Information Acquisition and the Pre-Announcement Drift

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Abstract: We present a dynamic Grossman-Stiglitz model with endogenous information acquisition to explain the pre-FOMC announcement drift. Because FOMC announcements reveal substantial information about the economy, investors' incentives to acquire information are particularly strong in days ahead of the announcements. Information acquisition partially resolves the uncertainty for uninformed traders, and under generalized risk sensitive preferences (Ai and Bansal, 2018), lowers the discount rate and results in a stock market run-up. Because our theory does not rely on the leakage of information, it can simultaneously explain the low realized volatility during the pre-FOMC announcement period and the lack of a positive correlation between pre- and post-announcement returns.

Keywords: Macroeconomic Announcement Premium, Pre-FOMC Announcement Drift, Asymmetric Information.

JEL Code: D83, D84, G11, G12, G14

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1 Introduction

In this paper, we present a general equilibrium model with endogenous information acquisition to explain the pre-FOMC announcement drift documented by Lucca and Moench (2015). Information is publicly available but costly to acquire. Because FOMC announcements resolve substantial uncertainty of the aggregate economy and have a significant impact on the stock market, informed traders have particularly large information advantages in trading over uninformed traders before announcements are made. As a result, it is optimal for uninformed traders to start to acquire information days ahead of the announcements. Due to generalized risk sensitivity (GRS) (Ai and Bansal, 2018) in preferences, as uncertainty resolves, equity market risk premium realizes shortly before announcements. More importantly, because the newly acquired information is from publicly available sources rather than leakage of the content of the upcoming announcement, our theory can simultaneously explain the low realized volatility during the pre-FOMC announcement period, and the lack of a positive correlation between pre- and post-announcement returns.

Stock market returns earned on FOMC announcement days account for almost 100% of the overall equity market risk premium since the mid-1990s. Ai and Bansal (2018) demonstrate that this phenomenon can be consistent with general equilibrium asset pricing models if investors have generalized risk sensitive preferences. However, the puzzling aspect of the FOMC announcement premium is that it is mostly realized in hours or one trading day before the actual announcements. If one is willing to assume that most of the time, the contents of FOMC announcements are leaked to the market in days before the announcements, the example in Ai and Bansal (2018), illustrated in Figure 4 of their paper, provides a direct explanation for the pre-FOMC announcement drift. However, the existing evidence for information leakage is mostly anecdotal, and this extreme form of information leakage is implausible from an institutional point of view.

We define information leakage as the arrival of new information that is correlated with the upcoming announcement but has not been incorporated in market prices. Information leakage-based models typically imply a counter-factually high level of realized volatility and high level of trading volume during the drift period. In representative agent models, the arrival of new information is typically associated with a higher realized volatility. In the presence of heterogenous information, it also triggers a higher trading volume. Empirically, however, the realized volatility of market returns and the trading volume during the pre-announcement period are slightly lower than their counterparts on non-FOMC announcement days.

We propose a theory for the pre-FOMC announcement drift based on the endogenous information acquisition in financial markets. The endogenous information acquisition in our model is consistent with the evidence documented by Fisher, Martineau, and Sheng (2020) that investor's attention peaks roughly three days before pre-scheduled FOMC announcements. Our theory does not rely on information leakage. Therefore it not only explains the existence of the pre-announcement drift but also the lack of positive correlation between pre- and post-announcement returns, as well as the low realized volatility during the pre-announcement period.

In our model, the long-run growth rate of the economy is governed by a latent state variable that is unobservable to all investors but periodically announced by the central bank. Information about the latent variable is available but costly to acquire. There are two groups of investors, informed and uninformed. Informed investors have zero cost of information acquisition and always observe noisy signals about the latent growth rate. Uninformed investors do not observe the signals unless they pay a cost to acquire them. We interpret informed investors as professional traders and uninformed investors as retail investors who normally pay less attention to stock market dynamics than professional traders but may choose to increase their attention when the benefit exceeds the cost of information acquisition.

In the above environment, uninformed investors have incentives to acquire information to avoid trading losses due to information asymmetry. This incentive is particularly strong and results in a sharp increase in information acquisition for uninformed investors in days ahead of FOMC announcements when the information advantage of informed traders peaks. As a result, our model provides a rational explanation for the pattern of investors' attentions ahead of macroeconomic announcements documented by Fisher, Martineau, and Sheng (2020).

As investors acquire more information, uncertainty resolves in days ahead of the FOMC announcements. Under generalized risk sensitive preferences, the discount rate drops, and the stock price rises. As in Ai and Bansal (2018), this mechanism produces a pre-FOMC announcement drift. Because the newly acquired information has already been in the public domain and incorporated into stock prices, realized market volatility is low during this period, consistent with the empirical evidence. As in the data, announcements reveal new information and result in a sudden increase in realized volatility and trading volume. In addition, the absence of leakage of information in our model implies that pre- and post- announcement returns are not positively correlated. These features of our model are broadly consistent with the empirical evidence of stock market dynamics presented in Lucca and Moench (2015).

In our model, the endogenously acquired information by uninformed investors is a noisy signal about information already known to informed investors. The newly acquired information in our model therefore differs from leakage because i) it is public available albeit costly to acquire, and, ii) it has already been incorporated into prices before information acquisition, albeit with noise. The costly acquired information has already been incorporated into prices because previous tradings of informed investors generate a price impact and influence the market price. This information is still valuable for uninformed investors to acquire because prices contain only noisy signals of informed investors' information. In our model, information acquisition, combined with the generalized risk sensitivity generate a risk premium due to the same mechanism as that in Ai and Bansal (2018). Because this risk premium is realized during the information acquisition period before the announcement, it manifests itself as a pre-FOMC announcement drift. The fact that newly acquired information is not leakage is important to account for the dynamics of realized volatility and trading volume around announcements. Because the newly acquired information is already contained in prices, the information acquisition period is a period in which noise gets eliminated from prices and as a result, realized volatility drops before the upcoming announcement.

Related Literature Our paper builds on the literature of macroeconomic announcement premium. Savor and Wilson (2013, 2014) are among the first to document the macroeconomic announcement premium. Ai and Bansal (2018) provide a reveal preference theory for the macroeconomic announcement premium. Wachter and Zhu (2020) develop a quantitative model of the macroeconomic announcement premium based on rare disasters. Ai, Bansal, Im, and Ying (2020) provide evidence for the impact of announcements on macroeconomic quantities as well as asset markets and develop a production-based asset pricing model to explain these facts. Ernst, Gilbert, and Hrdlicka (2019) present additional evidence for the macroeconomic announcement premium.

Within the above broader literature, our paper is more closely related to the FOMC announcement premium. Lucca and Moench (2015) document the pre-FOMC announcement drift and Cieslak, Morse, and Vissing-Jorgensen (2019) provide evidence for stock returns over the FOMC announcement cycles. Morse and Vissing-Jorgensen (2020) provide a study for the information transmission mechanism for Fed policies. Both Laarits (2020) and Ying (2020) provide models of pre-announcement drifts. Both papers rely on the arrival of new information during the pre-announcement period as in the example of Ai and Bansal (2018). Cocoma (2020) develops a general equilibrium with disagreement to explain the pre-FOMC announcement drift.

Several recent empirical work document important facts related to investor attention and trading activities around FOMC announcement which provide a basis for the development of the theoretical model in this paper. Fisher, Martineau, and Sheng (2020) develop a macroeconomic attention index and provide a systematic study of the pattern of investor attention around macroeconomic announcements. Boguth, Grégoire, and Martineau (2018) emphasize the importance of press conferences in shaping market expectations. Hu, Pan, Wang, and Zhu (2020) document the dynamics of implied volatility around FOMC announcements. Ai, Bansal, Guo, and Yaron (2020) link the dynamics of implied volatility around announcements to investors' preference for early resolution of uncertainty. Bollerslev, Li, and Xue (2018) study the relationship between realized volatility and trading volume around FOMC announcements.

From the theoretical point of view, this paper builds on the noisy rational expectations literature pioneered by Grossman and Stiglitz (1980), Grossman (1981), and Hellwig (1980). The continuous-time and dynamic setup are directly related to Wang (1993, 1994), and the setup of the macroeconomic announcement is related to Han (2020). An incomplete list of recent applications of the dynamic Grossman-Stiglitz models include Breon-Drish (2015), Bond and Goldstein (2015), Banerjee and Green (2015), Goldstein and Yang (2017), Albuquerque and Miao (2014), Avdis (2016), Andrei and Cujean (2017), Andrei, Cujean, and Wilson (2018), Sockin (2019), Buffa, Vayanos, and Woolley (2019).

This paper is also related to the literature on endogenous information acquisition and information choice. Veldkamp and Van Nieuwerburgh (2010) study a joint decison problem of portfolio choice and information acquisition. Banerjee and Breon-Drish (2020) analyze endogenous information acquisition problems in an environment with strategic trading. Veldkamp (2011) provides an exellent review of the literature of information choice and attention allocation.

From the perspective of general equilibrium asset pricing, this paper belongs to the large literature that studies various aspects of equity market risk and risk compensation based on preferences with generalized risk sensitivity. To incorporate generalized risk sensitivity in a tractable way in the Grossman-Stiglitz setup, we use the recursive multiple prior setup of Chen and Epstein (2002). See also, Epstein and Schneider (2007). This preference is also related to the robust control preference of Hansen and Sargent (2007, 2008, 2011). We do not attempt to survey this large literature but refer the readers to Ai and Bansal (2018) for the references of preferences that satisfy generalized risk sensitivity and their applications in asset pricing.

The rest of the paper is organized as follows. In Section 2, we summarize stylized facts related to the FOMC announcement premium and the pre-FOMC announcement drift. We present our model in Section 4 and study its implications in Section 5. Section 6 concludes.

2 Stylized Facts

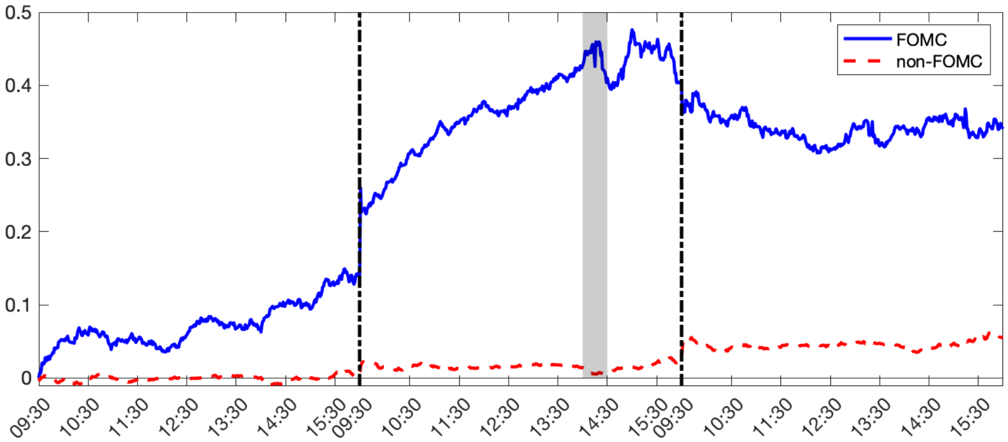
We begin by summarizing the four stylized facts about stock market dynamics around pre-scheduled FOMC announcements. All of the facts we list here are well established in the literature, and we simply use them as a guidance for the development of the model. See Appendix 6.1 for a detailed data description.

1. The aggregate stock market exhibits high average returns starting from the previous trading day until the release of the FOMC announcement. In Figure 1, we plot the cumulative return around FOMC announcement starting from one trading day before the announcement until one trading day afterwards. The solid line stands for announcement days and the dashed line is the non-FOMC announcement day cumulative returns. The shaded area, 14:00-14:30 p.m., depicts the timing of most prescheduled FOMC meetings. Consistent with Lucca and Moench (2015), we find that the 24-hour return before the pre-scheduled FOMC announcement during the period of January 1994 to September 2020 is about 32 basis points on average.

2. Investors' attention rises roughly three days before FOMC announcements and peaks right after FOMC announcements. Fisher, Martineau, and Sheng (2020) develop a macroeconomic attention index and show that investor attention rises roughly three days ahead of announcements. This is the motivating evidence for our endogenous information acquisition-based theory.

3. The realized volatility before announcement hour is not significantly different between FOMC announcement days and non-FOMC announcement days. The market realized volatility peaks right after FOMC announcements. The trading volume follows a similar pattern. In Figure 2, we plot the 30-minute realized volatility (top panel) and the 30-minute trading volume over the three days around FOMC announcements. The dotted lines stand for announcement days and the dashed lines are non-announcement days. Both realized volatility and trading volume

Figure 1: The Pre-FOMC Announcement Drift



This figure plots the average three-day cumulative return around FOMC and non-FOMC announcement days. The solid line displays the average cumulative return during regular trading hours from 9:30 a.m. on one trading days before the FOMC announcements to 16:00 p.m. on days afterward. The dashed line is the average cumulative return on all three trading consecutive days that do not include any FOMC announcement (Note that there exits overlapping among these three-day windows). The shaded area, 14:00-14:30 is the half an hour window containing most of the FOMC releases. The sample period is from January 1994 to September 2020.

are slightly lower before FOMC announcements, compared to non-FOMC announcement days. They peak right after the announcements.

This evidence is inconsistent with information leakage-based story, which will trigger stock market reactions and result in a high realized volatility and a high trading volume during the pre-FOMC announcement drift period.

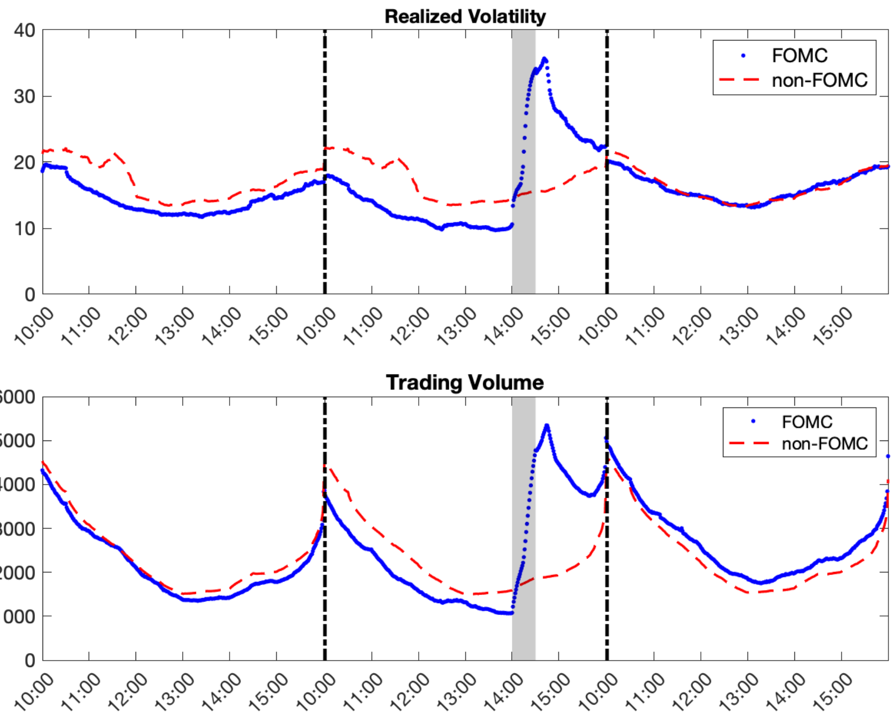
4. The return realized during the pre-FOMC announcement period does not display any significant positive correlation. We regress the one-hour return after FOMC announcements on the pre-announcement return, defined as the 24-hour return prior to 5 minutes before the pre-scheduled FOMC announcements and obtain a coefficient of -0.106, with a robust standard error of 0.048 and a t-statistic of -2.21. The first order auto-correlation of 15-minute returns during the pre-FOMC announcement period is 0.004 but with an insignificant t-statistic of 0.03.

In the following section, we show that a dynamic noisy rational expectations (NREE) model with endogenous information acquisition, after incorporating generalized risk sensitive preferences, provides a unified explanation for the above facts.

3 An Example of Pre-FOMC Announcement Drift

In this section, we reproduce the simple example in Figure 4 of Ai and Bansal (2018) to illustrate how combining generalized risk sensitivity and information leakage can generate a pre-FOMC announcement drift, and why this version of the Ai and Bansal (2018) model cannot explain the

Figure 2: Realized Volatility and Trading Volume



This figure plots the intraday average market realized volatility (top panel) and the average trading volume (bottom panel) during the three days around FOMC and non-FOMC announcement days. The dotted lines are realized volatility and trading volumes for FOMC announcement days, and the dashed lines are those for non-FOMC announcement days. Realized volatility (annualized in percentage) is the average rolling sum of squared log returns on the S&P 500 E-mini futures over the past 30 minutes. The trading volume is the average contracts traded on S&P 500 E-mini futures during the past 30 minutes. The dashed lines are the same calculation on all three consecutive trading days that do not include any FOMC announcement (Note that there exits overlapping among these three-day windows). We calculate the realized volatility and trading volume for each minute from 10:00 to 16:00. The sample period is from September 1997 to September 2020. The shaded area, 14:00-14:30 is the half an hour window containing most of the FOMC releases.

volatility dynamics around FOMC announcements.

The Ai and Bansal (2018) model assumes a continuous-time setup where the aggregate consumption follows $\frac{dC_t}{C_t} = [x_t dt + \sigma dB_{C,t}]$, and the expected consumption growth, x_t is given by:

$$dx_t = b\left(\bar{x} - x_t\right)dt + \sigma_x dB_{x,t}, \tag{1}$$

where $\bar{x}$ is the long-run mean of x_t, b is the rate of mean reversion, σ_x is the volatility of the hidden state x_t, and $B_{x,t}$ is a standard Brownian motion independent of $B_{C,t}$. At time $t = T, 2T, 3T, \cdots$, pre-scheduled FOMC announcements reveal the true values of x_t. To model information leakage, we assume that starting from time $\tau < T$, all investors observe an additional signal s_t, which carries information about the content of upcoming announcement x_t:

$$s_t = x_t dt + \sigma_s dB_{s,t}. \tag{2}$$

where σ_s is the inverse of signal precision and $B_{s,t}$ is a mutually independent Brownian motion noise. Ai and Bansal (2018) show that the posterior mean of x_t, denoted $\hat{x}_t$ can be written as:

$$d\hat{x}_t = b\left(\bar{x} - \hat{x}_t\right)dt + \frac{q_t}{\sigma}d\hat{B}_{C,t} + \frac{q_t}{\sigma_s}d\hat{B}_{i,t}, \tag{3}$$

where q_t is the posterior variance of x_t and $d\hat{B}_{C,t} = \frac{1}{\sigma}\left\{\frac{dC_t}{C_t} - \mathbb{E}_t\left[\frac{dC_t}{C_t}\right]\right\}$ and $d\hat{B}_{s,t} = \frac{1}{\sigma_s}\left\{ds_t - \mathbb{E}_t\left[ds_t\right]\right\}$ are innovations in the observation processes relative to investor's belief.

Assume that the investors have a multiplier robust control preference with a subjective discount rate of ρ, a unit IES, and a multiplier κ, the pricing kernel can be written as:

$$d\pi_t = -r_t dt - \sigma dB_{C,t} - \kappa\left[\left(1 + \frac{q_t}{(b+\rho)\sigma^2}\right)\sigma d\hat{B}_{C,t} + \frac{q_t}{(b+\rho)\sigma_s}d\hat{B}_{s,t}\right], \tag{4}$$

where the first term in the market price of risk, $\sigma dB_{C,t}$ comes from the standard log preference, and the term in the square bracket can be interpreted as probability distortions due to the preference for robustness.[1] As shown in Ai and Bansal (2018), the robust control preference satisfy generalized risk sensitivity and generates an announcement premium.

