Information-Driven Volatility

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Abstract: Standard asset pricing models with stochastic volatility predict a robust positive relationship between past realized volatility and future expected returns. Empirical work typically finds this relationship to be negative. We develop an asset pricing model where stock market volatility dynamics are driven by information. We show that under strong generalized risk sensitivity of preferences, information-driven volatility induces a negative correlation between past realized volatility and future expected returns. We provide empirical evidence for the unique implications of the information-driven volatility channel and demonstrate that our model can quantitatively replicate the evidence.

Keywords: Information, Return Volatility, Macroeconomic Announcements, Generalized Risk Sensitivity

JEL Code: D83, D84, G11, G12, G14

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1 Introduction

Leading consumption-based asset pricing models with stochastic volatility, such as the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the rare disaster model of Barro (2006) and Gabaix (2012), predict a robust positive relationship between past realized volatility and future expected returns. Empirically, many authors (e.g., Nelson (1991) and Glosten, Jagannathan, and Runkle (1993)) find a negative relationship between the two. More recently, Moreira and Muir (2017) demonstrate that the above negative relationship can be used to construct a volatility-managed portfolio that earns a higher average return than the market portfolio. In this paper, we develop a model with information-driven volatility. We show that information-driven volatility induces a negative relationship between past realized volatility and future expected returns. We provide empirical evidence for the information-driven volatility channel and demonstrate that our model can account for several volatility-related asset pricing puzzles.

There are two main motivations for our theory of information-driven volatility. First, high-frequency financial market fluctuations, such as those at a monthly, daily, or hourly frequency, are more likely to be driven by information rather than by the volatility of macroeconomic fundamentals. Standard consumption-based asset pricing models — such as Bansal, Kiku, Shaliastovich, and Yaron (2014) in the context of the long-run risk model and Wachter (2013) in the context of the rare disaster model — typically generate time-varying volatility of returns by assuming time-varying volatility of macroeconomic fundamentals. While these models can explain quite well asset pricing moments at lower frequencies (annual frequencies and above), the volatility of macroeconomic fundamentals in the data typically does not change substantially at high frequencies and is unlikely to explain high-frequency financial market volatilities. In contrast, information about the macroeconomy arrives continuously at financial markets and affects the stock return volatility at high frequencies.

Second, recent empirical evidence and theoretical work establish that significant risk compensation is associated with informational events on financial markets. Consistent with Savor and Wilson (2013) and Ai and Bansal (2018), we show that roughly 70% of the stock market risk premium during the 1961-2021 period is realized on a small number of days with significant macroeconomic announcements. Lucca and Moench (2015) find a similar pattern for FOMC announcements. Ai and Bansal (2018) demonstrate that expected utility models are inconsistent with the announcement premium, and the existence of such implies that investor preferences must satisfy generalized risk sensitivity (GRS). As we will show in this paper, under expected utility, information-driven volatility is irrelevant for risk compensation. We propose a concept of strong generalized risk sensitivity and demonstrate that under this condition, information-driven volatility induces a negative relationship between past realized volatility and future expected returns.

Our theory of information-driven volatility has two main theses. First, more precise public information about macroeconomic fundamentals is associated with a higher stock market reaction and a higher realized volatility upon the arrival of the information and a lower volatility afterward. Second, more precise public information about macroeconomic fundamentals is associated with higher stock market risk compensation upon the arrival of the information and lower risk compensation afterward.

To understand the first thesis, the link between the informativeness of public information and volatility, consider the following variance decomposition identity:

$$\mathrm{Var}\left[\theta_T\right] = \mathrm{Var}\left[\mathbb{E}\left(\theta_T|s_t\right)\right] + \mathbb{E}[\mathrm{Var}\left(\theta_T|s_t\right)]. \tag{1}$$

We interpret θ_T as macroeconomic fundamentals such as state variables that govern the dynamics of aggregate consumption and aggregate productivity, the value of which will be realized at the terminal time T. We interpret s_t as a public signal revealed at time $t < T$ that is informative about θ_T. One example of such public signals is a macroeconomic announcement. The above formula then decomposes the total variance of macroeconomic fundamentals, $\mathrm{Var}\left[\theta_T\right]$, into the variance realized on the announcement day t, $\mathrm{Var}\left[\mathbb{E}\left(\theta_T|s_t\right)\right]$, and the variance that will realize after the announcement at time T, $\mathbb{E}\left[\mathrm{Var}\left(\theta_T|s_t\right)\right]$.

