

Information-Driven Volatility

Hengjie Ai, Leyla Jianyu Han, and Lai Xu*

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Abstract: Standard asset pricing models with stochastic volatility predict a robust positive relationship between past realized volatility and future expected returns. Empirical work typically finds this relationship to be negative. We develop an asset pricing model where stock market volatility dynamics are driven by information. We show that information-driven volatility induces a negative correlation between past realized volatility and future expected returns. We provide empirical evidence for the unique implications of the information-driven volatility channel and demonstrate that our model can quantitatively replicate the evidence.

Keywords: Information, Return Volatility, Macroeconomic Announcements, Generalized Risk Sensitivity

JEL Code: D83, D84, G11, G12, G14

*Hengjie Ai (hengjie.ai@wisc.edu) is from Wisconsin School of Business, University of Wisconsin-Madison. Leyla Jianyu Han (leylahan@bu.edu) is from the Questrom School of Business, Boston University, and Lai Xu (lxu100@syr.edu) is at the Whitman School of Management, Syracuse University. We thank Bjorn Eraker, Christian Heyerdahl-Larsen, Zhongjin Lu, Charles Martineau, Tyler Muir, Xuhui (Nick) Pan, Liyan Yang, and seminar participants at Baruch college, University of Washington, University of Manitoba, University of Minnesota, University of Oklahoma, University of Texas at Dallas, University of Toronto, University of Wisconsin-Madison, Chinese University of Hong Kong–Shenzhen, Tsinghua University PBCSF, Wuhan University, Zhongnan University of Economics and Law, as well as participants at the Canadian Derivatives Institute conference, the JEDC SI conference, Midwest Finance Association, China International Risk Forum, and the SAFE asset pricing workshop for their helpful comments.

1 Introduction

Leading consumption-based asset pricing models with stochastic volatility, such as the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the rare disaster model of Barro (2006) and Gabaix (2012), predict a robust positive relationship between past realized volatility and future expected returns. Empirically, many authors find a negative relationship between the two, for example, Nelson (1991) and Glosten, Jagannathan, and Runkle (1993). More recently, Moreira and Muir (2017) cast the above negative relationship in terms of trading strategies. They construct a volatility-managed portfolio that invests more in the stock market when past realized volatility is low and less when past realized volatility is high and demonstrate that such a strategy earns a higher average return than the market portfolio. In this paper, we develop a model with information-driven volatility. We show that information-driven volatility induces a negative relationship between past realized volatility and future expected returns. We provide empirical evidence for the information-driven volatility channel and demonstrate that our model can account for several volatility-related asset pricing puzzles.

There are two main motivations for our theory of information-driven volatility. First, high frequency, such as monthly-, daily-, or hourly- frequency financial market fluctuations are more likely to be driven by information rather than the volatility of macroeconomic fundamentals. Both volatilities of macroeconomic fundamentals and information can affect financial market volatility. Standard consumption-based asset pricing models typically generate time-varying volatility of returns by assuming time-varying volatility of macroeconomic fundamentals, such as Bansal, Kiku, Shaliastovich, and Yaron (2014) in the context of the long-run risk model and Wachter (2013) in the context of the rare disaster model. While these models can explain quite well asset pricing moments at lower frequencies (annual frequencies and above), the volatility of macroeconomic fundamentals in the data typically does not change substantially at high frequencies and is unlikely to explain high-frequency financial market volatilities. In contrast, information about the macroeconomy arrives continuously at financial markets and affects the stock return volatility.

Second, recent empirical evidence and theoretical work establish a significant risk compensation associated with informational events on financial markets. As documented by Savor and Wilson (2013) and Ai and Bansal (2018), roughly 55% of the stock market risk premium is realized on a small number of days with significant macroeconomic announcements. Lucca and Moench (2015) find a similar pattern for FOMC announcements. Ai and Bansal (2018) demonstrate that expected utility models are inconsistent with the announcement premium, and the existence of such implies that investor preferences must satisfy generalized risk sen-

sitivity. As we will show in this paper, under expected utility, information-driven volatility is irrelevant for risk compensation. Generalized risk sensitivity provides a theoretical foundation for information-driven volatility to affect stock market risk premiums.

To understand the distinction between fundamental driven and information-driven volatility, consider the following variance decomposition identity:

$$\text{Var} [C_T] = \text{Var} [\mathbb{E} (C_T|X_t)] + \mathbb{E}[\text{Var} (C_T|X_t)]. \quad (1)$$

We interpret C_T as macroeconomic fundamentals such as aggregate consumption, the value of which will be realized at the terminal time T . We interpret X_t as a public signal revealed at time $t < T$ that is informative about C_T . One example of such public signals is a macroeconomic announcement. The above formula then decomposes the total variance of macroeconomic fundamentals, $\text{Var} [C_T]$, into the variance realized on the announcement day t , $\text{Var} [\mathbb{E} (C_T|X_t)]$, and the variance that will realize after the announcement day t , $\mathbb{E} [\text{Var} (C_T|X_t)]$.

Traditional models of stochastic volatility generate time-varying volatility of returns by time-varying volatility of macroeconomic fundamentals, $\text{Var} [C_T]$. Because the volatility of macroeconomic fundamentals such as aggregate consumption and aggregate productivity is positively autocorrelated over time, in asset pricing models with stochastic volatility of macroeconomic fundamentals, a high past realized volatility predicts a high expected future volatility and therefore a high expected future return.

Given that the volatility of macroeconomic fundamentals does not change substantially at high frequencies, consider now the exercise of varying the informativeness of the signal X_t by holding the total volatility $\text{Var} [C_T]$ fixed. A more accurate signal X_t triggers a larger response of investors' conditional expectation about future cash flow, which is captured by the term $\text{Var} [\mathbb{E} (C_T|X_t)]$, and is associated with a higher realized volatility on financial markets upon the arrival of information. The variance decomposition identity (1) then implies that such a high realized variance, $\text{Var} [\mathbb{E} (C_T|X_t)]$, must be associated with a lower expected future volatility, $\mathbb{E} [\text{Var} (C_T|X_t)]$. This impact of information on the intertemporal distribution of risk and risk compensation is the information-driven volatility channel.

To illustrate the information-driven volatility channel, we first present a three-period model to theoretically establish its two key implications under the assumption of generalized risk sensitivity:

- I1** a more informative macroeconomic announcement is associated with a higher risk compensation and higher volatility of return upon the announcement, and
- I2** a more informative macroeconomic announcement is associated with a lower risk premium

and lower expected volatility after the announcement.

We will refer to the above as Implication I1 and Implication I2 in the rest of the paper.

To empirically test the information-driven volatility channel and assess its quantitative importance, we calibrate a continuous-time asset pricing model with time-varying informativeness of macroeconomic announcements. We conduct several tests of the above two implications of the information-driven volatility channel and we replicate these tests in our model.

In the data, FOMC announcements are the most identifiable events that reveal information about the macroeconomy. To test the mechanism of information-driven volatility, we first develop a measure of informativeness of FOMC announcements. Using this measure, we show that higher informativeness predicts a larger FOMC announcement-day return and a large implied volatility reduction on announcement days. In addition, higher informativeness also predicts a lower expected return and a lower stock return volatility after the announcement.

To provide additional evidence for the information-driven volatility channel, using the stock market jump data constructed by Baker, Bloom, Davis, and Sammon (2021), we show that news-triggered stock market jumps negatively predict future market volatility and future stock returns, while there is no significant negative relationship between realized volatility and future expected returns on days without significant causes of news.

We calibrate our model and demonstrate that it matches well conventional asset pricing moments and stock market dynamics around macroeconomic announcements. We replicate the above statistical tests of the information-driven volatility in our calibrated model to demonstrate the consistency between our model and data and the quantitative significance of the information-driven volatility channel.

Furthermore, we show that our model provides an explanation for the variance risk premium predictability without relying on high-frequency variations in the volatility of macroeconomic fundamentals. As shown by Bollerslev, Tauchen, and Zhou (2009), in the data, the difference between implied volatility and realized volatility has strong predictive powers for future returns for up to three to six months. They provide an explanation for this predictability based on high-frequency variations in the volatility of aggregate consumption. Our model assumes a homoscedastic consumption growth process; however, the difference between implied volatility and realized volatility predicts returns because it reflects the informativeness of the upcoming information event: holding the past realized volatility constant, the anticipation of the arrival of highly informative news is associated with both high implied volatility right before and a larger risk compensation upon the arrival of the news.

Our results should not be interpreted as dismissing the role of the volatility of macroe-

conomic fundamentals in affecting stock market dynamics. In fact, our calibrated model features low-frequency movements in the volatility of macroeconomic fundamentals and exhibits an overall positive auto-correlation of stock market volatilities, as in the data. We emphasize the importance of identifying the driving force of volatility in understanding the volatility-expected return relationship in the data. The volatility of macroeconomic fundamentals varies at low frequencies and determines the long-run relationship between volatility and expected returns. Information arrives at the stock market continuously, and is more likely to be responsible for the relationship between past realized volatility and future expected returns at higher frequencies.

Related literature Our model builds on the literature on learning and information in financial markets. David (1997) and David (2008) develop learning models to study equity market risk compensations. Veronesi (2000) and Ai (2010) study how information quality affects the aggregate stock market risk premium. David and Veronesi (2013) estimate a regime-switching model with learning. Pastor and Veronesi (2009) develop a learning model to study the relationship between technological innovations and stock market valuations. Bansal and Shaliastovich (2010, 2011) and Shaliastovich (2015) develop models where learning results in asset price jumps. None of the above papers focus on the time-varying informativeness of macroeconomic news as we do.

Our paper is evidently related to the vast empirical literature on the expected return-volatility relationship. Both Nelson (1991) and Glosten, Jagannathan, and Runkle (1993) document empirical evidence that is supportive of a negative relationship between past realized volatility and future expected returns. Harvey (1989) finds mixed evidence of a time-varying relationship between expected excess returns and conditional variances. Consistent with our theory, Harrison and Zhang (1999) find a negative relationship and sometimes mixed evidence for the relationship between past realized volatility and future expected returns over short horizons, but a positive relationship over horizons longer than a year. More recently, Moreira and Muir (2017) demonstrate a positive average return on their volatility-managed portfolio relative to the market return, and their evidence is also consistent with a negative relationship between past realized volatility and future expected returns. Lochstoer and Muir (forthcoming) develop a model of extrapolative expectations of volatility shocks to explain the variance risk premium predictability and the negative relationship between volatility and expected returns.

Several recent papers provide empirical evidence consistent with the information-driven volatility channel emphasized in this paper. Baker, Bloom, Davis, Kost, Sammon, and Viratyosin (2020) document that a higher clarity of news is associated with lower realized

volatility in the future. Zhang and Zhao (2020) provide evidence that in periods where public information is imprecise, the realized macroeconomic announcement premium is low. Chaudhry (2021) shows that announcement days are typically associated with uncertainty reductions and lower expectations after announcements.

Our paper is also related to the variance risk premium predictability literature. Bollerslev, Tauchen, and Zhou (2009) document the predictability of stock market returns by the difference between implied and realized variance, and develop a model of variance risk premium predictability based on stochastic volatility in the volatility of macroeconomic fundamentals. Drechsler and Yaron (2011) develop a model with stochastic volatility and stochastic jumps to quantitatively explain the variance risk premium predictability. Eraker and Wang (2015) estimate a non-linear diffusion model and study the variance risk premium predictability. Zhou (2018) provides a thorough review of this literature. The above literature has interpreted the difference between implied and realized variance as the difference between variance under the physical measure and that under the risk neutral measure and hence defined as the variance risk premium. We show that the difference between implied and realized variance can predict returns without assuming a variance risk premium. In our model, the difference between the two reflects the informativeness of the upcoming announcement. It predicts returns because, in our model, resolution of uncertainty is associated with realizations of risk premium.

This paper is also closely related to the literature on generalized risk sensitivity and macroeconomic announcements. From the empirical perspective, pre-scheduled macroeconomic announcements are the most salient information events that are associated with significant realizations of market equity premiums and affect the volatility of stock market returns. The literature that documents a significant macroeconomic announcement premium, for example, Savor and Wilson (2013, 2014) and Lucca and Moench (2015) provide strong empirical support for the mechanism emphasized in this paper. From the theoretical point of view, Ai and Bansal (2018) demonstrate that the existence of the announcement premium implies generalized risk sensitivity in preferences. In our setup, as we will demonstrate in Section 3 of the paper, generalized risk sensitivity is also necessary for information quality to affect the intertemporal distribution of risk compensation.

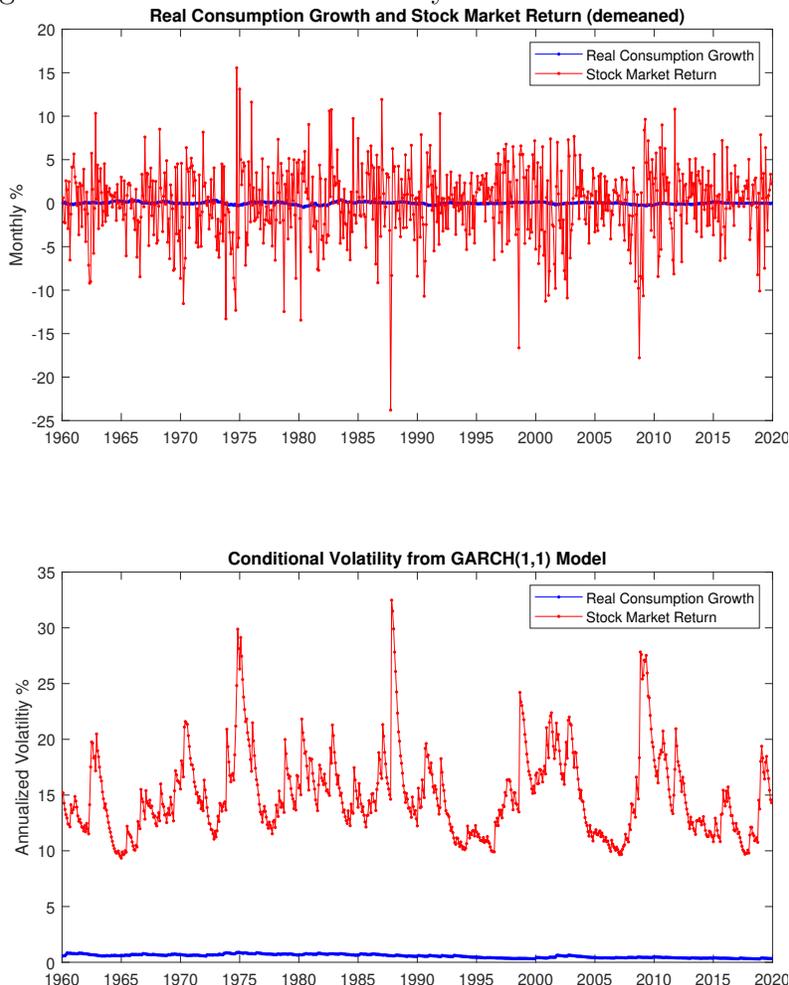
The rest of the paper is organized as follows. Section 2 summarizes the stylized facts between realized variance and expected returns. Section 3 presents a simple three-period model to illustrate the impact of informativeness of macroeconomic news on the intertemporal distribution of risk and risk compensation. Section 4 develops a dynamic model to account for the stylized facts, and Section 5 presents the quantitative results. Section 6 concludes.

2 Motivating Facts for Information-Driven Volatility

In this section, we provide details on several stylized facts on volatility and stock market returns that motivate the development of our theory of information-driven volatility.

1. The volatility of macroeconomic fundamentals does not exhibit significant variations at higher frequencies than monthly.

Figure 1: Macroeconomic volatility and stock market volatility



The top panel is the monthly consumption growth rates (solid line) and the S&P500 index returns (dash-dotted line) during 1960.02-2019.12. The bottom panel is the estimated conditional volatility of the two series from a GARCH(1,1) model during the same sample period.

In Figure 1, we plot the time series of monthly consumption growth and monthly stock market returns in the top panel. In the bottom panel, we plot the estimated conditional volatility of the two time series from a GARCH (1,1) model. Compared to stock market returns, the variations of consumption growth are much smaller. The estimated con-

ditional volatility of stock returns exhibits sharp variations over the monthly horizon, while that of aggregate consumption growth is virtually flat by comparison.

The above observation highlights the fact that variations in the volatility of macroeconomic fundamentals are unlikely to explain the high-frequency movements in financial market volatility.

2. A large fraction of equity premium is realized on a small fraction of trading days with significant macroeconomic announcements.

Table 1 reports the average market excess returns on macroeconomic announcement days and non-announcement days from 1961 to 2021.¹ In this period, on average, forty-eight trading days per year have significant macroeconomic announcements. At the daily level, the average stock market excess return is 14.05 bps on announcement days and 1.28 bps on days without major macroeconomic announcements. As a result, the cumulative stock market excess return on the thirty announcement days averages 6.75% per year, accounting for about 73% of the annual equity premium (9.29%).

As shown in Ai and Bansal (2018), the fact that information revelation is associated with realizations of risk compensation implies that investors’ preferences must satisfy generalized risk sensitivity. Generalized risk sensitivity provides a theoretical foundation for our model of information-driven volatility. As we demonstrate in this paper, it allows information-driven volatility to affect stock market risk compensation, whereas expected utility models do not.

Table 1: Macroeconomic Announcement Premium

	# of days p.a.	Daily prem.	Prem. p.a.	t-stat
Market	247	3.76 bp	9.29%	2.61
Ann	48	14.05 bp	6.75%	4.06
Non-Ann	199	1.28 bp	2.54%	0.81

This table documents the average excess return of the U.S. stock market during the 1961–2021 period. The column “# days p.a.” is the average number of trading days per annum. The second column is the daily market equity premium on all days, announcement days, and days without announcement. The column “premium p.a.” is the cumulative market excess returns within a year, computed by multiplying the daily premium by the number of event days and converting it into percentage points.

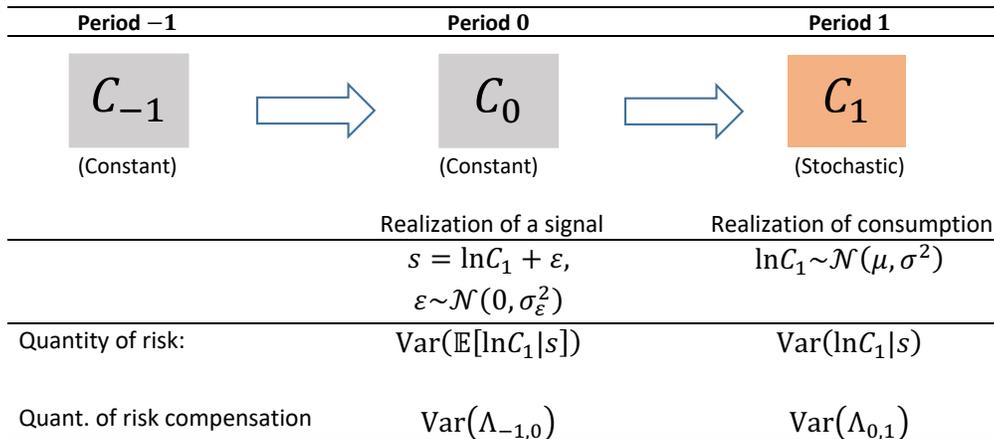
¹As in Ai and Bansal (2018), we focus on a relatively small set of pre-scheduled macroeconomic announcements that are released at monthly or lower frequencies. Within this category, we select the top five announcements ranked by investor attention from Bloomberg users. This procedure yields, on average, forty-eight announcement days per year for 1961–2021.

3 A Three-Period Model

In this section, we present a three-period model to illustrate the mechanism of information-driven volatility. We show how the informativeness of public signals affects the intertemporal distribution of risk and, under the condition of generalized risk sensitivity, also affects the intertemporal distribution of risk compensation.