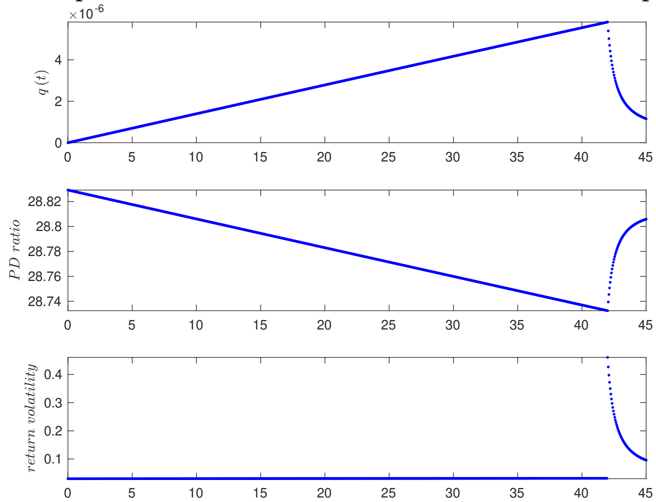
In the above example, between $t \in [\tau, T]$, investors observe an additional signal, s_t. In Figure 3 below, we plot the posterior variance (top panel), and average price-to-dividend ratio (middle panel) and the volatility of the market return (bottom panel) implied by the above model. To model leakage of information, we choose $\sigma_s = 0.01\%$ to be very small. Because the information is very precise, the posterior variance q_t drops sharply at $t = \tau$. At the same time, the average price-to-dividend ratio rises sharply. This is because leakage of information is associated with high volatility of the stochastic discount factor: the term $\frac{q_t}{(b+\rho)\sigma_s}$ in equation (4) is very large when σ_s is close to zero, generating a large risk premium in a short period ahead of announcements.

The high volatility of the stochastic discount factor, however, is associated with high volatility of the posterior belief $\frac{q_t}{\sigma_s}$ in equation (3). In fact, the high volatility of $\hat{x}_t$ is the reason for the high volatility of the stochastic discount factor. As shown in Figure 3, the realized volatility rises sharply and simultaneously as the price-to-dividend ratio increases with leakage of information.

The above example illustrates a key difficulty for models that generate a pre-FOMC announcement drift based on the arrival of new information to the market, or leakage of information. In the data, the average excess return during the pre-FOMC announcement period is roughly 40 bps per trading day, and that on non-announcement days is less than 2 bps. Holding the Sharpe ratio constant, to account for a 40 bps premium, the information leakage-based story requires a realized market volatility of twenty times higher during the pre-announcement period, whereas in the data, the realized market volatility in this period is slightly lower than that on non-announcement days. In the rest of the paper, we develop a noisy rational expectations model with information acquisition to resolve the above puzzle.

[1]Different from Ai and Bansal (2018), here we assume robust control preference as Hansen and Sargent (2008) and continuous information arrival, because these assumptions allow us to link and compare to the asymmetric information model we develop in the rest of the paper.

Figure 3: Equilibrium without and with Information Acquisition



This figure plots $\hat{q}_t$, the posterior variance for $\hat{x}_t$ (top panel), the average price-to-dividend ratio (middle panel), and the return volatility (bottom panel) over one announcement cycle. The information starts to leak at time $\tau < T$.

4 Dynamic Model

This section develops a continuous-time NREE model with periodic macroeconomic announcements and with endogenous information acquisition. The model is a continuous-time version of the Grossman-Stiglitz model with generalized risk-sensitive preferences.

4.1 Model Setup

The asset market Time is continuous and infinite. There is a unit measure of investors. An ω fraction of them are uninformed investors and $1-\omega$ fraction are informed. There are two assets available for trading, a stock and a risk-free bond. We assume that the risk-free return r is constant. The stock is the claim to the following dividend process:

$$dD_t = (x_t - D_t)\, dt + \sigma_D dB_{D,t}, \tag{5}$$

where D_t is the dividend flow, x_t is the long-run trend for the dividend flow, σ_D is the volatility of the dividend flow, and $B_{D,t}$ is an i.i.d. shock to the dividend payment modeled as a standard Brownian motion. We model the expected dividend flow as $x_t - D_t$, so that the dividend process is stationary. The assumption that the mean reversion rate equals to 1 is not important and can be relaxed without affecting most parts of the model. The long-run trend of the dividend flow, x_t, is itself mean reverting, modeled as an Ornstein-Uhlenbeck (OU) process as in equation (1). In addition, as is standard in the NREE literature, we assume that the total equity supply is a stochastic process and denote it as θ_t, where

$$d\theta_t = a\left(\bar{\theta} - \theta_t\right) dt + \sigma_\theta dB_{\theta,t}. \tag{6}$$

In the above equation, a is the rate of mean reversion, $\bar{\theta}$ is the long-run mean for θ_t, and σ_θ is the noisy supply volatility. We assume that Brownian motions $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$ are mutually independent.

Information and preference of informed investors We assume that the dividend process, D_t, is observable to all investors, but its long-run trend x_t and the total risky asset supply θ_t are not. At pre-scheduled times, $t = nT$, for $n = 1, 2, \cdots$, the monetary authority (central bank) makes periodic announcements that reveal the true value of x_t. Both the informed and the uninformed investors can observe D_t and the the pre-scheduled FOMC announcements and use them to update their beliefs about the latent variable that drives economic growth, x_t.

We assume that market research can produce a signal that is informative about x_t, denoted as s_t:

$$ds_t = x_t dt + \sigma_s dB_{s,t}, \tag{7}$$

where σ_s is the signal volatility and $B_{s,t}$ is a Brownian motion independent of $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$. We think of s_t as information available in the public domain but costly to acquire. We interpret informed investors as professional investors who have a comparative advantage or higher capacity in acquiring information s_t. For simplicity, they have zero information acquisition cost and observe s_t at all times.

Informed investors maximize CARA utilities represented by $\left[\mathbb{E} \int_0^\infty -e^{-\rho t - \gamma C_t} dt\right]$, where C_t is the consumption at time t, ρ is the subjective time discount rate and γ is the absolute risk aversion.

GRS through recursive multiple prior preferences In order to account for the equilibrium announcement premium, we assume that the uninformed investors are concerned about model uncertainty or ambiguity averse, which is modeled by a robust control preference. The robust control model, as shown by Ai and Bansal (2018), satisfies generalized risk sensitivity. Formally, let $(\Omega, \mathcal{F}, P)$ be the probability space where all uncertainty in this economy is generated. The agent's preference is specified by a CARA utility function with the absolute risk aversion of γ and a set of probability models defined on $(\Omega, \mathcal{F}, P)$, denoted as $\mathcal{P}$. That is, the agent computes his time-t utility using:

$$V_t = \inf_{\mathcal{Q} \in \mathcal{P}} \mathbb{E}_t^{\mathcal{Q}} \left[\int_t^\infty -e^{-\rho s - \gamma C_s} ds\right]. \tag{8}$$

Here, $\mathcal{P}$ captures model uncertainty as Hansen and Sargent (2008). That is, the agent is ambiguous about the true data generating process and entertains a set of probability models to compute the worse-case scenarios over $\mathcal{P}$ when ranking stochastic consumption streams.

Information and beliefs The informed investors observe three sources of information about the latent variable x_t that drives the economic growth: the dividend process D_t, pre-scheduled FOMC announcements at $t = nT, n = 1, 2, \cdots$, and the signal process s_t obtained from market research. Denote $\hat{x}_t \equiv \hat{\mathbb{E}}_t [x_t]$ and $\hat{q}(t) \equiv \hat{\mathbb{E}}_t \left[(\hat{x}_t - x_t)^2\right]$ as the posterior mean and variance of the informed investors about x_t. If the informed investors' prior for x_0 is a Gaussian distribution, then their posterior distribution for x_t is also Gaussian and can be characterized by the standard Kalman filter. We assume FOMC announcements convey information about the economic growth and fully reveal the true value of x_t, we then have $\hat{x}_t = x_t$ and $\hat{q}_t = 0$ at prescheduled announcements $t = nT$. After announcements, because $\hat{x}_t$ process evolves according to equation (9), $\hat{x}_t$ drifts away from the true value of x_t and $\hat{q}_t$ increases above zero, up until the next announcement. Standard Kalman filter implies that the dynamics of $\hat{x}_t$ can be computed by:

$$d\hat{x}_t = b\left(\bar{x} - \hat{x}_t\right)dt + \frac{\hat{q}\left(t\right)}{\sigma_D}d\hat{B}_{D,t} + \frac{\hat{q}\left(t\right)}{\sigma_s}d\hat{B}_{s,t}, \tag{9}$$

where $d\hat{B}_{D,t} = dD_t - \hat{\mathbb{E}}_t\left[dD_t\right]$ and $d\hat{B}_{s,t} = ds_t - \hat{\mathbb{E}}_t\left[ds_t\right]$ are innovations in the observation processes relative to informed investors' expectations.

In contrast, the uninformed investors do not observe s_t, until they pay a cost. To keep the structure simple, we assume that uninformed investors can choose to obtain information about s_t by paying a flow cost k per unit of time until the next announcement. Paying the cost allows all uninformed investors to observe a common noisy signal about the best forecast of x_t obtained by the market research. That is, it allows uninformed investors to observe a signal of the form:

$$ds_{i,t} = \hat{x}_t dt + \sigma_i\left(t\right)dB_{i,t}, \tag{10}$$

where $B_{i,t}$ is independent of $B_{s,t}$, $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$. We focus on the symmetric equilibrium where all uninformed investors start to acquire information at time τ. To save notation, assuming an uninformed investor chooses to observe the signal during the period $[\tau, \tau']$, we write $\sigma_i\left(t\right) = \sigma_i$ for $t \in [\tau, \tau']$ and $\sigma_i\left(t\right) = \infty$ otherwise.

It is convenient to denote the posterior mean of an uninformed investor as $\tilde{x}_t = \tilde{\mathbb{E}}_t\left[\hat{x}_t\right]$ and the posterior variance as $\tilde{q}\left(t\right) \equiv \tilde{\mathbb{E}}_t\left[\left(\tilde{x}_t - \hat{x}_t\right)^2\right]$, where $\tilde{\mathbb{E}}$ is the belief of an uninformed investor. We conjecture and later verify that the equilibrium price takes the following form

$$P_t = \phi\left(t\right) + \phi_D D_t - \phi_\theta\left(t\right)\theta_t + \phi_x\left(t\right)\hat{x}_t + \phi_\Delta\left(t\right)\tilde{x}_t, \tag{11}$$

where the sum of the two coefficients, $\phi_x\left(t\right) + \phi_\Delta\left(t\right) = \bar{\phi}_x$, is a constant.[2] Note that the equilibrium price contains information about the best prediction for x_t obtained by the market research and uninformed investors should learn from it. Clearly, if we define $\Delta_t \equiv \hat{x}_t - \tilde{x}_t$ to be the difference between the beliefs of the informed and uninformed investors, price can therefore be written as:

$$P_t = \phi\left(t\right) + \phi_D D_t - \phi_\theta\left(t\right)\theta_t + \bar{\phi}_x\hat{x}_t - \phi_\Delta\left(t\right)\Delta_t. \tag{12}$$

Learning from prices Here we describe the beliefs of uninformed investors in our model, which is the key to understanding the model's implications for the pre-FOMC announcement drift. It is

[2] As is standard in this literature, we use a guess-and-verify approach to prove the functional form of P_t and the property that $\phi_x\left(t\right) + \phi_\Delta\left(t\right) = \bar{\phi}_x$ is a constant.

convenient to define $\xi_t = \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t - \frac{\hat{q}_t}{\sigma_D^2}\phi_x(t)D_t$ as the information content of prices, as observing ξ_t is the same as observing the equilibrium price. The uninformed traders observe three sources of information about the informed investors' belief $\hat{x}_t$: the dividend process, the equilibrium price, and the signal $s_{i,t}$ after paying the information acquisition cost. Standard Kalman filter implies that the dynamics of $\tilde{x}_t$ can be written as:

$$d\tilde{x}_t = b(\bar{x} - \tilde{x}_t)dt + \frac{\hat{q}(t) + \tilde{q}(t)}{\sigma_D} d\tilde{B}_{D,t} + \nu(t)\sigma_\xi(t)d\tilde{B}_{\xi,t} + \frac{\tilde{q}(t)}{\sigma_i(t)} d\tilde{B}_{i,t}, \tag{13}$$

where $d\tilde{B}_{D,t} = dD_t - \tilde{\mathbb{E}}_t[dD_t]$, $d\tilde{B}_{\xi,t} = d\xi_t - \tilde{\mathbb{E}}_t[d\xi_t]$ and $d\tilde{B}_{i,t} = ds_{i,t} - \tilde{\mathbb{E}}_t[ds_{i,t}]$ are innovations in the observation processes relative to expectations. In the above expression, $\nu(t)$ is defined in equation (42) and the volatility of $d\xi_t$, $\sigma_\xi(t)$ is defined in (37) in Appendix 6.2.

Before τ, uninformed traders can only learn about $\hat{x}_t$ from the dividend process and the equilibrium price. After the information acquisition, they can also learn from the newly acquired information, $s_{i,t}$. Our notation in (13) incorporates both possibilities by using the convention $\sigma_i(t) = \sigma_i$ for $t \in [\tau, \tau']$ and $\sigma_i(t) = \infty$ otherwise. It is important to note that in our setup, the endogenously acquired signals, $s_{i,t}$ is about the information about $\hat{x}_t$, which is already in the market. $s_{i,t}$ is not informative about the difference between the true value of x_t and $\hat{x}_t$, which is only revealed through announcements. In other words, the content of an announcement is not revealed until right after the announcement. This feature of our model is important in accounting for the volatility dynamics around the announcements.

To incorporate the generalized risk-sensitive preference, we assume the uninformed investors have a preference for robustness or ambiguity aversion as Hansen and Sargent (2007; 2011). Under the worst case probability, $d\tilde{B}_{D,t} = d\tilde{B}^\kappa_{D,t} - \kappa\left[\bar{\phi}_x \frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D} + \phi_D\sigma_D\right]$, $d\tilde{B}_{\xi,t} = d\tilde{B}^\kappa_{\xi,t} - \kappa\bar{\phi}_x\nu(t)\sigma_\xi(t)$, and $d\tilde{B}_{i,t} = d\tilde{B}^\kappa_{i,t} - \kappa\bar{\phi}_x\frac{\tilde{q}(t)}{\sigma_i(t)}$ are Brownian motions with negative drift distortions, where $\tilde{B}^\kappa_{D,t}$, $\tilde{B}^\kappa_{\xi,t}$, and $\tilde{B}^\kappa_{i,t}$ are standard Brownian motions under the worst case probability. In Appendix 6.3, we show that the above probability distortions can be derived from a robust valuation problem, where κ is the preference for robustness, or the Lagrangian multiplier on the relative entropy constraint.

4.2 Equilibrium and Equilibrium Conditions

For simplicity, we will focus on stationary equilibria in which equilibrium prices satisfy $P_t = P_{t \ mod \ T}$ and so do equilibrium quantities. That is, all equilibria are identical across different announcement cycles. Without loss of generality, we can therefore focus on prices and quantities over the closed time interval $[0, T]$, because they repeat themselves within each announcement cycle. We use T^+ and T^- to denote the moment right after announcements and right before announcements, respectively. Whenever there is any confusion, time 0 should be understood as T^+ and T should be understood as T^-.

Below we construct an equilibrium in which there exists a $\tau \in (0, T)$ such that for all $t \leq \tau$, all uninformed investors find it suboptimal to acquire any information, and after $T \leq t > \tau$, all uninformed investor acquire the same information until the next announcement.

Definition of the equilibrium A stationary equilibrium consists of a collection of pricing functions $\{\phi(t), \phi_D, \phi_\theta(t), \phi_\Delta(t)\}$, demand functions of the informed, $\alpha(t, \theta_t, \Delta_t) = \alpha_0(t) + \alpha_\theta(t)\theta_t + \alpha_\Delta(t)\Delta_t$, and demand functions for uninformed investors, $\beta\left(t, \tilde{\theta}_t\right) = \beta_0(t) + \beta_\theta(t)\tilde{\theta}_t$ such that:

1. Given the pricing functions $\{\phi(t), \phi_D, \phi_\theta(t), \phi_x(t), \phi_\Delta(t)\}$, $\{\alpha_0(t), \alpha_\theta(t), \alpha_\Delta(t)\}$ represents the optimal portfolio demand for the informed investors.

2. Uninformed investors strictly prefer not to acquire information for all $t < \tau$. After time τ, uninformed investors prefer to acquire information.

3. Given uninformed investors' information set, $\{\beta_0(t), \beta_\theta(t)\}$ represents their optimal portfolio demand.

4. Markets clear, such that

$$(1-\omega)\left[\alpha_0(t) + \alpha_\theta(t)\theta_t + \alpha_\Delta(t)\Delta_t\right] + \left[\beta_0(t) + \beta_\theta(t)\tilde{\theta}_t\right] = \theta_t \tag{14}$$

   for all $t \in [0, T]$.

Equilibrium beliefs Given the pricing equation (12), we define the excess return process as $dQ_t = (D_t - rP_t) + dP_t$. As informed investors can distinguish Δ_t from θ_t, we can combine equations (9) and (13) to derive the difference in belief as a diffusion process:

$$d\Delta_t = -a_\Delta(t)\Delta_t dt - \frac{\tilde{q}(t)}{\sigma_D} d\hat{B}_{D,t} + \frac{\hat{q}(t)}{\sigma_s}\left[1 - \phi_x(t)\nu(t)\right] d\hat{B}_{s,t} + \phi_\theta(t)\nu(t)\sigma_\theta dB_{\theta,t} - \frac{\tilde{q}(t)}{\sigma_i(t)} dB_{i,t}, \tag{15}$$

where $a_\Delta(t)$ is defined in equation (65) in Appendix 6.2. Using the law of motion of the state variables, we can write the excess return as a diffusion process from the perspective of the informed investors:

$$dQ_t = \left[e_0(t) + e_\theta(t)\theta_t + e_\Delta(t)\Delta_t\right] dt + \varrho_D(t) d\hat{B}_{D,t} + \left[1 + \phi_\Delta(t)\nu(t)\right]\sigma_\xi(t) d\hat{B}_{\xi,t} + \varrho_i(t) dB_{i,t}, \tag{16}$$

where the coefficients $e_0(t)$, $e_\theta(t)$, $e_\Delta(t)$, $\varrho_D(t)$, $\varrho_i(t)$ are given in equation (74) in Appendix 6.2, and $\sigma_\xi(t) d\hat{B}_{\xi,t} = d\xi_t - \hat{\mathbb{E}}_t[d\xi_t]$ is the innovations of ξ_t relative to the informed investors' information.

