\leq K \leq \lambda W. \end{aligned} \tag{4}$$

Profits depend on wealth W due to the presence of the collateral constraints (3). The flow budget constraint of an entrepreneur is given by

$$\text{s.t. } dW_{i,t} = \left[rW_{i,t} + \Pi\left(\theta_{i,t}, W_{i,t}; w_t, r_t\right) - C_{i,t} \right] dt + W_{i,t} s_{i,t} dM_t \tag{5}$$

where $s_{i,t}$ is the exposure to idiosyncratic risk embedded in M_t after the insurance arrangements with the intermediary. Workers are assumed to be hand-to-mouth and their budget sets are given by

$$C_{L,t} = w.$$

Equilibrium At each point in time t, a density $\Phi_t\left(\theta, W\right)$ describes the mass of entrepreneurs over productivity and wealth. A stationary competitive equilibrium consists of prices processes $\{r_t, \ w_t\}$, distributions $\{\Phi_t\}$, and decisions $\{C_{i,}, s_{i,t}, K_{i,t}, \ L_{i,t}\}$ such that

1. Given the prices, entrepreneurs maximize (2), subject to (5) and where profits satisfy (4).

2. Capital and labor markets clear

$$\mathbf{K}_t \equiv \int K_t\left(\theta, W\right) d\Phi_t\left(\theta, W\right) = \int W_t d\Phi_t\left(\theta, W\right)$$

$$\int L_t\left(\theta, W\right) d\Phi_t\left(\theta, W\right) = \mathbf{L}$$

3. Aggregate consumption of entrepreneurs is given by

$$\int C_t(\theta, W)\, d\Phi_t(\theta, W) + \delta \int K_t(\theta, W)\, d\Phi_t(\theta, W) = \alpha \int y_t(\theta, W)\, d\Phi_t(\theta, W)$$

4. Stationarity conditions

$$d\Phi_t = 0, \quad d\mathbf{K}_t = 0.$$

   are satisfied.

The competitive equilibrium will be compared to an allocation where factors of production $\mathbf{K}$, and $\mathbf{L}$ are efficiently allocated given a distribution of θ_i. Define $Y^* \equiv \max_{\{K_i, L_i\}_i} \int y(\theta_i, K_i, L_i) di$ such that $\int K_i di = \mathbf{K}$ and $\int L_i di = \mathbf{L}$. We define an *efficiency wedge* as

$$EF = \frac{\int (\theta_i K_i)^{\alpha_K} L_i^{\alpha_L} di}{Y^*}.$$

In the next section, we will characterize EF as function of the degree of persistence in θ.

3 Main Result

We specialize the environment in section 2 by assuming constant returns to scale, $\alpha_K = \alpha = 1 - \alpha_L$, and that θ_t follows a two state Markov chain $\{\theta_H, \theta_L\}$ with an instantaneous switching rate of κ. Formally, the law of motion of θ can be described as

$$d\theta_t = (\theta_H - \theta_L) \left[-I_H(\theta_t)\, dN_{H,t} + I_L(\theta_t)\, dN_{L,t} \right]. \tag{6}$$

where $I_\theta\left(\tilde{\theta}_t\right)$ is the indicator function that takes value of 1 when $\tilde{\theta}_t = \theta$, and the processes $\{dN_{\theta,t}\}_\theta$ are independent Poisson processes with a common intensity κ. Given these assumptions, $1 - \kappa$ is a measure of one-dimensional measure of the persistence in θ. Furthermore, changing κ keeps the unconditional distribution of θ_t unchanged. This will be useful when we do comparative statics with persistence.

Given sequences of prices $\{r_t, w_t\}$, the entrepreneur whether or not to operate the technology, factor choices capital and labor, a (non-negative) consumption process , and a portfolio of Arrow securities $\{g_{HL,t}W_t,\ g_{LH,t}W_t\}$. The process $g_{HL,t}$ represents the fraction of wealth W_t that is carried from state $\theta_t = \theta_H$ to state $\theta_t = \theta_L$, and $g_{LH,t}$ is the fraction of wealth that is carried from state $\theta_t = \theta_L$ to state $\theta_t = \theta_H$. The entrepreneurs value

$$V_t(\theta, W) = \max_{C_t, g_{H,t}, g_{L,t}} E_t\left[\int_t^\infty e^{-\beta(s-t)} u(C_s)\, ds\right]$$

$$\text{s.t. } dW_t = [r_t W_t + \Pi(\theta, W; w_t, r_t) - C_t]\, dt + I_H(\theta_t) W_t g_{HL}[dN_{H,t} - \kappa dt] + I_L(\theta_t) W_t g_{LH}[dN_{L,t} - \kappa dt].$$

The drift ("dt") term budget constraint is the change in wealth coming from risk-free financial return, business profits net of entrepreneurial consumption and the state-contingent returns arise from the terms multiplying the martingale $[dN_{\theta,t} - \kappa]$. The next lemma summarizes the optimal consumption savings plan in a stationary equilibrium.