We consider a representative-agent economy with three periods -1 , 0 , and 1 . The agent is assumed to have recursive utility with the intertemporal elasticity of substitution (IES) ψ , the constant risk version γ and time discount rate ρ . We assume $\psi = 1$ for simplicity and $\gamma > 1/\psi = 1$ to reflect the condition of generalized risk sensitivity from (Ai and Bansal, 2018). Denote C_t as the aggregate consumption in period t , and we plot the event tree for the economy in Figure 2. For simplicity, we assume that both C_{-1} and C_0 are deterministic. Consumption in period 1, C_1 , follows a lognormal distribution with $\ln C_1 \sim \mathcal{N}(\mu, \sigma^2)$. At time 0, there is a public announcement that provides a noisy signal about $\ln C_1$ of the form $s = \ln C_1 + \varepsilon$ with $\text{Var}[\varepsilon] = \sigma_\varepsilon^2$. Here, σ_ε^{-2} represents the informativeness of the announcement.

Figure 2: A three-period model



This figure illustrates the timing of the three-period model. Consumption in period -1 and that in period 0 are constant, whereas the consumption in period 1 follows a lognormal distribution.

In the lognormal setup, we can explicitly compute $\text{Var}[\mathbb{E}[\ln C_1 | s]]$ and $\mathbb{E}[\text{Var}[\ln C_1 | s]]$ in the variance decomposition formula of Equation (1). The expressions for the stochastic discount factors (SDF) are also standard: $\Lambda_{-1,0} = \frac{\rho}{1-\rho} \left(\frac{C_0}{C_{-1}}\right)^{-1} \left(\frac{V_0}{m_{-1}}\right)^{1-\gamma}$ and $\Lambda_{0,1} = \frac{\rho}{1-\rho} \left(\frac{C_1}{C_0}\right)^{-1} \left(\frac{C_1}{m_0}\right)^{1-\gamma}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor that prices date- $t+1$ consumption goods in terms of date- t consumption goods. Here, we use $V_0 = C_0^{1-\rho} (\mathbb{E}[C_1^{1-\gamma}])^{\frac{\rho}{1-\gamma}}$ for the date-0 utility of the agent. $m_0 = (\mathbb{E}[C_1^{1-\gamma}])^{\frac{1}{1-\gamma}}$ is the date-0 certainty equivalent of

future utility and $m_{-1} = (\mathbb{E}[V_0^{1-\gamma}])^{\frac{1}{1-\gamma}}$ is the date-1 certainty equivalent of future utility. The following proposition demonstrates how the precision of the information, σ_ε^{-2} , affects the risk the risk premium upon and after the announcement.

Proposition 1. (*Intertemporal distribution of risk and risk compensation*)

Suppose $\ln C_1 \sim \mathcal{N}(\mu, \sigma^2)$, then the total variance, $\text{Var}[\mathbb{E}[\ln C_1|s]] + \text{Var}[\ln C_1|s] = \sigma^2$ is a constant, and the variance of the conditional expectation is:

$$\text{Var}[\mathbb{E}[\ln C_1|s]] = \lambda\sigma^2, \quad (2)$$

where $\lambda = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}$, and $\frac{\partial}{\partial(\sigma_\varepsilon^{-2})}\lambda > 0$.

Furthermore, suppose $\gamma > 1$, both $\ln \Lambda_{-1,0}$ and $\ln \Lambda_{0,1}$ are decreasing functions of C_1 . In addition, the total variance of the stochastic discount factor is $\text{Var}[\ln(\Lambda_{-1,0} \times \Lambda_{0,1})] = \gamma^2\sigma^2$, and the variance of the announcement stochastic discount factor is

$$\text{Var}[\ln \Lambda_{-1,0}] = \rho^2(1-\gamma)^2\lambda\sigma^2. \quad (3)$$

Equation (2) is derived from the variance decomposition formula assuming the total amount of variance is fixed. λ reflects the weight of intertemporal distribution between risks at time 0 and 1. The fact that $\frac{\partial}{\partial(\sigma_\varepsilon^{-2})}\lambda > 0$ implies that more precise signals are associated with a larger fraction of risk being released upon the announcement at time 0, measured by the variance of conditional expectation $\text{Var}[\mathbb{E}[\ln C_1|s]]$. If we think of stock price as reflecting agent's belief about $\ln C_1$, then higher signal precision should be associated with larger stock market reactions upon the announcement of s . At the same time, the announcement also reduces the expected future variance $\text{Var}[\ln C_1|s]$. Clearly, the release of the signal s does not change the total amount of risk, but affects the intertemporal distribution of it.

The second part of the above proposition is about the risk compensation upon and after the announcement. We assume $\gamma > 1$, which reflects the condition of generalized risk sensitivity from (Ai and Bansal, 2018). Under this condition, the announcement at time 0 is associated with a non-trivial variance of the stochastic discount factor: $\text{Var}[\ln \Lambda_{-1,0}]$. In addition, $\frac{\partial}{\partial\sigma_\varepsilon^{-2}}\text{Var}[\ln \Lambda_{-1,0}] > 0$, that is, higher precision of the information revealed at the announcement is associated with a higher variance of the announcement stochastic discount factor. Because the total variance of the stochastic discount factor, $\text{Var}[\ln(\Lambda_{-1,0} \times \Lambda_{0,1})]$ does not depend on the precision of information, σ_ε^{-2} , Equation (3) implies that a higher precision of the signal corresponds to a higher risk compensation at time 0, but a lower risk compensation at time 1. Information quality affects not only the intertemporal distribution of risk but also that of the risk compensation: when public announcements are informative,

the announcement premium is higher, but the risk premium going forward will be lower.

Note that if $\gamma = 1$, the representative agent has an expected utility, then the precision of the signal σ_ε^{-2} will still affect $\text{Var}[\mathbb{E}[\ln C_1|s]]$ so that the distribution of risk, but not the risk compensation at the announcement, that is, $\text{Var}[\ln \Lambda_{-1,0}]$ in Equation (3). The assumption $\gamma > 1/\psi$ reflects the condition of generalized risk sensitivity (Ai and Bansal, 2018). In general, under the condition of generalized risk sensitivity, the magnitude of announcement premium is increasing in the precision of signals.

4 A Dynamic Model

In this section, we develop a dynamic model with time-varying information quality. Our model provides a quantitative benchmark for us to replicate the empirical tests for the information-driven volatility channel which we develop in the next section, and to assess the relevance of this mechanism in affecting stock market return predictability. In order to provide a realistic model for stock market volatility dynamics, our setup incorporates both low-frequency variations in fundamental volatility and high-frequency movements in information-driven volatility. The presence of fundamental volatility allows our model to capture the well documented positive autocorrelation of realized volatility of stock returns in the data. The presence of information-driven volatility allows our model to explain the puzzling relationship between realized volatility and expected returns in the data and emphasize the important distinction between fundamental-driven volatility and information-driven volatility in affecting stock market returns. We provide details of the model solutions and derivations in Appendix 7.2.

Preferences and endowment We consider an endowment economy where the representative agent has a Duffie and Epstein (1992a) recursive preference with the time discount rate ρ , constant risk aversion γ and constant intertemporal elasticity of substitution (IES) ψ . We assume $\gamma > 1/\psi$ such that investors' preferences satisfy generalized risk sensitivity in (Ai and Bansal, 2018) so that high-frequency information about fundamentals could be priced. Consider the following diffusion process of the growth rate of the aggregate endowment

$$\frac{dY_t}{Y_t} = \theta_t dt + \sigma_Y dB_{Y,t}, \quad (4)$$

where $B_{Y,t}$ is a standard Brownian motion and $\{\theta_t\}_{t \geq 0}$ is a two-state Markov process with the state space $\Theta = \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L$. The transition probability for θ_t over an

infinitesimal interval Δ is given by

$$\begin{bmatrix} e^{-\lambda_H \Delta} & 1 - e^{-\lambda_H \Delta} \\ 1 - e^{-\lambda_L \Delta} & e^{-\lambda_L \Delta} \end{bmatrix}. \quad (5)$$

where the intensity λ_H is the rate of transition from high state to low state, and λ_L is the transition probability from low to high.

Information quality and announcements We assume that the latent state variable θ_t is unobservable to investors. However, information about θ_t continuously arrives at the financial markets. Investors observe two sources of information about θ_t . First, the aggregate endowment (4) itself contains information about θ_t . Second, pre-scheduled macroeconomic announcements are made at time $T, 2T, \dots, nT$ for $n = 1, 2, \dots$ ² We assume that the announcement at time nT carries a noisy signal s_n about θ_{nT} , which reveals the true value of θ_t with probability ν_n . The distribution of the signal is given as follows: if $\theta_{nT} = \theta_H$, i.e., the true state is the high growth state,

$$\begin{aligned} s_n = \theta_H & \text{ with prob. } \nu_n \\ s_n = \theta_L & \text{ with prob. } 1 - \nu_n \end{aligned},$$

and if $\theta = \theta_L$, i.e., the true state is the low growth state,

$$\begin{aligned} s_n = \theta_H & \text{ with prob. } 1 - \nu_n \\ s_n = \theta_L & \text{ with prob. } \nu_n \end{aligned}.$$

Here $\nu_n \in [\frac{1}{2}, 1]$ is the key parameter in our model that measures the *information quality*, or the time-varying *informativeness* of announcements. When $\nu_n = 1$, announcements carry perfectly accurate information because the signal reveals the true state with probability one. And $\nu_n = 0.5$ indicates that announcements are completely uninformative. Namely, the announcement discloses the true state with half probability, while with another half probability, the information is entirely wrong. For simplicity, we assume that $\nu_1, \nu_2, \dots, \nu_n$ are i.i.d. over time.

Now we want to characterize investors' beliefs about θ_t . We define $\pi_t = P_t(\theta_t = \theta_H)$ as the probability of $\theta_t = \theta_H$ and $\hat{\theta}_t = \mathbb{E}_t[\theta_t]$ to be the posterior mean of θ_t . That is, $\hat{\theta}_t = \pi_t \theta_H + (1 - \pi_t) \theta_L$. In the interior of $((n - 1)T, nT)$ without announcements, investors

²Our benchmark model matches the quantitative results based on FOMC announcements, that are pre-scheduled and periodically announced. However, our information-volatility theory does not rely on the assumption of fixed announcements. Instead, we show that if we allow announcements to be an unexpected Poisson process, our results still hold.

update their beliefs based on the observed consumption process (4). The posterior mean of θ_t satisfies

$$d\hat{\theta}_t = (\lambda_H + \lambda_L) (\bar{\theta} - \hat{\theta}_t) dt + (\theta_H - \hat{\theta}_t) (\hat{\theta}_t - \theta_L) \frac{1}{\sigma_Y} d\hat{B}_{Y,t}, \quad (6)$$

where $\bar{\theta} = \frac{\lambda_L \theta_H + \lambda_H \theta_L}{\lambda_L + \lambda_H}$ is the steady-state mean of θ_t , and $\hat{B}_{Y,t}$ is the innovation process defined by $d\hat{B}_{Y,t} = \frac{1}{\sigma_Y} \left(\frac{dY_t}{Y_t} - \hat{\theta}_t dt \right)$, which indicates the surprises from the difference between observed and expected consumption growth.

At the announcements nT , investors update their beliefs based on the second source of information s_n from the announcement. For notational convenience, we denote nT^+ as the moment right after the announcements, and nT^- refers to the moment right before the announcements.³ In Appendix 7.2.1, we show that given the prior belief $\hat{\theta}_{nT}^-$ before the announcement, the posterior distribution of $\hat{\theta}_{nT}^+$ after the announcement follows

$$\hat{\theta}_{nT}^+ = \begin{cases} \theta_L + \frac{(\hat{\theta}_{nT}^- - \theta_L) \nu_n (\theta_H - \theta_L)}{(\hat{\theta}_{nT}^- - \theta_L) \nu_n + (\theta_H - \hat{\theta}_{nT}^-) (1 - \nu_n)} & \text{w.p. } \frac{\hat{\theta}_{nT}^- - \theta_L}{\theta_H - \theta_L} \nu_n + \frac{\theta_H - \hat{\theta}_{nT}^-}{\theta_H - \theta_L} (1 - \nu_n) \\ \theta_L + \frac{(\hat{\theta}_{nT}^- - \theta_L) (1 - \nu_n) (\theta_H - \theta_L)}{(\hat{\theta}_{nT}^- - \theta_L) (1 - \nu_n) + (\theta_H - \hat{\theta}_{nT}^-) \nu_n} & \text{w.p. } \frac{\hat{\theta}_{nT}^- - \theta_L}{\theta_H - \theta_L} (1 - \nu_n) + \frac{\theta_H - \hat{\theta}_{nT}^-}{\theta_H - \theta_L} \nu_n \end{cases}. \quad (7)$$

The informativeness ν not only influences the posterior mean of investors' beliefs about the latent growth rate, but also governs the probability of observing the accurate signal of the underlying state. For example, it is straightforward to verify that if $\nu = 1$, the signal is perfectly informative, and the above becomes $\hat{\theta}^+ = \begin{cases} \theta_H & \text{w.p. } \frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} \\ \theta_L & \text{w.p. } \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} \end{cases}$. And if $\nu = 0.5$, the signal is absolutely uninformative and $\hat{\theta}^+ = \hat{\theta}^-$ with probability one.

Asset prices We assume the aggregate stock market is the claim to the following dividend process

$$\frac{dD_t}{D_t} = \left[\xi (\hat{\theta}_t - \bar{\theta}) + \bar{\theta} \right] dt + \sigma_Y d\hat{B}_{Y,t} + \sigma_{i,t} dB_{i,t}, \quad (8)$$

where ξ and is the leverage on expected consumption growth, and $dB_{i,t}$ is a Brownian motion uncorrelated with the consumption shock $d\hat{B}_{Y,t}$. Here we allow the idiosyncratic volatility $\sigma_{i,t}$ in dividend growth rate to be time varying and we assume it to follow a two-state Markov chain with state space $\{\sigma_i^H, \sigma_i^L\}$, where $\sigma_i^H > \sigma_i^L$. The transition matrix over an small interval Δ is $\begin{bmatrix} e^{-\lambda_i^H \Delta} & 1 - e^{-\lambda_i^H \Delta} \\ 1 - e^{-\lambda_i^L \Delta} & e^{-\lambda_i^L \Delta} \end{bmatrix}$, where λ_i^H (λ_i^L) denotes the probability of transition

³Without loss of generality, we focus on stationary equilibria across different announcement cycles. Therefore, prices and quantities are the same over difference cycles. For notational convenience, we focus on a representative cycle, $[0, T]$. T^+ (or nT^+) and T^- (or nT^-) denote the moment right after and before the announcement, respectively. Whenever there is any confusion, time 0 should be understood as T^+ and T is equivalent to T^- .

from high (low) to low (high) state of dividend idiosyncratic volatility. This specification allows our model to capture the low-frequency movements in fundamental volatility as shown in Figure 1 and generate a positive autocorrelation in stock market volatility, which is an important feature of the data.

The lifetime utility of the representative agent can be written as a function of state variables: $V(\hat{\theta}_t, t, Y_t) = \frac{1}{1-\gamma} H(\hat{\theta}_t, t) Y_t^{1-\gamma}$. As a result, changes in beliefs about θ_t are immediately reflected through the variations in the continuation utility $H(\hat{\theta}_t, t)$. We show the solutions of the value function in Lemma 1 in Appendix 7.2.1. Given the value function, we can construct the pricing kernel M_t . The law of motion of M_t in the interior of $((n-1)T, nT)$, $n = 1, 2, \dots$ without announcements can be written as

$$\frac{dM_t}{M_t} = -r(\hat{\theta}_t, t) dt - \sigma_M(\hat{\theta}_t, t) d\hat{B}_{Y,t}, \quad (9)$$

where the risk-free rate $r(\hat{\theta}_t, t)$ and the market price of risk $\sigma_M(\hat{\theta}_t, t)$ are given in Equations (54) and (55) in Appendix 7.2.2.

Denote $p(\hat{\theta}_t, t)$ as the price-to-dividend ratio so that the stock price is given by $p(\hat{\theta}_t, t)D_t$. By definition, the stock price is the discounted future cash flow:

$$p(\hat{\theta}_t, t) D_t = \mathbb{E} \left[\int_0^\infty \frac{M_{t+s}}{M_t} D_{t+s} ds \mid \hat{\theta}_t, t \right]. \quad (10)$$

We provide the expression for the partial differential equations (PDE) together with the boundary conditions that determine the solution of $p(\hat{\theta}_t, t)$ in Appendix 7.2.2. With the pricing kernel and the price-to-dividend ratio, the market risk premium is given by the following proposition:

Proposition 2. (*Equity premium*)

In the interior of $((n-1)T, nT)$, $n = 1, 2, \dots$, the instantaneous risk premium is given by:

$$\mathbb{E}_t \left[\frac{d \left[p(\hat{\theta}_t, t) D_t \right] + D_t dt}{p(\hat{\theta}_t, t)} \right] - r(\hat{\theta}_t, t) dt = \sigma_M(\hat{\theta}_t, t) \left(\frac{p_\theta}{p} \frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y \right), \quad (11)$$

where $\frac{p_\theta}{p} = \frac{\partial p(\hat{\theta}_t, t) / \partial \hat{\theta}_t}{p(\hat{\theta}_t, t)}$. At the announcements nT , the announcement premium is given by:

$$\mathbb{E}_{nT^-} \left[\frac{p(\hat{\theta}_{nT}^+, nT^+)}{p(\hat{\theta}_{nT}^-, nT^-)} \right] - 1 = \frac{\left(\mathbb{E}_{nT^-} \left[H(\hat{\theta}_{nT}^+, nT^+) \right] \right)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}} \mathbb{E}_{nT^-} \left[p(\hat{\theta}_{nT}^+, nT^+) \right]}{\mathbb{E}_{nT^-} \left[H(\hat{\theta}_{nT}^+, nT^+)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}} p(\hat{\theta}_{nT}^+, nT^+) \right]} - 1. \quad (12)$$

where $\hat{\theta}_{nT}^+$ is drawn from the distribution in Equation (7).

In Appendix 7.2.2, we show that $\frac{H(\hat{\theta}_{nT}^+, nT^+)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}}}{\mathbb{E}_{nT^-} \left[H(\hat{\theta}_{nT}^+, nT^+)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}} \right]}$ is the announcement stochastic discount factor. Note that the value function $H(\hat{\theta}_t, t)$ is decreasing in $\hat{\theta}_t$ while the price-to-dividend ratio $p(\hat{\theta}_t, t)$ is an increasing function of $\hat{\theta}_t$. Under the assumption $\gamma > 1/\psi$ required by the generalized risk sensitivity in preferences, the term $H(\hat{\theta}_{nT}^+, nT^+)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}}$ will be negatively correlated with $p(\hat{\theta}_{nT}^+, nT^+)$. As a result, $\text{Cov} \left[H(\hat{\theta}_{nT}^+, nT^+)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}}, p(\hat{\theta}_{nT}^+, nT^+) \right] < 0$ and the announcement requires a positive risk compensation. This gives rise to the macroeconomic announcement premium we observed in the data (Savor and Wilson, 2013). Mathematically, upon announcements, the investor's belief jumps from $\hat{\theta}_{nT}^-$ to $\hat{\theta}_{nT}^+$ according to Equation (7) so does the value function and price-to-dividend ratio. Intuitively, investors require compensations for risk because the arrival of information over a short time interval creates significant variations in the continuation utility. Risk premium starts to realize after the uncertainty resolution in beliefs. If $\gamma = 1/\psi$, the investor has the expected utility. The announcement stochastic discount factor becomes one; therefore, the information contained in the announcement will not be priced.