Uninformed investors, however, cannot distinguish Δ_t from θ_t. Because they observe the prices, rational expectations imply $P_t = \tilde{\mathbb{E}}_t[P_t]$. This allows us to write the Equilibrium price (12) as:

$$P_t = \phi(t) + \phi_D D_t - \phi_\theta(t)\tilde{\theta}_t + \bar{\phi}_x \tilde{x}_t. \tag{17}$$

The law of motion of $\tilde{x}_t$ is given in equation (13). To derivate a law of motion for $\tilde{\theta}_t$, recall that observing prices is equivalent to observing $\xi_t = \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t) D_t$. Taking conditional expectation $\tilde{\mathbb{E}}_t$ on both sides, we have $\xi_t = \tilde{\mathbb{E}}_t[\xi_t]$. Therefore,

$$\xi_t = \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t) D_t = \phi_x(t)\tilde{x}_t - \phi_\theta(t)\tilde{\theta}_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t) D_t. \tag{18}$$

We have: $\tilde{\theta}_t = \frac{\phi_x(t)}{\phi_\theta(t)}\tilde{x}_t - \frac{1}{\phi_\theta(t)}\xi_t - \frac{\hat{q}(t)}{\sigma_D^2}\frac{\phi_x(t)}{\phi_\theta(t)}D_t$. The law of motion of $\tilde{\theta}_t$ can therefore be written as:

$$d\tilde{\theta}_t = a\left(\bar{\theta} - \tilde{\theta}_t\right)dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}d\tilde{B}_{D,t} + \left[\phi_x(t)\nu(t) - 1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}d\tilde{B}_{\xi,t} + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)}d\tilde{B}_{i,t}. \quad (19)$$

This allows us to write the excess return process dQ_t in terms of a diffusion process adapted to the information set of the uninformed investors:

$$dQ_t = \left[e_0(t) + e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)\,d\tilde{B}_{D,t} + \left[1 + \phi_\Delta(t)\nu(t)\right]\sigma_\xi(t)\,d\tilde{B}_{\xi,t} + \varrho_i(t)\,d\tilde{B}_{i,t}. \quad (20)$$

Portfolio selection and information acquisition Informed investors in our model solve an optimal portfolio selection problem. At time t, they maximize life-time utility, $\hat{\mathbb{E}}_t\left[\int_0^\infty -e^{-\rho s - \gamma\hat{C}_{t+s}}ds\right]$ by choosing consumption and portfolio holdings, $\left\{\hat{C}_{t+s}, \alpha_{t+s}\right\}_{s=0}^\infty$, subject to the following law of motion of wealth:

$$d\hat{W}_t = \left(\hat{W}_t r - \hat{C}_t\right)dt + \alpha_t dQ_t, \quad (21)$$

where the excess return process dQ_t is given in equation (16). As a result, the value function for informed investors, denoted $\hat{V}(t, W, \theta, \Delta)$ satisfies the following HJB equation:

$$\hat{V}\left(t, \hat{W}, \theta, \Delta\right) = \max_{\hat{C},\alpha}\left\{u\left(\hat{C}\right) + \hat{\mathcal{L}}^{\hat{C},\alpha}\hat{V}\left(t, \hat{W}, \theta, \Delta\right)\right\}, \quad (22)$$

where the operator $\hat{\mathcal{L}}^{\hat{C},\alpha}$ is defined as:

$$\hat{\mathcal{L}}^{\hat{C},\alpha}\hat{V}\left(t, \hat{W}_t, \theta_t, \Delta_t\right) = \lim_{h\to 0}\frac{1}{h}\hat{\mathbb{E}}_t^{C,\alpha}\left[\hat{V}\left(t+h, \hat{W}_{t+h}, \theta_{t+h}, \Delta_{t+h}\right) - \hat{V}\left(t, \hat{W}_t, \theta_t, \Delta_t\right)\right],$$

and the notation $\hat{\mathbb{E}}_t^{C,\alpha}$ emphasizes that the law of motion of wealth, (21) depends on the consumption and portfolio choice decisions.

Uninformed investors solve both an optimal consumption-investment problem and an optimal information acquisition problem. Consider an announcement cycle, $[0, T]$. We assume that the uninformed trader has an option to start to acquire information at a flow cost of k per unit of time until T. He is free to exercise this option at any time $\tau \in [0, T]$, but once the option is exercised, he has to continue to pay the cost until T. Assume that the optimal stopping time τ is in the interior of $[0, T]$. We interpret τ as the moment when uninformed investors start to allocate their attention to do market research. Therefore, uninformed investors maximize the life-time utility, $\tilde{\mathbb{E}}_t\left[\int_0^\infty -e^{-\rho s - \gamma\tilde{C}_{t+s}}ds\right]$ by choosing the optimal stopping time τ, optimal consumption and portfolio holdings, $\left\{\tilde{C}_{t+s}, \beta_{t+s}\right\}_{s=0}^\infty$, subject to the following law of motion of wealth:

$$d\tilde{W}_t = \left(\tilde{W}_t r - \tilde{C}_t\right)dt + \beta_t dQ_t. \quad (23)$$

Let $\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)$ be the value function of uninformed investors, then it must satisfy that, for $t\leq\tau$,

$$\rho\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)=\max_{\tilde{C},\beta}\left[u\left(\tilde{C}\right)+\tilde{\mathcal{L}}^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)\right], \tag{24}$$

where the operator $\tilde{\mathcal{L}}^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)$ is defined as:

$$\tilde{\mathcal{L}}^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)=\lim_{h\to 0}\frac{1}{h}\tilde{\mathbb{E}}_t^{\tilde{C},\beta}\left[\tilde{V}\left(t+h,\tilde{q}_{t+h},\tilde{W}_{t+h},\tilde{\theta}_{t+h}\right)-\tilde{V}\left(t,\tilde{q}_t,\tilde{W}_t,\tilde{\theta}_t\right)\right],$$

and for $t>\tau$, the uninformed investors pay the information acquisition cost to observe the signals:

$$\rho\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)=\max_{\tilde{C},\beta}\left[u\left(\tilde{C}-k\right)+\tilde{\mathcal{L}}_i^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)\right]. \tag{25}$$

Here, the operator $\mathcal{L}^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)$ is defined similarly as above, except that the expectation operator contains the newly acquired information s_i. The fact that τ is the optimal stopping time implies for $t\leq\tau$,

$$\max_{\tilde{C},\beta}\left[u\left(\tilde{C}\right)+\tilde{\mathcal{L}}^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)\right]\geq\max_{\tilde{C},\beta}\left[u\left(\tilde{C}-k\right)+\tilde{\mathcal{L}}_i^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)\right],$$

and $t>\tau$, the investor strictly prefers to acquire information:

$$\max_{\tilde{C},\beta}\left[u\left(\tilde{C}-k\right)+\tilde{\mathcal{L}}_i^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)\right]>\max_{\tilde{C},\beta}\left[u\left(\tilde{C}\right)+\tilde{\mathcal{L}}^{\tilde{C},\beta}\tilde{V}\left(t,\tilde{q},\tilde{W},\tilde{\theta}\right)\right].$$

Market clearing In our model, the equilibrium price is pinned down by the market clearing condition in equation (14). Using equation (18), $\phi_x(t)\hat{x}_t-\phi_\theta(t)\theta_t=\phi_x(t)\tilde{x}_t-\phi_\theta(t)\tilde{\theta}_t$, that is, $\tilde{\theta}_t=\theta_t-\frac{\phi_x(t)}{\phi_\theta(t)}\Delta_t$. Intuitively, because uninformed investors observe prices, they can make mistakes about $\hat{x}_t$ and θ_t separately, but will not make a mistake about $\phi_x(t)\hat{x}_t-\phi_\theta(t)\theta_t$. This restriction implies that the only reason for the uninformed to be pessimistic about $\hat{x}_t$ is that they believe that the level of price is not justified by high fundamentals, $\hat{x}_t$, but by a lower supply θ_t. That is, $\hat{x}_t-\tilde{x}_t$ and $\theta_t-\tilde{\theta}_t$ must have the same sign.

Using $\tilde{\theta}_t=\theta_t-\frac{\phi_x(t)}{\phi_\theta(t)}\Delta_t$ to replace $\tilde{\theta}_t$ in the market clearing condition (14), we obtain the following restrictions on the portfolio decisions:

$$(1-\omega)\,\alpha_0(t)+\omega\beta_0(t)=0, \tag{26}$$

$$(1-\omega)\,\alpha_\theta(t)+\omega\beta_\theta(t)=1, \tag{27}$$

$$(1-\omega)\,\alpha_\Delta(t)-\omega\frac{\phi_x(t)}{\phi_\theta(t)}\beta_\theta(t)=0. \tag{28}$$

In Appendix 6.2, we show that investors' optimality problems and the above market clearing conditions jointly pin down the pricing functions $\{\phi(t),\phi_\theta(t),\phi_\Delta(t)\}$.

5 Model Implications

We show details of our model solutions and derivations in Appendix 6.4. We then calibrate our model to match the overall market equity premium and evaluate its implications on the FOMC announcement premium, pre-FOMC announcement drift, and the pattern of realized volatility and trading volume around FOMC announcements. We provide details of the parameter calibrations in the Appendix 6.6 and focus on its implications here.

The key mechanism in our model is that after each pre-scheduled announcement, the uncertainty (defined as the posterior variances $\hat{q}_t$ and $\tilde{q}_t$) of the economy starts to build up over time. Uninformed investors find it optimal to acquire information ahead of the next announcement. In this section, we focus on the following four implications of the endogenous information acquisition problem.

1. Uninformed investors' incentive to acquire information increases monotonically over time and peaks before the announcements. Because information acquisition is costly, it is optimal to acquire information shortly before announcements.

2. As uninformed investors start to acquire information, stock returns and their future financial wealth become more correlated. Under generalized risk sensitivity, this higher correlation translates into a higher risk premium and leads to an increase in expected returns, or pre-FOMC announcement drift.

3. Because newly acquired information (about $\hat{x}_t$) has already been incorporated into the market price through informed investors' beliefs, information acquisition does not lead to an increase in the realized volatility of the market.

4. Upon the announcement, the true value of x_t is revealed. As a result, realized volatility spikes and so does the trading volume.

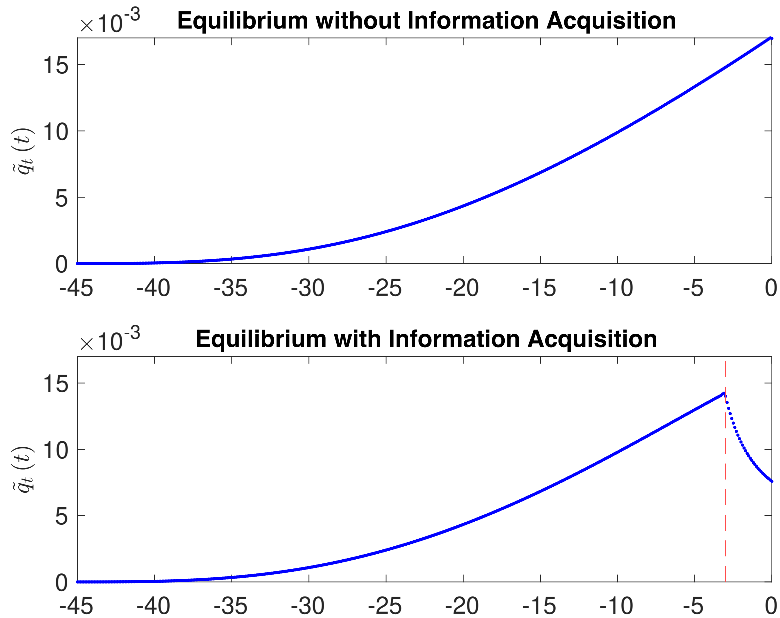
We begin by analyzing the incentives for endogenous information acquisition.

Timing of the information acquisition In our model, as in the data, periodical announcements are pre-scheduled. Uninformed investors do not find it optimal to acquire information until close to the upcoming announcements for two reasons. First, because announcements fully reveal the true value of x_t, initially after the previous announcement, both the informed and the uninformed investors have little uncertainty about x_t so that there is no need to acquire additional information. As t increases from 0, x_t drifts away from its previous value due to lack of information. From the perspective of uninformed traders, uncertainty slowly builds up and the benefit of information acquisition rises over time.

Second, information disadvantage for uninformed investors is relatively small on non-announcement days and spikes right before announcements. On non-announcement days, uncertainty about x_t resolves slowly over time as the Brownian motions $B_{x,t}$ and $B_{s,t}$ evolve continuously. As information flows in continuously, the trading loss due to information disadvantage also increases continuously over time.

At the announcement, however, the true value of x_t is revealed and a large amount of new information arrives at the market in a short period. The posterior variance for both informed and uninformed investors jumps discontinuously. Therefore, information acquisition prior to announcements is particularly important for uninformed investors because the information disadvantage is particularly costly right before the announcements.

Figure 4: Uninformed Investors' Posterior Variance, $\tilde{q}_t$

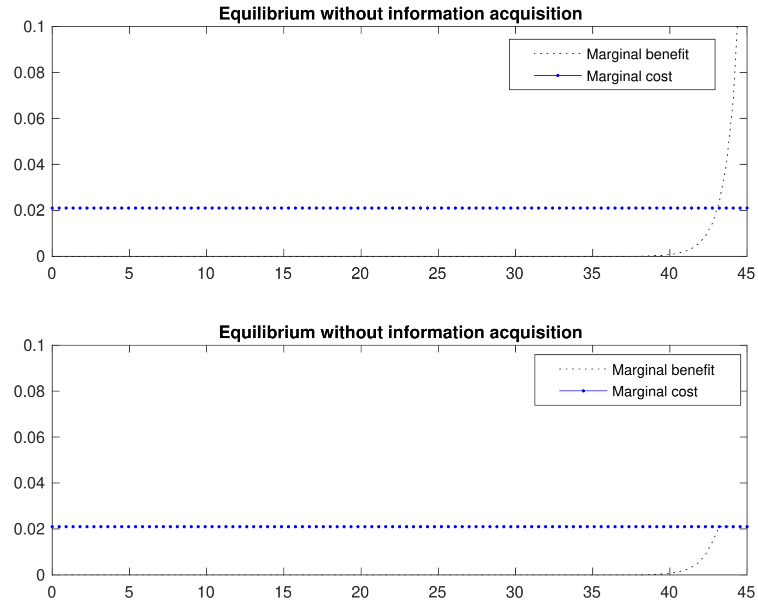


This figure plots $\tilde{q}_t$, the posterior variance of the uninformed investor's belief of $\hat{x}_t$ over one announcement cycle. The top panel is a model without information acquisition and the bottom penal is our benchmark economy with endogenous information acquisition. The vertical line indicates the timing when uninformed investors start to acquire information. The horizon axis is the number of days before the upcoming announcement, which is normalized to 0. A -5, for example, stands for five days before the announcement.

In Figure 4, we plot the uninformed investors' posterior variance $\tilde{q}_t$, which is the uninformed investors' posterior variance of $\hat{x}_t$. As in the data, our calibration features eight FOMC announcements per year and therefore each announcement cycle is 45 days. The top panel is the path of $\tilde{q}_t$ in equilibrium without information acquisition, where $\tilde{q}_t$ increases monotonically from day -45 to day 0, the announcement day. The bottom panel of Figure 4 is $\tilde{q}_t$ in our benchmark model with endogenous information acquisition, in which the uninformed decide to allocate attention to do market research to acquire information is made around 3 days before the announcement, consistent with Fisher, Martineau, and Sheng (2020). As uninformed investors start to acquire information, the price becomes more informative, and $\tilde{q}_t$ drops sharply from day -3 to day 0.

In Figure 5, we plot the marginal cost and marginal benefit for the uninformed investors to acquire information in equilibrium without information acquisition (top panel) and those in the equilibrium with information acquisition (bottom panel). In both panels, the marginal benefit of information acquisition, as measured in consumption equivalent terms sharply increases in days ahead of the announcement. In the model without information acquisition, it keeps increasing until

Figure 5: Incentive for Information Acquisition



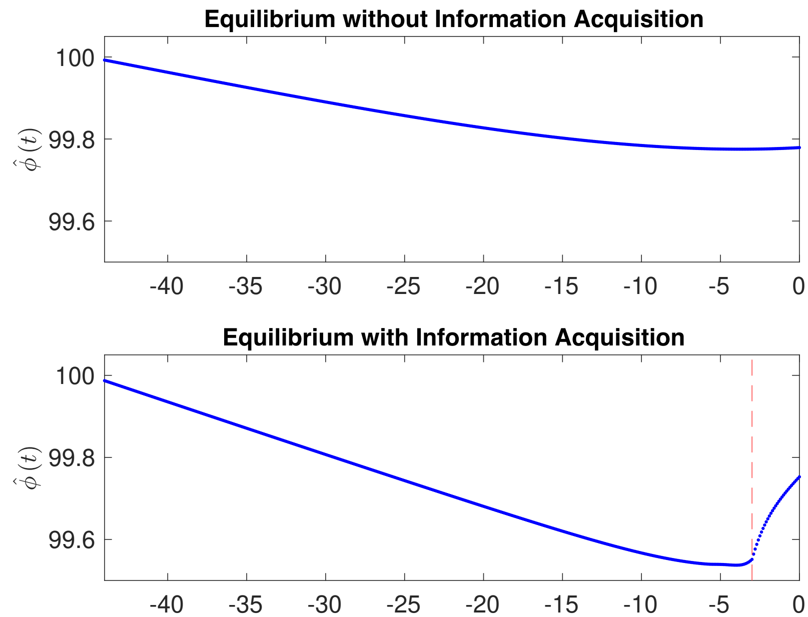
This figure plots the marginal benefit and marginal cost for the uninformed investors to acquire information over one announcement cycle. The top panel is a model without information acquisition and the bottom penal is our benchmark economy with endogenous information acquisition. The horizon axis is the number of days after the upcoming announcement, which is normalized as 0. A 5, for example, stands for five days after the announcement.

the announcement day. In the model with endogenous information acquisition, as more investors acquire information, the equilibrium price becomes more informative, and the marginal benefit of information acquisition equals its marginal cost after day 42, when investors become indifferent towards information acquisition. The fact that investors start to acquire information endogenously in our model days ahead of the FOMC announcement provides a rational explanation for the increasing patterns of investors' attentions around macroeconomic announcements documented by Fisher, Martineau, and Sheng (2020).

Pre-FOMC announcement drift To understand the model's implications on pre-FOMC announcement drift, in Figure 6, we plot the unconditional expectation of equilibrium price: $\hat{\phi}(t) = \mathbb{E}[P_t] = \phi(t) + \left[\bar{\phi}_x + \phi_D\right]\bar{x}$ as a function of time for a model without information acquisition (top panel) and that for a model with information acquisition (bottom panel). To illustrate the quantitative implication on the magnitude of the pre-announcement drift, we normalize the level of $\hat{\phi}(t)$ at time -45 to 100. Therefore, an increase of $\hat{\phi}(t)$ from 99 to 100, for instance, corresponds to 100 basis points of return.

In the model without information acquisition, the expected level of price monotonically decreases until T and jumps upwards upon the next announcement. The fact that generalized risk sensitivity produces an announcement premium is the same as in Ai and Bansal (2018). In the model with information acquisition, the function $\hat{\phi}(t)$ reaches its minimum at time τ, as the uninformed investors start to acquire information. From its minimum at time $\tau = -3$ to the announcement time,

Figure 6: Expected level of price, $\hat{\phi}(t)$



This figure plots the expected level of price, $\hat{\phi}(t)$ of a model without information acquisition (top panel) and that for our benchmark economy with endogenous information acquisition (bottom panel). The vertical line indicates the timing when uninformed investors start to acquire information. The horizon axis is the number of days before the upcoming announcement, which is normalized as 0. A -5 for example, stands for five days before the announcement.