Lemma 1. *The value function $V(\theta, W)$ and the profit function $\Pi(\theta, W; w, r)$ satisfy*

$$V(\theta, W) = \frac{1}{1-\gamma} H(\theta) W^{1-\gamma}$$

$$\Pi(\theta, W; w, r) = W\pi(\theta; r, w)$$

with

$$H(\theta) = \left\{\frac{1}{\gamma}\left[\beta + (\gamma - 1)(r + \pi_\theta(r, w)) + \gamma\kappa(1-\omega)\right]\right\}^{-\gamma}$$

$$\pi_\theta(r, w) = \left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}\theta - r - \delta\right]^+ \lambda$$

and the optimal consumption and portfolio rules are given by

$$\frac{C(\theta, W)}{W} = x(\theta) = H(\theta)^{-\frac{1}{\gamma}}, \quad g_{HL} = \omega - 1, \quad g_{LH} = \omega^{-1} - 1$$

with $\omega = \left[\frac{H(\theta_H)}{H(\theta_L)}\right]^{-\frac{1}{\gamma}}$.

Proof. See Appendix □

The derivation of the optimal policies comes from a standard Merton-like problem. The explicit solutions arise thanks to the homotheticity properties of preferences and technology. Since $\pi_H > \pi_L$, consumption smoothing across states is achieved by committing more wealth to states with low business income, i.e., set $g_{HL} > g_{LH}$.

Let us suppose for now that both types produce in the stationary equilibrium. The next lemma shows that a sufficient (distributional) statistic for the degree of misallocation is the ratio of aggregate wealth of entrepreneurs with type θ_H to entrepreneurs with type θ_L.

Lemma 2. *Suppose a stationary equilibrium exists with $\pi_L(r,w) = 0$. The degree of misallocation is given by*

$$1 - EF = 1 - \left[\frac{\theta_L\left(\eta(r,w) + 1 - \lambda\eta(r,w)\right) + \theta_H\lambda\eta(r,w)}{\theta_H\left(\eta(r,w)+1\right)}\right]^{\alpha} \tag{7}$$

where $\eta(r,w) \equiv \frac{\int W d\Phi(W|\theta_H;r,w)}{\int W\varphi d\Phi(W|\theta_L;r,w)}$.

Proof. For an entrepreneur who is operating with non-zero capital, factor demands are linear in wealth and given by

$$K\left(W,\theta\right) = \lambda W$$

$$L\left(W,\theta\right) = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}\theta\lambda W$$

Let φ be the fraction of θ_L entrepreneurs who are active. Substituting the factor demands, we get

$$
\begin{aligned}
EF &= \frac{\int (\theta_i K_i)^{\alpha} L_i^{1-\alpha} di}{\theta_H^{\alpha} \left(\int K_i di\right)^{\alpha} \left(\int L_i di\right)^{1-\alpha}} \\
&= \frac{\int \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda \theta_L W_i I_{\{\theta_{i,t}=\theta_L\}} \varphi di + \int \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda \theta_H W_i I_{\{\theta_{i,t}=\theta_H\}} di}{\theta_H^{\alpha} \left[\int \lambda W_i I_{\{\theta_{i,t}=\theta_L\}} \varphi di + \int \lambda W_i I_{\{\theta_{i,t}=\theta_H\}} di\right]^{\alpha} \cdot 1} \\
&= \frac{\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda \left(\theta_L W_L \varphi + \theta_H W_H\right)}{\theta_H^{\alpha} \lambda^{\alpha} \left(W_L \varphi + W_H\right)^{\alpha}} = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda^{1-\alpha} \frac{\theta_L W_L \varphi + \theta_H W_H}{\theta_H^{\alpha} \left(W_L \varphi + W_H\right)^{\alpha}} \\
&= \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda^{1-\alpha} W_L^{1-\alpha} \frac{\theta_L \varphi + \theta_H \eta}{\theta_H^{\alpha} \left(\varphi + \eta\right)^{\alpha}} \\
&= \left[\frac{\theta_L \left(\eta + 1 - \lambda \eta\right) + \theta_H \lambda \eta}{\theta_H \left(\eta + 1\right)}\right]^{\alpha}.
\end{aligned}
$$