Traditional models of stochastic volatility generate time-varying volatility of returns by time-varying volatility of macroeconomic fundamentals, $\mathrm{Var}\left[\theta_T\right]$. Because the volatility of macroeconomic fundamentals such as that of the aggregate consumption and that of aggregate productivity is positively autocorrelated over time, high past realized volatility predicts high expected future volatility and therefore high expected future returns.

Given that the volatility of macroeconomic fundamentals does not change substantially at high frequencies, now consider the exercise of varying the informativeness of the signal s_t by holding the total variance $\mathrm{Var}\left[\theta_T\right]$ fixed. A more accurate signal s_t triggers a larger response by investors regarding their conditional expectations about future cash flow, which is captured by the term $\mathrm{Var}\left[\mathbb{E}\left(\theta_T|s_t\right)\right]$ and is associated with a higher realized volatility in financial markets upon the arrival of information. The variance decomposition identity (1) then implies that such a high realized variance, $\mathrm{Var}\left[\mathbb{E}\left(\theta_T|s_t\right)\right]$, must be associated with a lower expected future variance, $\mathbb{E}\left[\mathrm{Var}\left(\theta_T|s_t\right)\right]$.

While the relationship between volatility and information is merely a consequence of the variance decomposition identity, (1), how information affects risk compensation depends on investors' risk preferences. Ai and Bansal (2018) demonstrate that information-induced volatility requires risk compensation if and only if investors' preferences satisfy GRS. In this paper, we extend the Ai and Bansal (2018) result and show that under strong GRS, more precise information is associated with a higher expected return upon the informational event and a lower expected return afterward. Intuitively, GRS requires the certainty-equivalent functional to satisfy a concavity condition; that is, it must be increasing in second-order stochastic dominance. The concept of strong GRS imposes a lower bound on the Arrow-Pratt measure of absolute risk aversion of the certainty-equivalent functional and is a stronger concavity requirement. Our main theorem implies that under strong GRS, higher information-driven volatility is associated not only with lower future expected volatility but also with lower future expected returns.

To empirically test the above implications of the information-driven volatility theory and assess their quantitative importance, we calibrate a continuous-time asset pricing model with time-varying informativeness of macroeconomic announcements. We conduct several tests of the information-driven volatility channel in the data and replicate these tests in our model.

In the data, FOMC announcements are the most identifiable events that reveal information about the macroeconomy. To test the mechanism of information-driven volatility, we first develop a measure of the informativeness of FOMC announcements based on the option-implied variance. Using this measure, we show that higher informativeness predicts a larger FOMC announcement-day return and a larger implied variance reduction on announcement days. In addition, higher informativeness also predicts a lower expected return and a lower stock return variance after the announcement.

To provide additional evidence for the information-driven volatility channel, using the stock market jump data constructed by Baker, Bloom, Davis, and Sammon (2021), we show that news-triggered stock market jumps negatively predict future market volatility and future stock returns, whereas there is no significant negative relationship between realized volatility and future expected returns on days without significant causes of news.

We calibrate our model and demonstrate that it matches well with conventional asset pricing moments and stock market dynamics around macroeconomic announcements. We replicate the above statistical tests of the information-driven volatility in our calibrated model to demonstrate the consistency between our model and data and the quantitative significance of the information-driven volatility channel.

Furthermore, we show that our model provides an explanation for the variance risk premium predictability without relying on high-frequency variations in the volatility of macroeconomic fundamentals. As shown by Bollerslev, Tauchen, and Zhou (2009), in the data, the difference between implied variance and realized variance has strong predictive powers for future returns for up to three to six months. They provide an explanation for this predictability based on high-frequency variations in the volatility of aggregate consumption. Our model assumes a homoscedastic consumption growth process; however, the difference between implied variance and realized variance predicts returns because it reflects the informativeness of the upcoming informational event: holding the past realized volatility constant, the anticipation of the arrival of highly informative news is associated with both high implied variance right before the news and larger risk compensation upon the arrival of the news.