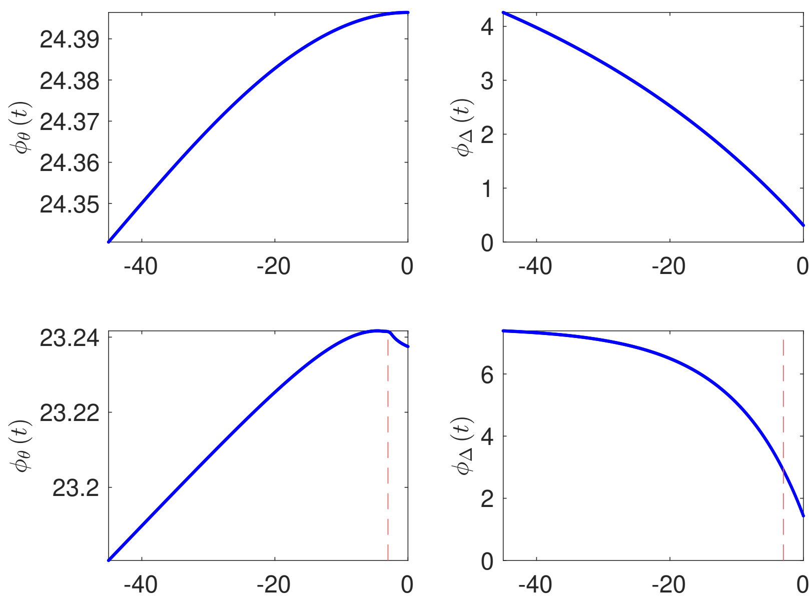
the drift is about 20 basis points, which is similar in magnitude to the pre-FOMC announcement drift we computed in Section 2. At the announcement, the expected return increases again as the new information arrives, giving an announcement premium about 22 basis points.

Starting from time τ, uninformed investors begin to acquire information about the frontier research $\hat{x}_t$. The newly arrived information induces a positive covariance between stock returns and their financial wealth. Under the robust control preference, this positive covariance translates into a higher required return from the perspective of uninformed investors. This mechanism is similar to the robust control model (4). However, different from the simple model in Section 3, the newly arrived information is already incorporated into prices through informed investors' tradings and does not trigger spikes in realized volatility.

From the perspective of the uninformed traders, information acquisition dramatically amplifies the positive correlation between stock return and their financial wealth and therefore their continuation utility. Under the robust control in preferences, which satisfy generalized risk sensitivity, marginal utility is a decreasing function of continuation utility. The positive correlation between marginal utility and stock return implies a negative hedging demand. As a result, uniformed investors' valuation of the asset drops sharply and they start to sell the stock. From the perspective of informed investors, higher expected return induces more purchase. However, risk aversion implies that they cannot purchase an unlimited amount to fully offset the price impact of the uninformed investors. In equilibrium, a heightened risk premium is associated with a sharpe increase in $\hat{\phi}(t)$

from time τ, producing a pre-announcement drift during the period of endogenous information acquisition.

Figure 7: Pricing functions



This figure plots the pricing functions $\phi_\theta(t)$ and $\phi_\Delta(t)$ of a model without information acquisition (the top panels) and those for our benchmark economy with endogenous information acquisition (the bottom panels). The vertical line indicates the timing when uninformed investors start to acquire information. The horizon axis is the number of days before the upcoming announcement, which is normalized as 0. A -5 for example, stands for five days before announcements.

We plot pricing functions $\phi_\theta(t)$ and $\phi_\Delta(t)$ in Figure 7 for an economy without information acquisition in top panels and those for our benchmark model with endogenous information acquisition in bottom panels. As shown in Figure 7, the function $\phi_\theta(t)$ monotonically increases in the model without information acquisition. $\phi_\theta(t)$ is the impact of noisy supply on the stock price from equation (11). In our model, stock price decreases in θ_t for two reasons. First, increases in supply lower the equilibrium price due to a downward sloping demand curve as in standard equilibrium models. This effect does not depend on the uncertainty or the asymmetric information. Second, the information asymmetry and learning amplify the responses of prices to supply shocks, therefore an increase in θ_t further lowers the price. Because the uninformed investors cannot infer the true value of θ_t and x_t from prices, they attribute part of the price drop as deteriorations in fundamentals and downwardly revise their beliefs about x_t. The uninformed investors reduce their holdings of the stock because of their distorted pessimistic beliefs. This lowers the demand of the asset and the price has to drop further to clear the market.

Clearly, the second effect is stronger when uninformed investors are more uncertain about x_t. At time $t = -45$, right after an announcement, uninformed investors know the true value of x_0 and the information asymmetry is temporarily eliminated. As t increases, the uncertainty about x_t builds up, and changes in prices have stronger impacts on uninformed investors' beliefs because they have to rely more and more on learning from prices. Therefore, prices become more sensitive to supply shocks, θ_t.

In the economy with information acquisition, after time $\tau = -3$, as the uninformed investors acquire more information, their uncertainty drops and the amplification effect from information asymmetry reduces. As a result, the function $\phi_\theta(t)$ starts to drop until time 0. The drop of $\phi_\theta(t)$ function after τ is important for our model to account for the lower realized volatility during the pre-announcement drift period. After time τ, the impact of noise traders reduces, and so does the realized volatility of stock returns.

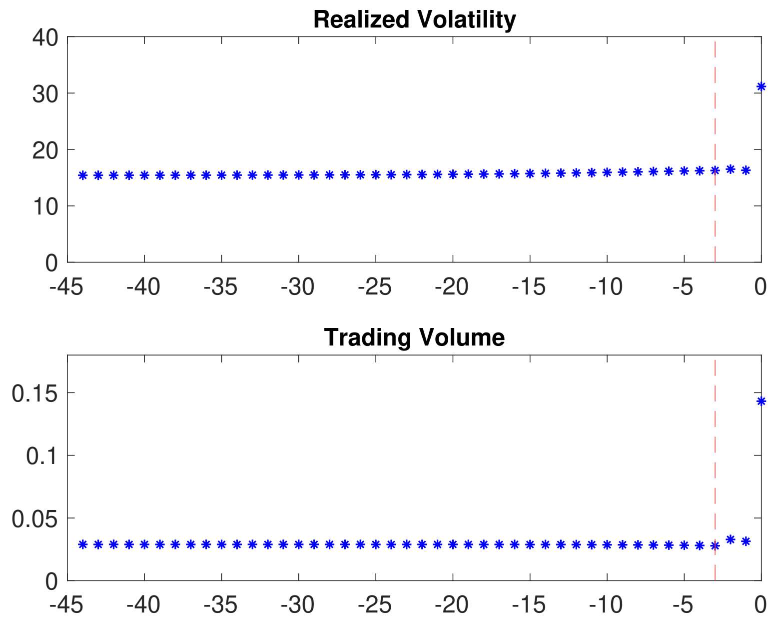
In both models, with and without information acquisition, $\phi_\Delta(t)$ monotonically decreases over time. The function $\phi_\Delta(t)$ has a clear interpretation in equation (11): it is the price impact of uninformed investors. In the economy without information acquisition, because the posterior variance, $\tilde{q}(t)$, monotonically increases over time, uninformed investors become more and more uncertain about $\hat{x}_t$ and, as a result, they trade less and less aggressively, and exert a lower and lower price impact over time.

In the model with information acquisition, $\phi_\Delta(t)$ decreases monotonically over time until time -3, when information acquisition starts. The information acquisition at time $\tau = -3$ has two effects on uninformed investors' price impact, $\phi_\Delta(t)$. On one hand, information acquisition lowers uncertainty and uninformed traders have an incentive to trade more aggressively. On the other hand, information acquisition creates an additional correlation between stock returns and the wealth of the uninformed investors. As a result, they trade less aggressively due to the hedging demand channel. In Figure (7), the hedging demand channel dominates, and the price impact $\phi_\Delta(t)$ keeps falling after τ.

Realized volatility and trading volume As we emphasize in Section 2, a key feature of the data is the low realized volatility and the low trading volume during the pre-announcement drift period. While the low realized volatility during the pre-announcement drift period is difficult to reconcile with an information leakage-based explanation, it is a robust feature of our model with endogenous information acquisition. Because the newly acquired information the uninformed investors obtained is about the market research, or informed investors' beliefs $\hat{x}_t$, which is known to informed investors and has already been incorporated into market prices. As shown in Figure 7, before time τ, the price impact of noise traders, $\phi_\theta(t)$ is monotonically increasing over time; that is, noise accumulates into prices. After time τ, $\phi_\theta(t)$ drops and noise is gradually eliminated from prices. As a result, the realized volatility during the period of pre-announcement drift in our model does not significantly increase.

In Figure 8, we plot the realized volatility of price (top panel) and the trading volume (bottom panel) implied by our model (see Appendix 6.5 for model solutions). There is no significant change in the realized volatility or the trading volume during the information acquisition period, or equivalently, the pre-announcement drift period. This is because the newly acquired information is not a leakage of new information about the upcoming announcement, but rather, it is the information that has already been incorporated into the market price due to informed trading. As a result, the

Figure 8: Realized Volatility and Trading Volume



The top panel plots the realized volatility (annualized in percentage) and the bottom panel depicts the trading volume or our benchmark economy with endogenous information acquisition (the bottom panels). The vertical line indicates the timing when uninformed investors start to acquire information. The horizon axis is the number of days before the upcoming announcement, which is normalized as 0. A -5 for example, stands for five days before announcements.

realized volatility of stock returns during the pre-announcement period is very similar to that on non-announcement days, as in the data. In addition, realized volatility and trading spike right after the announcement, as shown in the same figure, which is also consistent with the empirical evidence that we document in Section 2.

Correlation between pre- and post- announcement return Another stylized fact about the pre-FOMC announcement drift is the lack of correlation between pre- and post- announcement returns. Given the functional form of price in equation (12), the announcement return can be written as:

$$P_T^+ - P_T^- = \left[\phi\left(T^+\right) - \phi\left(T^-\right)\right] + \bar{\phi}_x\left(x_T - \tilde{x}_T^-\right) + \left[\phi_\theta\left(T^-\right) - \phi_\theta\left(T^+\right)\right]\tilde{\theta}_T. \tag{29}$$

The term $\phi\left(T^+\right) - \phi\left(T^-\right)$ is the announcement premium and is deterministic. The term $x_T - \tilde{x}_T$ is the innovation of the true value of x_T relative to its expectation, and therefore cannot be predictable by publicly available information. Announcement return will not be predictable unless the term $\tilde{\theta}_T$ is. The return realized during the pre-announcement period can be written as:

$$P_T - P_\tau = \left[\phi\left(T\right) - \phi\left(\tau\right)\right] + \bar{\phi}_x\left(\tilde{x}_T - \tilde{x}_\tau\right) + \left[\phi_\theta\left(\tau\right)\tilde{\theta}_\tau - \phi_\theta\left(T\right)\tilde{\theta}_T\right]. \tag{30}$$

In the above expression, the term $\phi(T) - \phi(\tau)$ is the pre-announcement drift, and the term $\tilde{x}_T - \tilde{x}_\tau$ is the innovation in the rational expectation about x_T. The last term is the noisy supply in prices. Because the process θ_t is mean reverting, the pre- and post- announcement return in the above expressions are actually slightly negatively correlated. In our calibrated example, this correlation is -0.003. This feature of our model also matches the empirical evidence well.

6 Conclusion

In this paper, we develop a noisy rational expectations model with endogenous information acquisition and periodic announcements to account for the pre-FOMC announcement drift puzzle. We show that the endogenous information acquisition together with the generalized risk sensitive preference not only allow us to provide an equilibrium interpretation of the pre-FOMC announcement drift but also the stylized facts of the volatility dynamics and the trading volume around the FOMC announcements. We argue that models with information leakage have counter-factual implications on the volatility dynamics around announcements, and on the correlation between pre- and post-announcement returns. Our model does not assume information leakage and matches the empirical patterns of the FOMC announcement returns and volatility dynamics in the data quite well.

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Appendix

6.1 Data

We obtain the pre-scheduled FOMC announcement days from Bloomberg. It includes both the dates and the exact release time. Following Lucca and Moench (2015), we focus on pre-scheduled FOMC meetings, and extend the sample period to September 2020. There are in total 213 scheduled FOMC meetings between January 1994 and September 2020. Before 2011, most FOMC announcements were scheduled around 14:15 p.m. Between 2011 to 2012, there were eight FOMC meetings arranged around 12:30 p.m. After March 2013, all the FOMC announcements were scheduled around 14:00.

We use high frequency data on E-mini S&P 500 index futures from the Chicago Mercantile Exchange (CME) which start from 11:30 a.m. EST, September 9, 1997.[3,4] We focus on the Emini data because it reports the trading volume and it is tradable over 24 hours. Before that, we use S&P 500 index futures instead from CME, available from April 21, 1982. On each day of the E-mini futures, there may be multiple contract delivery dates. We choose the delivery date with the highest volume within each calendar day as the most active futures contract, which is usually the nearest-term contract and occasionally the next contract during rolling forward weeks. We then convert the time zone to EST as the original time stamp is in CST. The raw data are cleaned following the standard procedures described in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009). First, we delete those entries outside regular trading hours (9:30 to 16:00). Second, we delete the invalid prices with missing values or equal to zero (or 9999.75). Third, we delete entries with canceled or corrected prices (exclude "CAN" = "C" or "X"). Last, within each time stamp (in seconds), we use the median price. If there are two median prices, we use the mean of these two medians. In this way, we obtain the time series of prices in one second. Fourth, in order to mitigate the microstructure noise, we sample the price into one minute frequency. The sampling method follows the "Last" scheme of Brownlees and Gallo (2006), where we pick the last entry of the period ending immediately prior to the timestamp. For example, 10:30 represents the last data from 10:29:00 to 10:29:59. After September 1997 when the Emini S&P 500 futures are available, we obtain the trading volume as the total contracts traded within the 1-min sampling interval. We delete the all the entries with 0 or missing trading volume.

We use log return on the futures from 24 hours before to five minutes before FOMC announcements as the pre-announcement drift. To measure the post-announcement return, we use log return from five minutes before FOMC announcements to one hour afterwards. For instance, the pre-announcement drift for the meeting at 14:00 on 2019 Dec.11 is defined as the log return from 13:55 on 2019 Dec.10 to 13:55 on 2019 Dec.11, whereas the post-announcement return is calculated as the log return from 13:55 to 14:55 on 2019 Dec.11. We report the summary statistics for pre- and post- announcement return in Table 1.

[3] https://www.cmegroup.com/confluence/display/EPICSANDBOX/Time+and+Sales. We use the calendar date (Entry Date) instead of the adjusted trading date (Trade Date).

[4] There are three missing dates from E-mini future data: October 29,1997, January 28 and 29, 2014. The last one is a pre-scheduled FOMC release day. We exclude these days in our analysis.

The average pre-FOMC announcement drift is 32.28 basis points with a Newey-West t-stat of 4.88. The coefficient of regressing pre-announcement drift on post-announcement return is -0.106, with a robust standard error of 0.048 (t-stat of -2.21). The ex-ante return is significantly negative correlated with ex-post announcement return.

We calculate realized variance as the sum of 30-minutes log squared return. The realized volatility at the minute t can be estimated by $\sigma_t = \sqrt{\frac{1}{N}\sum_{j=0}^{N-1} r_{t-j}^2}$, where N equals 30 if there is no missing values and r_t is the log return at time t (defined as the log price difference). For the trading volume M, we simply average over past 30 minutes total number of contracts traded: $M_t = \frac{1}{N}\sum_{j=0}^{N-1} volm_{t-j}$. We calculate the rolling realized volatility and trading volume for each minute.

Table 1: Summary Statistics

	Mean	St.Dev.	Min	Max	Obs.	Time
Pre-Ann Drift (%)	0.323	0.965	-3.098	8.639	212	1994Jan-2020Sep
Post-Ann Return (%)	0.056	0.056	-2.131	2.901	212	1994Jan-2020Sep
Realized Volatility (annualized in %)	13.360	9.634	1.865	144.628	184	1997Sep-2020Sep
Trading Volume (1000 shares)	2.473	2.397	0.004	26.011	184	1997Sep-2020Sep

This table reports summary statistics of pre-announcement drift, post-announcement return, realized volatility and trading volume on FOMC announcement days. We obtain log returns on S&P 500 futures during regular trading hours (9:30-16:00) from January 1994 to September 2020. Pre-Ann Drift stands for the log return in 24-hour windows from one day before the FOMC announcement to five minutes before the meeting. Post-Ann Return is the log return from 5 minutes before the FOMC announcement to one hour afterward. Realized volatility (annualized in percentage) is the average sum of squared returns over the past 30 minutes ($t = [-29, 0]$) and the trading volume is the average contracts traded during the past 30 minutes on FOMC days. We calculate the rolling realized volatility and trading volume for each minute from 10:00 to 16:00. The sample period is from September 1997 to September 2020.

6.2 Equilibrium Beliefs

The Filtering Problem of Informed The optimal learning for the informed investor is a standard Kalman filter problem with the unobserved state variable given in (1) and the observed processes (5), (6), and (7). Applying Theorem 10.3 from Liptser and Shiryaev (2001), it is straightforward to show that the law of motion of the posterior mean satisfies (9) where the innovation processes for (5) and (7) are given by

$$d\hat{B}_{D,t} = \frac{1}{\sigma_D}\left[dD_t - (\hat{x}_t - D_t)\,dt\right], \text{and } d\hat{B}_{s,t} = \frac{1}{\sigma_s}\left(ds_t - \hat{x}_t dt\right). \tag{31}$$

The law of motion of the conditional variance $\hat{q}_t$ must satisfy the Riccati equation

$$d\hat{q}(t) = \left[\sigma_x^2 - 2b\hat{q}(t) - \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\hat{q}^2(t)\right] dt. \tag{32}$$

We can solve $\hat{q}(t) = \frac{\sigma_x^2\left(1-e^{-2\hat{b}(t+t^*)}\right)}{\left(\hat{b}-b\right)e^{-2\hat{b}(t+t^*)}+b+\hat{b}}$, where $\hat{b} = \sqrt{b^2 + \sigma_x^2\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)}$ and $t^* = \frac{1}{2\hat{b}}\ln\frac{\sigma_x^2+\left(\hat{b}-b\right)\hat{q}(0)}{\sigma_x^2-\left(\hat{b}+b\right)\hat{q}(0)}$. We assume announcements fully resolve the uncertainty, so that $\hat{q}(0) = 0$.

Information Content of Prices In addition to observing the dividend, the uninformed trader also observes the prices process. We have assumed that the price process takes the form of equation (11).