□

In view of Lemma 2, we only need to characterize how the aggregate wealth shares evolve in the stationary equilibrium. Let $W_{H,t} = \int W_i I_{\{\theta_{i,t}=\theta_H\}} di$ and $W_{L,t} = \int W_i I_{\{\theta_{i,t}=\theta_L\}} di$. A heuristic derivation of the dynamics of $dW_{\theta,t}$ is as follows. At time $t+\Delta$, $e^{-\kappa\Delta}$ fraction of entrepreneurs will remain at $\theta_{t+\Delta} = \theta_H$. Conditional on being in state θ_H, that is, conditioning on $dN_{Ht} = 0$ over the interval $(t, t+\Delta)$,

$$dW_{H,t} = W_{H,t} \left[r + \pi \left(\theta; r, w\right) - x\left(\theta_H\right) - \kappa \left(\omega - 1\right)\right] dt$$

from the flow budget constraint. At the same time, $\left(1 - e^{-\kappa_L \Delta}\right)$ fraction experience a regime switch and transition from $\theta_L \to \theta_H$ and when they become θ_H, their wealth $W_{L,t} \to \omega^{-1} W_{L,t}$. Putting both of these together, and taking limits as $\Delta \to 0$, we get

$$dW_{H,t} = \left[r + \pi \left(\theta_H; r, w\right) - x\left(\theta_H\right) - \kappa \omega\right] W_{H,t} dt + \kappa \omega^{-1} W_{L,t} dt \tag{8}$$

The law of motion of the wealth share of type θ_L is obtained analogously and we have

$$dW_{L,t} = \left[r + \pi_L \left(\theta_L; r, w\right) - x\left(\theta_L\right) - \kappa \omega^{-1}\right] W_{L,t} dt + \kappa \omega W_{H,t} dt \tag{9}$$

Since we study the steady-state equilibrium, stationarity requires

$$\frac{dW_{H,t}}{W_{H,t}} = \frac{dW_{L,t}}{W_{L,t}} = 0. \tag{10}$$

From the capital market clearing condition, $d\mathbf{K}_t = dW_{H,t} + dW_{L,t}$ and therefore we get

$$0 = \left[r + \pi\left(\theta_H; r, w\right) - x\left(\theta_H\right)\right] W_{H,t}dt + \left[r + \pi\left(\theta_L; r, w\right) - x\left(\theta_L\right)\right] W_{L,t}dt$$

which can be simplified to obtain

$$\eta(r, w) = -\frac{r + \pi\left(\theta_L; r, w\right) - x\left(\theta_L\right) - \kappa\omega^{-1}}{\kappa\omega}.$$

In the final part of this section, we show that for sufficiently high risk aversion, misllocation is increasing in persistence. Given our discussion above, this is true if and only if $\frac{\partial\eta}{\partial\kappa} > 0$. To formally prove our claim, we need to make the following technical assumption that restrict the parameter space for the economy:

Assumption 1. $\lambda \in \left(1, \min\left\{2, \frac{2\beta}{\delta\left(\frac{\theta_H}{\theta_L} - 1\right)}\right\}\right)$ *and* $\frac{2\beta}{\delta\left(\frac{\theta_H}{\theta_L} - 1\right)} > 1.$

The next theorem states our main result.

Theorem 1. *Under assumption 1,* $\exists\,\bar{\gamma}$ *such that for* $\forall\gamma > \bar{\gamma}$, $\frac{\partial\eta}{\partial\kappa} > 0$ *for all* $\kappa > 0$ *.*

Proof. See Appendix □

The intuition for the result is as follows. Our economy features a trade off between production efficiency and insurance. To maximize production in presence of a collateral constraint, the entrepreneur would want to bring more wealth in the high productivity states, i.e., states with $\theta_t = \theta_H$. This would make the constraint less likely to bind. On the other hand, to smooth consumption across states, the entrepreneur would like to borrow against states of the world with $\theta_t = \theta_H$ and transfer wealth to states with $\theta_t = \theta_L$. Thus insurance concerns force an allocation of wealth across states in a way opposite of what is desired for production efficiency.