Finally, we define the implied variance of the stock return at time t with maturity $\tau - t$ is defined as the expected variance of the log return from t to τ , where the expiration date $\tau \geq t$, and $h = \tau - t$ is the day-to-maturity:

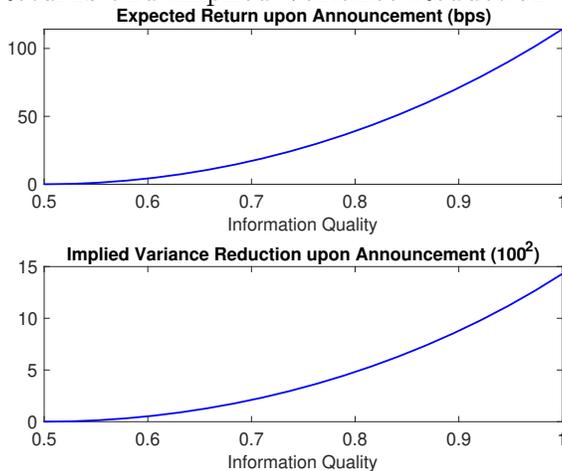
$$IV_{t,\tau}(\hat{\theta}_t, t) = \text{Var}_t \left[\ln \left[p(\hat{\theta}_\tau, \tau) D_\tau \right] - \ln \left[p(\hat{\theta}_t, t) D_t \right] \right]. \quad (13)$$

Because the log price $\ln \left[p(\hat{\theta}_t, t) D_t \right]$ is known at time t , we can simply write $IV_{t,\tau}(\hat{\theta}_t, t) = \text{Var}_t \left[\ln \left[p(\hat{\theta}_\tau, \tau) D_\tau \right] \right]$. We present solutions to our model implied IV in Lemma 4 and 5 in Appendix 7.2.3.

Comparative statics with respect to informativeness of announcements The key mechanism in our model is that the arrival of information affects the intertemporal distribution of both risk and risk compensation. In this section, we plot policy functions from our calibrated model to illustrate the two unique implications of the information-driven volatility channel, I1 and I2. We provide the formula in computing policy functions in Appendix 7.2.4.

To illustrate the Implication I1, in Figure 3, we plot the announcement premium (Equation (12)) in the top panel and the implied variance reduction in the bottom panel as functions of the information quality ν . Instead of plotting the variance of announcement return, we plot the implied variance reduction as the latter is a market expected-based measure of the variance of announcement return.

Figure 3: Expected Returns and Implied Variance Reduction upon Announcements



The top panel is the expected announcement return as a function of the information quality ν , where expected returns are measured in basis points. The bottom panel is the option implied variance reduction upon announcements (in percentage square) implied by our model as a function of informativeness ν .

Applying the general definition in (13), the implied variance at time T^+ right after an announcement is $IV_{T,\tau}^+(\hat{\theta}_T^+, T^+) = \text{Var}_T^+ \left[\ln \left[p \left(\hat{\theta}_\tau, \tau \right) D_\tau \right] \right]$, and the implied variance before the announcement and right after the value of ν is revealed is $IV_{T,\tau}^-(\hat{\theta}_T^-, T^-, \nu) = \text{Var}_T^- \left[\ln \left[p \left(\hat{\theta}_\tau, \tau \right) D_\tau \right] \right]$. We define implied variance reduction upon the announcement as the difference between implied variance before and after the announcement,

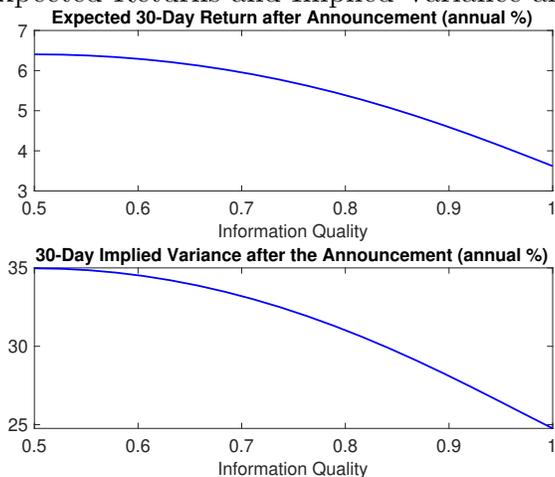
$$\Delta IV_T = IV_{T,\tau}^-(\hat{\theta}_T^-, T^-, \nu) - IV_{T,\tau}^+(\hat{\theta}_T^+, T^+). \quad (14)$$

In Appendix 7.2.3, we show that ΔIV_T can be used to compute the variance of the announcement return: $\mathbb{E} \left[\Delta IV_T | \nu, \hat{\theta}_T^-, T^- \right] = \text{Var} \left(\ln p \left(\hat{\theta}_T^+, T^+ \right) - \ln p \left(\nu, \hat{\theta}_T^-, T^- \right) | \nu, \hat{\theta}_T^-, T^- \right)$.

As shown in Figure 3, at $\nu = 0.5$, the announcement is completely uninformative. The expected announcement return and the implied variance reduction upon the announcement

are both zero. As ν increases from 0.5 to 1, the expected return and the implied variance reduction rise monotonically. Consistent with the intuition in the three-period model, the expected return and expected variance on announcement days are both increasing functions of the informativeness of the announcement. Because of the monotonic relationship between the informativeness ν and the implied variance reduction ΔIV_T , the latter provides a market price-based measure of the informativeness of announcements, which we use to exploit in the empirical exercises in the following section.

Figure 4: Future Expected Returns and Implied Variance after Announcements



The top panel is the expected 30-day return (annual percentage) during the post-announcement period as a function of the information quality ν implied by our model. The bottom panel is the 30-day implied variance (annual percentage) after the announcement implied by our model.

In Figure 4, we plot the model-implied expected return during the 30-day period after the announcement (top panel), and the 30-day implied variance during the same period (bottom panel) as a function of the information quality ν . Clearly, as the informativeness of announcements increases, the expected return after the announcement in the future reduces, and so does the expected variance of the market return during the same period. More informative announcements resolve a larger fraction of uncertainty about future consumption growth and are associated with lower expected returns and lower expected variance after the announcement going forward.

5 Quantitative Results

In this section, we first calibrate our model and demonstrate that it can quantitatively match a broad set of moments of the aggregate market, especially announcement-related moments. We then develop empirical tests for the information-driven volatility channel and replicate

these tests in our model. Finally, we show that our model provides an explanation for the variance risk premium return predictability without assuming high-frequency variations in the volatility of macroeconomic fundamentals.

Parameter values We choose a discount rate $\rho = 1.5\%$, a risk aversion $\gamma = 20$, an IES $\psi = 2$, a leverage parameter $\xi = 3$ in line with the standard long-run risk literature. We set the volatility of consumption growth $\sigma_Y = 3\%$ to match the volatility of annual consumption growth in the U.S. in our sample period from 1929.01-2019.12. We calibrate the value of the two Markov states $\theta_H = 4.4\%$, $\theta_L = -1.7\%$ and the transition probabilities to match the mean, standard deviation, and autocorrelation of the aggregate consumption data. For simplicity, we assume $\lambda_H = \lambda_L$. We set the dividend volatility parameters σ_i^H and σ_i^L and the transition probabilities $\lambda_i^H = 0.06$ and $\lambda_i^L = 0.025$ by estimating a regime switching model of dividend growth rates. We provide details for the data sources and estimation procedures in the Appendix.

Table 2: Calibrated Parameters

<i>Panel A. Preferences</i>					
ρ	Time discount rate	0.015	γ	Relative risk aversion	20
ψ	IES	2			
<i>Panel B. Consumption and dividend dynamics</i>					
σ_Y	Endowment growth volatility	0.03	ξ	Leverage	3
θ_H	High endowment growth state	0.044	θ_L	Low endowment growth state	-0.017
λ_H	θ transition prob. (high to low)	0.8	λ_L	θ transition prob. (low to high)	0.8
σ_i^H	High dividend idiosyncratic vol.	0.14	σ_i^L	Low dividend idiosyncratic vol.	0.04
λ_i^H	σ_i transition prob. (high to low)	0.06	λ_i^L	σ_i transition prob. (low to high)	0.025
<i>Panel C: Information</i>					
ν_H	High informativeness of ann.	0.999	ν_L	Low informativeness of ann.	0.6
$\frac{1}{T}$	Frequency of announcements	8			

This table displays the calibrated annual parameters in our model.

The parameters ν_H and ν_L govern the informativeness of the announcements. We set $\nu_H = 0.999$ and $\nu_L = 0.60$ so that our model matches the mean and standard deviation of implied variance reduction on announcement days. In particular, $\nu_L > 0.5$ so that our model can also generate implied variance *increase* upon the announcement, as shown in the 5th quantile in Table 4. Otherwise, if $\nu_L = 0.5$, the announcement is completely uninformative, the implied variance reduction will be zero instead of a negative number as in the data. Finally, we choose $T = \frac{1}{8}$ so that there are eight announcements per year in our model,

matching the frequency of FOMC announcements in the data. All calibrated parameters are listed in Table 2.

Basic statistics of model-implied unconditional moments We list the asset pricing moments in the data and the corresponding statistics for our calibrated model in Table 3. We simulate our continuous-time model at daily frequencies and aggregate it quantitatively to appropriate frequencies to compare with the data.

Table 3: Asset Pricing Moments

<i>Panel A: Aggregate market moments</i>		Data	Model
$\mathbb{E}[R] - r_f$	Equity premium	7.46%	5.69%
Std $[R]$	Vol of market return	18.55%	12.03%
$\mathbb{E}[r_f]$	Average risk-free rate	0.26%	0.42%
Std $[r_f]$	Vol of risk-free rate	1.08%	0.92%
$\mathbb{E}[dY/Y]$	Consumption growth rate	1.74%	1.33%
Std $[dY/Y]$	Vol of consumption growth rate	2.72%	3.36%
AC $[dY/Y]$	AC(1) of consumption growth rate	0.38	0.36
$\mathbb{E}[dD/D]$	Dividend growth rate	1.55%	1.24%
Std $[dD/D]$	Vol of dividend growth rate	10.66%	9.12%
AC $[dD/D]$	AC(1) of dividend growth rate	0.17	0.35
Corr $(dY/Y, dD/D)$	Corr b.t. consumption and dividend	0.43	0.64
AC $[RV]$	AC(1) of RV	0.65	0.79
<i>Panel B: Announcement moments</i>		Data	Model
$\mathbb{E}[R^A]$	A-day average return	26 bps	29 bps
$\mathbb{E}[R^N]$	NA-day average return	2 bps	1 bps
$\mathbb{E}[\Delta IV_T]$	Av. IV reduction on A-days	2.14	4.18
Std $[\Delta IV_T]$	Std. of IV reduction on A-days	8.86	4.87

This table displays the asset pricing moments in the data and those implied by the model. The data in Panel A contains the period of 1929.01-2019.12. The data in Panel B contains the period of 1994.09-2019.12.

Our model matches well both the return and volatility moments in the data. The average equity market premium in the model is 5.69% per year, and the volatility of annual market return is 12%. Our model generates an average level of the risk-free rate of 0.42%, with a standard deviation of 0.92% per year. Both moments are close to their data counterparts in the data. Our model also captures the dynamics of consumption and dividend growth in the data. The first two moments and autocorrelation of consumption growth in the data (model) are 1.74% (1.33%), 2.72% (3.36%) and 0.38 (0.36). The model-based mean, volatility and autocorrelation of dividend growth are 1.24%, 9.12% and 0.35, fairly close to 1.55%, 10.66% and 0.17 in the data. Our model produces a significant announcement premium. The average

announcement-day return is 29 bps and the average non-announcement day return is 1 bps, which are very close to the same moments (26 bps and 2 bps) reported in the data.

Several key moments in the data are particularly important in assessing the quantitative importance of the information-driven volatility channel. First, as we explain earlier, changes in fundamental volatility and information-driven volatility both affect the dynamics of stock return volatility. Fundamental-driven volatility induces a positive auto-correlation of return volatility, and information-driven volatility induces a negative autocorrelation. The auto-correlation of stock market return volatility in the data therefore provides an upbound on how strong the information driven volatility channel can be: a large magnitude of information driven volatility may result in a counterfactually negative auto-correlation of return volatility. Thanks to the stochastic volatility in dividend growth, our calibration matches the autocorrelation of stock market return volatility quite well. The first-order autocorrelation of realized volatility of annual stock market returns is 0.65 in the data, and the same moment is 0.79 in our model.

Second, our model successfully matches moments of implied variance reduction on announcement days. As we remarked earlier, ΔIV_T provides a market-based measure of the informativeness of announcements. In Table 3, implied variance reduction is significantly positive on announcement days and averages 2.14 (monthly percentage squared) in the data. Our model features a similar magnitude of implied variance reduction and a similar standard deviation as in the data.

Table 4: Implied Variance Reduction

	Q5	Q25	Q50	Q75	Q95
Data	-5.54	-0.26	1.14	3.88	10.00
Model	-0.29	0.24	1.82	7.07	14.63

This table displays the quantile of 30-day implied variance drop on announcement days in the data and that in the model. The data contains the period of 1994.09-2019.12. Implied variance reductions are in monthly percentage squared unit.

In Table 4, we tabulate the histogram of ΔIV_T in the data and the same in our model. In the data, implied variance typically drops after announcements, but occasionally *increases*, as shown in the 5th and 25th percentiles in Table 4. Our model shows a similar pattern. An average positive reduction in implied variance after arrivals of information is a general property of Bayes' updating. The fact that implied variance can occasionally increase after the information release requires a deviation from the Gaussian distribution. Because investors' posterior variance mainly determines the implied variance in our model, we illustrate the

above pattern of implied variance reduction using the variance decomposition identity.

Because $\text{Var} [\mathbb{E} (C_T|X_t)] \geq 0$, Equation (1) implies that $\text{Var} [C_T] \geq \mathbb{E} [\text{Var} (C_T|X_t)]$. That is, *on average*, the conditional variance must decline after information arrival and, like enough, the implied variance. However, in general, $\text{Var} [C_T] \geq \mathbb{E} [\text{Var} (C_T|X_t)]$ allows the posterior variance to rise above the unconditional variance, $\text{Var} [C_T] < \text{Var} (C_T|X_t)$ for some realizations of information. In our model’s context of the two-state Markov chain, signals are wrong with the probability $1 - \nu$. An error in the announcement may trigger an increase in posterior variance and the implied variance of returns. Note that $\text{Var} [C_T] < \text{Var} (C_T|X_t)$ cannot happen under the Kalman filter with Gaussian distribution, because $\text{Var} (C_T|X_t)$ will be a constant and does not depend on the realization of information.

Testing the information-driven volatility channel This section conducts statistical tests for Implications I1 and I2 of the information-driven volatility channel. We first present these tests using the U.S. stock market return data and then replicate them with the simulated data from our model.

Testing Implication I1 requires a measure of the informativeness of FOMC announcements so that we can test how informativeness predicts FOMC returns. We cannot use implied variance reduction directly, as it requires the knowledge of implied variance after the announcement. It is subject to the bias induced by the well-known correlation between realized returns and realized variance. An ideal measure of informativeness should only use ex-ante information before announcements, and be a good predictor for implied variance reduction after announcements.

Our construction of the informativeness measure is based on the following intuition. Suppose implied variance reflects information arrival, and suppose FOMC announcement days have more information arrivals than non-announcement days. Denote the variance of information on the upcoming announcement day as *Info* and the variance of information on non-announcement days as *InfoN*. Before the announcement at time $T - 1$, suppose we observe an implied variance with 9 days to maturity ($IV_{T-1,T+8}$) and another implied variance with 30 days to maturity ($IV_{T-1,T+29}$). Then the two market-implied variances allow us to back out the two unknowns, *Info* and *InfoN*:

$$IV_{T-1,T+8} = \text{Info} + 8 \times \text{InfoN}, \tag{15}$$

$$IV_{T-1,T+29} = \text{Info} + 29 \times \text{InfoN}. \tag{16}$$

Info reflects the information content of the upcoming announcement. Empirically, we make two modifications to the above simple construction. First, we find that normalizing the

implied variance on the left-hand side of (15) and (16) by the realized variance with the same maturity yields a more effective measure of informativeness. This is intuitive, because a high implied variance may either be due to a period of heightened fundamental volatility or anticipation of more informative events in the future. Normalizing by realized variance allows us to control for the fundamental volatility. Given the nature of the stochastic volatility in dividend growth, this correction also yields a better measure of informativeness in our model. Second, whenever we have more than two maturities of implied variance, we use all available maturities by finding a *Info* and *InfoN* that minimize the mean squared error of similar equations of (15) and (16) constructed from all maturities.

In Table 5, we present the results of the following two regressions on the informativeness measure constructed above:

$$R_t^A = \alpha + \beta \times Info_{t-1} + \varepsilon_t, \quad (17)$$

and

$$\Delta IV_t = \alpha + \beta \times Info_{t-1} + \varepsilon_t, \quad (18)$$

where R_t^A is the announcement-day return, earned from the beginning of the announcement day to the end of the announcement day; $\Delta IV_t = IV_{t-1,t+29} - IV_{t,t+30}$ is the difference between the 30-day option implied variance (VIX index squared) on the day before the announcement and the announcement day; $Info_{t-1}$ is the informativeness measure we construct using option prices on the day before the announcement at time $t - 1$.

Table 5: Announcement return and IV reduction predictability by informativeness

		R_t^A	ΔIV_t
$Info_{t-1}$	Data	65.04 (4.70)	6.32 (3.25)
	R^2 (%)	15.44	22.20
ν_t	Model	22.22	3.98

This table presents the results of the return predictability regressions defined in (17) and the implied variance reduction predictability regression defined in (18). The bottom panel is the model-implied regression coefficients of the informativeness of announcements ν_t . Both informativeness in the data and model, $info_{t-1}$ and ν_t , are normalized by the mean and standard deviation. Returns are in daily basis points; implied variance reductions are in monthly percentage squared unit. Newey-West t -statistics are in parentheses.

First, our measure of informativeness has strong predictive power for implied variance reduction on announcement days. The coefficient is significantly positive with a t -statistic

of 3.25 and an R^2 of 22.20%. This result indicates that $Info_{t-1}$ is an effective measure of the informativeness of the upcoming announcement perceived by the market.

Second, as shown in the table, in the return predictability regression, the regression coefficient on $Info_{t-1}$ is positive and significant with a t -statistic of 4.70. The above regression has an R^2 of 15.44%. This provides a strong support for Implication I1 especially given that daily returns are notoriously hard to be predicted in the data.

In the model, we run the exactly same regressions as in the data. Our model has a precise measurement of informativeness: ν_t , which captures the information quality of the announcements. The announcement return and implied variance reduction are defined in Equations (12) and (14), respectively. Our model produces significantly positive regression coefficients of 22.22 and 3.98 in predicting the announcement returns and implied variance reduction upon the announcements, close to the counterparts in the data. Therefore, we confirm our intuition that higher informativeness is associated with higher expected return and higher expected variance upon the arrival of the information.

Our second set of tests are predictability regressions for post-announcement returns and post announcement realized variance by the informativeness of the announcement. Because our interest is to predict the returns and realized variance *after* the announcement, we can directly use the announcement-day drops in implied variance ΔIV_t as the measure of the informativeness instead of the ex-ante measure of $Info_{t-1}$. We consider the following regression specification:

$$RV_{t,t+h} = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \varepsilon_{t,t+h}, \quad (19)$$

for the realized variance predictability, and

$$R_{t,t+h} - r_f = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \varepsilon_{t,t+h}, \quad (20)$$

for the realized excess return predictability. In the above regressions, $RV_{t,t+h}$ and $R_{t,t+h}$ are the realized variance and realized returns, respectively, from the announcement day t (not including the announcement day itself) to h days after the announcement, for various choices of h : $h = 1, 2, 3, 4, 5, 30, 60$ up to two months.⁴ We control for $RV_{t-2,t-1}$, the realized variance on the day before announcement days as a measure of the level of uncertainty before the announcement.⁵ Our main interest is the regression coefficients on the measure of the informativeness of the announcement, ΔIV_t . We report our regression results in Table 6.

⁴Note that h indicates the calendar days. In the data, we use [1, 2, 3, 4, 5, 21, 42] trading days.

⁵In the Online Appendix, we show our predictability results hold if we also include the implied variance on the announcement day, IV_t , that is known to predict realized variance. Specifically, we consider the

We conduct the same regressions in the model, where we provide our detailed solutions of $R_{t,t+h}$ and $RV_{t,t+h}$ in the Online Appendix. The reason that we control for the realized variance before the announcement is as follows. In addition to the information-driven volatility channel, the level of pre-announcement posterior variance, a measure of the level of risk, is another channel that induces a high expected return and expected variance in the future. Holding the implied variance reduction as constant, higher levels of posterior variance must be associated with higher expected returns and variance in the future because realized variance is positively autocorrelated and investors require larger risk premium in the future. This is a standard result from the stochastic volatility type of models, that is independent of our information-driven volatility. Therefore, in order to uniquely identify our information-driven volatility channel, we control for the realized variance before the announcement.