Because the uninformed know D_t and $\tilde{x}_t$, observing the price is equivalent to observing $\zeta_t \equiv \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t$. Here, ζ_t can be represented as a Markov process given the state variable $\hat{x}_t, \zeta_t$:

$$\begin{aligned} d\zeta_t &= \left[b\bar{x}\phi_x(t) - a\bar{\theta}\phi_\theta(t) + \left(\left(a - b - \frac{\phi_\theta'(t)}{\phi_\theta(t)}\right)\phi_x(t) + \phi_x'(t)\right)\hat{x}_t + \left(\frac{\phi_\theta'(t)}{\phi_\theta(t)} - a\right)\zeta_t\right]dt \\ &\quad + \frac{\hat{q}(t)}{\sigma_D}\phi_x(t)\,d\hat{B}_{D,t} + \frac{\hat{q}(t)}{\sigma_s}\phi_x(t)\,d\hat{B}_{s,t} - \sigma_\theta\phi_\theta(t)\,dB_{\theta,t}, \end{aligned} \tag{33}$$

It is convenient to define $\xi_t = \zeta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t)D_t$ so that $(\hat{x}_t, D_t, \xi_t)$ has a state space representation. We first derive the law of motion of ξ_t under the objective measure, and then translate it into the subjective measure. The dynamics of ξ_t is

$$d\xi_t = \left[b\bar{x}\phi_x(t) - a\bar{\theta}\phi_\theta(t) + m_x(t)\hat{x}_t + \left(\frac{\phi_\theta'(t)}{\phi_\theta(t)} - a\right)\xi_t + m_D(t)D_t\right]dt + \sigma_\xi(t)\,d\hat{B}_{\xi,t}, \tag{34}$$

where the coefficients,

$$m_x(t) = \left(a - b - \frac{\phi_\theta'(t)}{\phi_\theta(t)} - \frac{\hat{q}(t)}{\sigma_D^2}\right)\phi_x(t) + \phi_x'(t), \tag{35}$$

$$m_D(t) = \frac{1}{\sigma_D^2}\left[\hat{q}(t)\phi_x(t)\left(1 - a + \frac{\phi_\theta'(t)}{\phi_\theta(t)}\right) - \hat{q}'(t)\phi_x(t) - \hat{q}(t)\phi_x'(t)\right], \tag{36}$$

and the volatility of ξ_t is

$$\sigma_\xi(t) = \sqrt{\frac{\hat{q}^2(t)}{\sigma_s^2}\phi_x^2(t) + \sigma_\theta^2\phi_\theta^2(t)} \tag{37}$$

and $\hat{B}_{\xi,t}$ is a standard Brownian motion that is independent of $\hat{B}_{D,t}$:

$$d\hat{B}_{\xi,t} = \frac{1}{\sigma_\xi(t)}\left[\frac{\hat{q}(t)}{\sigma_s}\phi_x(t)\,d\hat{B}_{s,t} - \sigma_\theta\phi_\theta(t)\,dB_{\theta,t}\right]. \tag{38}$$

We will call ξ_t the information content of price, as observing price is equivalent to observing ξ_t. From the informed investor's perspective, dividend flow follows

$$dD_t = (\hat{x}_t - D_t)\,dt + \sigma_D d\hat{B}_{D,t}. \tag{39}$$

The uninformed investors' belief can be characterized as follows

$$d\tilde{x}_t = b(\bar{x} - \tilde{x}_t)\,dt + \frac{\hat{q}(t) + \tilde{q}(t)}{\sigma_D}d\tilde{B}_{D,t} + \nu(t)\sigma_\xi(t)\,d\tilde{B}_{\xi,t} + \frac{\tilde{q}(t)}{\sigma_i(t)}d\tilde{B}_{i,t}, \tag{40}$$

$$d\tilde{\theta}_t = a\left(\bar{\theta} - \tilde{\theta}_t\right)dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}d\tilde{B}_{D,t} + [\phi_x(t)\nu(t) - 1]\frac{\sigma_\xi(t)}{\phi_\theta(t)}d\tilde{B}_{\xi,t} + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)}d\tilde{B}_{i,t}. \tag{41}$$

where

$$\nu\left(t\right)=\frac{1}{\sigma_{\xi}^{2}\left(t\right)}\left[\frac{\phi_{x}\left(t\right)}{\sigma_{s}^{2}}\hat{q}^{2}\left(t\right)+m_{x}\left(t\right)\tilde{q}\left(t\right)\right]. \tag{42}$$

and $d\tilde{B}_{D,t}=\frac{1}{\sigma_D}\left(dD_t-\tilde{\mathbb{E}}_t\left[dD_t\right]\right)$, $d\tilde{B}_{\xi,t}=\frac{1}{\sigma_\xi(t)}\left(d\xi_t-\tilde{\mathbb{E}}_t\left[d\xi_t\right]\right)$ and $d\tilde{B}_{i,t}=\frac{1}{\sigma_i(t)}\left(ds_{i,t}-\tilde{\mathbb{E}}_t\left[ds_{i,t}\right]\right)$ are innovations in the observation processes relative to expectations. More specifically,

$$d\tilde{B}_{D,t} = \frac{1}{\sigma_D}\left[dD_t-\left(\tilde{x}_t-D_t\right)dt\right] \tag{43}$$

$$d\tilde{B}_{\xi,t} = \frac{1}{\sigma_\xi\left(t\right)}\left[d\xi_t-\tilde{\mu}_\xi\left(t\right)dt\right] \tag{44}$$

$$d\tilde{B}_{i,t} = \frac{1}{\sigma_i\left(t\right)}\left[ds_{i,t}-\tilde{x}_t dt\right]. \tag{45}$$

where $\tilde{\mu}_\xi\left(t\right)=b\bar{x}\phi_x\left(t\right)-a\bar{\theta}\phi_\theta\left(t\right)+m_x\left(t\right)\tilde{x}_t+\left(\frac{\phi'_\theta(t)}{\phi_\theta(t)}-a\right)\xi_t+m_D\left(t\right)D_t$.

6.3 Robust valuation for the uninformed

Consider the following robust valuation problem for the uninformed investors.

$$V\left(\tilde{x}\right)=\min_{\mathcal{Q}}\tilde{\mathbb{E}}^{\mathcal{Q}}\left[\int_0^\infty e^{-\rho t}D_t dt\right]+\frac{1}{\kappa}\mathcal{R}\left(\mathcal{Q}\right),$$

where $\tilde{\mathbb{E}}$ represent the information set of the uninformed investors, $\mathcal{Q}$ represents the probability distortion, and $\mathcal{R}\left(Q\right)$ is the relative entropy of the probability distortion. Note that under the objective probability measure, the law of motion of the state variables in the above model include:

$$d\tilde{x}_t=b\left(\bar{x}-\tilde{x}_t\right)dt+\frac{\hat{q}\left(t\right)+\tilde{q}\left(t\right)}{\sigma_D}d\tilde{B}_{D,t}+\nu\left(t\right)\sigma_\xi\left(t\right)d\tilde{B}_{\xi,t}+\frac{\tilde{q}\left(t\right)}{\sigma_i\left(t\right)}d\tilde{B}_{i,t},$$

and,

$$dD_t=\left(\tilde{x}_t-D_t\right)dt+\sigma_D d\tilde{B}_{D,t}.$$

In the interior, the underlying uncertainty is generated by a vector of three Brownian motions, $d\tilde{B}_t=\left[d\tilde{B}_{D,t},d\tilde{B}_{\xi,t},\tilde{B}_{i,t}\right]$. To use a compact notation, we denote $\sigma_x\left(t\right)=\left[\frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D},\nu\left(t\right)\sigma_\xi\left(t\right),\frac{\tilde{q}(t)}{\sigma_i(t)}\right]$, and $\sigma_D\left(t\right)=\left[\sigma_D,0,0\right]$. This way the law of motion of the state variables can be written compactly: $d\tilde{x}_t=b\left(\bar{x}-\tilde{x}_t\right)dt+\sigma_x\left(t\right)d\tilde{B}_t$, and $dD_t=\left(\tilde{x}_t-D_t\right)dt+\sigma_D d\tilde{B}_t$. The probability distortion can be constructed from the Girsanov theorem. That is, the density of the distorted probability can be constructed by the exponential martingale. In addition, under the distorted probability measure, $d\tilde{B}_t^h=d\tilde{B}_t-h_t dt$ is a vector of standard Brownian motion. The minimizing probability is pinned down once we determine the adapted h process, which solves the following HJB:

$$\rho V(t,\tilde{x},D)=\min_{h}\Big\{D+\frac{\theta}{2}h^{T}h+V_{t}(t,\tilde{x},D)+\left[b(\bar{x}-\tilde{x})+h^{T}\sigma_{x}(t)\right]V_{x}(t,\tilde{x},D)$$
$$+\left[(\tilde{x}-D)+h^{T}\sigma_{D}\right]V_{D}(t,\tilde{x},D)+\frac{1}{2}\sigma_{x}^{T}(t)\,\sigma_{x}(t)\,V_{xx}(t,\tilde{x},D)$$
$$+\frac{1}{2}\sigma_{D}^{T}\sigma_{D}V_{DD}(t,\tilde{x},D)+\left[\tilde{q}(t)+\hat{q}(t)\right]V_{x,D}(t,\tilde{x},D)\Big\}$$

We conjecture and verify that the value function must be of the following form:

$$V(t,\tilde{x},D)=\phi(t)+\bar{\phi}_{x}\tilde{x}+\phi_{D}D, \tag{46}$$

where $\bar{\phi}_x$ and ϕ_D are the same as in the main model, and $\phi(t)$ is to be determined by the HJB equations. The resulting minimizing probability measure is given by the probability distortions:

$$h=-\kappa\left[\bar{\phi}_{x}\frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_{D}}+\phi_{D}\sigma_{D},\bar{\phi}_{x}\nu(t)\,\sigma_{\xi}(t)\,,\bar{\phi}_{x}\frac{\tilde{q}(t)}{\sigma_{i}(t)}\right].$$

We summarize our results as follows.

Claim 1. Under the distorted probability measure,

$$\left[d\tilde{B}_{D,t},d\tilde{B}_{\xi,t},\tilde{B}_{i,t}\right]+\kappa\left[\bar{\phi}_{x}\frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_{D}}+\phi_{D}\sigma_{D},\bar{\phi}_{x}\nu(t)\,\sigma_{\xi}(t)\,,\bar{\phi}_{x}\frac{\tilde{q}(t)}{\sigma_{i}(t)}\right]$$

is a standard BM.

On the boundary, the robust valuation problem can simply be written as the following:

$$V\left(T^{-},\tilde{x}_{T}^{-},D\right)=\min_{m}\left\{\mathbb{E}\left[m\left\{V\left(T^{+},x_{T},D\right)+\frac{1}{\kappa}\ln m\right\}\right]\right\}.$$

The solution to this problem is of course

$$m^{*}=\frac{e^{-\frac{V\left(T^{+},x_{T},D\right)}{1/\kappa}}}{\mathbb{E}\left[e^{-\frac{V\left(T^{+},x_{T},D\right)}{1/\kappa}}\right]}. \tag{47}$$

Given the functional form of value function in (46), we have:

$$m^{*}=\frac{e^{-\kappa\bar{\phi}_{x}x_{T}}}{\mathbb{E}\left[e^{-\kappa\bar{\phi}_{x}x_{T}}\right]}. \tag{48}$$

We can easily show that x_T is normally distributed with mean $-\kappa\bar{\phi}_x\left[\hat{q}(T)+\tilde{q}(T)\right]$ and variance $\hat{q}(T)+\tilde{q}(T)$ using the following lemma. In addition, if we break the above valuation problem into two steps, we have, in the first step, $\hat{x}$ is revealed:

$$V\left(T^{-},\tilde{x}_{T}^{-},D\right)=\min\left\{\mathbb{E}\left[m\left\{V^{1}\left(\hat{T},\hat{x}_{T},D\right)+\frac{1}{\kappa}\ln m\right\}\right]\right\}.$$

The above problem allows us to calculate the distribution of $\hat{x}$ given $\tilde{x}$. In the second step, conditioning on $\hat{x}$, we have:

$$V^{1}\left(\hat{T},\hat{x}_{T},D\right)=\min\left\{\mathbb{E}\left[m\left\{V\left(T^{+},x_{T},D\right)+\frac{1}{\kappa}\ln m\right\}\middle|\,\hat{x}\right]\right\}.$$

This allows us to calculation the conditional distribution of x_T given $\hat{x}$. We first summarize our results as follows.

Claim 2. Under the distorted probability measure,

1. $\hat{x}_T$ is normally distributed with mean $\tilde{x}_T^{-}-\kappa\bar{\phi}_x\tilde{q}\left(T\right)$ and variance $\tilde{q}\left(T\right)$.

2. x_T conditioning on $\hat{x}_T$ is normally distributed with mean $\hat{x}_T-\kappa\bar{\phi}_x\hat{q}\left(T\right)$ and variance $\hat{q}\left(T\right)$.

3. x_T is normally distributed with mean $\tilde{x}_T^{-}-\kappa\bar{\phi}_x\left[\hat{q}\left(T\right)+\tilde{q}\left(T\right)\right]$ and variance $\hat{q}\left(T\right)+\tilde{q}\left(T\right)$.

The Distorted Belief of Uninformed Because of the ambiguity aversion, the uninformed investors' beliefs are distorted. They think

$$d\tilde{B}_{D,t} = d\tilde{B}_{D,t}^{\kappa}-\kappa_D\left(t\right)dt \tag{49}$$

$$d\tilde{B}_{\xi,t} = d\tilde{B}_{\xi,t}^{\kappa}-\kappa_\xi\left(t\right)dt \tag{50}$$

$$d\tilde{B}_{i,t} = d\tilde{B}_{i,t}^{\kappa}-\kappa_i\left(t\right)dt \tag{51}$$

where

$$\kappa_D\left(t\right) = \kappa\left(\bar{\phi}_x\frac{\hat{q}\left(t\right)+\tilde{q}\left(t\right)}{\sigma_D}+\phi_D\sigma_D\right) \tag{52}$$

$$\kappa_\xi\left(t\right) = \kappa\bar{\phi}_x\nu\left(t\right)\sigma_\xi\left(t\right) \tag{53}$$

$$\kappa_i\left(t\right) = \kappa\bar{\phi}_x\frac{\tilde{q}\left(t\right)}{\sigma_i\left(t\right)} \tag{54}$$

where $dB_{D,t}^{\kappa}$, $dB_{\xi,t}^{\kappa}$, $dB_{i,t}^{\kappa}$, are mutually independent Brownian motions.

To summarize, under the distorted beliefs, the law of motion of the state variables should follow

Lemma 1. *The uninformed trader's learning problem can be written as follows. The state variables* *are*

$$d\tilde{x}_t = [b(\bar{x}-\tilde{x}_t)-\kappa_x(t)]\,dt + \frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D} d\tilde{B}^{\kappa}_{D,t} + \nu(t)\sigma_\xi(t)\,d\tilde{B}^{\kappa}_{\xi,t} + \frac{\tilde{q}(t)}{\sigma_i(t)} d\tilde{B}^{\kappa}_{i,t}, \tag{55}$$

$$\begin{aligned} d\tilde{\theta}_t &= \left[a\left(\bar{\theta}-\tilde{\theta}_t\right)-\kappa_\theta(t)\right]dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D} d\tilde{B}^{\kappa}_{D,t} \\ &\quad + [\phi_x(t)\nu(t)-1]\frac{\sigma_\xi(t)}{\phi_\theta(t)} d\tilde{B}^{\kappa}_{\xi,t} + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)} d\tilde{B}^{\kappa}_{i,t}, \end{aligned} \tag{56}$$

where

$$\kappa_x(t) = \kappa_D(t)\frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D} + \kappa_\xi(t)\nu(t)\sigma_\xi(t) + \kappa_i(t)\frac{\tilde{q}(t)}{\sigma_i(t)} \tag{57}$$

$$\kappa_\theta(t) = \kappa_D(t)\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D} + \kappa_\xi(t)[\phi_x(t)\nu(t)-1]\frac{\sigma_\xi(t)}{\phi_\theta(t)} + \kappa_i(t)\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)} \tag{58}$$

and the observation processes are:

$$dD_t = [\tilde{x}_t - D_t - \sigma_D\kappa_D(t)]\,dt + \sigma_D d\tilde{B}^{\kappa}_{D,t} \tag{59}$$

$$d\xi_t = [\tilde{\mu}_\xi(t) - \sigma_\xi(t)\kappa_\xi(t)]\,dt + \sigma_\xi(t)\,d\tilde{B}^{\kappa}_{\xi,t} \tag{60}$$

$$ds_{i,t} = [\tilde{x}_t - \sigma_i(t)\kappa_i(t)]\,dt + \sigma_i(t)\,d\tilde{B}_{i,t}. \tag{61}$$

The posterior variance is

$$d\tilde{q}(t) = \left[\left(\frac{1}{\sigma_D^2}+\frac{1}{\sigma_s^2}\right)\hat{q}_t^2 - 2b\tilde{q}_t - \frac{1}{\sigma_i^2(t)}\tilde{q}_t^2 - \frac{(\hat{q}_t+\tilde{q}_t)^2}{\sigma_D^2} - \left(\frac{m_x(t)\tilde{q}_t + \frac{\phi_x(t)\hat{q}_t^2}{\sigma_s^2}}{\sigma_\xi}\right)^2\right]dt. \tag{62}$$

Joint Distributions From the perspective of the informed, $x_t|s^t \sim N(\hat{x}_t, \hat{q}_t)$, and both $\hat{x}_t$ and θ_t are observable. Below, we derive the joint distribution of $[x_t, \hat{x}_t, \theta_t]$ from the perspective of the uninformed investors.

We deal with the interior and the boundary separately. In the interior, beliefs are continuous and there is no probability distortion over an infinitesimal interval. Obviously, under the belief of the uninformed, $\hat{x}_t|s^t \sim \mathcal{N}(\tilde{x}_t, \tilde{q}_t)$. By law of iterated expectation, $\tilde{\mathbb{E}}(x_t) = \tilde{x}_t$. In addition, $\tilde{V}ar[x_t] = \tilde{q}_t + \hat{q}_t$. That is, from the perspective of the uninformed, $x_t|s^t \sim \mathcal{N}(\tilde{x}_t, \tilde{q}_t + \hat{q}_t)$.

We are now ready to derive the conditional distribution for θ_t. Note that the learning identity implies $\zeta_t \equiv \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t$, that is, $\theta_t = \frac{1}{\phi_{\theta,t}}(\phi_{x,t}\hat{x}_t - \zeta_t)$. Hence, $\tilde{\mathbb{E}}(\theta_t) = \frac{1}{\phi_{\theta,t}}(\phi_{x,t}\tilde{x}_t - \zeta_t)$. In addition, $\tilde{V}ar[\theta_t] = \left(\frac{\phi_{x,t}}{\phi_{\theta,t}}\right)^2 \tilde{q}_t$.

We can also compute covariance.

$$\tilde{C}ov(x_t, \hat{x}_t) = \tilde{C}ov(x_t, \hat{x}_t) = \tilde{q}_t.$$

In addition,

$$\tilde{Cov}\left(x_t,\theta_t\right)=\tilde{Cov}\left[x_t,\frac{1}{\phi_{\theta,t}}\left(\phi_{x,t}\hat{x}_t-\zeta_t\right)\right]=\frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t, \tag{63}$$

and $\tilde{Cov}\left(\hat{x}_t,\theta_t\right)=\frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t$.

On the boundary, however, there is probability distortion. As we have demonstrated, from the perspective of the uninformed,

$$\hat{x}_T|s^T\sim\mathcal{N}\left(\tilde{x}_T-\kappa\bar{\phi}_x\tilde{q}_T,\tilde{q}_T\right);\ x_T|s^T\sim\mathcal{N}\left(\tilde{x}_T-\kappa\bar{\phi}_x\left(\tilde{q}_T+\hat{q}_T\right),\tilde{q}_T+\hat{q}_T\right).$$

To compute the distribution of θ_T, we have $\tilde{\mathbb{E}}\left(\theta_T|\,\hat{x}_T^-\right)=\theta_T$ because once the uninformed know $\hat{x}_t$, they will know θ_t from the learning identity, and

$$\tilde{\mathbb{E}}\left[\tilde{\mathbb{E}}\left(\theta_T|\,\hat{x}_T^-\right)\right]=\frac{1}{\phi_{\theta,T}}\tilde{\mathbb{E}}\left(\phi_{x,T}\hat{x}_T^--\zeta_T\right).$$

This is to say, there is probability distortion in the first step, but no probability distortion after we conditioning on $\hat{x}$. In the first step, we have:

$$\begin{aligned}\tilde{\mathbb{E}}\left[\tilde{\mathbb{E}}\left(\theta_T|\,\hat{x}_T^-\right)\right]&=\frac{1}{\phi_{\theta,T}}\tilde{\mathbb{E}}\left(\phi_{x,T}\hat{x}_T^--\zeta_T\right)\\&=\tilde{\theta}_T^--\frac{\phi_{x,T}}{\phi_{\theta,T}}\kappa\bar{\phi}_x\tilde{q}\left(T\right).\end{aligned}$$

The last equality is true, because in the interior, the equality $\zeta_T^-\equiv\phi_x\left(T^-\right)\tilde{x}_T^--\phi_\theta\left(T^-\right)\tilde{\theta}_T^-$.