For a given level of risk aversion, as shocks become more persistent, they amplify insurance requirements. The entrepreneur would want to bring a larger fraction of wealth to $\theta_t = \theta_L$ to insure against the adverse shock in a present value sense. Since this means a lower wealth in states which $\theta_t = \theta_H$, more entrepreneurs will be constrained and aggregate misallocation will be larger. We term this the "stock effect" of increasing persistence. There is another "flow effect" which has to be acknowledged. More persistent shocks, mean that productive entrepreneurs earn high profits for longer duration and on this account will accumulate more wealth. Theorem 1 asserts that the flow effect is not sufficiently strong for large values of risk aversion. More generally, the degree of misallocation will be an inverse U-shaped function of persistence. For a given value of risk aversion and low levels of persistence, misallocation increases with persistence due to the stock effect but eventually when shocks are very persistent, the flow effect dominates. The persistence that generates the highest degree of misallocation is a function of the level of risk aversion and it moves towards $\kappa = 0$ (or when θ_t approaches a unit root) for a finite but "large" value of γ.

A natural question is how large is "large". We approach using numerical analysis in two ways. Here, we study how the two shock calibrated economy behaves for a range of (κ, γ) and in section 4, we use a Ornstein-Uhlenbeck process with a reflecting barrier as in Moll (2014) and compare allocations in both complete markets and incomplete markets (bond) economy. We set $\beta = \delta = 0.05$. We calibrate $\theta_H = 1.2$, $\theta_L = 0.8$ to hit match the unconditional volatility of productivity to 4%. Then for $\lambda = 1.2$, we plot $1 - EF$ for several values of κ and γ.

In the left panel of Figure 1, we see that for risk aversion of as low as 5, misllocation is increasing for almost all feasible values of $1-\kappa$. The magnitude of misallocation ranges between 2% – 6%. These results and insights can be contrasted with an economy where only the only asset is a risk-free bond. In

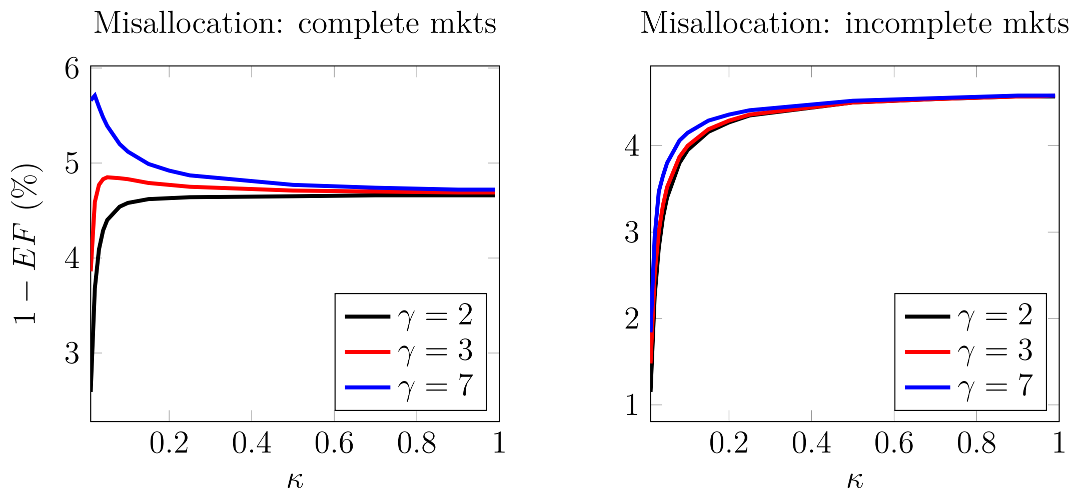

Figure 1: Misallocation for several values of risk aversion, $\gamma \in \{2, 3, 7\}$ in the two-shock CRS economy. The left panel has a complete market structure and the right panel has a single risk-free bond market structure.

our setup, this would mean adding an additional constraint

$$g_{HL,t} = g_{LH,t} = 0.$$

Keeping all the parameters the same, we solve for the stationary equilibrium and measure misllocation using the same way, i.e., $1-EF$ in the bond economy. In right panel of Figure 1, we plot the misallocation as a function of persistence for several values of γ. The plots show that patterns are quite different. In the bond-economy steady-state misallocation is always decreasing in persistence. This should be expected given the analysis in [CTE Moll]. In absence of state-contingent returns, the only response to more persistent shocks is to self-insure and save more. But more wealth also relaxes the collateral constraints. Thus the insurance requirements go in the same direction as production efficiency.

4 Numerical Example

In the previous section, we used a simplified shock process where θ_t took only two values. This helped us with closed-form characterization of the wealth distribution and all equilibrium quantities. However, the insights are more general. In this section, we study them numerically using a more "standard" Ornstein-Uhlenbeck process for θ_t. This process has been used in Moll (2014) and others. We demonstrate that our main take-away that for reasonable levels of risk aversion, the patterns of misallocation in the complete markets version are diametrically opposite to those in the incomplete markets version.