Table 6: Model-Implied Return and Variance Predictability by IV Reduction

Number of days		1	2	3	4	5	30	60
$RV_{t,t+h}$	Data	-0.12	-0.08	-0.05	-0.05	-0.05	-0.07	-0.04
		(-3.83)	(-3.43)	(-2.41)	(-2.57)	(-2.70)	(-4.15)	(-2.54)
R^2 (%)		65.30	58.80	44.94	43.63	43.70	33.16	23.61
$RV_{t,t+h}$	Model	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.04
$R_{t,t+h} - r_f$	Data	-2.55	-2.02	-1.08	-1.26	-0.61	-0.19	-0.30
		(-1.03)	(-1.62)	(-1.92)	(-2.08)	(-1.61)	(-1.25)	(-2.03)
R^2 (%)		4.55	3.83	3.13	7.83	1.66	1.09	1.73
$R_{t,t+h} - r_f$	Model	-0.56	-0.48	-0.34	-0.25	-0.31	-0.16	-0.23

This table presents the results of the realized variance predictability regression (19) and the excess return predictability regression (20) in the data and those in our model. The 3-9 columns represent the horizon of returns and variances on the left hand side of Equations (19) and (20), respectively, with $h = 1, 2, 3, 4, 5, 30, 60$ calendar days. We normalize right-hand-side variables in the model. Returns and realized variances are in daily basis points. Implied variance reductions are in monthly percentage squared unit. Newey-West t -statistics are in parentheses.

Consistent with the policy functions we plot in Figure 4, in our model, more informative announcements are associated with lower realized variance and lower expected returns after the announcements going forward. As a result, the betas in both regressions, (19) and (20),

alternative regressions:

$$RV_{t,t+h} = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \beta_3 IV_t + \varepsilon_{t,t+h},$$

for the realized variance predictability, and

$$R_{t,t+h} - r_f = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \beta_3 IV_t + \varepsilon_{t,t+h},$$

for the realized excess return predictability.

are negative in the model simulated data. Consistent with our model, these regressions show a similar pattern in the data. The drops of implied variance on announcement days negatively predict post-announcement day variances at 1-5 days horizon. This pattern extends to the one-month horizon, but dissipates over time and becomes insignificant over the two-month horizon. The announcement-day drop in implied variance can also negatively predict post-announcement day returns up to the two-month horizon. The decreasing patterns of predictabilities over time further establish our intuition that information-driven volatility is crucial in explaining the expected return and variance, especially in the short run over higher frequencies. All of the above evidence confirms the basic mechanism of the information-driven volatility mechanism we illustrate in Figures 3 and 4.

Additional tests for the information-driven volatility channel In search for major events with large public information release about the economy, we identify informational days on which the S&P 500 Index daily returns (close to close) are above 2.5 percent or below -2.5 percent. According to Baker, Bloom, Davis, and Sammon (2021), all these informational days are well matched with at least one type of news.⁶ In our 30 years sample, we have in total 295 informational days. We then classify a month as “informational” if at least one of the trading days within this month is an informational day. Finally, we identify 95 months as informational months. To test the model implication, that is, informational events are followed by lower future expected returns, we run the following regression,

$$R_{t,t+h} - r_f = \alpha + \beta RV_{t-h,t} + \varepsilon_{t,t+h}, \quad (21)$$

where $h = 30$ so that $RV_{t-h,t}$ is the realized variance in the current month, and $R_{t,t+h}$ is the return in the future month.

In the model, because the announcement is pre-scheduled eight times per year, the informational month is naturally the month with one announcement. We again run the same regressions as in the data and report the regression results in Table 7.

The first column in Table 7 uses all months available in our sample. We find a significantly negative relationship between the current month’s realized variance and the next month’s return. This finding is consistent with that of Nelson (1991), Glosten, Jagannathan, and Runkle (1993), and Moreira and Muir (2017). Suppose we only keep the informational months in the sample and use the realized variance in these informational months to predict the returns in the next month. In that case, we find the negative relationship is even stronger

⁶In Baker, Bloom, Davis, and Sammon (2021), the categories for news causes are Macroeconomic News & Outlook, Corporate Earnings & Outlook, Sovereign Military & Security Actions, Government Spending, Commodities, etc.

Table 7: Monthly Return Predictability by Monthly Realized Variance

		Full Sample	Informational Month t	The Rest
$R_{t,t+h} - r_f$	Data	-6.00 (-2.11)	-15.07 (-4.12)	-0.46 (-0.22)
R^2 (%)		1.48	4.91	0.01
$R_{t,t+h} - r_f$	Model	-0.46	-0.44	0.32

This table presents the results of the monthly return predictability regression (21). The 1-3 columns represent the monthly returns (in percentage) and realized variances on the left hand side of Equations (21) for different samples with $h = 30$ within a month. Returns are in annual percentage points and realized variances are in monthly percentage squared unit. Newey-West t -statistics are in parentheses.

with a much higher degree of significance with a t -statistic of -4.12 and an R^2 of 4.91 in the data. Since these informational months are months with large information releases, there will be less risk and therefore less risk premium in the future. In addition, the last column elaborates that our model predicts an even positive correlation of realized variance and future returns conditional on months with no announcements. We also see no predictability of returns conditional on months that have no news event in the data. This subsample analysis with informational months showcases the information-driven volatility mechanism in our model.

We next propose a more rigorous test on whether the informativeness of an informational event can predict risk and risk premium in the future. To perform these tests, we adopt the realized variance predictability regression (19) and the return predictability regression (20). In this analysis, the $RV_{t,t+h}$ and $R_{t,t+h}$ are the realized variance and realized returns, respectively, from the informational day t (not including the informational day itself) to h days after the informational day. We report our regression results in Table 8.

Predictability regression results in Table 8 again provide consistent support for our information-driven volatility mechanism. The drops of implied variance on informational days significantly negatively predict post-informational day variances and returns at 1-5 days horizon. Overall, our analysis on informational days echoes the analysis on FOMC announcement days. The informational event days of higher informativeness usually predict lower risk and lower risk premium in the subsequent days because of the information-driven volatility channel.

Variance risk premium predictability In this section, we report our model’s implications on variance risk premium (VRP) return predictability. Previous literature has documented robust empirical evidence on return predictability by the difference between option-

Table 8: Expected Return and Variance Predictability by IV Reduction around Jump Days

Number of days	1	2	3	4	5	21	42
$RV_{t,t+h}$	-0.06 (-3.59)	-0.06 (-4.41)	-0.05 (-5.05)	-0.05 (-4.53)	-0.04 (-4.20)	-0.03 (-2.63)	-0.02 (-2.35)
R^2 (%)	37.12	41.56	41.08	40.32	39.45	37.49	34.26
$R_{t,t+h} - r_f$	-1.05 (-3.93)	-1.07 (-6.09)	-0.46 (-3.14)	-0.51 (-3.62)	-0.58 (-4.71)	-0.09 (-1.69)	-0.00 (-0.05)
R^2 (%)	3.88	7.20	2.19	3.62	6.07	3.27	4.65

This table presents the results of the realized variance predictability regression (19) and the return predictability regression (20) in the data. The 3-9 columns represent the horizon of returns (in percentage) and variances on the left hand side of Equations (19) and (20), respectively, with $h = 1, 2, 3, 4, 5, 21, 42$ trading days. Returns and realized variances are in daily basis points. Implied variance reductions are in monthly percentage squared unit. Newey-West t -statistics are in parentheses.

implied variance and realized variance up to six-month horizons. The stochastic volatility models developed to address this empirical phenomenon typically relied on high-frequency variations in the volatility of aggregate consumption, for example, Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011). However, in our model, information-driven volatility creates a wedge between implied and realized variance. The reason is that the implied variance before the announcement includes investors’ expectations about the upcoming announcements, whereas the past realized variance does not. Therefore, the difference between implied and realized variance in our model immediately reflects the informativeness of the upcoming announcement. Because implied variance is a forward-looking measure of variance, it increases when the upcoming announcement is expected to be informative. The difference between implied and realized variance predicts returns because more informative announcements are associated with higher realizations of announcement premiums.

In Table 9, we report the results of the return predictability regression:

$$R_{t,t+h} - r_f = \alpha + \beta [IV_{t,t+30} - RV_{t-30,t}] + \varepsilon_{t,t+h}, \tag{22}$$

where $R_{t,t+h}$ is the cumulative market return from time t to time $t + h$. Here t denotes one day before the announcement so that $R_{t,t+1}$ indicates the announcement-day return. $IV_{t,t+30}$ is the forward-looking 30-day implied variance (VIX index squared) at time t , and $RV_{t-30,t}$ is the past 30-day realized variance. The regression coefficients are statistically significant and increase up to six months. As in the data, in our model, returns are predictable by the differences between IV and RV . The regression coefficients of returns on VRP are significant up to the six-month horizon in our model as well. Note that the return predictability is

Table 9: Return Predictability by Variance Risk Premium

Number of calendar days		1	30	60	90	120	150	180
$IV_{t,t+30} - RV_{t-30,t}$	Data	30.21	4.14	5.64	4.27	5.71	3.60	2.93
		(2.68)	(1.44)	(3.42)	(2.83)	(3.05)	(2.68)	(2.35)
R^2 (%)		7.58	0.72	2.55	2.11	4.36	2.32	1.67
$IV_{t,t+30} - RV_{t-30,t}$	Model	12.02	1.77	1.06	0.65	0.51	0.43	0.38

This table shows the results of the return predictability regression (22) using U.S. stock market return data and those using data simulated from the model. Columns 2 to 7 represent returns on the left hand side of (22) with $h = [1, 30, 60, 90, 120, 150, 180]$ calendar days ($[1, 21, 42, 63, 84, 105, 126]$ trading days in the data) including the upcoming announcement day. The regression includes returns and VRPs for every announcement day during the period of 1994.09-2019.12. Newey-West t -statistics with 1-6 lags are in parentheses. We normalize the VRP both in the data and in the model. Returns are in daily basis points and VRPs are in monthly percentage squared unit.

strongest for the announcement-day return because the announcement premium immediately reflects the risk compensation required by our information-driven volatility.

6 Conclusion

In this paper, we present a model of information-driven volatility. Traditional stochastic volatility models typically imply a positive relationship between the realized variance of past returns and the forward-looking future returns. However, empirical evidence often favors a negative relationship between the two. We develop a model of information-driven volatility. We show that when variations in stock market volatility are driven by information, high realized volatility of past return predict lower future volatility and lower future returns. We show that our model can account for several stylized facts on the variance-expected return relationships in the data.

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7 Appendix

7.1 The Three-Period Model

Below, we provide the proof for Proposition 1.

Proof. Standard Bayesian updating implies that the posterior mean $\mathbb{E}[\ln C_1|s] = \frac{\sigma^{-2}}{\sigma^{-2} + \sigma_\varepsilon^{-2}}\mu + \frac{\sigma_\varepsilon^{-2}}{\sigma^{-2} + \sigma_\varepsilon^{-2}}s$. In addition, $\text{Var}[\mathbb{E}[\ln C_1|s]] = \lambda\sigma^2$, where $\lambda = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}$, and $\text{Var}[\ln C_1|s] = (1 - \lambda)\sigma^2$, which comes from the variance decomposition formula in (1).

The expressions of the stochastic discount factor are standard. The stochastic discount factor that prices date-0 consumption goods in terms of date-1 consumption goods is $\Lambda_{-1,0} = \frac{\rho}{1-\rho} \left(\frac{C_0}{C_{-1}}\right)^{-1} \left(\frac{V_0}{m_{-1}}\right)^{1-\gamma}$, and the stochastic discount factor that prices date-1 consumption goods in terms of date-0 consumption goods is given by: $\Lambda_{0,1} = \frac{\rho}{1-\rho} \left(\frac{C_1}{C_0}\right)^{-1} \left(\frac{C_1}{m_0}\right)^{1-\gamma}$, where $V_0 = C_0^{1-\rho} (\mathbb{E}[C_1^{1-\gamma}])^{\frac{\rho}{1-\gamma}}$, is the date-0 utility of the agent, $m_0 = (\mathbb{E}[C_1^{1-\gamma}])^{\frac{1}{1-\gamma}}$ is the certainty equivalent of future at time 0 and $m_{-1} = (\mathbb{E}[V_0^{1-\gamma}])^{\frac{1}{1-\gamma}}$ is the date-1 certainty equivalent of future utility.

Therefore, $\text{Var}[\ln \Lambda_{0,1}] = \gamma^2 \text{Var}[\ln C_1|s] = \gamma^2 (1 - \lambda)\sigma^2$. Note also, $\text{Var}[\ln \Lambda_{-1,0}] = (1 - \gamma)^2 \text{Var}[\ln V_0]$, where

$$\begin{aligned} \ln V_0 &= (1 - \rho) \ln C_0 + \frac{\rho}{1 - \gamma} \ln \mathbb{E}[C_1^{1-\gamma}] \\ &= (1 - \rho) \ln C_0 + \rho \left\{ \mathbb{E}[\ln C_1|s] + \frac{1}{2} (1 - \gamma) \text{Var}[\ln C_1|s] \right\}. \end{aligned} \quad (23)$$

This implies that $\text{Var}[\ln \Lambda_{-1,0}] = (1 - \gamma)^2 \rho^2 \text{Var}[\mathbb{E}[\ln C_1|s]] = (1 - \gamma)^2 \rho^2 \lambda \sigma^2$. \square

7.2 The Infinite-horizon Model Solutions

7.2.1 Preferences and beliefs

In this subsection, we start by deriving the posterior belief of a representative agent with two sources of information. We first solve for learning from the observable consumption using optimal filtering. Then we derive belief updating upon the announcements. Finally, we present solutions to the value functions and the associated boundary conditions at the announcement.

Learning in the interior: optimal filtering The two-state Markov chain process can be conveniently represented as an integration with respect to a Poisson process. In particular,

let $\{N_{j,t}\}_{t \geq 0}$ be a Poisson process with intensity λ_j , for $j = H, L$. Let $I_{\{x\}}$ be an indicator function, that is,

$$I_{\{\theta_j\}}(\theta_t) = \begin{cases} 1 & \text{if } \theta_t = \theta_j \\ 0 & \text{if } \theta_t \neq \theta_j \end{cases}, \quad (24)$$

Then $\{\theta_t\}$ can be represented as the following compound Poisson process:

$$d\theta_t = (\theta_H - \theta_L) \eta(\theta_t^-) dN_t \quad (25)$$

and $\eta(\theta)$, and $N(t)$ are vectors:

$$\eta(\theta_t) = [-I_{\{\theta_H\}}(\theta_t), I_{\{\theta_L\}}(\theta_t)], \text{ and } N(t) = [N_{H,t}, N_{L,t}]^\top. \quad (26)$$

Here we use the convention that $\{\theta_t\}$ is right-continuous with left limit, and use the notation $\theta_t^- = \lim_{s \rightarrow t, s < t} \theta_s$. And $N_{j,t}$ is the counting processes with intensity λ_j . That is,

$$dN_{j,t} = \begin{cases} 1 & \text{with prob. } \lambda_j dt \\ 0 & \text{with prob. } 1 - \lambda_j dt \end{cases}. \quad (27)$$

For example, if the true state is the high growth state $\theta_t^- = \theta_H$, and the transition from high state to low state happens, i.e., $dN_{H,t} = 1$. Then $\eta(\theta_t) = [-1, 1]$ gives rise to $d\theta_t = (\theta_H - \theta_L) (-I_{\{\theta_H\}}(\theta_t) dN_{H,t} + I_{\{\theta_L\}}(\theta_t) dN_{L,t}) = \theta_L - \theta_H$. Therefore, $\theta_t^+ = \theta_t^- + d\theta_t = \theta_L$.

Applying optimal filtering (see Chapter 9 of Liptser and Shiryaev (2001)), we could obtain

$$d\pi_t = [\lambda_L - (\lambda_H + \lambda_L) \pi_t] dt + \pi_t (1 - \pi_t) (\theta_H - \theta_L) \frac{1}{\sigma_Y} d\hat{B}_{Y,t}, \quad (28)$$

where $\hat{B}_{Y,t}$ is the innovation process defined in the main text. Note that the mapping between $\hat{\theta}_t$ and π_t is one-to-one, so we can equivalently use $\hat{\theta}_t$ as the state variable. By definition, $\hat{\theta}_t = \pi_t \theta_H + (1 - \pi_t) \theta_L$. This recovers π_t from $\hat{\theta}_t$

$$\pi_t = \frac{\hat{\theta}_t - \theta_L}{\theta_H - \theta_L}. \quad (29)$$

Applying Ito's lemma, we get Eq.(6) in the main text.

Learning upon the announcements At the announcement, investors observe a noisy signal s_n where the distribution is given as

$$\begin{aligned} P(s_H|\theta_H) &= \nu, \quad P(s_L|\theta_H) = 1 - \nu \\ P(s_H|\theta_L) &= 1 - \nu, \quad P(s_L|\theta_L) = \nu. \end{aligned}$$

Here we denote s_j as investors receive a signal of θ_j upon the announcement. Given the prior distribution $P^-(\theta_H) = \pi^-$, we need to compute the posterior distribution of π^+ . Applying Bayes' rule,

$$P^+(\theta_i|s_j) = \frac{P(s_j|\theta_i) P^-(\theta_i)}{\sum_{\theta_i \in \Theta} P(s_j|\theta_i) P^-(\theta_i)}.$$

That is, given that $P^-(\theta_H) = \pi^-$,

$$P^+(\theta_H|s_H) = \frac{\pi^- \nu}{\pi^- \nu + (1 - \pi^-)(1 - \nu)}; \quad P^+(\theta_L|s_H) = 1 - P^+(\theta_H|s_H) = \frac{(1 - \pi^-)(1 - \nu)}{\pi^- \nu + (1 - \pi^-)(1 - \nu)} \quad (30)$$

$$P^+(\theta_H|s_L) = \frac{\pi^- (1 - \nu)}{\pi^- (1 - \nu) + (1 - \pi^-) \nu}; \quad P^+(\theta_L|s_L) = 1 - P^+(\theta_H|s_L) = \frac{(1 - \pi^-) \nu}{\pi^- (1 - \nu) + (1 - \pi^-) \nu} \quad (31)$$

This is equivalent to say, if we see s_H , then, $\pi^+ = \frac{\pi^- \nu}{\pi^- \nu + (1 - \pi^-)(1 - \nu)}$, and if we see s_L , $\pi^+ = \frac{\pi^- (1 - \nu)}{\pi^- (1 - \nu) + (1 - \pi^-) \nu}$. So π^+ only has two possible realizations: the probability of seeing s_H and the probability of seeing s_L . Denote h_{s_H} and h_{s_L} as the probability of seeing s_H and s_L *after* the announcement, respectively,

$$h_{s_H} = P(s_H|\theta_H) P^-(\theta_H) + P(s_H|\theta_L) P^-(\theta_L) = \pi^- \nu + (1 - \pi^-)(1 - \nu) \quad (32)$$

$$h_{s_L} = P(s_L|\theta_H) P^-(\theta_H) + P(s_L|\theta_L) P^-(\theta_L) = \pi^- (1 - \nu) + (1 - \pi^-) \nu. \quad (33)$$

For notational convenience, we denote

$$\pi_{s_H}^+ = \frac{\pi^- \nu}{\pi^- \nu + (1 - \pi^-)(1 - \nu)} \quad \text{and} \quad \pi_{s_L}^+ = \frac{\pi^- (1 - \nu)}{\pi^- (1 - \nu) + (1 - \pi^-) \nu} \quad (34)$$

as the probability of $\theta^+ = \theta_H$ *after* the announcement if we see s_H and s_L , respectively.

Note that our signal generates the “correct” result with probability ν and produces a “wrong” signal with probability $1 - \nu$. That is to say, the conditional distribution of π^+ is given as follows. If the true state is θ_H , then $\pi^+ = \begin{cases} \pi_{s_H}^+ & \text{w.p. } \nu \\ \pi_{s_L}^+ & \text{w.p. } 1 - \nu \end{cases}$, and if the true state

is θ_L , $\pi^+ = \begin{cases} \pi_{s_L}^+ & \text{w.p. } \nu \\ \pi_{s_H}^+ & \text{w.p. } 1 - \nu \end{cases}$.