Difference in Beliefs Define the difference in beliefs $\Delta_t\equiv\hat{x}_t-\tilde{x}_t$. Because the informed do not have ambiguity, the law of motion of Δ_t under the informed investors information set is

$$d\Delta_t=-a_\Delta\left(t\right)\Delta_tdt-\sigma_{\Delta D}\left(t\right)d\hat{B}_{D,t}+\sigma_{\Delta s}\left(t\right)d\hat{B}_{s,t}+\sigma_{\Delta\theta}\left(t\right)dB_{\theta,t}-\sigma_{\Delta i}\left(t\right)dB_{i,t}, \tag{64}$$

where the coefficients are:

$$a_\Delta\left(t\right)=b+\frac{\hat{q}\left(t\right)+\tilde{q}\left(t\right)}{\sigma_D^2}+\nu\left(t\right)m_x\left(t\right)+\frac{\tilde{q}_t}{\sigma_i\left(t\right)}, \tag{65}$$

$$\sigma_{\Delta D}\left(t\right)=\frac{\tilde{q}\left(t\right)}{\sigma_D},$$

$$\sigma_{\Delta s}\left(t\right)=\frac{\hat{q}\left(t\right)}{\sigma_s}\left[1-\phi_x\left(t\right)\nu\left(t\right)\right],$$

$$\sigma_{\Delta\theta}\left(t\right)=\phi_\theta\left(t\right)\nu\left(t\right)\sigma_\theta.$$

$$\sigma_{\Delta i}\left(t\right)=\frac{\tilde{q}\left(t\right)}{\sigma_i\left(t\right)}. \tag{66}$$

Note that compared to Han (2020), $d\Delta_t$ has a downward trend. The uninformed think that the informed are over optimistic, and therefore, $\tilde{x}_t$ typically increases faster than what $\hat{x}_t$ actually is.

Excess Returns In this subsection, we use the results from the filtering problem derived above to derive the excess return of the stock as diffusion processes under three types of investors' beliefs. We have conjectured that the equilibrium price is of the form (11). In order to solve for the optimal portfolio choice, we need to compute investors' belief about the return process. In the interior, this means we need to represent instantaneous excess return $dQ_t = dP_t + D_t dt - rP_t dt$ as functions of investors' own Brownian motions. On the boundary, we need to compute the conditional distribution of $P_T^+ - P_T^-$ from investors' own beliefs. Consider first the informed investors. Equations (39), (6), (9), and (15) represent the variables D_t, θ_t, $\hat{x}_t$, and Δ_t in terms of Brownian motions with respect to their information set. These give

$$\begin{aligned} dQ_t &= \{e_0(t) + [1-(1+r)\phi_D(t)]D_t + e_\theta(t)\theta_t + [\phi_D - (b+r)\phi_x]\hat{x}_t + e_\Delta(t)\Delta_t\}dt \\ &\quad + \varrho_D(t)d\hat{B}_{D,t} + \varrho_s(t)d\hat{B}_{s,t} + \varrho_\theta(t)dB_{\theta,t} + \varrho_i(t)dB_{i,t}, \end{aligned} \tag{67}$$

where

$$e_0(t) = \phi'(t) - r\phi(t) + b\bar{x}\bar{\phi}_x - a\bar{\theta}\phi_\theta(t) \tag{68}$$

$$e_\theta(t) = (a+r)\phi_\theta(t) - \phi'_\theta(t), \tag{69}$$

$$e_\Delta(t) = (a_\Delta(t)+r)\phi_\Delta(t) - \phi'_\Delta(t), \tag{70}$$

$$\varrho_D(t) = \phi_D\sigma_D + \bar{\phi}_x\frac{\hat{q}(t)}{\sigma_D} + \phi_\Delta(t)\sigma_{\Delta D}(t), \tag{71}$$

$$\varrho_s(t) = [1+\phi_\Delta(t)\nu(t)]\phi_x(t)\frac{\hat{q}_t}{\sigma_s}, \tag{72}$$

$$\varrho_\theta(t) = -[1+\phi_\Delta(t)\nu(t)]\phi_\theta(t)\sigma_\theta, \tag{73}$$

$$\varrho_i(t) = \phi_\Delta(t)\frac{\tilde{q}_t}{\sigma_i(t)}. \tag{74}$$

Note that before the information acquisition, all uninformed have homogeneous information, and price is of the form (17). Further define the variance of excess return as

$$\sigma_P(t) = \varrho_D^2(t) + \varrho_s^2(t) + \varrho_\theta^2(t) + \varrho_i^2(t). \tag{75}$$

The market clearing condition implies that the expected return of the stock cannot depend on D_t, $\hat{x}_t$ and the constant. As a result, the coefficients them must be 0, implying

$$\phi_D = \frac{1}{1+r}, \text{ and } \bar{\phi}_x = \frac{\phi_D}{b+r}. \tag{76}$$

Similarly, we can use equations (19), and (13) to write the excess return in terms of Brownian motions with respect to the uninformed investor's information set. This gives

$$dQ_t = \left[e_0(t) + e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)d\tilde{B}_{D,t} + \varrho_\xi(t)d\tilde{B}_{\xi,t} + \varrho_i(t)d\tilde{B}_{i,t}. \tag{77}$$

where

$$\varrho_\xi(t) = -\frac{\sigma_\xi(t)}{\sigma_\theta \phi_\theta(t)} \varrho_\theta(t). \tag{78}$$

It is equivalent to write the excess return under the distorted beliefs of uninformed investors as

$$dQ_t = \left[e_0(t) - e_1(t) + e_\theta(t)\tilde{\theta}_t\right] dt + \varrho_D(t)\, d\tilde{B}^\kappa_{D,t} + \varrho_\xi(t)\, d\tilde{B}^\kappa_{\xi,t} + \varrho_i(t)\, d\tilde{B}^\kappa_{i,t}, \tag{79}$$

where

$$e_1(t) = \varrho_D(t)\kappa_D(t) + \varrho_\xi(t)\kappa_\xi(t) + \varrho_i(t)\kappa_i(t). \tag{80}$$

6.4 Optimal Portfolio Choice Decisions

Portfolio Demand for the Informed: Interior The optimization problem for the informed investor in the interior is written as

$$\begin{aligned}
\hat{V}\left(t, \hat{W}, \theta, \Delta\right) &= \max_{\alpha, \hat{C}_t} \hat{\mathbb{E}}\left[\int_0^{T-t} -e^{-\rho s - \gamma \hat{C}_{t+s}} ds + e^{-\rho(T-t)} V^-\left(T, \hat{W}_T, \theta_T, \Delta_T\right)\right] \\
s.t.\ d\hat{W}_t &= \left(\hat{W}_t r - \hat{C}_t\right) dt + \alpha_t dQ_t \\
dQ_t &= \left[e_0(t) + e_\theta(t)\theta_t + e_\Delta(t)\Delta_t\right] dt + \varrho_D(t)\, d\hat{B}_{D,t} + \varrho_s(t)\, d\hat{B}_{s,t} + \varrho_\theta(t)\, dB_{\theta,t} + \varrho_i(t)\, dB_{i,t}, \\
d\theta_t &= a\left(\bar{\theta} - \theta_t\right) dt + \sigma_\theta dB_{\theta,t}, \\
d\Delta_t &= -a_\Delta(t)\Delta_t dt - \sigma_{\Delta D}(t)\, d\hat{B}_{D,t} + \sigma_{\Delta s}(t)\, d\hat{B}_{s,t} + \sigma_{\Delta\theta}(t)\, dB_{\theta,t} - \sigma_{\Delta i}(t)\, dB_{i,t}.
\end{aligned}$$

Conjecture the informed investor's value function takes the form of $\hat{V}\left(t, \hat{W}, \theta, \Delta\right) = -e^{-r\gamma \hat{W} - g(t,\theta,\Delta)}$, where

$$g(t, \theta, \Delta) = g(t) + g_\theta(t)\theta_t + \frac{1}{2} g_{\theta\theta}(t)\theta_t^2 + g_\Delta(t)\Delta_t + \frac{1}{2} g_{\Delta\Delta}(t)\Delta_t^2 + g_{\theta\Delta}(t)\theta_t\Delta_t. \tag{81}$$

Using Ito's Lemma, the HJB equation is:

$$\begin{aligned}
\rho J &= -e^{-\gamma \hat{C}} + \hat{V}_t + \hat{V}_W\left[r\hat{W} - \hat{C} + \alpha\left(e_0(t) + e_\theta(t)\theta + e_\Delta(t)\Delta\right)\right] + \frac{1}{2}\hat{V}_{WW}\alpha^2\sigma_P(t) + \alpha\hat{V}_{W\theta}\sigma_\theta\varrho_\theta(t) \\
&\quad + \alpha\hat{V}_{W\Delta}\sigma_{Q\Delta}(t) + \hat{V}_\theta a\left(\bar{\theta} - \theta\right) - \hat{V}_\Delta\left(a_\Delta(t)\Delta + b_\Delta(t)\right) + \frac{1}{2}\hat{V}_{\theta\theta}\sigma_\theta^2 + \frac{1}{2}\hat{V}_{\Delta\Delta}\sigma_\Delta(t) + \hat{V}_{\Delta\theta}\sigma_\theta\sigma_{\Delta\theta}(t),
\end{aligned}$$

where

$$\begin{aligned}
\sigma_\Delta(t) &= \sigma_{\Delta D}^2(t) + \sigma_{\Delta s}^2(t) + \sigma_{\Delta\theta}^2(t) + \sigma_{\Delta i}^2(t) \\
\sigma_{Q\Delta}(t) &= -\varrho_D(t)\sigma_{\Delta D}(t) + \varrho_s(t)\sigma_{\Delta s}(t) + \varrho_\theta(t)\sigma_{\Delta\theta}(t) - \varrho_i(t)\sigma_{\Delta i}(t),
\end{aligned} \tag{82}$$

Under the guessed value function form, the first order conditions (FOCs) with respect to $\hat{C}$ and α are

$$\hat{C}_t = r\hat{W} + \frac{1}{\gamma}\left[g\left(t,\theta,\Delta\right) - \ln r\right], \tag{83}$$

$$\alpha_t = \frac{\left[\begin{array}{c} e_0\left(t\right) + e_\theta\left(t\right)\theta + e_\Delta\left(t\right)\Delta - \left(g_\theta\left(t\right) + g_{\theta\theta}\left(t\right)\theta_t + g_{\theta\Delta}\left(t\right)\Delta_t\right)\sigma_\theta\varrho_\theta\left(t\right) \\ -\left(g_\Delta\left(t\right) + g_{\Delta\Delta}\left(t\right)\Delta_t + g_{\theta\Delta}\left(t\right)\theta_t\right)\sigma_{Q\Delta}\left(t\right) \end{array}\right]}{r\gamma\sigma_P\left(t\right)} \tag{84}$$

substituting expressions in (82) yields the demand function of the form:

$$\alpha_t = \alpha_0\left(t\right) + \alpha_\theta\left(t\right)\theta_t + \alpha_\Delta\left(t\right)\Delta_t, \tag{85}$$

where

$$\alpha_0\left(t\right) = \frac{e_0\left(t\right) - g_\theta\left(t\right)\sigma_\theta\varrho_\theta\left(t\right) - \sigma_{Q\Delta}\left(t\right)g_\Delta\left(t\right)}{r\gamma\sigma_P\left(t\right)} \tag{86}$$

$$\alpha_\theta\left(t\right) = \frac{e_\theta\left(t\right) - \varrho_\theta\left(t\right)\sigma_\theta g_{\theta\theta}\left(t\right) - \sigma_{Q\Delta}\left(t\right)g_{\theta\Delta}\left(t\right)}{r\gamma\sigma_P\left(t\right)} \tag{87}$$

$$\alpha_\Delta\left(t\right) = \frac{e_\Delta\left(t\right) - \varrho_\theta\left(t\right)\sigma_\theta g_{\theta\Delta}\left(t\right) - \sigma_{Q\Delta}\left(t\right)g_{\Delta\Delta}\left(t\right)}{r\gamma\sigma_P\left(t\right)}. \tag{88}$$

Matching coefficients of the value function, and use $\alpha_0\left(t\right)$, $\alpha_\theta\left(t\right)$ and $\alpha_\Delta\left(t\right)$ to simplify, we have the following odes system,

$$\begin{aligned} g'\left(t\right) &= r - \rho - r\ln r + rg\left(t\right) - \frac{1}{2}r^2\gamma^2\sigma_P\left(t\right)\alpha_0^2\left(t\right) + \frac{1}{2}\sigma_\theta^2\left[g_\theta^2\left(t\right) - g_{\theta\theta}\left(t\right)\right] \\ &\quad + \frac{1}{2}\sigma_\Delta\left(t\right)\left[g_\Delta^2\left(t\right) - g_{\Delta\Delta}\left(t\right)\right] + \sigma_\theta\sigma_{\Delta\theta}\left(t\right)\left[g_\theta\left(t\right)g_\Delta\left(t\right) - g_{\theta\Delta}\left(t\right)\right] - a\bar{\theta}g_\theta\left(t\right), \end{aligned} \tag{89}$$

$$g'_{\theta\theta}\left(t\right) = rg_{\theta\theta}\left(t\right) - r^2\gamma^2\sigma_P\left(t\right)\alpha_\theta^2\left(t\right) + 2ag_{\theta\theta}\left(t\right) + \sigma_\theta^2 g_{\theta\theta}^2\left(t\right) + \sigma_\Delta\left(t\right)g_{\theta\Delta}^2\left(t\right) + 2\sigma_\theta\sigma_{\Delta\theta}\left(t\right)g_{\theta\theta}\left(t\right)g_{\theta\Delta}\left(t\right), \tag{90}$$

$$\begin{aligned} g'_{\Delta\Delta}\left(t\right) &= rg_{\Delta\Delta}\left(t\right) - r^2\gamma^2\sigma_P\left(t\right)\alpha_\Delta^2\left(t\right) + 2a_\Delta\left(t\right)g_{\Delta\Delta}\left(t\right) + \sigma_\theta^2 g_{\theta\Delta}^2\left(t\right) + \sigma_\Delta\left(t\right)g_{\Delta\Delta}^2\left(t\right) \\ &\quad + 2\sigma_\theta\sigma_{\Delta\theta}\left(t\right)g_{\theta\Delta}\left(t\right)g_{\Delta\Delta}\left(t\right), \end{aligned} \tag{91}$$

$$\begin{aligned} g'_{\theta\Delta}\left(t\right) &= rg_{\theta\Delta}\left(t\right) - r^2\gamma^2\sigma_P\left(t\right)\alpha_\theta\left(t\right)\alpha_\Delta\left(t\right) + ag_{\theta\Delta}\left(t\right) + a_\Delta\left(t\right)g_{\theta\Delta}\left(t\right) + \sigma_\theta^2 g_{\theta\theta}\left(t\right)g_{\theta\Delta}\left(t\right) \\ &\quad + \sigma_\Delta\left(t\right)g_{\Delta\Delta}\left(t\right)g_{\theta\Delta}\left(t\right) + \sigma_\theta\sigma_{\Delta\theta}\left(t\right)\left[g_{\theta\theta}\left(t\right)g_{\Delta\Delta}\left(t\right) + g_{\theta\Delta}^2\left(t\right)\right]; \end{aligned} \tag{92}$$

$$\begin{aligned} g'_\theta\left(t\right) &= rg_\theta\left(t\right) - r^2\gamma^2\sigma_P\left(t\right)\alpha_0\left(t\right)\alpha_\theta\left(t\right) + ag_\theta\left(t\right) + \sigma_\theta^2 g_\theta\left(t\right)g_{\theta\theta}\left(t\right) \\ &\quad + \sigma_\Delta\left(t\right)g_\Delta\left(t\right)g_{\theta\Delta}\left(t\right) + \sigma_\theta\sigma_{\Delta\theta}\left(t\right)\left[g_\theta\left(t\right)g_{\theta\Delta}\left(t\right) + g_{\theta\theta}\left(t\right)g_\Delta\left(t\right)\right] - a\bar{\theta}g_{\theta\theta}, \end{aligned} \tag{93}$$

$$\begin{aligned} g'_\Delta\left(t\right) &= rg_\Delta\left(t\right) - r^2\gamma^2\sigma_P\left(t\right)\alpha_0\left(t\right)\alpha_\Delta\left(t\right) + a_\Delta\left(t\right)g_\Delta\left(t\right) + \sigma_\theta^2 g_\theta\left(t\right)g_{\theta\Delta}\left(t\right) \\ &\quad + \sigma_\Delta\left(t\right)g_\Delta\left(t\right)g_{\Delta\Delta}\left(t\right) + \sigma_\theta\sigma_{\Delta\theta}\left(t\right)\left[g_\theta\left(t\right)g_{\Delta\Delta}\left(t\right) + g_{\theta\Delta}\left(t\right)g_\Delta\left(t\right)\right] - a\bar{\theta}g_{\theta\Delta}, \end{aligned} \tag{94}$$

Portfolio demand for the uninformed investors who never acquire information: interior
The optimization problem of the uninformed investors is characterized as:

$$\begin{aligned}
\tilde{V}\left(t,\tilde{W},\tilde{\theta}\right) &= \max_{\beta_t,\tilde{C}_t}\tilde{\mathbb{E}}\left[\int_0^{T-t} -e^{-\rho s-\gamma\tilde{C}_{t+s}}ds + e^{-\rho(T-t)}\tilde{V}^-\left(T,\tilde{W}_T,\tilde{\theta}_T\right)\right] \\
s.t.\ d\tilde{W}_t &= \left(\tilde{W}_t r - \tilde{C}_t\right)dt + \beta_t dQ_t \\
dQ_t &= \left[e_0(t) - e_1(t) + e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)\,d\tilde{B}^\kappa_{D,t} + \varrho_\xi(t)\,d\tilde{B}^\kappa_{\xi,t} + \varrho_i(t)\,d\tilde{B}^\kappa_{i,t}, \\
d\tilde{\theta}_t &= \left[a\left(\bar{\theta}-\tilde{\theta}_t\right) - \kappa_\theta(t)\right]dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}d\tilde{B}^\kappa_{D,t} + \left[\phi_x(t)\nu(t)-1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}d\tilde{B}^\kappa_{\xi,t} + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)}d\tilde{B}^\kappa_{i,t}.
\end{aligned}$$

Conjecture the uninformed investor's value function would be of the form: $\tilde{V}\left(t,\tilde{W},\tilde{\theta}\right) = -e^{-r\gamma\tilde{W}-f\left(t,\tilde{\theta}\right)}$, where

$$f\left(t,\tilde{\theta}\right) = f(t) + f_\theta(t)\tilde{\theta}_t + \frac{1}{2}f_{\theta\theta}(t)\tilde{\theta}_t^2. \tag{95}$$

The HJB of the above problem is written as:

$$\begin{aligned}
\rho\tilde{V} &= -e^{-\gamma\tilde{C}} + \tilde{V}_t + \tilde{V}_W\left[r\tilde{W} - \tilde{C} + \beta\left(e_0(t) - e_1(t) + e_\theta(t)\tilde{\theta}\right)\right] \\
&\quad + \frac{1}{2}\tilde{V}_{WW}\beta^2\sigma_P(t) + \beta\tilde{V}_{W\theta}\sigma_{Q\theta}(t) + \tilde{V}_\theta\left[a\left(\bar{\theta}-\tilde{\theta}\right) - \kappa_\theta(t)\right] + \frac{1}{2}\tilde{V}_{\theta\theta}\sigma_{\theta\theta}(t)
\end{aligned}$$

where

$$\sigma_{Q\theta}(t) = \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}\varrho_D(t) + \left(\phi_x(t)\nu(t)-1\right)\frac{\sigma_\xi(t)}{\phi_\theta(t)}\varrho_\xi(t) + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)}\varrho_i(t) \tag{96}$$

$$\sigma_{\theta\theta}(t) = \left[\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}\right]^2 + \left[\left(\phi_x(t)\nu(t)-1\right)\frac{\sigma_\xi(t)}{\phi_\theta(t)}\right]^2 + \left[\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_i(t)}\right]^2. \tag{97}$$