Our results should not be interpreted as dismissing the role of the volatility of macroeconomic fundamentals in affecting stock market dynamics. In fact, our calibrated model features low-frequency movements in the volatility of macroeconomic fundamentals and exhibits an overall positive autocorrelation of stock market volatilities, as in the data. We emphasize the importance of identifying the driving force of volatility in understanding the volatility-expected return relationship in the data. The volatility of macroeconomic fundamentals varies at low frequencies and determines the long-run relationship between volatility and expected returns. Information arrives at the stock market continuously and is more likely to be responsible for the relationship between past realized volatility and future expected returns at higher frequencies.

Related literature This paper is closely related to the literature on generalized risk sensitivity and macroeconomic announcements. From an empirical perspective, pre-scheduled macroeconomic announcements are the most salient informational events that are associated with significant realizations of the equity risk premium and volatility. The literature that documents a significant macroeconomic announcement premium — for example, Savor and Wilson (2013, 2014), Lucca and Moench (2015), and Cieslak, Morse, and Vissing-Jorgensen (2019) — provides strong empirical support for the information-driven volatility mechanism emphasized in this paper. From a theoretical point of view, Ai and Bansal (2018) establish that the existence of the macroeconomic announcement premium implies that preferences must satisfy generalized risk sensitivity. We extend the theorem of generalized risk sensitivity of Ai and Bansal (2018) and provide conditions under which more precise announcements are associated with a higher expected return upon the announcement and a lower expected return afterward.

Our main theoretical result relates to the literature on non-expected utility analysis in economics and finance — for example, the recursive preference of Kreps and Porteus (1978) and Epstein and Zin (1989), the robust control preference of Hansen and Sargent (2005, 2008), and the related multiplier preference of Strzalecki (2011). The long-run risk model of Bansal and Yaron (2004), Bansal (2007), and Hansen, Heaton, and Li (2008) builds on recursive preferences. Borovicka and Stachurski (2020) provide necessary and sufficient conditions for the existence and uniqueness of recursive utility, and Borovicka and Stachurski (2021) study the stability of equilibrium in asset pricing models. Epstein and Schneider (2010) provide a review of asset pricing studies with ambiguity-averse preferences. Ju and Miao (2012) study the asset pricing implications of the smooth ambiguity preference, and Routledge and Zin (2010) focus on a model with the disappointment aversion. Skiadas (2009) provides an excellent textbook treatment of asset pricing theory based on recursive preferences.

Our model builds on the literature on learning and information in financial markets in general. David (1997) and David (2008) develop learning models to study equity market risk compensation. Veronesi (2000) and Ai (2010) study how information quality affects the aggregate stock market risk premium. Pastor and Veronesi (2009b) develop a learning model to study the relationship between technological innovations and stock market valuations. David and Veronesi (2013) estimate a regime-switching model with learning. Bansal and Shaliastovich (2010, 2011) and Shaliastovich (2015) develop models in which learning results in asset price jumps. Pastor and Veronesi (2009a) provide an excellent review of the literature on learning and financial markets. Different from the above papers, we provide theoretical conditions under which the time-varying informativeness of macroeconomic news affects risk compensation and evaluate its quantitative importance.

Our paper is related to the vast empirical literature on the expected return volatility relationship. Both Nelson (1991) and Glosten, Jagannathan, and Runkle (1993) document empirical evidence that is supportive of a negative relationship between past realized volatility and future expected returns. Harvey (1989) finds mixed evidence of a time-varying relationship between expected excess returns and conditional variances. Consistent with our theory, Harrison and Zhang (1999) find a negative relationship and sometimes mixed evidence for the relationship between past realized volatility and future expected returns over short horizons, but a positive relationship over horizons longer than a year. More recently, Moreira and Muir (2017) demonstrate a positive average return on their volatility-managed portfolio relative to the market return, and their evidence is also consistent with a negative relationship between past realized volatility and future expected returns. Lochstoer and Muir (2022) develop a model of extrapolative expectations of volatility shocks to explain the variance risk premium predictability and the negative relationship between volatility and expected returns. We provide a rational explanation of the negative relationship between past realized volatility and future expected returns based on strong generalized risk sensitivity.

Several recent papers provide empirical evidence that is consistent with the information-driven volatility channel emphasized in this paper. Baker, Bloom, Davis, Kost, Sammon, and Viratyosin (2020) document that higher news clarity is associated with lower realized volatility in the future. Zhang and Zhao (2020) provide evidence that in periods when public information is imprecise, the realized macroeconomic announcement premium is low. Chaudhry (2021) shows that announcement days are typically associated with uncertainty reductions and lower expected returns afterward.