The productivity θ follows

$$d\theta_t = \kappa(\mu_\theta - \theta_t)dt + \sigma\sqrt{\kappa}dB_t \tag{11}$$

and we impose that θ_t has a reflecting barrier at $\bar{\theta}$. The flow budget constraint for the entrepreneur is expressed as

$$dW_t = \left[r_t W_t + \Pi\left(\theta, W\right) - C_t\right] dt + W_t g_{\theta,t}\sigma\left(\theta\right) dB_t$$

The process $g_{\theta,t}$ measures the exposure of wealth to the idiosyncratic risk. Using same steps as in the derivation of Lemma 1, we can show that the

$$V\left(\theta, W\right) = \frac{1}{1-\gamma} H\left(\theta\right) W^{1-\gamma}$$

and the optimal consumption and portfolio rules are given by

$$\frac{C\left(\theta, W\right)}{W} = x(\theta) = H\left(\theta\right)^{-\frac{1}{\gamma}}, \quad g(\theta) = \frac{1}{\gamma}\frac{H_\theta\left(\theta\right)}{H\left(\theta\right)}.$$

The wealth distribution now follows a Kolmogorov forward equation with the joint density $\Phi_t\left(\theta, W\right)$ evolving as

$$\begin{aligned}\frac{\partial \Phi_t\left(\theta, W\right)}{\partial t} &= -\frac{\partial}{\partial \theta}\left[\mu\left(\theta\right)\Phi_t\left(\theta, W\right)\right] - \frac{\partial}{\partial W}\left[\left(r_t W_t + \Pi\left(\theta, W\right) - C_t\right)\Phi_t\left(\theta, W\right)\right] \\ &+ \frac{1}{2}\frac{\partial^2}{\partial \theta^2}\left[\sigma^2\left(\theta\right)\Phi_t\left(\theta, W\right)\right] + \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[\left(W_t g_{\theta,t}\sigma\left(\theta\right)\right)^2\Phi_t\left(\theta, W\right)\right] \\ &+ \frac{\partial^2}{\partial \theta \partial W}\left[W_t g_{\theta,t}\sigma^2\left(\theta\right)\Phi_t\left(\theta, W\right)\right].\end{aligned}$$

In Appendix C, we describe the algorithm to compute the stationary competitive equilibrium.

We follow Moll (2014) and set $\left\{\mu_\theta, \sigma, \bar{\theta}\right\}$ to $\{1, 0.25, 1.5\}$. The rest of the parameters are same as before. In figure [tba], we plot the resulting misallocation as a function κ for several values of γ for our economy and the economy with only a risk-free bond (or the market structure assumed in Moll (2014)). Qualitatively we see the same patterns as in figure 1, confirming our main insights.

5 Conclusion

This paper shows that factor misallocation is closely tied to the risk-sharing avenues available to firm owners. In contrast to the commonly studied bond-only economy with collateral constraints (for example Moll (2014)), we find that keeping fixed the nature of financial frictions, the degree of misallocation is *increasing* in persistence of the idiosyncratic risk when firms have access to state-contingent contracts. Allowing the possibility to transfer wealth from states where productivity is high to states where productivity is low generates a force that works against efficient allocation of capital. We show that for reasonable values of risk aversion, insurance needs more than offset efficiency concerns. A rigorous empirical examination of the extent of explicit and implicit insurance available to entrepreneurs will require us to study consumption patterns of firm-owners. We leave this for future work.

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A Proof for Lemma 1

With the constant returns to scale technology, given the interest rate r and wage w, entreprenurs' profits are

$$\begin{aligned} \Pi\left(\theta, W; r, w\right) &= \max_{K,L} \left\{ \left(\theta K\right)^{\alpha} L^{1-\alpha} - wL - \left(r+\delta\right) K \right\} \\ 0 &\leq K \leq \lambda W. \end{aligned} \tag{12}$$

where the first order conditions imply that

$$L\left(W,\theta\right) = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} \theta K\left(W,\theta\right), \quad K\left(W,\theta\right) = \lambda W \text{ or } 0.$$

Therefore, the profits are linear in wealth:

$$\Pi\left(\theta, W; r, w\right) = W\pi\left(\theta; r, w\right)$$

where

$$\pi\left(\theta; r, w\right) = \left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \theta - r - \delta\right]^{+} \lambda.$$

Due to the homogeneity of value function, we assume

$$V\left(\theta, W\right) = \frac{1}{1-\gamma} H\left(\theta\right) W^{1-\gamma}$$