Under the guessed value function form, the FOCs are

$$\tilde{C}_t = r\tilde{W} + \frac{1}{\gamma}\left[f\left(t,\tilde{\theta}_t\right) - \ln r\right]. \tag{98}$$

$$\beta_t = \frac{e_0(t) - e_1(t) + e_\theta(t)\tilde{\theta} - \left(f_\theta(t) + f_{\theta\theta}(t)\tilde{\theta}_t\right)\sigma_{Q\theta}(t)}{r\gamma\sigma_P(t)} \tag{99}$$

We then obtain the demand function of the form

$$\beta_t = \beta_0(t) + \beta_\theta(t)\tilde{\theta}_t. \tag{100}$$

where

$$\beta_0(t) = \frac{e_0(t) - e_1(t) - \sigma_{Q\theta}(t)f_\theta(t)}{r\gamma\sigma_P(t)} \tag{101}$$

$$\beta_\theta(t) = \frac{e_\theta(t) - \sigma_{Q\theta}(t)f_{\theta\theta}(t)}{r\gamma\sigma_P(t)}. \tag{102}$$

Substituting this into HJB and matching coefficients of the value function, we have

$$
\begin{aligned}
f'(t) &= r-\rho-r\ln r+rf(t)-r\gamma\beta_0-\frac{1}{2}r^2\gamma^2\sigma_P(t)\beta_0^2(t)+\frac{1}{2}\sigma_{\theta\theta}\left[f_\theta^2(t)-f_{\theta\theta}(t)\right] \\
&\quad -\left(a\bar{\theta}-\kappa_\theta(t)\right)f_\theta(t) && (103)\\
f'_{\theta\theta}(t) &= rf_{\theta\theta}(t)-r^2\gamma^2\sigma_P(t)\beta_\theta^2(t)+2af_{\theta\theta}(t)+\sigma_{\theta\theta}f_{\theta\theta}^2(t) && (104)\\
f'_\theta(t) &= rf_\theta(t)-r^2\gamma^2\sigma_P(t)\beta_0(t)\beta_\theta(t)+af_\theta(t)+\sigma_{\theta\theta}f_\theta(t)f_{\theta\theta}(t)-\left(a\bar{\theta}-\kappa_\theta(t)\right)f_{\theta\theta}(t) && (105)
\end{aligned}
$$

Market Clearing Conditions Market clearing condition is

$$(1-\omega)\alpha_t+\omega\beta_t=\theta_t, \tag{106}$$

Using the relationship $\tilde{\theta}_t=\theta_t-\frac{\phi_x(t)}{\phi_\theta(t)}\Delta_t$ gives

$$
\begin{aligned}
0 &= (1-\omega)\alpha_0(t)+\omega\beta_0(t) && (107)\\
1 &= (1-\omega)\alpha_\theta(t)+\omega\beta_\theta(t), && (108)\\
0 &= (1-\omega)\alpha_\Delta(t)-\omega\beta_\theta(t)\frac{\phi_x(t)}{\phi_\theta(t)}. && (109)
\end{aligned}
$$

The ODEs for the pricing functions $\phi(t)$, $\phi_\theta(t)$ and $\phi_\Delta(t)$ can be characterized as follows:

$$
\begin{aligned}
\phi'(t) &= r\phi(t)+a\bar{\theta}\phi_\theta(t)-b\bar{x}\bar{\phi}_x+(1-\omega)\left(g_\theta(t)\sigma_\theta\varrho_\theta(t)+g_\Delta(t)\sigma_{Q\Delta}(t)\right)+\omega\left(e_1(t)+\sigma_{Q\theta}(t)f_\theta(t)\right) && (110)\\
\phi'_\theta(t) &= (a+r)\phi_\theta(t)-r\gamma\sigma_P(t)-(1-\omega)\left(g_{\theta\theta}(t)\sigma_\theta\varrho_\theta(t)+g_{\theta\Delta}(t)\sigma_{Q\Delta}(t)\right)-\omega\sigma_{Q\theta}(t)f_{\theta\theta}(t) && (111)
\end{aligned}
$$

or simplify the term $e_\theta(t)=(a+r)\phi_\theta(t)-\phi'_\theta(t)$ in $\beta_\theta(t)$ in (109) gives

$$
\begin{aligned}
\phi'_\Delta(t) &= (a_\Delta+r)\phi_\Delta(t)-\left(g_{\theta\Delta}(t)\sigma_\theta\varrho_\theta(t)+g_{\Delta\Delta}(t)\sigma_{Q\Delta}(t)\right)\\
&\quad -\frac{\bar{\phi}_x-\phi_\Delta(t)}{\phi_\theta(t)}\left[\frac{\omega}{1-\omega}r\gamma\sigma_P(t)+\omega\left(g_{\theta\Delta}(t)\sigma_{Q\Delta}(t)+g_{\theta\theta}(t)\sigma_\theta\varrho_\theta(t)-f_{\theta\theta}(t)\sigma_{Q\theta}(t)\right)\right] && (112)
\end{aligned}
$$

Portfolio Demand for the Informed: Boundary First, we derive boundary conditions for the informed investor's value function coefficients. The informed investor's optimization problem at the boundary can be written as

$$
\begin{aligned}
-e^{-r\gamma\hat{W}^- -g(T,\theta_T,\Delta_T)} &= \max_{\alpha_T}\left\{-\hat{\mathbb{E}}_T\left[e^{-r\gamma\hat{W}^+ -g(0,\theta_T,0)}\right]\right\}\\
&= e^{-r\gamma\hat{W}^-}\max_{\alpha_T}\left\{-\hat{\mathbb{E}}_T\left[e^{-r\gamma\alpha_T\left(P_T^+-P_T^-\right)-g(0,\theta_T,0)}\right]\right\}, && (113)
\end{aligned}
$$

where $x_T\sim\mathcal{N}(\hat{x}_T,\hat{q}_T)$. Solving the exponent part within the expectation operator yields:

$$-r\gamma\alpha_T\left(P_T^+-P_T^-\right)-g(0,\theta_T,0)=-\Phi_0-\Phi_1 x_T,$$

where $\Phi_0 = r\gamma\alpha_T \left\{ [\phi(0) - \phi(T)] - [\phi_\theta(0) - \phi_\theta(T)]\theta_T - \bar{\phi}_x \hat{x}_T + \phi_\Delta(t)\Delta_T \right\} + g(0) + g_\theta(0)\theta_T + \frac{1}{2} g_{\theta\theta}(0)\theta_T^2$ and $\Phi_1 = r\gamma\alpha_T\bar{\phi}_x$. Then

$$\hat{\mathbb{E}}_T \left[e^{-r\gamma\alpha_T\left(P_T^+ - P_T^-\right) - g(0,\theta_T,0)} \right] = e^{-\Phi_0 - \left(\Phi_1 \hat{x}_T - \frac{1}{2}\Phi_1^2 \hat{q}_T\right)} = e^{Term^i}, \tag{114}$$

where

$$\begin{aligned} Term^i &= -r\gamma\alpha_T \left\{ [\phi(0) - \phi(T)] - [\phi_\theta(0) - \phi_\theta(T)]\theta_T + \phi_\Delta(t)\Delta_T \right\} \\ &\quad - g(0) - g_\theta(0)\theta_T - \frac{1}{2} g_{\theta\theta}(0)\theta_T^2 + \frac{1}{2} r^2\gamma^2\alpha_T^2\bar{\phi}_x^2\hat{q}_T. \end{aligned} \tag{115}$$

Optimization implies

$$\alpha_T = \alpha_0(T) + \alpha_\theta(T)\theta_T + \alpha_\Delta(T)\Delta_T, \tag{116}$$

where

$$\alpha_0(T) = \frac{\phi(0) - \phi(T)}{r\gamma\bar{\phi}_x^2\hat{q}_T}, \ \alpha_\theta(T) = \frac{\phi_\theta(T) - \phi_\theta(0)}{r\gamma\bar{\phi}_x^2\hat{q}_T}, \text{ and } \alpha_\Delta(T) = \frac{\phi_\Delta(T)}{r\gamma\bar{\phi}_x^2\hat{q}_T}. \tag{117}$$

Therefore, $g(T, \theta_T, \Delta_T) = -Term^i$ gives

$$\begin{aligned} & g(T) + g_\theta(T)\theta_T + \frac{1}{2} g_{\theta\theta}(T)\theta_T^2 + g_\Delta(T)\Delta_T + \frac{1}{2} g_{\Delta\Delta}(T)\Delta_T^2 + g_{\theta\Delta}(T)\theta_T\Delta_T \\ = & \frac{[\phi(0) - \phi(T) + \phi_\Delta(T)\Delta_T + (\phi_\theta(T) - \phi_\theta(0))\theta_T]^2}{\hat{q}_T\bar{\phi}_x^2} + \frac{1}{2} g_{\theta\theta}(0)\theta_T^2 + g_\theta(0)\theta_T + g(0). \end{aligned} \tag{118}$$

Matching the coefficients yields the boundary conditions summarized as follows

$$\begin{aligned} g(T) - g(0) &= \frac{[\phi(T) - \phi(0)]^2}{2\hat{q}_T\bar{\phi}_x^2}, \ g_{\theta\theta}(T) - g_{\theta\theta}(0) = \frac{[\phi_\theta(T) - \phi_\theta(0)]^2}{\hat{q}_T\bar{\phi}_x^2}, \\ g_\theta(T) - g_\theta(0) &= \frac{-[\phi(T) - \phi(0)][\phi_\theta(T) - \phi_\theta(0)]}{\hat{q}_T\bar{\phi}_x^2}, \ g_{\Delta\Delta}(T) = \frac{\phi_\Delta^2(T)}{\hat{q}_T\bar{\phi}_x^2}, \\ g_\Delta(T) &= -\frac{[\phi(T) - \phi(0)]\phi_\Delta(T)}{\hat{q}_T\bar{\phi}_x^2}, \ g_{\theta\Delta}(T) = \frac{\phi_\Delta(T)[\phi_\theta(T) - \phi_\theta(0)]}{\hat{q}_T\bar{\phi}_x^2}. \end{aligned} \tag{119}$$

Portfolio Demand for the Uninformed: Boundary Second, we derive boundary conditions for the uninformed investor's value function coefficients. The uninformed investor's optimization problem at the boundary is

$$\begin{aligned} -e^{-r\gamma\tilde{W}^- - f\left(T,\tilde{\theta}_T\right)} &= \max_{\beta_T} \left\{ \tilde{\mathbb{E}}_T \left[-e^{-r\gamma\tilde{W}_T^+ - f(0,\theta_T)} \right] \right\} \\ &= e^{-r\tilde{W}^-} \max_{\beta_T} \tilde{\mathbb{E}}_T \left[-e^{-r\gamma\beta_T\left(P_T^+ - P_T^-\right) - f(0,\theta_T)} \right], \end{aligned} \tag{120}$$

where $\begin{pmatrix} x_T \\ \theta_T \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \tilde{x}_T - \kappa\bar{\phi}_x\left(\tilde{q}_T + \hat{q}_T\right) \\ \tilde{\theta}_T - \frac{\phi_{x,T}}{\phi_{\theta,T}}\kappa\bar{\phi}_x\tilde{q}_T \end{pmatrix}, \begin{pmatrix} \hat{q}_T + \tilde{q}_T & \frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T \\ \frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T & \frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T \end{pmatrix}\right)$, in which we use the variance-covariance relationship derived before. For simplicity, denote $\tilde{x}_T^\kappa = \tilde{x}_T - \kappa\bar{\phi}_x\left(\tilde{q}_T + \hat{q}_T\right)$, $\tilde{\theta}_T^\kappa = \tilde{\theta}_T - \kappa_\theta\left(T\right)$, where

$$\kappa_\theta\left(T\right) \equiv \frac{\phi_{x,T}}{\phi_{\theta,T}}\kappa\bar{\phi}_x\tilde{q}_T \tag{121}$$

$$\Omega \equiv \frac{\phi_x^2\left(T\right)}{\phi_\theta^2\left(T\right)}\tilde{q}_T. \tag{122}$$

Note, $x_T - \tilde{x}_T = x_T - \hat{x}_T + \hat{x}_T - \tilde{x}_T$. Recall the learning identity: $\phi_x\left(t\right)\hat{x}_t - \phi_\theta\left(t\right)\theta_t = \phi_x\left(t\right)\tilde{x}_t - \phi_\theta\left(t\right)\tilde{\theta}_t$. Use this to write $\hat{x}_T - \tilde{x}_T$ as a function of $\theta_T - \tilde{\theta}_T$, we obtain: $\hat{x}_T - \tilde{x}_T = \frac{\phi_{\theta,T}}{\phi_{x,T}}\left(\theta_T - \tilde{\theta}_T\right)$. Therefore,

$$\begin{aligned} P_T^+ - P_T^- &= \left[\phi\left(0\right) - \phi\left(T\right)\right] - \left[\phi_\theta\left(0\right)\theta_T - \phi_\theta\left(T\right)\tilde{\theta}_T\right] + \bar{\phi}_x\left(x_T - \tilde{x}_T\right) \\ &= \left[\phi\left(0\right) - \phi\left(T\right)\right] - \left(\phi_{\theta,0} - \frac{\phi_{\theta,T}}{\phi_{x,T}}\bar{\phi}_x\right)\left(\theta_T - \tilde{\theta}_T\right) + \left(\phi_{\theta,T} - \phi_{\theta,0}\right)\tilde{\theta}_T + \bar{\phi}_x\left(x_T - \hat{x}_T\right). \end{aligned}$$

Note that $x_T \sim \mathcal{N}\left(\hat{x}_T^\kappa, \hat{q}_T\right)$ where $\hat{x}_T^\kappa \equiv \hat{x}_T - \kappa\bar{\phi}_x\hat{q}_T$. Or $x_T - \hat{x}_T^\kappa \sim \mathcal{N}\left(0, \hat{q}_T\right)$. We also know that $\theta_T - \tilde{\theta}_T^\kappa \sim \mathcal{N}\left(0, \Omega\right)$. The joint distribution $\begin{pmatrix} x_T - \hat{x}_T^\kappa \\ \theta_T - \tilde{\theta}_T^\kappa \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \hat{q}_T & 0 \\ 0 & \Omega \end{pmatrix}\right)$. Hence, we would like to rewrite the above equation in terms of $x_T - \hat{x}_T^\kappa$ and $\theta_T - \tilde{\theta}_T^\kappa$:

$$P_T^+ - P_T^- = m_0\left(T\right) + \mu_1\left(T\right)\left(\theta_T - \tilde{\theta}_T^\kappa\right) + n_\theta\left(T\right)\tilde{\theta}_T^\kappa + \bar{\phi}_x\left(x_T - \hat{x}_T^\kappa\right)$$

where

$$n_0\left(T\right) \equiv \phi\left(0\right) - \phi\left(T\right) \tag{123}$$

$$m_0\left(T\right) \equiv n_0\left(T\right) - \bar{\phi}_x^2\kappa\left(\hat{q}_T + \tilde{q}_T\right) + \kappa_\theta\left(T\right)\phi_\theta\left(T\right) \tag{124}$$

$$n_\theta\left(T\right) \equiv \phi_\theta\left(T\right) - \phi_\theta\left(0\right) \tag{125}$$

$$\mu_1\left(T\right) \equiv \bar{\phi}_x\frac{\phi_\theta\left(T\right)}{\phi_x\left(T\right)} - \phi_\theta\left(0\right). \tag{126}$$

Also,

$$\begin{aligned} f\left(0, \theta_T\right) &= f\left(0\right) + f_\theta\left(0\right)\theta_T + \frac{1}{2}f_{\theta\theta}\left(0\right)\theta_T^2 \\ &= \nu_0\left(T\right) + \nu_1\left(T\right)\left(\theta_T - \tilde{\theta}_T^\kappa\right) + \frac{1}{2}f_{\theta\theta,0}\left(\theta_T - \tilde{\theta}_T^\kappa\right)^2 \end{aligned}$$

where

$$\nu_0(T) \equiv f(0) + f_\theta(0)\,\tilde{\theta}_T^\kappa + \frac{1}{2} f_{\theta\theta}(0)\,\tilde{\theta}_T^{\kappa 2} \tag{127}$$

$$\nu_1(T) \equiv f_{\theta,0} + f_{\theta\theta,0}\tilde{\theta}_T^\kappa. \tag{128}$$

Solving the exponent part within the expectation operator in the optimization problem gives:

$$-r\gamma\beta_T\left[m_0(T) + \mu_1(T)\left(\theta_T - \tilde{\theta}_T^\kappa\right) + n_\theta(T)\,\tilde{\theta}_T^\kappa + \bar{\phi}_x\left(x_T - \hat{x}_T^\kappa\right)\right] - \left[\nu_0 + \nu_1\left(\theta_T - \tilde{\theta}_T^\kappa\right) + \frac{1}{2} f_{\theta\theta,0}\left(\theta_T - \tilde{\theta}_T^\kappa\right)^2\right]$$

Then we need to compute the expectation. We first integrate out

$$\tilde{\mathbb{E}}_T\left[e^{-r\gamma\beta_T\bar{\phi}_x\left(x_T - \hat{x}_T^\kappa\right)}\right] = e^{\frac{1}{2}(r\gamma)^2\bar{\phi}_x^2\hat{q}_T\beta_T^2}.$$

Hence,

$$\begin{aligned} &\tilde{\mathbb{E}}_T\left[e^{-r\gamma\beta_T\left(P_T^+ - P_T^-\right) - f(0,\theta_T)}\right] \\ = \; & e^{-r\gamma\beta_T\mu_0 - \nu_0 + \frac{1}{2}(r\gamma)^2\bar{\phi}_x^2\hat{q}_T\beta_T^2}\tilde{\mathbb{E}}_T\left[e^{-(r\gamma\beta_T\mu_1 + \nu_1)\left(\theta_T - \tilde{\theta}_T^\kappa\right) - \frac{1}{2} f_{\theta\theta,0}\left(\theta_T - \tilde{\theta}_T^\kappa\right)^2}\right], \end{aligned}$$

where

$$\mu_0(T) \equiv m_0(T) + n_\theta(T)\,\tilde{\theta}_T^\kappa. \tag{129}$$

Lemma 2. *Let $X \sim \mathcal{N}(0, \Omega)$, then*

$$\mathbb{E}\left[e^{-\frac{1}{2}aX^2 + bX}\right] = \frac{1}{\sqrt{1 + a\Omega}} e^{\frac{1}{2}\frac{b^2\Omega}{1 + a\Omega}} = e^{\frac{1}{2}\left[\frac{b^2\Omega}{1 + a\Omega} - \ln(1 + a\Omega)\right]}. \tag{130}$$