Since the mapping between $\hat{\theta}_t$ and π_t is one-to-one, we could instead use $\hat{\theta}$ as the state variable. Substituting Eq.(29) back, we can derive the probability of seeing s_H and s_L after the announcement respectively as

$$h_{s_H} = \frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} \nu + \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} (1 - \nu), \quad (35)$$

$$h_{s_L} = \frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} (1 - \nu) + \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} \nu. \quad (36)$$

Also, given $\pi^+ = \{\pi_{s_H}^+, \pi_{s_L}^+\}$, we can compute $\hat{\theta}^+ = \theta_L + \pi^+ (\theta_H - \theta_L)$ using Eq.(34). Therefore, with probability h_{s_H} and h_{s_L} , the posterior mean becomes

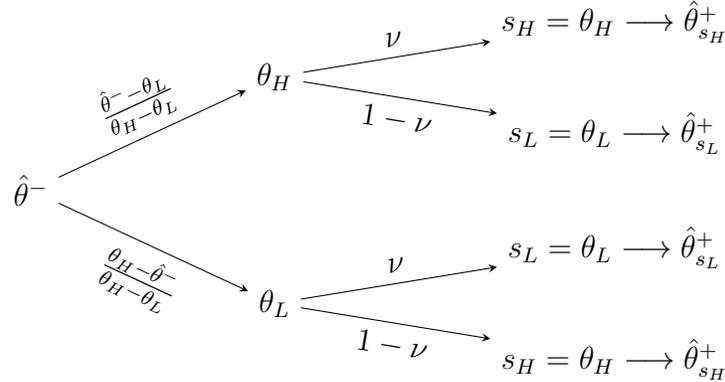
$$\hat{\theta}_{s_H}^+ = \theta_L + \frac{(\hat{\theta}^- - \theta_L) \nu (\theta_H - \theta_L)}{(\hat{\theta}^- - \theta_L) \nu + (\theta_H - \hat{\theta}^-) (1 - \nu)}, \quad (37)$$

$$\hat{\theta}_{s_L}^+ = \theta_L + \frac{(\hat{\theta}^- - \theta_L) (1 - \nu) (\theta_H - \theta_L)}{(\hat{\theta}^- - \theta_L) (1 - \nu) + (\theta_H - \hat{\theta}^-) \nu}. \quad (38)$$

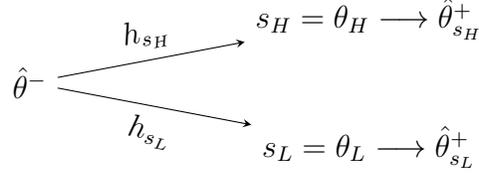
Here $\hat{\theta}_{s_H}^+$ and $\hat{\theta}_{s_L}^+$ indicate the posterior mean of $\hat{\theta}^+$ after the announcement if we observe s_H and s_L , respectively.

In summary, the *conditional* distribution of $\hat{\theta}^+$ is: if the underlying state is θ_H , $\hat{\theta}^+ = \begin{cases} \hat{\theta}_{s_H}^+ & \text{w.p. } \nu \\ \hat{\theta}_{s_L}^+ & \text{w.p. } 1 - \nu \end{cases}$. If the true state is θ_L , then $\hat{\theta}^+ = \begin{cases} \hat{\theta}_{s_L}^+ & \text{w.p. } \nu \\ \hat{\theta}_{s_H}^+ & \text{w.p. } 1 - \nu \end{cases}$. The *unconditional* distribution of $\hat{\theta}^+$ after the announcement follows: $\hat{\theta}^+ = \begin{cases} \hat{\theta}_{s_H}^+ & \text{w.p. } h_{s_H} \\ \hat{\theta}_{s_L}^+ & \text{w.p. } h_{s_L} \end{cases}$.

The following plot summarizes both distributions.



which is equivalent to



As a matter of notation, for any function f , we denote

$$\begin{aligned}
f(\nu, \hat{\theta}^-, T) &= \mathbb{E} \left[f(\hat{\theta}^+, 0) \mid \nu, \hat{\theta}^-, T \right] = h_{s_H} f(\hat{\theta}_{s_H}^+, 0) + h_{s_L} f(\hat{\theta}_{s_L}^+, 0) \quad (39) \\
&= \left[\frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} \nu + \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} (1 - \nu) \right] f \left(\theta_L + \frac{(\hat{\theta}^- - \theta_L) \nu (\theta_H - \theta_L)}{(\hat{\theta}^- - \theta_L) \nu + (\theta_H - \hat{\theta}^-) (1 - \nu)}, 0 \right) \\
&\quad + \left[\frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} (1 - \nu) + \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} \nu \right] f \left(\theta_L + \frac{(\hat{\theta}^- - \theta_L) (1 - \nu) (\theta_H - \theta_L)}{(\hat{\theta}^- - \theta_L) (1 - \nu) + (\theta_H - \hat{\theta}^-) \nu}, 0 \right) \quad (40)
\end{aligned}$$

The value function of the representative agent The representative consumer's preference is specified by a pair of aggregators (f, \mathcal{A}) such that:

$$dV_t = [-f(Y_t, V_t) - \frac{1}{2} \mathcal{A}(V_t) \|\sigma_V(t)\|^2] dt + \sigma_V(t) dB_t \quad (41)$$

for some square-integrable process $\sigma_V(t)$. We adopt the convenient normalization $\mathcal{A}(V_t) = 0$ (Duffie and Epstein, 1992b), and denote \bar{f} as the normalized aggregator. Under this normalization, for $\psi \neq 1$, $\bar{f}(Y_t, V_t)$ is,

$$\bar{f}(Y_t, V_t) = \frac{\rho}{1 - 1/\psi} \frac{Y_t^{1-1/\psi} - ((1 - \gamma) V_t)^{\frac{1-1/\psi}{1-\gamma}}}{((1 - \gamma) V_t)^{\frac{1-1/\psi}{1-\gamma} - 1}}. \quad (42)$$

The Hamilton–Jacobi–Bellman (HJB) equation for the recursive utility is

$$\bar{f}(Y_t, V(\hat{\theta}, t, Y)) + \mathcal{L} [V(\hat{\theta}, t, Y)] = 0, \quad (43)$$

where \mathcal{L} is the infinitesimal generator defined as

$$\mathcal{L}(V_t) = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \mathbb{E}_t [V_{t+\Delta} - V_t]. \quad (44)$$

Consider the following homogeneous form of

$$V(\hat{\theta}_t, t, Y_t) = \frac{1}{1-\gamma} H(\hat{\theta}_t, t) Y_t^{1-\gamma} \quad (45)$$

where

$$\frac{dY_t}{Y_t} = \hat{\theta}_t dt + \sigma_Y d\hat{B}_{Y,t}, \quad (46)$$

$$d\hat{\theta}_t = \mu_{\theta,t} dt + \frac{\sigma_{\theta,t}}{\sigma_Y} d\hat{B}_{Y,t}, \quad (47)$$

where $\mu_{\theta,t} = (\lambda_H + \lambda_L)(\bar{\theta} - \hat{\theta}_t)$, $\sigma_{\theta,t} = (\theta_H - \hat{\theta}_t)(\hat{\theta}_t - \theta_L)$. The following lemma summarizes the solution to the value function. We provide details for numerical solutions in the Online Appendix.

Lemma 1. *In the interior $(0, T)$, $H(\hat{\theta}_t, t)$ satisfies the following HJB equation*

$$\begin{aligned} 0 = & \frac{1}{(1-\gamma)H} \left\{ H_t + H_\theta [\mu_{\theta,t} + (1-\gamma)\sigma_{\theta,t}] + \frac{\sigma_{\theta,t}^2}{2\sigma_Y^2} H_{\theta\theta} \right\} \\ & + \frac{\rho}{1-\frac{1}{\psi}} \left(H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} - 1 \right) + \left(\hat{\theta}_t - \frac{1}{2}\gamma\sigma_Y^2 \right). \end{aligned} \quad (48)$$

where we use notations $H_t = \frac{\partial H(\hat{\theta}_t, t)}{\partial t}$, $H_\theta = \frac{\partial H(\hat{\theta}_t, t)}{\partial \hat{\theta}_t}$, and $H_{\theta\theta} = \frac{\partial^2 H(\hat{\theta}_t, t)}{\partial \hat{\theta}_t^2}$.

Assume the informativeness of the announcement ν takes m values, i.e., $\nu_1, \nu_2, \nu_3, \dots, \nu_m$ with the associated probabilities $q_1, q_2, q_3, \dots, q_m$. Then the boundary condition at the announcement satisfies

$$\tilde{H}(\nu, \hat{\theta}^-, T) = \mathbb{E} \left[H(\hat{\theta}^+, 0) \mid \nu, \hat{\theta}^-, T \right] = h_{s_H} H(\hat{\theta}_{s_H}^+, 0) + h_{s_L} H(\hat{\theta}_{s_L}^+, 0), \quad (49)$$

$$\text{and } H(\hat{\theta}^-, T) = \mathbb{E} \left[\tilde{H}(\nu, \hat{\theta}^-, T) \mid \hat{\theta}^-, T \right] = \sum_{j=1}^m q_j \tilde{H}(\nu_j, \hat{\theta}^-, T), \quad (50)$$

where h_{s_H} , h_{s_L} , $\hat{\theta}_{s_H}^+$, and $\hat{\theta}_{s_L}^+$ are defined in Eqs. (35) to (38).

Proof. The form of value function implies: $\bar{f}(Y, V) = \frac{\rho}{1-\frac{1}{\psi}} Y^{1-\gamma} \left(H^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} - H \right)$. Using Ito's

lemma, we have

$$\begin{aligned} \frac{\mathcal{L} \left[V \left(\hat{\theta}_t, t, Y_t \right) \right]}{Y_t^{1-\gamma}} &= \frac{\mathcal{L} \left[H \left(\hat{\theta}_t, t \right) Y_t^{1-\gamma} \right]}{(1-\gamma) Y_t^{1-\gamma}} \\ &= H \left(\hat{\theta}_t - \frac{1}{2} \gamma \sigma_Y^2 \right) + \frac{1}{1-\gamma} \left(H_t + H_{\theta} \mu_{\theta,t} + \frac{1}{2 \sigma_Y^2} H_{\theta\theta} \sigma_{\theta,t}^2 \right) + H_{\theta} \sigma_{\theta,t} \end{aligned} \quad (51)$$

Therefore, HJB equation is written as Eq.(48).

The boundary condition upon the announcement satisfies

$$H \left(\hat{\theta}^-, T \right) = \mathbb{E} \left[\mathbb{E} \left[H \left(\hat{\theta}^+, 0 \right) \mid \nu, \hat{\theta}^-, T \right] \mid \hat{\theta}^-, T \right]. \quad (52)$$

We could understand the above boundary condition in two steps, and we summarize the timeline in the table below.

Time	T (or nT^-)		0 (or nT^+)
Information set	don't know ν $\{\hat{\theta}^-, T\}$	draw ν $\{\nu, \hat{\theta}^-, T\}$	after announcement $\{\nu, \hat{\theta}^+, 0\}$
Continuation utility	$H \left(\hat{\theta}^-, T \right)$	$\tilde{H} \left(\nu, \hat{\theta}^-, T \right)$	$H \left(\hat{\theta}^+, 0 \right)$
Steps	Step2: $H \left(\hat{\theta}^-, T \right) = \mathbb{E} \left[\tilde{H} \left(\nu, \hat{\theta}^-, T \right) \mid \hat{\theta}^-, T \right]$		Step1: $\tilde{H} \left(\nu, \hat{\theta}^-, T \right) = \mathbb{E} \left[H \left(\hat{\theta}^+, 0 \right) \mid \nu, \hat{\theta}^-, T \right]$

Step 1, we draw the i.i.d. random variable ν . Because investors know the distributions of ν so that they could update their beliefs about the associated realized values and probabilities of $\hat{\theta}^+$ according to (7) conditioning on each ν . It is useful to denote this intermediate step as $\tilde{H} \left(\nu, \hat{\theta}^-, T \right) = \mathbb{E} \left[H \left(\hat{\theta}^+, 0 \right) \mid \nu, \hat{\theta}^-, T \right]$, where $\mathbb{E} \left[H \left(\hat{\theta}^+, 0 \right) \mid \nu, \hat{\theta}^-, T \right]$ is defined in (40). In this step, we calculate the expected value of the continuation utility right after the announcement conditional on the information set $\{\nu, \hat{\theta}^-, T\}$, as defined in Eq. (49).

Step 2, we compute the unconditional expectation by integrating over all possible realizations of ν to get Eq. (50). This step allows us to derive the expected value function based on the information set $\{\hat{\theta}^-, T\}$ right before the announcement. If, for example, ν only takes only two values, ν_H and ν_L with probability q and $1 - q$. Then,

$$H \left(\hat{\theta}^-, T \right) = \mathbb{E} \left[\tilde{H} \left(\nu, \hat{\theta}^-, T \right) \mid \hat{\theta}^-, T \right] = q \tilde{H} \left(\nu_H, \hat{\theta}^-, T \right) + (1 - q) \tilde{H} \left(\nu_L, \hat{\theta}^-, T \right). \quad (53)$$

□

7.2.2 Asset Prices

In this subsection, we first derive the pricing kernel for the representative investor. We then derive the risk-free rate and the partial differential equation (PDE) for the price-to-dividend ratio with the boundary condition at the announcement. Finally, we calculate the cumulative return and the risk premium.

Pricing kernel and the risk-free rate We first provide proof for the law of motion of the pricing kernel M_t , which satisfies the stochastic differential equation (SDE) of Eq. (9), and

$$r(\hat{\theta}_t, t) = \rho + \frac{1}{\psi} \hat{\theta}_t - \frac{1 + \frac{1}{\psi}}{2} \gamma \sigma_Y^2 + \frac{(\frac{1}{\psi} - \gamma)(1 - \frac{1}{\psi})}{2(1 - \gamma)^2} \left(\frac{H_\theta}{H}\right)^2 \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_\theta}{H} \sigma_{\theta,t}, \quad (54)$$

$$\sigma_M(\hat{\theta}_t, t) = \gamma \sigma_Y - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_\theta}{H} \frac{\sigma_{\theta,t}}{\sigma_Y}, \quad (55)$$

where $\sigma_{\theta,t} = (\theta_H - \hat{\theta}_t)(\hat{\theta}_t - \theta_L)$, and we use notations $\frac{H_\theta}{H} = \frac{\partial H(\hat{\theta}_t, t)/\partial \hat{\theta}_t}{H(\hat{\theta}_t, t)}$ and $\frac{H_{\theta\theta}}{H} = \frac{\partial^2 H(\hat{\theta}_t, t)/\partial \hat{\theta}_t^2}{H(\hat{\theta}_t, t)}$.

Proof. The pricing kernel is defined as

$$\frac{dM_t}{M_t} = \frac{d\bar{f}_Y(Y, V)}{\bar{f}_Y(Y, V)} + \bar{f}_V(Y, V) dt \quad (56)$$

where $\bar{f}_Y(Y, V) = \rho H^{\frac{1}{\psi} - \gamma} Y^{-\gamma}$, and $\bar{f}_V(Y, V) = \rho^{\frac{1}{\psi} - \gamma} H^{-\frac{1}{\psi} - \gamma} - \rho^{\frac{1 - \gamma}{1 - \frac{1}{\psi}}}$. Applying Ito's lemma,

$$\begin{aligned} \frac{d\bar{f}_Y(Y, V)}{\bar{f}_Y(Y, V)} &= \frac{d[H^{\frac{1}{\psi} - \gamma} Y_t^{-\gamma}]}{H^{\frac{1}{\psi} - \gamma} Y_t^{-\gamma}} = \left\{ -\gamma \hat{\theta}_t + \frac{1}{2} \gamma (\gamma + 1) \sigma_Y^2 + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \left(\frac{H_t}{H} + \frac{H_\theta}{H} \mu_{\theta,t} \right) \right. \\ &\quad + \frac{1}{2} \left[\frac{\left(\frac{1}{\psi} - \gamma \right) \left(\frac{1}{\psi} - 1 \right)}{(1 - \gamma)^2} \left(\frac{H_\theta}{H} \right)^2 + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_{\theta\theta}}{H} \right] \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \\ &\quad \left. - \gamma \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_\theta}{H} \sigma_{\theta,t} \right\} dt + \left(-\gamma \sigma_Y + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_\theta}{H} \frac{\sigma_{\theta,t}}{\sigma_Y} \right) d\hat{B}_{Y,t}. \end{aligned} \quad (57)$$

Matching the drift and diffusion of Eq.(9), we can get (55) and the risk-free rate

$$\begin{aligned}
r_t &= -\frac{\frac{1}{\psi} - \gamma}{(1 - \gamma)H} \left[H_t + H_\theta \mu_{\theta,t} + \frac{1}{2} \left(\frac{\frac{1}{\psi} - 1}{1 - \gamma} \frac{H_\theta^2}{H} + H_{\theta\theta} \right) \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} - \gamma H_\theta \sigma_{\theta,t} \right] \\
&\quad + \gamma \hat{\theta}_t - \frac{1}{2} \gamma (\gamma + 1) \sigma_Y^2 - \rho \frac{\frac{1}{\psi} - \gamma}{1 - \frac{1}{\psi}} H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \rho \frac{1 - \gamma}{1 - \frac{1}{\psi}}.
\end{aligned} \tag{58}$$

Using the HJB equation to simplify r_t by multiplying $\frac{1}{\psi} - \gamma$ on both sides of (48),

$$\begin{aligned}
0 &= \frac{\frac{1}{\psi} - \gamma}{(1 - \gamma)H} \left\{ H_t + H_\theta [\mu_{\theta,t} + (1 - \gamma) \sigma_{\theta,t}] + \frac{1}{2} H_{\theta\theta} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \right\} \\
&\quad + \frac{\rho \left(\frac{1}{\psi} - \gamma \right)}{1 - \frac{1}{\psi}} \left(H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} - 1 \right) + \left(\hat{\theta}_t - \frac{1}{2} \gamma \sigma_Y^2 \right) \left(\frac{1}{\psi} - \gamma \right),
\end{aligned} \tag{59}$$

and adding up with (58), we get the instantaneous risk-free rate in the main text. \square

Price-to-dividend ratio We show the solution for $p(\hat{\theta}_t, t)$ in the following lemma. We provide details for numerical solutions in the Online Appendix.