By applying Ito's Lemma, the HJB equation can be written as

$$\begin{aligned}
\beta\frac{1}{1-\gamma}H\left(\theta_H\right)W^{1-\gamma} &= \frac{1}{1-\gamma}{C_H}^{1-\gamma}+H\left(\theta_H\right)W^{1-\gamma}\left[r+\pi_H-\frac{C_H}{W}-\kappa g_{HL}\right] \\
&\quad +\kappa\left[\frac{1}{1-\gamma}H\left(\theta_L\right)\left(\left(1+g_{HL}\right)W\right)^{1-\gamma}-\frac{1}{1-\gamma}H\left(\theta_H\right)W^{1-\gamma}\right];
\end{aligned}$$

$$\begin{aligned}
\beta\frac{1}{1-\gamma}H\left(\theta_L\right)W^{1-\gamma} &= \frac{1}{1-\gamma}{C_L}^{1-\gamma}+H\left(\theta_L\right)W^{1-\gamma}\left[r+\pi_L-\frac{C_L}{W}-\kappa g_{LH}\right] \\
&\quad +\kappa\left[\frac{1}{1-\gamma}H\left(\theta_H\right)\left(\left(1+g_{LH}\right)W\right)^{1-\gamma}-\frac{1}{1-\gamma}H\left(\theta_L\right)W^{1-\gamma}\right];
\end{aligned}$$

Denote $x\left(\theta\right)$ as the consumption-wealth ratio

$$x\left(\theta\right)=\frac{C\left(\theta,W\right)}{W},$$

and the first order condition implies

$$x\left(\theta\right)=H\left(\theta\right)^{-\frac{1}{\gamma}};\quad 1+g_{HL}=\left[\frac{H\left(\theta_H\right)}{H\left(\theta_L\right)}\right]^{-\frac{1}{\gamma}}\qquad 1+g_{LH}=\left[\frac{H\left(\theta_L\right)}{H\left(\theta_H\right)}\right]^{-\frac{1}{\gamma}}.$$

Combing the policy functions and HJB equations, we can derive

$$\omega=\frac{\beta+\left(\gamma-1\right)\left[r+\pi_H\right]+2\gamma\kappa}{\beta+\left(\gamma-1\right)\left[r+\pi_L\right]+2\gamma\kappa}\tag{13}$$

where we define $\omega=\left[\frac{H(\theta_H)}{H(\theta_L)}\right]^{-\frac{1}{\gamma}}$.

Given r, the value function and policy function can be constructed as:

$$\begin{aligned}
H\left(\theta_H\right) &= \left\{\frac{1}{\gamma}\left[\beta+\left(\gamma-1\right)\left(r+\pi_H\left(r\right)\right)+\gamma\kappa\left(1-\omega\right)\right]\right\}^{-\gamma}; \\
H\left(\theta_L\right) &= \left\{\frac{1}{\gamma}\left[\beta+\left(\gamma-1\right)\left(r+\pi_L\left(r\right)\right)+\gamma\kappa\left(1-\omega^{-1}\right)\right]\right\}^{-\gamma};
\end{aligned}$$

$$x\left(\theta\right)=H\left(\theta\right)^{-\frac{1}{\gamma}};\quad g_{HL}=\omega-1;\quad g_{LH}=\omega^{-1}-1.\tag{14}$$

where ω is defined in equation (13).

B Proof for Theorem 1

Lemma 3. *Under assumption 1, among the firms which receive θ_L, the fraction of producing belongs to* $(0,1)$ *for* $\forall\gamma > \bar{\gamma}$, *where* $\bar{\gamma}$ *is defined in the theorem 1.*

We will verify the Lemma 3 later during the proof of Theorem 1. Now assume Lemma 3 holds, then the economy is determined by the following five equations:

$$r = \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \theta_L - \delta$$

$$\omega = \frac{\beta + (\gamma-1)\left[r + \pi_H\right] + 2\gamma\kappa}{\beta + (\gamma-1)\left[r + \pi_L\right] + 2\gamma\kappa}$$

$$\eta = -\frac{r + \pi\left(\theta_L; r, w\right) - x\left(\theta_L\right) - \kappa\omega^{-1}}{\kappa\omega}$$

$$\varphi = \frac{\eta + 1 - \lambda\eta}{\lambda}$$

where η is fraction of firms produce when they receive θ_L.

$$x\left(\theta_H\right)\eta + x\left(\theta_L\right) + \delta\left(\eta + 1\right) = \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \lambda\left[\theta_H\eta + \theta_L\varphi\right]$$