Applying the above lemma, we have

$$\tilde{\mathbb{E}}_T\left[e^{-(r\gamma\beta_T\mu_1 + \nu_1)\left(\theta_T - \tilde{\theta}_T^\kappa\right) - \frac{1}{2} f_{\theta\theta,0}\left(\theta_T - \tilde{\theta}_T^\kappa\right)^2}\right] = e^{\frac{1}{2}(r\gamma\beta_T\mu_1 + \nu_1)^2\frac{\Omega}{1 + f_{\theta\theta,0}\Omega} - \frac{1}{2}\ln\left(1 + f_{\theta\theta,0}\Omega\right)}.$$

$$\tilde{\mathbb{E}}_T\left[e^{-r\beta_T\left(P_T^+ - P_T^-\right) - f(0,\theta_T)}\right] = e^{Term^u},$$

where

$$\begin{aligned} Term^u &= -r\gamma\beta_T\mu_0 - \nu_0 + \frac{1}{2}(r\gamma)^2\bar{\phi}_x^2\hat{q}_T\beta_T^2 + \frac{\Omega}{2\left(1 + f_{\theta\theta,0}\Omega\right)}(r\gamma\beta_T\mu_1 + \nu_1)^2 - \frac{1}{2}\ln\left(1 + f_{\theta\theta,0}\Omega\right) \\ &= \frac{1}{2}(r\gamma)^2\Lambda\beta_T^2 - r\gamma\left(\mu_0 - \Gamma\mu_1\nu_1\right)\beta_T + \left[-\nu_0 + \frac{\Gamma}{2}\nu_1^2 - \frac{1}{2}\ln\left(1 + f_{\theta\theta,0}\Omega\right)\right] \end{aligned}$$

where

$$\Gamma \equiv \frac{\Omega}{1+f_{\theta\theta}(0)\,\Omega} \tag{131}$$

$$\Lambda \equiv \bar{\phi}_x^2\hat{q}_T + \Gamma\mu_1^2(T). \tag{132}$$

The FOC with respect to β_T gives

$$\begin{aligned}\beta_T &= \frac{1}{r\gamma\Lambda}(\mu_0 - \Gamma\mu_1\nu_1)\\ &= \frac{1}{r\gamma\Lambda}\left[\tilde{m}_0(T) + \tilde{m}_\theta(T)\,\tilde{\theta}_T\right]\end{aligned}$$

where

$$\tilde{m}_\theta(T) \equiv n_\theta(T) - \Gamma\mu_1(T)\,f_{\theta\theta,0} \tag{133}$$

$$\tilde{m}_0(T) \equiv m_0(T) - \Gamma\mu_1(T)\,f_{\theta,0} - \tilde{m}_\theta(T)\,\kappa_\theta(T). \tag{134}$$

Denote

$$\beta_T = \beta_0(T) + \beta_\theta(T)\,\tilde{\theta}_T, \tag{135}$$

where

$$\beta_0(T) = \frac{1}{r\gamma\Lambda}\tilde{m}_0(T) \tag{136}$$

$$\beta_\theta(T) = \frac{1}{r\gamma\Lambda}\tilde{m}_\theta(T). \tag{137}$$

Then,

$$Term^u = \left[-\nu_0 + \frac{\Gamma}{2}\nu_1^2 - \frac{1}{2}\ln(1+f_{\theta\theta,0}\Omega)\right] - \frac{1}{2\Lambda}\left(\tilde{m}_0 + \tilde{m}_\theta\tilde{\theta}_T\right)^2.$$

Rewrite $\nu_0(T)$ and $\nu_1(T)$ in terms of $\tilde{\theta}_T$:

$$\begin{aligned}\nu_0(T) &= f(0) + f_\theta(0)\,\tilde{\theta}_T^\kappa + \frac{1}{2}f_{\theta\theta}(0)\,\tilde{\theta}_T^{\kappa 2}\\ &= f(0) + f_\theta(0)\left(\tilde{\theta}_T - \kappa_\theta\right) + \frac{1}{2}f_{\theta\theta}(0)\left(\tilde{\theta}_T - \kappa_\theta\right)^2\\ \nu_1(T) &= f_\theta(0) + f_{\theta\theta}(0)\,\tilde{\theta}_T^\kappa = f_\theta(0) + f_{\theta\theta}(0)\left(\tilde{\theta}_T - \kappa_\theta\right)\end{aligned}$$

Substituting these into $f\left(T,\tilde{\theta}_T\right) = -Term^u$ gives

$$\begin{aligned}f(T) + f_\theta(T)\,\tilde{\theta}_T + \frac{1}{2}f_{\theta\theta}(T)\,\tilde{\theta}_T^2 &= f(0) + f_\theta(0)\left(\tilde{\theta}_T - \kappa_\theta\right) + \frac{1}{2}f_{\theta\theta}(0)\left(\tilde{\theta}_T - \kappa_\theta\right)^2\\ &\quad \frac{\Gamma}{2}\left[f_\theta(0) - \kappa_\theta f_{\theta\theta}(0) + f_{\theta\theta}(0)\,\tilde{\theta}_T\right]^2 - \frac{1}{2}\ln(1+f_{\theta\theta,0}\Omega)\end{aligned}$$

and matching the coefficients yields the boundary conditions summarized as follows

$$
\begin{aligned}
f(T)-f(0) &= -\frac{\Gamma}{2}f_\theta^2(0)+\Gamma\frac{\phi_\theta(T)}{\phi_x(T)}\kappa\bar{\phi}_x\left[\frac{1}{2}f_{\theta\theta}(0)\kappa_\theta(T)-f_\theta(0)\right]+\frac{1}{2\Lambda}\tilde{m}_0^2(T)+\frac{1}{2}\ln(1+f_{\theta\theta}(0)) && (138)\\
f_\theta(T)-f_\theta(0) &= -\Gamma f_{\theta\theta}(0)\left[f_\theta(0)+\frac{\phi_\theta(T)}{\phi_x(T)}\kappa\bar{\phi}_x\right]+\frac{1}{\Lambda}\tilde{m}_0(T)\tilde{m}_\theta(T) && (139)\\
f_{\theta\theta}(T)-f_{\theta\theta}(0) &= -\Gamma f_{\theta\theta}^2(0)+\frac{1}{\Lambda}\tilde{m}_\theta^2(T). && (140)
\end{aligned}
$$

Market Clearing Note that market clearing requires: $(1-\omega)\alpha_T+\omega\beta_T=\theta_T$ at the announcement. This implies

$$
\begin{aligned}
(1-\omega)\alpha_0(T)+\omega\beta_0(T) &= 0,\\
(1-\omega)\alpha_\theta(T)+\omega\beta_\theta(T) &= 1,\\
(1-\omega)\alpha_\Delta(T)-\omega\beta_\theta(T)\frac{\phi_{x,T}}{\phi_{\theta,T}} &= 0.
\end{aligned}
$$

Substituting expressions in equations (117) and (135) gives

$$
\begin{aligned}
(1-\omega)\frac{\phi_0-\phi_T}{\bar{\phi}_x^2\hat{q}_T}+\omega\frac{\tilde{m}_0}{\Lambda} &= 0,\\
(1-\omega)\frac{\phi_{\theta,T}-\phi_{\theta,0}}{\bar{\phi}_x^2\hat{q}_T}+\omega\frac{\tilde{m}_\theta}{\Lambda} &= r\gamma,\\
(1-\omega)\frac{\phi_{\Delta,T}}{\bar{\phi}_x^2\hat{q}_T}-\omega\frac{\tilde{m}_\theta}{\Lambda}\frac{\phi_{x,T}}{\phi_{\theta,T}} &= 0.
\end{aligned}
$$

Eventually we can pin down the boundary conditions for the pricing function coefficients at the announcements.

$$
\begin{aligned}
\phi(T)-\phi(0) &= -\frac{\omega\bar{\phi}_x^2\hat{q}_T}{(1-\omega)\Lambda+\omega\bar{\phi}_x^2\hat{q}_T}\left[\bar{\phi}_x^2\kappa(\hat{q}_T+\tilde{q}_T)+\Gamma\mu_1(T)f_\theta(0)-\kappa_\theta(T)(\phi_\theta(0)+\Gamma\mu_1(T)f_{\theta\theta}(0))\right] && (141)\\
\phi_\theta(T)-\phi_\theta(0) &= \frac{\Lambda\bar{\phi}_x^2\hat{q}_T}{(1-\omega)\Lambda+\omega\bar{\phi}_x^2\hat{q}_T}\left[r\gamma+\omega\frac{\Gamma}{\Lambda}\mu_1(T)f_{\theta\theta}(0)\right] && (142)\\
\phi_\Delta(T) &= \frac{\omega}{(1-\omega)\Lambda}\bar{\phi}_x^2\hat{q}_T\frac{\phi_x(T)}{\phi_\theta(T)}\tilde{m}_\theta(T). && (143)
\end{aligned}
$$

6.5 Implied Volatility and Trading Volume

Implied Variance We would like to compute $Var_0[P_t-P_0]=Var_0[P_t]$. First consider the case in which $t<T$. We solve the three components separately. First, we solve for $\tilde{x}_t$. Using the law of motion (13), we have:

$$
\tilde{x}_t=e^{-bt}\int_0^t e^{bs}b\bar{x}ds+e^{-bt}\int_0^t e^{bs}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D}d\tilde{B}_{D,s}+e^{-bt}\int_0^t e^{bs}\nu(s)\sigma_\xi(s)d\tilde{B}_{\xi,s}+e^{-bt}\int_0^t e^{bs}\frac{\tilde{q}_s}{\sigma_{i,s}}d\tilde{B}_{i,s}.
$$

Therefore, with an abuse of notation, we use $DF[X]$ to denote the diffusion part of X, we have:

$$DF\left[\bar{\phi}_x \tilde{x}_t\right] = \bar{\phi}_x \int_0^t e^{b(s-t)} \frac{\hat{q}_s + \tilde{q}_s}{\sigma_D} d\tilde{B}_{D,s} + \bar{\phi}_x \int_0^t e^{b(s-t)} \nu(s)\, \sigma_\xi(s)\, d\tilde{B}_{\xi,s} + \bar{\phi}_x \int_0^t e^{b(s-t)} \frac{\tilde{q}_s}{\sigma_{i,s}} d\tilde{B}_{i,s}. \tag{144}$$

Next, we compute D_t.

$$dD_t = (\tilde{x}_t - D_t)\, dt + \sigma_D d\tilde{B}_{D,t},$$

and therefore

$$D_t = e^{-t} \int_0^t e^s \tilde{x}_s ds + e^{-t} \int_0^t e^s \sigma_D d\tilde{B}_{D,s}.$$

The term

$$\int_0^t e^u \tilde{x}_u du = \int_0^t e^{(1-b)u} \int_0^u \left\{ e^{bs} b\bar{x} ds + \int_0^u e^{bs} \frac{\hat{q}_s + \tilde{q}_s}{\sigma_D} d\tilde{B}_{D,s} + \int_0^u e^{bs} \nu(s)\, \sigma_\xi(s)\, d\tilde{B}_{\xi,s} + \int_0^u e^{bs} \frac{\tilde{q}_s}{\sigma_{i,s}} d\tilde{B}_{i,s} \right\} du.$$

We focus on the diffusion part:

$$\begin{aligned}\int_0^t \int_0^u e^{bs+(1-b)u} \frac{\hat{q}_s + \tilde{q}_s}{\sigma_D} d\tilde{B}_{D,s} du &= \int_0^t \int_s^t e^{bs+(1-b)u} \frac{\hat{q}_s + \tilde{q}_s}{\sigma_D} du d\tilde{B}_{D,s} \\ &= \frac{1}{(1-b)\,\sigma_D} \int_0^t \left[e^{(1-b)t+bs} - e^s \right] (\hat{q}_s + \tilde{q}_s)\, d\tilde{B}_{D,s}.\end{aligned}$$

Similarly, we have:

$$\int_0^t \int_0^u e^{bs+(1-b)u} \nu(s)\, \sigma_\xi(s)\, d\tilde{B}_{\xi,s} du = \frac{1}{(1-b)} \int_0^t \left[e^{(1-b)t+bs} - e^s \right] \nu(s)\, \sigma_\xi(s)\, d\tilde{B}_{\xi,s}.$$

$$\int_0^t \int_0^u e^{bs+(1-b)u} \frac{\tilde{q}_s}{\sigma_{i,s}} d\tilde{B}_{i,s} du = \frac{1}{(1-b)} \int_0^t \left[e^{(1-b)t+bs} - e^s \right] \frac{\tilde{q}_s}{\sigma_{i,s}} d\tilde{B}_{i,s}.$$

We have:

$$D_t = e^{-t} \left[\int_0^t e^s \tilde{x}_s ds + \int_0^t e^s \sigma_D d\tilde{B}_{D,s} \right].$$

The diffusion part is

$$\begin{aligned} DF\left[\phi_D D_t\right] &= \phi_D \int_0^t \Bigg[\left(e^{b(s-t)} - e^{s-t} \right) \left(\frac{\hat{q}_s + \tilde{q}_s}{(1-b)\,\sigma_D} + e^{s-t} \sigma_D \right) d\tilde{B}_{D,s} \\ &\quad + \frac{\nu(s)\, \sigma_\xi(s)}{1-b} d\tilde{B}_{\xi,s} + \frac{\tilde{q}_s}{(1-b)\,\sigma_{i,s}} d\tilde{B}_{i,s} \Bigg] \end{aligned} \tag{145}$$

Finally, we deal with $\tilde{\theta}_t$:

$$DF\left[-\phi_\theta(t)\tilde{\theta}_t\right] = -\phi_\theta(t)\left\{\int_0^t e^{a(s-t)}\frac{\phi_x(s)}{\phi_\theta(s)}\frac{\tilde{q}(s)}{\sigma_D}d\tilde{B}_{D,s} + \int_0^t e^{a(s-t)}\left[\phi_x(s)\nu(s)-1\right]\frac{\sigma_\xi(s)}{\phi_\theta(s)}d\tilde{B}_{\xi,s} + \int_0^t e^{a(s-t)}\frac{\phi_x(s)}{\phi_\theta(s)}\frac{\tilde{q}(s)}{\sigma_i(s)}d\tilde{B}_{i,s}\right\} \tag{146}$$

Summing up (144), (145), and (146), we can represent the price in the form of

$$DF[P_t] = \int_0^t Term_D(s)\,d\tilde{B}_{D,s} + \int_0^t Term_\xi(s)\,d\tilde{B}_{\xi,s} + \int_0^t Term_i(s)\,d\tilde{B}_{i,s}, \tag{147}$$

where

$$Term_D(s) = \phi_D\left[\left(e^{b(s-t)}-e^{s-t}\right)\frac{\hat{q}_s+\tilde{q}_s}{(1-b)\sigma_D}+e^{s-t}\sigma_D\right] - \phi_\theta(t)e^{a(s-t)}\frac{\phi_x(s)}{\phi_\theta(s)}\frac{\tilde{q}(s)}{\sigma_D} + \bar{\phi}_x e^{b(s-t)}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D} \tag{148}$$

$$Term_\xi(s) = \phi_D\left[e^{b(s-t)}-e^{s-t}\right]\frac{\nu(s)\sigma_\xi(s)}{1-b} - \phi_\theta(t)e^{a(s-t)}\left[\phi_x(s)\nu(s)-1\right]\frac{\sigma_\xi(s)}{\phi_\theta(s)} + \bar{\phi}_x e^{b(s-t)}\nu(s)\sigma_\xi(s) \tag{149}$$

$$Term_i(s) = \phi_D\left[e^{b(s-t)}-e^{s-t}\right]\frac{\tilde{q}_s}{(1-b)\sigma_{i,s}} - \phi_\theta(t)e^{a(s-t)}\frac{\phi_x(s)}{\phi_\theta(s)}\frac{\tilde{q}(s)}{\sigma_i(s)} + \bar{\phi}_x e^{b(s-t)}\frac{\tilde{q}_s}{\sigma_{i,s}}. \tag{150}$$

and compute the variance as:

$$Var_0[P_t] = \int_0^t Term_D^2(s)\,ds + \int_0^t Term_\xi^2(s)\,ds + \int_0^t Term_i^2(s)\,ds. \tag{151}$$

Next, consider the general case where we need to compute $Var_t[P_{t+\tau}]$. If $t+\tau<T$, that is, if we compute implied variance within an announcement cycle, we use the above formula. If $t+\tau>T$. We first compute $Var_t[P_{T^-}]$ using the above formula. It is also easy to compute $Var_{T^-}[P_{T^+}-P_{T^-}]$. We can then compute $Var_{T^+}[P_{t+\tau}]$. The reason we can just add up variance is because these different components are independent.

$$Var_t[P_{t+\tau}] = \int_t^{t+\tau}\left[Term_D^2(s)\,ds + Term_\xi^2(s) + Term_i^2(s)\right]ds. \tag{152}$$

$$\begin{aligned} P_T^+ - P_T^- &= \bar{\phi}_x(x_T-\tilde{x}_T) - \phi_\theta(0)\left[\theta_T-\tilde{\theta}_T\right] - \left[\phi_\theta(0)-\phi_\theta(T)\right]\tilde{\theta}_T, \\ &= \bar{\phi}_x x_T - \phi_\theta(0)\theta_T - \bar{\phi}_x\tilde{x}_T + \phi_\theta(T)\tilde{\theta}_T, \end{aligned}$$

and

$$Var_{T^-}[P_{T^+}-P_{T^-}] = \bar{\phi}_x^2(\hat{q}_T+\tilde{q}_T) + \phi_\theta^2(0)\frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T - 2\bar{\phi}_x\phi_\theta(0)\frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T. \tag{153}$$

Therefore, the total variance is obtained by

$$\begin{aligned} & Var_t\left[P_{T^-}\right] + Var_{T^-}\left[P_{T^+} - P_{T^-}\right] + Var_{T^+}\left[P_{t+\tau}\right] \\ = & \int_t^{T^-} \left[Term_D^2(s) + Term_\xi^2(s) + Term_i^2(s)\right] ds + \int_{T^+}^{t+\tau} \left[Term_D^2(s) + Term_\xi^2(s) + Term_i^2(s)\right] ds \\ & + \bar{\phi}_x^2\left(\hat{q}_T + \tilde{q}_T\right) + \phi_\theta^2(0)\frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T - 2\bar{\phi}_x\phi_\theta(0)\frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T \end{aligned} \tag{154}$$

Trading volume Since the two types of investors trade against each other, the trading volume can be calculated as changes in the portfolio holdings of either group of investors. Define the trading volume from t to $t+\delta$ as the turnover rate of uninformed investors (see Wang, 1994):

$$M(t, t+\delta) = \omega\left|\beta_{t+\delta} - \beta_t\right| \tag{155}$$

At the announcement, the trading volume can be calculated as

$$M\left(T^-, T^+\right) = \omega\left|\beta_T^+ - \beta_T^-\right|. \tag{156}$$

6.6 Policy Functions and Calibration

First, the calibrated parameters are:

Table 2: Parameters

Para.	Value	Description	Para.	Value	Description
r	0.028	risk-free rate	σ_x	0.5	volatility of hidden state
ρ	0.03	time discount factor	σ_θ	0.68	volatility of total equity supply
$\bar{x}$	20	mean level of dividend flow	σ_i	0.01	inverse of acquired information precision
b	0.1	persistence of hidden state	κ	0.1	ambiguity aversion
a	0.01	persistence of total equity supply	γ	1	risk aversion
σ_d	1	dividend flow volatility	$\bar{\theta}$	0	unconditional mean of aggregate supply
σ_s	0.06	inverse of signal precision	ω	0.99	fraction of uninformed investor

This table displays annualized parameter values used in the simulations.

The unconditional capital gain at the announcement is

$$\mathbb{E}\left[P_T^+ - P_T^-\right] = \left[\hat{\phi}(0) - \hat{\phi}(T)\right] - \left[\phi_\theta(0) - \phi_\theta(T)\right]\bar{\theta}. \tag{157}$$

We define the unconditional level of price before announcement to be

$$\bar{P}_T = \hat{\phi}(T) + \left(\phi_D + \bar{\phi}_x\right)\bar{x} - \phi_\theta(T)\bar{\theta}. \tag{158}$$

We can similarly define the unconditional level of price after announcement as

$$\bar{P}_0 = \hat{\phi}(0) + \left(\phi_D + \bar{\phi}_x\right)\bar{x} - \phi_\theta(0)\,\bar{\theta}. \tag{159}$$

The unconditional announcement premium is roughly:

$$\frac{1}{\bar{P}_T}\mathbb{E}\left[P_T^+ - P_T^-\right] = \frac{1}{\bar{P}_T}\left\{\left[\hat{\phi}(0) - \hat{\phi}(T)\right] - \left[\phi_\theta(0) - \phi_\theta(T)\right]\bar{\theta}\right\}. \tag{160}$$