Lemma 2. *In the interior $(0, T)$, the price-to-dividend ratio $p(\hat{\theta}_t, t)$ satisfies the PDE of*

$$\varpi \left(\hat{\theta}_t, t \right) p = p_t + p_\theta \varrho \left(\hat{\theta}_t, t \right) + \frac{1}{2} p_{\theta\theta} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + 1, \tag{60}$$

where we use notations $p_t = \frac{\partial p(\hat{\theta}_t, t)}{\partial t}$, $p_\theta = \frac{\partial p(\hat{\theta}_t, t)}{\partial \hat{\theta}_t}$, $p_{\theta\theta} = \frac{\partial^2 p(\hat{\theta}_t, t)}{\partial \hat{\theta}_t^2}$, and

$$\begin{aligned}
\varpi \left(\hat{\theta}_t, t \right) &= (\xi - 1) \bar{\theta} + \rho - \frac{1}{2} \gamma \sigma_Y^2 \left(\frac{1}{\psi} + 1 \right) + \gamma \sigma_Y^2 - \left(\xi - \frac{1}{\psi} \right) \hat{\theta}_t + \frac{(\frac{1}{\psi} - \gamma)(1 - \frac{1}{\psi})}{2(1 - \gamma)^2} \left(\frac{H_\theta}{H} \right)^2 \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \\
\varrho \left(\hat{\theta}_t, t \right) &= \mu_{\theta,t} + (1 - \gamma) \sigma_{\theta,t} + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_\theta}{H} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2}
\end{aligned} \tag{61}$$

with the boundary condition at the announcement satisfying

$$\begin{aligned} \tilde{p}(\nu, \hat{\theta}_T^-, T) &= \mathbb{E} \left[\frac{H(\hat{\theta}_T^+, 0)^{\frac{1}{\psi}-\gamma} p(\hat{\theta}_T^+, 0)}{\mathbb{E}[H(\hat{\theta}_T^+, 0) | \nu, \hat{\theta}_T^-, T]^{\frac{1}{\psi}-\gamma}} \middle| \nu, \hat{\theta}_T^-, T \right] \\ &= \frac{h_{s_H} H(\hat{\theta}_{s_H}^+, 0)^{\frac{1}{\psi}-\gamma} p(\hat{\theta}_{s_H}^+, 0) + h_{s_L} H(\hat{\theta}_{s_L}^+, 0)^{\frac{1}{\psi}-\gamma} p(\hat{\theta}_{s_L}^+, 0)}{[h_{s_H} H(\hat{\theta}_{s_H}^+, 0) + h_{s_L} H(\hat{\theta}_{s_L}^+, 0)]^{\frac{1}{\psi}-\gamma}}, \end{aligned} \quad (62)$$

$$\text{and } p(\hat{\theta}_T^-, T) = \mathbb{E} \left[\frac{\tilde{H}(\nu, \hat{\theta}_T^-, T)^{\frac{1}{\psi}-\gamma} \tilde{p}(\nu, \hat{\theta}_T^-, T)}{\mathbb{E}[\tilde{H}(\nu, \hat{\theta}_T^-, T) | \hat{\theta}_T^-, T]^{\frac{1}{\psi}-\gamma}} \middle| \hat{\theta}_T^-, T \right] = \frac{\sum_{j=1}^m q_j \tilde{H}(\nu_j, \hat{\theta}_T^-, T)^{\frac{1}{\psi}-\gamma} \tilde{p}(\nu_j, \hat{\theta}_T^-, T)}{[\sum_{j=1}^m q_j \tilde{H}(\nu_j, \hat{\theta}_T^-, T)]^{\frac{1}{\psi}-\gamma}} \quad (63)$$

where h_{s_H} , h_{s_L} , $\hat{\theta}_{s_H}^+$, and $\hat{\theta}_{s_L}^+$ are defined in Eqs.(35) to (38).

Proof. The present value relationship (10) implies

$$M_t D_t dt + \mathcal{L} [M_t p(\hat{\theta}_t, t) D_t] = 0. \quad (64)$$

This gives $\frac{\mathcal{L}[M_t p(\hat{\theta}_t, t) D_t]}{M_t p(\hat{\theta}_t, t) D_t} + \frac{1}{p(\hat{\theta}_t, t)} = 0$. Applying Ito's lemma and using Eqs.(8) and (9),

$$\begin{aligned} \frac{\mathcal{L} [M_t p(\hat{\theta}_t, t) D_t]}{M_t p(\hat{\theta}_t, t) D_t} &= -r_t + \frac{1}{p} \left(p_t + p_\theta \mu_{\theta,t} + \frac{1}{2} p_{\theta\theta} \frac{\sigma_{\hat{\theta}_t}^2}{\sigma_Y^2} \right) \\ &\quad + \xi \left(\hat{\theta}_t - \bar{\theta} \right) + \bar{\theta} + \frac{p_\theta}{p} \sigma_{\theta,t} - \sigma_{M,t} \left(\sigma_Y + \frac{p_\theta}{p} \frac{\sigma_{\theta,t}}{\sigma_Y} \right). \end{aligned} \quad (65)$$

Put in r_t from Eq. (54) would give the PDE for $p(\hat{\theta}_t, t)$.

We next solve the boundary condition. Another way to write Eq. (56) is: $M_t = f_Y(Y_t, V_t) e^{\int_0^t f_V(Y_s, V_s) ds}$. From this formula, we could derive the announcement SDF as $\frac{H(\hat{\theta}_T^+, 0)^{\frac{1}{\psi}-\gamma}}{\mathbb{E}[H(\hat{\theta}_T^+, 0) | \hat{\theta}_T^-, T]^{\frac{1}{\psi}-\gamma}}$.

The intuition is as follows. Upon the announcement, Y_t is continuous while the continuation utility $H(\hat{\theta}_t, t)$ jumps when new information about $\hat{\theta}_t$ arrives because of generalized risk sensitivity in preferences (Ai and Bansal, 2018). Therefore, using the announcement SDF, the

boundary condition for $p(\hat{\theta}_t, t)$ is

$$p\left(\hat{\theta}_T^-, T\right) = \mathbb{E} \left[\frac{H(\hat{\theta}_T^+, 0)^{\frac{1}{\psi}-\gamma} p(\hat{\theta}_T^+, 0)}{\mathbb{E} \left[H(\hat{\theta}_T^+, 0) \middle| \hat{\theta}_T^-, T \right]^{\frac{1}{\psi}-\gamma}} \middle| \hat{\theta}_T^-, T \right]. \quad (66)$$

We again understand the above boundary condition in two steps. First, conditioning on a given ν , the distribution of $\hat{\theta}_T^+$ is given by Eqs.(35)–(38). This intermediate step $\tilde{p}\left(\nu, \hat{\theta}_T^-, T\right)$ can be computed in Eq.(62). Next, we compute the unconditional expectation by averaging over all possible realizations of ν , as shown in Eq.(63). \square

Risk premium Here we provide proof for Proposition 2. For notational convenience, denote $\mu_{R,t} = \mathbb{E} \left[\frac{d(p(\hat{\theta}_t, t)D_t) + D_t dt}{p(\hat{\theta}_t, t)D_t} \right]$ as the expected return and $\sigma_{R,t}$ as the instantaneous return volatility. We show the following lemma holds.

Lemma 3. *The cumulated return takes the following form*

$$\frac{dR_t}{R_t} = \mu_{R,t} dt + \sigma_{R,t} d\hat{B}_{Y,t} + \sigma_{i,t} dB_{i,t}, \quad (67)$$

where

$$\mu_{R,t} = \frac{1}{p} \left(1 + p_t + p_\theta \mu_{\theta,t} + \frac{1}{2} p_{\theta\theta} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \right) + \xi \left(\hat{\theta}_t - \bar{\theta} \right) + \bar{\theta} + \frac{p_\theta}{p} \sigma_{\theta,t} \quad (68)$$

$$\sigma_{R,t} = \frac{p_\theta}{p} \frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y. \quad (69)$$

Proof. The cumulative return can be computed as

$$\frac{dR_t}{R_t} = \frac{d\left(p\left(\hat{\theta}_t, t\right) D_t\right) + D_t dt}{p\left(\hat{\theta}_t, t\right) D_t} = \frac{1}{p} dt + \frac{d(pD_t)}{pD_t} \quad (70)$$

Applying Ito's lemma and using (8),

$$\begin{aligned} \frac{d\left(p\left(\hat{\theta}_t, t\right) D_t\right)}{p\left(\hat{\theta}_t, t\right) D_t} &= \left[\frac{1}{p} \left(p_t + p_\theta \mu_{\theta,t} + \frac{1}{2} p_{\theta\theta} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \right) + \xi \left(\hat{\theta}_t - \bar{\theta} \right) + \bar{\theta} + \frac{p_\theta}{p} \sigma_{\theta,t} \right] dt \\ &\quad + \left(\frac{p_\theta}{p} \frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y \right) d\hat{B}_{Y,t} + \sigma_{i,t} dB_{i,t} \end{aligned} \quad (71)$$

Together with the expression of the pricing kernel (9), the risk premium is therefore

$$\begin{aligned}\mu_{R,t} - r_t &= -\text{Cov}_t \left[\frac{dM_t}{M_t}, \frac{dR_t}{R_t} \right] \\ &= \sigma_{M,t} \sigma_{R,t} = \left(\gamma \sigma_Y - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_\theta}{H} \frac{\sigma_{\theta,t}}{\sigma_Y} \right) \left(\frac{p_\theta}{p} \frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y \right),\end{aligned}\quad (72)$$

which corresponds to Eq.(11) in Proposition 2.

Upon the announcement, since dividend flow is continuous, the announcement premium is given by $\left(\mathbb{E} \left[p \left(\hat{\theta}_T^+, 0 \right) / p \left(\hat{\theta}_T^-, T \right) \mid \hat{\theta}_T^-, T \right] - 1 \right)$. Using the boundary condition of the price-to-dividend ratio (66), we can therefore obtain Eq.(12). \square

7.2.3 Forward Looking Measure of Implied Variance

In this session, we compute the model implied variance defined in Eq.(13). We first simplify the log return. The law of motion of dividend from Eq.(8) implies

$$\ln D_\tau = \ln D_t + \int_t^\tau \left(\xi \hat{\theta}_s + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) ds + \int_t^\tau \left(\sigma_Y d\hat{B}_{Y,s} + \sigma_{i,s} dB_{i,s} \right). \quad (73)$$

For simplicity, we define

$$\delta(t) = \int_0^t \left(\xi \hat{\theta}_s + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) ds + \int_0^t \sigma_Y d\hat{B}_{Y,s}, \quad (74)$$

or equivalently,

$$d\delta(t) = \left[\xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right] dt + \sigma_Y d\hat{B}_{Y,t}. \quad (75)$$

Therefore, the log return could be simplified as

$$\ln \left\{ p \left(\hat{\theta}_\tau, \tau \right) D_\tau \right\} - \ln \left\{ p \left(\hat{\theta}_t, t \right) D_t \right\} = \ln p \left(\hat{\theta}_\tau, \tau \right) - \ln p \left(\hat{\theta}_t, t \right) + \delta(\tau) - \delta(t) + \int_t^\tau \sigma_{i,s} dB_{i,s} \quad (76)$$

Then the implied variance could be decomposed as

$$\begin{aligned}IV_{t,\tau} &= \text{Var}_t \left[\ln p \left(\hat{\theta}_\tau, \tau \right) \right] + 2\text{Cov}_t \left[\ln p \left(\hat{\theta}_\tau, \tau \right), \delta(\tau) - \delta(t) \right] \\ &\quad + \text{Var}_t \left[\delta(\tau) - \delta(t) \right] + \text{Var}_t \left[\int_t^\tau \sigma_{i,s} dB_{i,s} \right],\end{aligned}\quad (77)$$

where we use the fact that $\ln p(\hat{\theta}_t, t)$ is known at time t , and $dB_{i,t}$ is uncorrelated with others. The first three terms in Eq.(77) can be further decomposed into:

$$\text{Var}_t \left[\ln p(\hat{\theta}_\tau, \tau) \right] = \mathbb{E}_t \left[\ln^2 p(\hat{\theta}_\tau, \tau) \right] - \left(\mathbb{E}_t \left[\ln p(\hat{\theta}_\tau, \tau) \right] \right)^2, \quad (78)$$

$$\begin{aligned} \text{Cov}_t \left[\ln p(\hat{\theta}_\tau, \tau), \delta(\tau) - \delta(t) \right] &= \mathbb{E}_t \left[\ln p(\hat{\theta}_\tau, \tau) (\delta(\tau) - \delta(t)) \right] \\ &\quad - \mathbb{E}_t \left[\ln p(\hat{\theta}_\tau, \tau) \right] \mathbb{E}_t \left[(\delta(\tau) - \delta(t)) \right], \end{aligned} \quad (79)$$

$$\text{Var}_t \left[\delta(\tau) - \delta(t) \right] = \mathbb{E}_t \left[\{\delta(\tau) - \delta(t)\}^2 \right] - \mathbb{E}_t \left[\delta(\tau) - \delta(t) \right]^2. \quad (80)$$

For the last term,

$$\text{Var}_t \left[\int_t^\tau \sigma_{i,s} dB_{i,s} \right] = \mathbb{E}_t \left[\left(\int_t^\tau \sigma_{i,s} dB_{i,s} \right)^2 \right] + \left(\mathbb{E}_t \left[\int_t^\tau \sigma_{i,s} dB_{i,s} \right] \right)^2 = \mathbb{E}_t \left[\int_t^\tau \sigma_{i,s}^2 ds \right] \quad (81)$$

where we applied Ito's isometry for the last equivalence. Using simplified notations defined above, we show the following lemma holds. We provide details for numerical solutions in the Online Appendix.

Lemma 4. *For $0 \leq t \leq \tau$, the implied variance is given by*

$$\begin{aligned} IV_{t,\tau}(\hat{\theta}_t, \sigma_{i,t}, t) &= w_2(\hat{\theta}_t, t) - w_1(\hat{\theta}_t, t)^2 + 2 \left[a_3(\hat{\theta}_t, t) - w_1(\hat{\theta}_t, t) a_1(\hat{\theta}_t, t) \right] \\ &\quad + a_2(\hat{\theta}_t, t) - a_1(\hat{\theta}_t, t)^2 + g(\sigma_{i,t}, t), \end{aligned} \quad (82)$$

where

$$w_1(\hat{\theta}_t, t) = \mathbb{E}_t \left[\ln p(\hat{\theta}_\tau, \tau) \right] \quad (83)$$

$$w_2(\hat{\theta}_t, t) = \mathbb{E}_t \left[\ln^2 p(\hat{\theta}_\tau, \tau) \right] \quad (84)$$

$$a_1(\hat{\theta}_t, t) = \mathbb{E}_t \left[\delta(\tau) - \delta(t) \right], \quad (85)$$

$$a_2(\hat{\theta}_t, t) = \mathbb{E}_t \left[\{\delta(\tau) - \delta(t)\}^2 \right], \quad (86)$$

$$a_3(\hat{\theta}_t, t) = \mathbb{E}_t \left[\ln p(\hat{\theta}_\tau, \tau) \{\delta(\tau) - \delta(t)\} \right], \quad (87)$$

and $g(\sigma_{i,t}, t) = \mathbb{E}_t \left[\int_t^\tau \sigma_{i,s}^2 ds \right]$ has the following closed form solution

$$g(\sigma_i^H, t) = \frac{\lambda_i^H (\sigma_i^L)^2 + \lambda_i^L (\sigma_i^H)^2}{\lambda_i^H + \lambda_i^L} (\tau - t) + \lambda_i^H \frac{(\sigma_i^H)^2 - (\sigma_i^L)^2}{(\lambda_i^H + \lambda_i^L)^2} \left[1 - e^{-(\lambda_i^H + \lambda_i^L)(\tau - t)} \right] \quad (88)$$

$$g(\sigma_i^L, t) = \frac{\lambda_i^H (\sigma_i^L)^2 + \lambda_i^L (\sigma_i^H)^2}{\lambda_i^H + \lambda_i^L} (\tau - t) - \lambda_i^L \frac{(\sigma_i^H)^2 - (\sigma_i^L)^2}{(\lambda_i^H + \lambda_i^L)^2} \left[1 - e^{-(\lambda_i^H + \lambda_i^L)(\tau - t)} \right]. \quad (89)$$

Note that all of them also depend on the expiration date τ , but we dropped τ to save notations. We first solve for $g(\sigma_{i,t}, t)$, where $\sigma_{i,t} \in \{\sigma_i^H, \sigma_i^L\}$ is a two-state Markov chain. We construct the martingale

$$\mathbb{E}_t \left[\int_0^\tau \sigma_{i,s}^2 ds \right] = \mathbb{E}_t \left[\int_t^\tau \sigma_{i,s}^2 ds \right] + \mathbb{E}_t \left[\int_0^t \sigma_{i,s}^2 ds \right] = g(\sigma_{i,t}, t) + \sigma_{i,t}^2.$$

Because $\mathbb{E}_t \left[\mathbb{E}_t \left[\int_0^\tau \sigma_{i,s}^2 ds \right] \right] = \mathbb{E}_t \left[\int_0^\tau \sigma_{i,s}^2 ds \right]$, $\mathbb{E}_t \left[\int_0^\tau \sigma_{i,s}^2 ds \right]$ is a martingale, we have

$$\mathcal{L}[g(\sigma_{i,t}, t)] + \sigma_{i,t}^2 = 0, \quad (90)$$

with the boundary condition $g(\sigma_{i,\tau}, \tau) = 0$. The above can be written as

$$\begin{aligned} (\sigma_i^H)^2 + \frac{\partial}{\partial t} g(\sigma_i^H, t) + \lambda_i^H [g(\sigma_i^L, t) - g(\sigma_i^H, t)] &= 0, \\ (\sigma_i^L)^2 + \frac{\partial}{\partial t} g(\sigma_i^L, t) + \lambda_i^L [g(\sigma_i^H, t) - g(\sigma_i^L, t)] &= 0, \end{aligned}$$

which further gives Eqs.(88) and (89) in Lemma 4.

Next, we solve for $w_j(\hat{\theta}_t, t)$, $j = 1, 2$, and $a_j(\hat{\theta}_t, t)$ where $j = 1, 2, 3$ separately. In general, there are two cases. In the first case, $\tau < T$ so that there will be no announcement before expiration τ . We construct martingales and use martingale property to determine the PDE and the associated boundary condition for each variable. In the second case, $\tau \geq T$, and the forward-looking implied variance will cover one announcement before expiration. If so, we solve $w_j(\hat{\theta}_t, t)$ and $a_j(\hat{\theta}_t, t)$ using backward induction in three steps: after, upon, and before the announcement. We summarize the results in the following lemma.

Lemma 5. *If $0 \leq t \leq \tau < T$, there is no announcement before expiration. The PDEs for*

$w_j(\hat{\theta}_t, t)$, $j = 1, 2$ and $a_j(\hat{\theta}_t, t)$, $j = 1, 2, 3$ are

$$0 = \frac{\partial}{\partial t} w_j(\hat{\theta}_t, t) + \frac{\partial}{\partial \hat{\theta}} w_j(\hat{\theta}_t, t) \mu_{\theta,t} + \frac{1}{2} \frac{\partial^2}{\partial \hat{\theta}^2} w_j(\hat{\theta}_t, t) \frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \quad (91)$$

$$0 = \frac{\partial}{\partial t} a_1(\hat{\theta}_t, t) + \frac{\partial}{\partial \hat{\theta}} a_1(\hat{\theta}_t, t) \mu_{\theta,t} + \frac{1}{2} \frac{\partial^2}{\partial \hat{\theta}^2} a_1(\hat{\theta}_t, t) \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + \xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2, \quad (92)$$

$$0 = 2a_1(\hat{\theta}_t, t) \left(\xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) + 2 \frac{\partial}{\partial \hat{\theta}} a_1(\hat{\theta}_t, t) \sigma_{\theta,t} + \sigma_Y^2 \\ + \frac{\partial}{\partial t} a_2(\hat{\theta}_t, t) + \frac{\partial}{\partial \hat{\theta}} a_2(\hat{\theta}_t, t) \mu_{\theta,t} + \frac{1}{2} \frac{\partial^2}{\partial \hat{\theta}^2} a_2(\hat{\theta}_t, t) \frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \quad (93)$$

$$0 = w_1(\hat{\theta}_t, t) \left(\xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) + \frac{\partial}{\partial \hat{\theta}} w_1(\hat{\theta}_t, t) \sigma_{\theta,t} + \frac{\partial}{\partial t} a_3(\hat{\theta}_t, t) \\ + \frac{\partial}{\partial \hat{\theta}} a_3(\hat{\theta}_t, t) \mu_{\theta,t} + \frac{1}{2} \frac{\partial^2}{\partial \hat{\theta}^2} a_3(\hat{\theta}_t, t) \frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \quad (94)$$

with the associated boundary conditions of

$$w_1(\hat{\theta}_\tau, \tau) = \ln p(\hat{\theta}_\tau, \tau) \text{ and } w_2(\hat{\theta}_\tau, \tau) = \ln p^2(\hat{\theta}_\tau, \tau), \quad (95)$$

$$a_1(\hat{\theta}_\tau, \tau) = 0, \quad a_2(\hat{\theta}_\tau, \tau) = 0 \text{ and } a_3(\hat{\theta}_\tau, \tau) = 0. \quad (96)$$

If $0 \leq t < T \leq \tau$, there will be an announcement before expiration. For $t \in [T^+, \tau]$, we use the above PDEs and boundary conditions (95) and (96) to compute $w_j(\hat{\theta}_{T^+}, T^+)$ and $a_j(\hat{\theta}_{T^+}, T^+)$. For $t \in [T^-, T^+]$, we update the boundary condition according to

$$w_j(\hat{\theta}_T^-, T^-) = \sum_{s=1}^m q_s \tilde{w}_j(\nu_s, \hat{\theta}_T^-, T^-), \quad \tilde{w}_j(\nu, \hat{\theta}_T^-, T^-) = h_{sH} w_j(\hat{\theta}_{sH}^+, T^+) + h_{sL} w_j(\hat{\theta}_{sL}^+, T^+) \quad (97)$$

$$a_j(\hat{\theta}_T^-, T^-) = \sum_{s=1}^m q_s \tilde{a}_j(\nu_s, \hat{\theta}_T^-, T^-), \quad \tilde{a}_j(\nu, \hat{\theta}_T^-, T^-) = h_{sH} a_j(\hat{\theta}_{sH}^+, T^+) + h_{sL} a_j(\hat{\theta}_{sL}^+, T^+). \quad (98)$$

For $t \in [t, T^-]$, we use the PDEs again with the above boundary conditions (97) and (98).