After the algebra, we can find the economy is determined by the following three equations:

$$\omega = 1 + \frac{(\gamma-1)(r+\delta)\left(\frac{\theta_H}{\theta_L} - 1\right)\lambda}{\beta + (\gamma-1)r + 2\gamma\kappa} \tag{15}$$

$$\eta = \frac{\beta - r + \gamma\kappa}{\gamma\kappa\omega} \tag{16}$$

$$r = \beta - (r+\delta)\left(\frac{\theta_H}{\theta_L} - 1\right)\lambda + \frac{\gamma\kappa(\beta - r)}{\beta - r + \gamma\kappa} \tag{17}$$

Assume the limit of ω, r and η exists and finite. Taking the limit of (15), (16), and (17) with respect to γ,

$$\bar{\omega} = 1 + \frac{(\bar{r}+\delta)\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}{\bar{r}+2\kappa}$$
$$\bar{\eta} = \frac{1}{\bar{\omega}}$$
$$\bar{r} = \frac{2\beta - \delta\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}{2+\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}$$

Later on we are going to characterize the *sufficient* condition when this economy is well defined.

Taking the partial derivative with respect to κ for equations (15), (16), and (17) :

$$\frac{\partial\omega}{\partial\kappa} = \frac{(\gamma-1)\left(\frac{\theta_H}{\theta_L}-1\right)\lambda\left[(\beta+2\gamma\kappa-(\gamma-1)\delta)\frac{\partial r}{\partial\kappa}-(r+\delta)(\beta+2\gamma)\right]}{(\beta+(\gamma-1)r+2\gamma\kappa)^2} \quad (18)$$

$$\frac{\partial\eta}{\partial\kappa} = \frac{-\gamma\kappa\omega\frac{\partial r}{\partial\kappa}-(\beta-r+\gamma\kappa)\gamma\kappa\frac{\partial\omega}{\partial\kappa}-(\beta-r)\gamma\omega}{(\gamma\kappa\omega)^2} \quad (19)$$

$$\frac{\partial r}{\partial\kappa} = \frac{\gamma(\beta-r)^2}{\left[1+\left(\frac{\theta_H}{\theta_L}-1\right)\lambda+\frac{\gamma^2\kappa^2}{(\beta-r+\gamma\kappa)^2}\right](\beta-r+\gamma\kappa)^2} > 0 \quad (20)$$

Assume the limit of $\frac{\partial\omega}{\partial\kappa}$, $\frac{\partial\eta}{\partial\kappa}$ and $\frac{\partial r}{\partial\kappa}$ exists and finite. Taking the limit, and we can find

$$\lim_{\gamma\to\infty}\frac{\partial\omega}{\partial\kappa} = \frac{1}{(\bar{r}+2\kappa)^2}\left[\left(\frac{\theta_H}{\theta_L}-1\right)\lambda(2\kappa-\delta)\lim_{\gamma\to\infty}\frac{\partial r}{\partial\kappa}-2(\bar{r}+\delta)\left(\frac{\theta_H}{\theta_L}-1\right)\lambda\right]$$
$$\lim_{\gamma\to\infty}\frac{\partial\eta}{\partial\kappa} = \frac{1}{\bar{\omega}^2}\lim_{\gamma\to\infty}\frac{\partial\omega}{\partial\kappa}$$
$$\lim_{\gamma\to\infty}\frac{\partial r}{\partial\kappa} = 0$$

Therefore,

$$\lim_{\gamma\to\infty}\frac{\partial\eta}{\partial\kappa} = \frac{2(\bar{r}+\delta)\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}{\bar{\omega}^2(\bar{r}+2\kappa)^2} > 0$$

since

$$\bar{r}+\delta=\frac{2\beta+2\delta}{2+\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}>0$$

Therefore, $\exists\,\bar{\gamma}_1$ such that for $\forall\gamma>\bar{\gamma}_1$, $\frac{\partial\eta}{\partial\kappa}>0$ for all κ.

Now we are going to show the economy is well defined. First, the wage is positive since

$$\bar{w}=\frac{1-\alpha}{\left(\frac{\bar{r}+\delta}{\alpha\theta_L}\right)^{\frac{\alpha}{1-\alpha}}}>0$$

Next, we show the fraction of firms which receive θ_L and produce in the economy belongs to $(0,1)$.

$$\bar{\varphi}=\frac{\bar{\eta}+1-\lambda\bar{\eta}}{\lambda}$$

It's easy to show

$$\bar{\varphi}=\frac{\bar{\eta}+1-\lambda\bar{\eta}}{\lambda}=\frac{\bar{\eta}+1-\lambda\bar{\eta}-\lambda}{\lambda}+1=\frac{(1-\lambda)\left(\eta+1\right)}{\lambda}+1<1$$

when $\lambda>1$.