Proof. Case 1: IV for $0 \leq t \leq \tau < T$: no announcement before expiration.

$w_j(\hat{\theta}_t, t)$ is a martingale because $\mathbb{E}_t [w_j(\hat{\theta}_\tau, \tau)] = \mathbb{E}_t (\mathbb{E}_t [\ln p^j(\hat{\theta}_\tau, \tau)]) = w_j(\hat{\theta}_t, t)$ for $t \leq \tau$, where $j = 1, 2$, using law of iterated expectations. Therefore, $w_j(\hat{\theta}_t, t)$ is determined by the PDE

$$\mathcal{L}w_1(\hat{\theta}_t, t) = 0 \text{ and } \mathcal{L}w_2(\hat{\theta}_t, t) = 0 \quad (99)$$

which further determines Eq.(91) with the boundary condition at time τ described in (95).

Similarly, $\mathbb{E}_t [\delta(\tau)] = a_1(\hat{\theta}_t, t) + \delta(t)$ is a martingale. Therefore we must have the PDE

$$\mathcal{L} \left[a_1(\hat{\theta}_t, t) + \delta(t) \right] = 0. \quad (100)$$

together with the boundary condition $a_1(\hat{\theta}_\tau, \tau) = \mathbb{E}_\tau [\delta(\tau) - \delta(\tau)] = 0$. Using the law of motion of $\hat{\theta}_t$ from (47), δ_t from (75) and $a_j(\hat{\theta}_t, t)$, $j = 1, 2, 3$ from below

$$da_j(\hat{\theta}_t, t) = \left(\frac{\partial a_j}{\partial t} + \frac{\partial a_j}{\partial \hat{\theta}} \mu_{\theta,t} + \frac{1}{2} \frac{\partial^2 a_j}{\partial \hat{\theta}^2} \frac{\sigma_{\hat{\theta},t}^2}{\sigma_Y^2} \right) dt + \frac{\partial a_j}{\partial \hat{\theta}} \frac{\sigma_{\theta,t}}{\sigma_Y} d\hat{B}_{Y,t}, \quad (101)$$

Eq. (100) immediately gives (92).

Next, consider $a_2(\hat{\theta}_t, t)$. Note that

$$\begin{aligned} \mathbb{E}_t [\delta^2(\tau)] &= \mathbb{E}_t [(\delta(\tau) - \delta(t) + \delta(t))^2] \\ &= \delta(t)^2 + 2a_1(\hat{\theta}_t, t) \delta(t) + a_2(\hat{\theta}_t, t). \end{aligned} \quad (102)$$

The fact that $\mathbb{E}_t [\delta^2(\tau)]$ is a martingale implies that

$$\mathcal{L} \left[\delta(t)^2 + 2a_1(\hat{\theta}_t, t) \delta(t) + a_2(\hat{\theta}_t, t) \right] = 0, \quad (103)$$

together with the boundary condition $a_2(\hat{\theta}_\tau, \tau) = \mathbb{E}_\tau [\{\delta(\tau) - \delta(\tau)\}^2] = 0$. This gives

$$\begin{aligned} &2\delta_t \left(\xi \hat{\theta}_t + \bar{\theta}(1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) + 2\delta_t \left(\frac{\partial a_1}{\partial t} + \frac{\partial a_1}{\partial \hat{\theta}} \mu_{\theta,t} + \frac{1}{2} \frac{\partial a_1^2}{\partial \hat{\theta}^2} \frac{\sigma_{\hat{\theta},t}^2}{\sigma_Y^2} \right) + \sigma_Y^2 \\ &+ 2a_1 \left(\xi \hat{\theta}_t + \bar{\theta}(1 - \xi) - \frac{1}{2} \eta^2 \sigma_Y^2 \right) + 2 \frac{\partial a_1}{\partial \hat{\theta}} \sigma_{\theta,t} + \left(\frac{\partial a_2}{\partial t} + \frac{\partial a_2}{\partial \hat{\theta}} \mu_{\theta,t} + \frac{1}{2} \frac{\partial a_2^2}{\partial \hat{\theta}^2} \frac{\sigma_{\hat{\theta},t}^2}{\sigma_Y^2} \right) = 0 \end{aligned} \quad (104)$$

Because the above equation must hold for all δ_t , matching the coefficient on δ_t and the rest, we obtain Eq.(92) and (93).

Last, we compute $a_3(\hat{\theta}_t, t)$. By definition, $\mathbb{E}_t [\ln p(\hat{\theta}_\tau, \tau) \delta(\tau)] = a_3(\hat{\theta}_t, t) + \mathbb{E}_t [\ln p(\hat{\theta}_\tau, \tau) \delta(t)]$ is also a martingale. Therefore,

$$\mathcal{L} \left[w_1(\hat{\theta}_t, t) \delta(t) + a_3(\hat{\theta}_t, t) \right] = 0, \quad (105)$$

with the boundary condition $a_3(\hat{\theta}_\tau, \tau) = \mathbb{E}_\tau \left[\ln p(\hat{\theta}_\tau, \tau) \{ \delta(\tau) - \delta(\tau) \} \right] = 0$. Further,

$$\begin{aligned} 0 &= \delta_t \left(\frac{\partial w_1}{\partial t} + \frac{\partial w_1}{\partial \hat{\theta}} \mu_{\theta,t} + \frac{1}{2} \frac{\partial w_1^2}{\partial \hat{\theta}^2} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \right) + w_1 \left(\xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) \\ &\quad + \frac{\partial w_1}{\partial \hat{\theta}} \sigma_{\theta,t} + \frac{\partial a_3}{\partial t} + \frac{\partial a_3}{\partial \theta} \mu_{\theta,t} + \frac{1}{2} \frac{\partial a_3^2}{\partial \theta^2} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \end{aligned}$$

we finally obtain Eq.(94).

Case 2: IV for $\tau \geq T$: one announcement before expiration.

Case 1 computes the expectations only for $\tau < T$. If $\tau \geq T$, which is often the case if we want to talk about implied variance across an announcement, we need to deal with the *announcement boundary* separately, and we do it in three steps backward induction: we start from T^+ , go back to (T^-, ν) , and go back to T^- without knowing ν . Note that the output in each step will be the input for the next step.

Step 1: from $T^+ \rightarrow \tau$, we use the PDEs and boundary conditions (95) in Lemma 5 to calculate $w_j(\hat{\theta}_{T^+}, T^+)$, $j = 1, 2$ and $a_j(\hat{\theta}_{T^+}, T^+)$, $j = 1, 2, 3$.

Step 2: from $T^- \rightarrow T^+$, we calculate the boundary conditions. Because $\delta(t)$ is a stochastic integral, it must be continuous. We have $\delta(T^-) = \mathbb{E}_{T^-}[\delta(T^+)] = \delta(T^+)$. Using Eqs.(99), (100), (103) and (105), the martingale property implies

$$\begin{aligned} w_1(\hat{\theta}_T^-, T^-) &= \mathbb{E}_{T^-} \left[w_1(\hat{\theta}_T^+, T^+) \right] \text{ and } w_2(\hat{\theta}_T^-, T^-) = \mathbb{E}_{T^-} \left[w_2(\hat{\theta}_T^+, T^+) \right] \\ a_1(\hat{\theta}_T^-, T^-) + \delta_{T^-} &= \mathbb{E}_{T^-} \left[a_1(\hat{\theta}_T^+, T^+) + \delta_{T^+} \right] \\ \Rightarrow a_1(\hat{\theta}_T^-, T^-) &= \mathbb{E}_{T^-} \left[a_1(\hat{\theta}_T^+, T^+) \right] \end{aligned} \tag{106}$$

$$\begin{aligned} \mathbb{E}_{T^-} \left[\delta_{T^+}^2 + 2a_1(\hat{\theta}_T^+, T^+) \delta_{T^+} + a_2(\hat{\theta}_T^+, T^+) \right] &= \delta_{T^-}^2 + 2\mathbb{E}_{T^-} \left[a_1(\hat{\theta}_T^+, T^+) \right] \delta_{T^-} + \mathbb{E}_{T^-} \left[a_2(\hat{\theta}_T^+, T^+) \right] \\ \Rightarrow a_2(\hat{\theta}_T^-, T^-) &= \mathbb{E}_{T^-} \left[a_2(\hat{\theta}_T^+, T^+) \right] \end{aligned} \tag{107}$$

$$\begin{aligned} \mathbb{E}_{T^-} \left[w_1(\hat{\theta}_T^+, T^+) \delta_{T^+} + a_3(\hat{\theta}_T^+, T^+) \right] &= w_1(\hat{\theta}_T^-, T^-) \delta_{T^-} + a_3(\hat{\theta}_T^-, T^-) \\ \Rightarrow a_3(\hat{\theta}_T^-, T^-) &= \mathbb{E}_{T^-} \left[a_3(\hat{\theta}_T^+, T^+) \right]. \end{aligned} \tag{108}$$

Then we follow the procedures in the proof of Lemma 1 to compute the boundary condition

$$w_j \left(\hat{\theta}_T^-, T^- \right) = \mathbb{E} \left[\mathbb{E} \left[w_j \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right] \mid \hat{\theta}_T^-, T^- \right], \quad j = 1, 2; \quad (109)$$

$$a_j \left(\hat{\theta}_T^-, T^- \right) = \mathbb{E} \left[\mathbb{E} \left[a_j \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right] \mid \hat{\theta}_T^-, T^- \right] \quad j = 1, 2, 3, \quad (110)$$

which correspond to Eqs.(97) and (98) in Lemma 5.

Step 3: from $t \rightarrow T^-$, we use the PDEs in Step 1 to solve the entire paths with the boundary conditions $w_j \left(\hat{\theta}_T^-, T^- \right)$ and $a_j \left(\hat{\theta}_T^-, T^- \right)$ obtained from Step 2. \square

Implied variance reduction upon the announcement With the implied variance from Lemma 4, we could calculate the implied variance reduction upon the announcement. Right after the announcement, the IV is

$$\begin{aligned} IV_{T^+, \tau} \left(\hat{\theta}_T^+, T^+ \right) &= w_2 \left(\hat{\theta}_T^+, T^+ \right) - w_1 \left(\hat{\theta}_T^+, T^+ \right)^2 + a_2 \left(\hat{\theta}_T^+, T^+ \right) - a_1 \left(\hat{\theta}_T^+, T^+ \right)^2 \\ &\quad + 2 \left[a_3 \left(\hat{\theta}_T^+, T^+ \right) - w_1 \left(\hat{\theta}_T^+, T^+ \right) a_1 \left(\hat{\theta}_T^+, T^+ \right) \right] + g \left(\sigma_{i, T^+}^+ \right) \end{aligned} \quad (111)$$

Right before the announcement, conditional on a given informativeness ν , the IV is

$$\begin{aligned} \tilde{IV}_{T^-, \tau} \left(\nu, \hat{\theta}_T^-, T^- \right) &= \mathbb{E} \left[IV_{T^+, \tau} \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right] \\ &= \tilde{w}_2 \left(\nu, \hat{\theta}_T^-, T^- \right) - \tilde{w}_1 \left(\nu, \hat{\theta}_T^-, T^- \right)^2 + \tilde{a}_2 \left(\nu, \hat{\theta}_T^-, T^- \right) - \tilde{a}_1 \left(\nu, \hat{\theta}_T^-, T^- \right)^2 \\ &\quad + 2 \left[\tilde{a}_3 \left(\nu, \hat{\theta}_T^-, T^- \right) - \tilde{w}_1 \left(\nu, \hat{\theta}_T^-, T^- \right) \tilde{a}_1 \left(\nu, \hat{\theta}_T^-, T^- \right) \right] + g \left(\sigma_{i, T^-}^- \right). \end{aligned} \quad (112)$$

Therefore, the implied variance reduction is defined as

$$\Delta IV_T = \tilde{IV}_{T^-, \tau} \left(\nu, \hat{\theta}_T^-, T^- \right) - IV_{T^+, \tau} \left(\hat{\theta}_T^+, T^+ \right), \quad (113)$$

which corresponds to Eq.(14) in the main text. Note that the first term denotes the IV before the announcement. It is captured by the conditional expectation of IV after the announcement, conditional on information right before the announcement at T^- and the distribution of ν . Instead of using unconditional expectation before the announcement, we use conditional expectation because it immediately reflects investors' belief updating conditional on the informativeness ν . The conditional expectation better measures information-induced changes in implied variance.

7.2.4 Policy functions

In all of the computations below, we plot policy functions for different choices of $\nu \in [0.5, 1]$. We assume that $\hat{\theta}_T^- = \bar{\theta} = \frac{\lambda_L \theta_H + \lambda_H \theta_L}{\lambda_L + \lambda_H}$. Given $\hat{\theta}_T^- = \bar{\theta}$ and given a ν , the distribution of $\hat{\theta}_T^+$ is given by Eq.(7), or equivalently, Eqs.(35) to (38).

Announcement premium Define the return earned upon the announcement as $R^A = \frac{p(\hat{\theta}_T^+, 0)}{\tilde{p}(\nu, \hat{\theta}_T^-, 0)}$. For a given ν , the announcement premium in the model is

$$\mathbb{E} \left[R^A \mid \nu, \hat{\theta}_T^-, T^- \right]_{-1} = \frac{\mathbb{E} \left[p \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right]_{-1}}{\tilde{p} \left(\nu, \hat{\theta}_T^-, T^- \right)} = \frac{h_{s_H} p \left(\hat{\theta}_{s_H}^+, T^+ \right) + h_{s_L} p \left(\hat{\theta}_{s_L}^+, T^+ \right)}{\tilde{p} \left(\nu, \hat{\theta}_T^-, T^- \right)}_{-1}, \quad (114)$$

where $\tilde{p} \left(\nu, \hat{\theta}_T^-, T^- \right)$ is defined in Eq.(62).

Implied variance reduction upon the announcement Because the dividend is continuous upon the announcement, the IV reduction can be computed as

$$\begin{aligned} \Delta IV_T &= \text{Var} \left[\ln p \left(\hat{\theta}_T^+, T^+ \right) - \ln \tilde{p} \left(\nu, \hat{\theta}_T^-, T^- \right) \mid \nu, \hat{\theta}_T^-, T^- \right] = \text{Var} \left[\ln p \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right] \quad (115) \\ &= \mathbb{E} \left[\ln p \left(\hat{\theta}_T^+, T^+ \right)^2 \mid \nu, \hat{\theta}_T^-, T^- \right] - \left(\mathbb{E} \left[\ln p \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right] \right)^2 \\ &= \left[h_{s_H} \ln p \left(\hat{\theta}_{s_H}^+, T^+ \right)^2 + h_{s_L} \ln p \left(\hat{\theta}_{s_L}^+, T^+ \right)^2 \right] - \left[h_{s_H} \ln p \left(\hat{\theta}_{s_H}^+, T^+ \right) + h_{s_L} \ln p \left(\hat{\theta}_{s_L}^+, T^+ \right) \right]^2 \end{aligned}$$

30-day expected variance after the announcement going forward We first compute the 30-day implied variance after the announcement:

$$IV_{T^+, T^+ \Delta} \left(\hat{\theta}_T^+, T^+ \right) = \text{Var} \left[\ln p \left(\hat{\theta}_{T^+ \Delta}^+, T^+ \Delta \right) + \ln D_{T^+ \Delta} \mid \hat{\theta}_T^+, T^+ \right]. \quad (117)$$

where $\Delta = \frac{1}{12}$ for monthly return. Note that $IV_{T^+, T^+ \Delta} \left(\hat{\theta}_T^+, T^+ \right)$ is a random variable that depends on the realization of $\hat{\theta}_T^+$. This is the same as the implied variance defined in (111). To compute expected value, we compute

$$\mathbb{E} \left[IV_{T^+, T^+ \Delta} \left(\hat{\theta}_T^+, T^+ \right) \mid \nu, \hat{\theta}_T^-, T^- \right] = h_{s_H} IV_{T^+, T^+ \Delta} \left(\hat{\theta}_{s_H}^+, T^+ \right) + h_{s_L} IV_{T^+, T^+ \Delta} \left(\hat{\theta}_{s_L}^+, T^+ \right). \quad (118)$$

Expected 30-day return after the announcement going forward Using the expression (76), the expected log return is given by:

$$\begin{aligned}
ER_{t,T+\Delta}(\hat{\theta}_T^+, T^+) &= \mathbb{E} \left[\ln \left[\frac{p(\hat{\theta}_{T+\Delta}, T+\Delta) D_{T+\Delta} + \int_{T^+}^{T+\Delta} D_s ds}{p(\hat{\theta}_T^+, T^+) D_{T^+}} \right] \middle| \hat{\theta}_T^+, T^+ \right] \\
&\approx \mathbb{E} \left[\ln \left[\frac{p(\hat{\theta}_{T+\Delta}, T+\Delta) D_{T+\Delta} + D_{T+\Delta} \Delta}{p(\hat{\theta}_T^+, T^+) D_{T^+}} \right] \middle| \hat{\theta}_T^+, T^+ \right] \\
&= \mathbb{E} \left[\ln \left[p(\hat{\theta}_{T+\Delta}, T+\Delta) + \Delta \right] \middle| \hat{\theta}_T^+, T^+ \right] \\
&\quad - \ln p(\hat{\theta}_T^+, T^+) + a_1(\hat{\theta}_T^+, T^+) + g(\sigma_{i,T}^+, T^+), \tag{119}
\end{aligned}$$

where $\Delta = \frac{1}{12}$ for monthly return. The only unknown in the above equation is $w_3(\hat{\theta}_{T+\Delta}, T+\Delta) = \mathbb{E} \left[\ln \left[p(\hat{\theta}_{T+\Delta}, T+\Delta) + \Delta \right] \middle| \hat{\theta}_T^+, T^+ \right]$. We apply the same procedure in computing $w_1(\hat{\theta}_T^+, T^+)$ with the only difference of the boundary condition $w_3(\hat{\theta}_{T+\Delta}, T+\Delta) = \ln p(\hat{\theta}_{T+\Delta}, T+\Delta) + \Delta$.

Then we compute the average expected return after announcement as

$$\mathbb{E} \left[ER_{t,\tau}(\hat{\theta}_T^+, T^+) \middle| \nu, \hat{\theta}_T^-, T^- \right] = h_{s_H} ER_{t,\tau}(\hat{\theta}_{s_H}^+, T^+) + h_{s_L} ER_{t,\tau}(\hat{\theta}_{s_L}^+, T^+). \tag{120}$$

Online Appendix

In this online appendix, we describe the numerical solutions we used to solve for our dynamic model. We first describe how we solve the daily returns, realized variance using the simulated sample paths. Then we demonstrate the finite difference method to solve for the value function, price-to-dividend ratio, as well as the implied variance.

Computing returns The return earned from t to $t + \Delta$ is defined as

$$R_{t,t+\Delta} = \frac{p(\hat{\theta}_{t+\Delta}, t+\Delta) D_{t+\Delta} + \int_t^{t+\Delta} D_s ds}{p(\hat{\theta}_t, t) D_t}. \tag{121}$$

Numerically, $\Delta = 1/360$ for daily return, we approximate

$$\begin{aligned}
R_{t,t+\Delta} &= \frac{p\left(\hat{\theta}_{t+\Delta}, t + \Delta\right) \frac{D_{t+\Delta}}{D_t} + \int_t^{t+\Delta} \frac{D_s}{D_t} ds}{p\left(\hat{\theta}_t, t\right)} = \frac{p\left(\hat{\theta}_{t+\Delta}, t + \Delta\right) \left(1 + \frac{dD_t}{D_t}\right) + \int_t^{t+\Delta} \left(1 + \frac{dD_{s-t}}{D_t}\right) ds}{p\left(\hat{\theta}_t, t\right)} \\
&= \frac{p\left(\hat{\theta}_{t+\Delta}, t + \Delta\right) \left(1 + \frac{dD_t}{D_t}\right) + \left(1 + 1 + \frac{dD_t}{D_t}\right) \frac{\Delta}{2}}{p\left(\hat{\theta}_t, t\right)} = \frac{p\left(\hat{\theta}_{t+\Delta}, t + \Delta\right) \left(1 + \frac{dD_t}{D_t}\right) + \left(\Delta + \frac{dD_t}{D_t} \frac{\Delta}{2}\right)}{p\left(\hat{\theta}_t, t\right)} \quad (122)
\end{aligned}$$

Computing realized variance Over a small interval dt , dividend is negligible, we have

$$r_{t,t+dt} = \ln R_{t+dt} - \ln R_t, \quad (123)$$

where the law of motion of R_t in the interior follows Lemma 3. In the data, we compute

$$RV_{t,t+\Delta}(dt) = \sum_{j=1}^{\Delta/dt} r_{t+(j-1)dt,t+jdt}^2. \quad (124)$$

Over a 5-minute interval subtracting a mean is quantitatively irrelevant. It can be shown as the $dt \rightarrow 0$, the above quantitative result converges to the quadratic variation of the return process:

$$\lim_{\Delta \rightarrow 0} RV_{t,t+\Delta}(dt) = \int_t^{t+dt} (\sigma_{R,s}^2 + \sigma_{i,s}^2) ds + \sum_{t \leq s \leq t+\Delta} I_{\{s\}} \left[\ln p\left(\hat{\theta}_s^+, s^+\right) - \ln p\left(\hat{\theta}_s^-, s^-\right) \right]^2 \quad (125)$$

where $I_{\{t\}}$ is the announcement indicator, $I = 1$ if $(t \bmod T = 0)$. Therefore, numerically, over a day, that is, $\Delta = \frac{1}{360}$, we simply compute

$$\begin{aligned}
RV_{t,t+\Delta} &\approx \frac{1}{2} \left[\sigma_R^2 \left(\hat{\theta}_t, t\right) + \sigma_R^2 \left(\hat{\theta}_{t+\Delta}, t + \Delta\right) + \sigma_i^2 \left(\hat{\theta}_t, t\right) + \sigma_i^2 \left(\hat{\theta}_{t+\Delta}, t + \Delta\right) \right] \Delta \\
&\quad + \sum_{t \leq s \leq t+\Delta} I_{\{s\}} \left[\ln p\left(\hat{\theta}_s^+, s^+\right) - \ln p\left(\hat{\theta}_s^-, s^-\right) \right]^2. \quad (126)
\end{aligned}$$

Calculating the risk-free rate Take the expression of the risk-free rate in (54), the cumulative risk-free return from t to $t + \Delta$ is:

$$e^{\int_t^{t+\Delta} r(\hat{\theta}_s, s) ds} \approx \exp \left\{ \sum_{j=1}^{\Delta/dt} r\left(\hat{\theta}_{t+(j-1)dt}, t + (j-1)dt\right) dt \right\}. \quad (127)$$

This way we can accumulate daily risk-free return to compute annual risk-free rate for $\Delta = 1$.

Solve for H function We use finite difference method to solve for the value function. The HJB equation in Eq.(48) can be rewritten as:

$$(1 - \gamma) \left(\frac{\rho}{1 - \frac{1}{\psi}} - \hat{\theta}_t + \frac{1}{2} \gamma \sigma_Y^2 \right) H = H_t + H_\theta [\mu_{\theta,t} + (1 - \gamma) \sigma_{\theta,t}] + \frac{1}{2} H_{\theta\theta} \frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + \frac{\rho(1 - \gamma)}{1 - \frac{1}{\psi}} H^{\frac{1}{\psi} - \gamma}.$$

Use finite difference method and approximate the functions $H(\hat{\theta}_t, t)$ at I discrete points in the space dimensions, $\hat{\theta}_i$, $i = 1, 2, \dots, I$. Denote $H_i^n = H(\hat{\theta}_i, t^n)$, where time dimension $n = 0, 1, 2, \dots, N$. Denote

$$\beta_i = (1 - \gamma) \left(\frac{\rho}{1 - \frac{1}{\psi}} - \hat{\theta}_i + \frac{1}{2} \gamma \sigma_Y^2 \right), \quad (128)$$

$$u_i^{n+1} = \frac{\rho(1 - \gamma)}{1 - \frac{1}{\psi}} (H_i^{n+1})^{\frac{1}{\psi} - \gamma}. \quad (129)$$

Use implicit method to update the value function,

$$\begin{aligned} \beta_i H_i^n &= \frac{H_i^{n+1} - H_i^n}{\Delta t} + u_i^{n+1} + \frac{1}{2} \partial_{\theta\theta} H_i^n \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \\ &+ \partial_{\theta,F} H_i^n [\mu_{\theta,i} + (1 - \gamma) \sigma_{\theta,i}]^+ + \partial_{\theta,B} H_i^n [\mu_{\theta,i} + (1 - \gamma) \sigma_{\theta,i}]^-. \end{aligned} \quad (130)$$

Use upwind scheme to approximate the derivatives $\partial_\theta H_i^n$ and $\partial_{\theta\theta} H_i^n$,

$$\begin{aligned} \beta_i H_i^n &= \frac{H_i^{n+1} - H_i^n}{\Delta t} + u_i^{n+1} + \frac{1}{2} \frac{H_{i+1}^n - 2H_i^n + H_{i-1}^n}{(\Delta \hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \\ &+ \frac{H_{i+1}^n - H_i^n}{\Delta \hat{\theta}} [\mu_{\theta,i} + (1 - \gamma) \sigma_{\theta,i}]^+ + \frac{H_i^n - H_{i-1}^n}{\Delta \hat{\theta}} [\mu_{\theta,i} + (1 - \gamma) \sigma_{\theta,i}]^-. \end{aligned} \quad (131)$$

Collecting terms and rewrite HJB equation,

$$\beta_i H_i^n = \frac{H_i^{n+1} - H_i^n}{\Delta t} + u_i^{n+1} + H_{i-1}^n x_i + H_i^n y_i + H_{i+1}^n z_i \quad (132)$$

where

$$x_i = -\frac{[\mu_{\theta,i} + (1-\gamma)\sigma_{\theta,i}]^-}{\Delta\hat{\theta}} + \frac{1}{2(\Delta\hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (133)$$

$$y_i = -\frac{[\mu_{\theta,i} + (1-\gamma)\sigma_{\theta,i}]^+}{\Delta\hat{\theta}} + \frac{[\mu_{\theta,i} + (1-\gamma)\sigma_{\theta,i}]^-}{\Delta\hat{\theta}} - \frac{1}{(\Delta\hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (134)$$

$$z_i = \frac{[\mu_{\theta,i} + (1-\gamma)\sigma_{\theta,i}]^+}{\Delta\hat{\theta}} + \frac{1}{2(\Delta\hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (135)$$

Rewrite in the matrix notation,

$$\beta H^n = u^{n+1} + \mathbf{A}^{n+1} H^n + \frac{H^{n+1} - H^n}{\Delta t}, \quad (136)$$

where

$$\mathbf{A}^{n+1} = \begin{bmatrix} y_1 & z_1 & 0 & \cdots & 0 \\ x_2 & y_2 & z_2 & 0 & \vdots \\ 0 & x_3 & y_3 & z_3 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & x_I & y_I \end{bmatrix}, H^n = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \vdots \\ H_I \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 & 0 & & \cdots \\ 0 & \beta_2 & & \\ & & \beta_3 & \\ & & & \ddots & 0 \\ & & & & \beta_I \end{bmatrix}, u^{n+1} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_I \end{bmatrix}.$$

The system can be written as

$$\mathbf{B}^{n+1} H^n = b^{n+1}, \quad \mathbf{B}^{n+1} = \left(\frac{1}{\Delta t} + \beta \right) - \mathbf{A}^{n+1}, \quad b^{n+1} = u^{n+1} + \frac{1}{\Delta t} H^{n+1}. \quad (137)$$

with the boundary condition

$$H(\hat{\theta}^-, T) = \mathbb{E} \left[\tilde{H}(\nu, \hat{\theta}^-, T) \mid \hat{\theta}^-, T \right]; \quad \tilde{H}(\nu, \hat{\theta}^-, T) = \mathbb{E} \left[H(\hat{\theta}^+, 0) \mid \nu, \hat{\theta}^-, T \right],$$

where from (50) and (49),

$$\begin{aligned} \tilde{H}(\nu, \hat{\theta}^-, T) &= h_{s_H} H(\hat{\theta}_{s_H}^+, 0) + h_{s_L} H(\hat{\theta}_{s_L}^+, 0) \\ &= \left[\frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} \nu + \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} (1 - \nu) \right] H \left(\theta_L + \frac{(\hat{\theta}^- - \theta_L) \nu (\theta_H - \theta_L)}{(\hat{\theta}^- - \theta_L) \nu + (\theta_H - \hat{\theta}^-) (1 - \nu)}, 0 \right) \\ &\quad + \left[\frac{\hat{\theta}^- - \theta_L}{\theta_H - \theta_L} (1 - \nu) + \frac{\theta_H - \hat{\theta}^-}{\theta_H - \theta_L} \nu \right] H \left(\theta_L + \frac{(\hat{\theta}^- - \theta_L) (1 - \nu) (\theta_H - \theta_L)}{(\hat{\theta}^- - \theta_L) (1 - \nu) + (\theta_H - \hat{\theta}^-) \nu}, 0 \right). \end{aligned} \quad (138)$$

$$H(\hat{\theta}^-, T) = \mathbb{E} \left[\tilde{H}(\nu, \hat{\theta}^-, T) \mid \hat{\theta}^-, T \right] = \sum_{j=1}^m q_j \tilde{H}(\nu_j, \hat{\theta}^-, T) \quad (139)$$

Solve for price-to-dividend ratio Similarly, the PDE for $p(\hat{\theta}_t, t)$ is defined in Lemma 2. Use finite difference method and approximate the functions $p(\hat{\theta}_t, t)$ at I discrete points in the space dimensions, $\hat{\theta}_i$, $i = 1, 2, \dots, I$. Denote $p_i^n = p(\hat{\theta}_i, t^n)$, where time dimension $n = 0, 1, 2, \dots, N$. Denote

$$\begin{aligned} \varpi_i^{n+1} &= (\xi - 1)\bar{\theta} + \rho - \frac{1}{2}\gamma\sigma_Y^2\left(\frac{1}{\psi} + 1\right) + \gamma\sigma_Y^2 - \left(\xi - \frac{1}{\psi}\right)\hat{\theta}_i + \frac{(\frac{1}{\psi} - \gamma)(1 - \frac{1}{\psi})}{2(1 - \gamma)^2} \left(\frac{H_{\theta,i}^{n+1}}{H_i^{n+1}}\right)^2 \frac{\sigma_{\hat{\theta}_i}^2}{\sigma_Y^2} \\ \varrho_i^{n+1} &= \mu_{\theta,i} + (1 - \gamma)\sigma_{\theta,i} + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \frac{H_{\theta,i}^{n+1}}{H_i^{n+1}} \frac{\sigma_{\hat{\theta}_i}^2}{\sigma_Y^2}. \end{aligned} \quad (141)$$

Use implicit method to update the price-to-dividend ratio (60),

$$\varpi_i^{n+1} p_i^n = \frac{p_i^{n+1} - p_i^n}{\Delta t} + 1 + \frac{1}{2} \partial_{\theta\theta} p_i^n \frac{\sigma_{\hat{\theta}_i}^2}{\sigma_Y^2} + \partial_{\theta,F} p_i^n (\varrho_i^{n+1})^+ + \partial_{\theta,B} p_i^n (\varrho_i^{n+1})^-. \quad (142)$$

Use upwind scheme to approximate the derivatives $\partial_{\theta} p_{i,j}^n$ and $\partial_{\theta\theta} p_{i,j}^n$,

$$\begin{aligned} \varpi_i^{n+1} p_i^n &= \frac{p_i^{n+1} - p_i^n}{\Delta t} + 1 + \frac{1}{2} \frac{p_{i+1}^n - 2p_i^n + p_{i-1}^n}{(\Delta\hat{\theta})^2} \frac{\sigma_{\hat{\theta}_i}^2}{\sigma_Y^2} \\ &\quad + \frac{p_{i+1}^n - p_i^n}{\Delta\hat{\theta}} (\varrho_i^{n+1})^+ + \frac{p_i^n - p_{i-1}^n}{\Delta\hat{\theta}} (\varrho_i^{n+1})^-. \end{aligned} \quad (143)$$

Collecting terms and rewrite the PDE,

$$\varpi_i^{n+1} p_i^n = \frac{p_i^{n+1} - p_i^n}{\Delta t} + 1 + p_{i-1}^n x_i^{n+1} + p_i^n y_i^{n+1} + p_{i+1}^n z_i^{n+1} \quad (144)$$

where

$$x_i^{n+1} = -\frac{(\varrho_i^{n+1})^-}{\Delta\hat{\theta}} + \frac{1}{2(\Delta\hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (145)$$

$$y_i^{n+1} = -\frac{(\varrho_i^{n+1})^+}{\Delta\hat{\theta}} + \frac{(\varrho_i^{n+1})^-}{\Delta\hat{\theta}} - \frac{1}{(\Delta\hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (146)$$

$$z_i^{n+1} = \frac{(\varrho_i^{n+1})^+}{\Delta\hat{\theta}} + \frac{1}{2(\Delta\hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (147)$$

Rewrite in the matrix notation,

$$\varpi^{n+1} p^n = 1 + \mathbf{A}^{n+1} p^n + \frac{p^{n+1} - p^n}{\Delta t}, \quad (148)$$

The system can be written as

$$\mathbf{B}^{n+1} p^n = b^{n+1}, \quad \mathbf{B}^{n+1} = \left(\frac{1}{\Delta t} + \varpi^{n+1} \right) - \mathbf{A}^{n+1}, \quad b^{n+1} = 1 + \frac{1}{\Delta t} p^{n+1}. \quad (149)$$

where

$$p^n = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_I \end{bmatrix}, \quad \varpi^{n+1} = \begin{bmatrix} \varpi_1 & 0 & & \cdots \\ 0 & \varpi_2 & & \\ & & \varpi_3 & \\ & & & \ddots & 0 \\ \vdots & & & & \varpi_I \end{bmatrix}.$$

At the boundary,

$$\tilde{p}(\nu, \hat{\theta}_T^-, T) = \frac{h_{s_H} H(\hat{\theta}_{s_H}^+, 0)^{\frac{1}{\psi}-\gamma} p(\hat{\theta}_{s_H}^+, 0) + h_{s_L} H(\hat{\theta}_{s_L}^+, 0)^{\frac{1}{\psi}-\gamma} p(\hat{\theta}_{s_L}^+, 0)}{\left[h_{s_H} H(\hat{\theta}_{s_H}^+, 0) + h_{s_L} H(\hat{\theta}_{s_L}^+, 0) \right]^{\frac{1}{\psi}-\gamma}} \quad (150)$$

$$p(\hat{\theta}^-, T) = \frac{\sum_{j=1}^m q_j \tilde{H}(\nu_j, \hat{\theta}^-, T)^{\frac{1}{\psi}-\gamma} \tilde{p}(\nu_j, \hat{\theta}^-, T)}{\left[\sum_{j=1}^m q_j \tilde{H}(\nu_j, \hat{\theta}^-, T) \right]^{\frac{1}{\psi}-\gamma}}. \quad (151)$$

Solve for implied variance The PDEs are in general as the form of

$$0 = \frac{w_i^{n+1} - w_i^n}{\Delta t} + u_i^{n+1} + \frac{1}{2} \frac{w_{i+1}^n - 2w_i^n + w_{i-1}^n}{(\Delta \hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} + \frac{w_{i+1}^n - w_i^n}{\Delta \hat{\theta}} \mu_{\theta,i}^+ + \frac{w_i^n - w_{i-1}^n}{\Delta \hat{\theta}} \mu_{\theta,i}^-. \quad (152)$$

where $u_i^{n+1} = 0$ for $w_1 = \mathbb{E}_t \left[\ln p \left(\hat{\theta}_\tau, \tau \right) \right]$ and $w_2 = \mathbb{E}_t \left[\ln^2 p \left(\hat{\theta}_\tau, \tau \right) \right]$, whereas for a_1, a_0 and a_3 ,

$$u_i^{n+1} = \xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \quad (153)$$

$$u_i^{n+1} = 2a_{1,i}^{n+1} \left(\xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) + 2 \frac{a_{1,i+1}^{n+1} - a_{1,i}^{n+1}}{\Delta \hat{\theta}} \sigma_{\theta,t} + \sigma_Y^2 \quad (154)$$

$$u_i^{n+1} = w_{1,i}^{n+1} \left(\xi \hat{\theta}_t + \bar{\theta} (1 - \xi) - \frac{1}{2} \sigma_Y^2 \right) + \frac{w_{1,i+1}^{n+1} - w_{1,i}^{n+1}}{\Delta \hat{\theta}} \sigma_{\theta,t} \quad (155)$$

Therefore, rewrite the PDE as

$$0 = \frac{w_i^{n+1} - w_i^n}{\Delta t} + u_i^{n+1} + w_{i-1}^n x_i + w_i^n y_i + w_{i+1}^n z_i \quad (156)$$

where

$$x_i = -\frac{\mu_{\theta,i}^-}{\Delta \hat{\theta}} + \frac{1}{2} \frac{\sigma_{\theta,i}^2}{(\Delta \hat{\theta})^2 \sigma_Y^2} \quad (157)$$

$$y_i = -\frac{\mu_{\theta,i}^+}{\Delta \hat{\theta}} + \frac{\mu_{\theta,i}^-}{\Delta \hat{\theta}} - \frac{1}{(\Delta \hat{\theta})^2} \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (158)$$

$$z_i = \frac{\mu_{\theta,i}^+}{\Delta \hat{\theta}} + \frac{1}{2} \frac{\sigma_{\theta,i}^2}{(\Delta \hat{\theta})^2 \sigma_Y^2}. \quad (159)$$

Rewrite in the matrix notation,

$$0 = u^{n+1} + \mathbf{A}^{n+1} w^n + \frac{w^{n+1} - w^n}{\Delta t}. \quad (160)$$

The system can be written as

$$\mathbf{B}^{n+1} w^n = b^{n+1}, \quad \mathbf{B}^{n+1} = \frac{1}{\Delta t} \mathbf{I} - \mathbf{A}^{n+1}, \quad b^{n+1} = u^{n+1} + \frac{1}{\Delta t} w^{n+1}. \quad (161)$$

with the boundary condition for $\mathbb{E}_t \left[\ln p \left(\hat{\theta}_\tau, \tau \right) \right]$ is:

$$w_1 \left(\hat{\theta}, \tau \right) = \ln p \left(\hat{\theta}_\tau, \tau \right) \quad (162)$$

and boundary condition for $\mathbb{E}_t \left[\ln^2 p \left(\hat{\theta}_\tau, \tau \right) \right]$ is:

$$w_2 \left(\hat{\theta}, \tau \right) = \ln^2 p \left(\hat{\theta}_\tau, \tau \right) \quad (163)$$

and the rest boundary conditions

$$a_1 \left(\hat{\theta}_\tau, \tau \right) = 0, \quad a_0 \left(\hat{\theta}_\tau, \tau \right) = 0 \quad \text{and} \quad a_3 \left(\hat{\theta}_\tau, \tau \right) = 0. \quad (164)$$