Now we need to show $\bar{\varphi}=\frac{\bar{\eta}+1-\lambda\bar{\eta}}{\lambda}>0\Leftrightarrow\bar{\eta}<\frac{1}{\lambda-1}$. Since $\lambda\in\left(1,\min\left\{2,\ \frac{2\beta}{\delta\left(\frac{\theta_H}{\theta_L}-1\right)}\right\}\right)$ and $\kappa>0$,

$$\bar{r}+2\kappa=\frac{2\beta-\delta\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}{2+\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}+2\kappa>\frac{2\beta-\delta\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}{2+\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}>0$$

, which implies

$$\bar{\omega}=1+\frac{\left(\bar{r}+\delta\right)\left(\frac{\theta_H}{\theta_L}-1\right)\lambda}{\bar{r}+2\kappa}>1$$

. Therefore,

$$\bar{\eta}=\frac{1}{\bar{\omega}}<1<\frac{1}{\lambda-1},\ \forall\lambda\in\left(1,\min\left\{2,\ \frac{2\beta}{\delta\left(\frac{\theta_H}{\theta_L}-1\right)}\right\}\right)$$

which implies that $\bar{\varphi}\in(0,1)$. Therefore, $\exists\,\bar{\gamma}_2$ such that for $\forall\gamma>\bar{\gamma}_2$, $\varphi\in$

$(0, 1)$ and $w > 0$, i.e. the economy is well defined for all $\kappa > 0$ when $\lambda \in \left(1, \min\left\{2, \frac{2\beta}{\delta\left(\frac{\theta_H}{\theta_L}-1\right)}\right\}\right)$ and $\frac{2\beta}{\delta\left(\frac{\theta_H}{\theta_L}-1\right)} > 1$. The condition is exactly the assumption 1, which proves Lemma 3.

We can conclude, under assumption 1, $\exists\,\bar{\gamma} = \max\{\bar{\gamma}_1, \bar{\gamma}_2\}$ such that for $\forall \gamma > \bar{\gamma}$, $\frac{\partial \eta}{\partial \kappa} > 0$ for all $\kappa > 0$.

C Details for Numerical Algorithm

1. An initial guess r
2. An initial guess wage w
3. Find the cutoff by $\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \underline{\theta} = r + \delta$ and the the policy functions: the labor, capital, marginal profit function
4. Solve the HJB equation and find the optimal consumption and portfolio choice.

$$\beta V(\theta, W) = \max_{C_t,\, g_{\theta,t}} \frac{C^{1-\gamma}}{1-\gamma} + V_\theta(\theta, W)\,\mu(\theta) + V_W(\theta, W)\left(r_t W_t + \Pi(\theta, W) - C_t\right)$$
$$+\frac{1}{2}V_{\theta\theta}(\theta, W)\,\sigma^2(\theta) + \frac{1}{2}V_{WW}(\theta, W)\left(W g_{\theta,t}\sigma(\theta)\right)^2 + V_{\theta W}(\theta, W)\, W g_{\theta,t}\sigma^2(\theta) \tag{21}$$

The first order condition implies that

$$C = \left(V_W(\theta, W)\right)^{-\frac{1}{\gamma}};\; g_{\theta,t} = -\frac{V_{\theta W}(\theta, W)}{V_{WW}(\theta, W)\, W}$$

5. Solve the stationary distribution $\Phi(\theta, W)$ from

$$0 = -\frac{\partial}{\partial \theta}\left[\mu(\theta)\,\Phi(\theta, W)\right] - \frac{\partial}{\partial W}\left[\left(rW + \Pi(\theta, W) - C\right)\Phi(\theta, W)\right]$$
$$+\frac{1}{2}\frac{\partial^2}{\partial \theta^2}\left[\sigma^2(\theta)\,\Phi(\theta, W)\right] + \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[\left(W g_\theta \sigma(\theta)\right)^2 \Phi(\theta, W)\right] + \frac{\partial^2}{\partial \theta \partial W}\left[W g_\theta \sigma^2(\theta)\,\Phi(\theta, W)\right]$$

6. Compute the aggregate consumption/capital/labor/output.

7. Check whether the labor market clears:

$$\int L_t(\theta, W)\, d\Phi_t(\theta, W) = \mathbf{L}$$

if not, go to step 2; until the labor market clears.

8. Check whether the consumption goods market clears

$$\int C_t(\theta, W)\, d\Phi_t(\theta, W) + \delta \int K_t(\theta, W)\, d\Phi_t(\theta, W) = \alpha \int y_t(\theta, W)\, d\Phi_t(\theta, W)$$

if not, go to step 1; until consumption goods market clears.