Our paper is also related to the literature on the variance risk premium predictability. Bollerslev, Tauchen, and Zhou (2009) document the predictability of stock market returns by the difference between the implied and realized variance, and develop a model of the variance risk premium predictability based on stochastic volatility in the volatility of macroeconomic fundamentals. Drechsler and Yaron (2011) develop a model with stochastic volatility and stochastic jumps to quantitatively explain the variance risk premium predictability. Eraker and Wang (2015) estimate a non-linear diffusion model and study the variance risk premium predictability. Zhou (2018) provides a thorough review of this literature. The above literature has interpreted the difference between the implied and realized variance as the difference between variance under the risk-neutral measure and that under the physical measure hence defined as the variance risk premium. We show that the difference between the implied and realized variance can predict returns without assuming high-frequency variations in volatility. In our model, the difference between the two reflects the informativeness of the upcoming announcement, which predicts returns through the information-driven volatility channel.

The rest of the paper is organized as follows. Section 2 summarizes the stylized facts between the realized variance and expected returns. Section 3 presents the main theorem of the paper and demonstrates that under the assumption of strong GRS, more informative announcements are associated with higher expected returns upon announcements and lower expected returns afterward. Section 4 develops a dynamic model to account for the stylized facts, and Section 5 presents the quantitative results. Section 6 concludes.

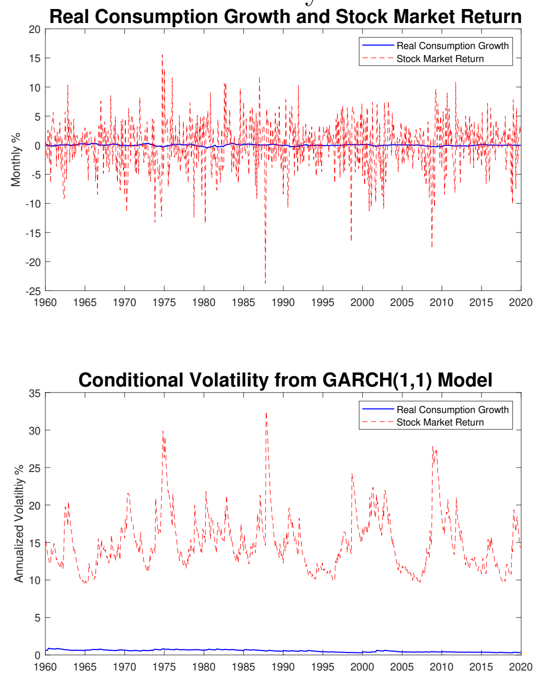
2 Motivating Facts for Information-Driven Volatility

In this section, we present two facts on volatility and stock market returns that motivate the development of our theory of information-driven volatility. The first fact motivates information as the main driver of financial market volatility, and the second fact motivates information as the main determinant of risk compensation.

1. The volatility of macroeconomic fundamentals does not exhibit significant variation at higher than monthly frequencies.

   In Figure 1, we plot the time series of monthly consumption growth (solid line) and monthly stock market returns (dashed line) in the top panel. In the bottom panel,

Figure 1: Macroeconomic volatility and stock market volatility



The top panel is the monthly consumption growth rates (solid line) and the total stock market index returns (dashed line) during 1960.02-2019.12. The bottom panel is the estimated conditional volatility of the two series from a GARCH(1,1) model during the same sample period.

we plot the estimated conditional volatility of the two time series from a GARCH (1,1) model. Compared to stock market returns, the variations in consumption growth are much smaller. The estimated conditional volatility of stock returns exhibits sharp variations over the monthly horizon, whereas that of aggregate consumption growth is virtually flat by comparison.

The above observation highlights the fact that variations in the volatility of macroeconomic fundamentals are unlikely to explain the high-frequency movements in financial market volatility. Information is much more likely to be the main driver of financial market volatility at high frequencies.

2. A large fraction of the equity premium is realized on a small fraction of trading days with significant macroeconomic announcements.

   Table 1 reports the average excess market returns on macroeconomic announcement days and non-announcement days from 1961 to 2021.[1] In this period, on average, 48 trading days per year have significant macroeconomic announcements. At the daily

[1] As in Ai and Bansal (2018), we focus on a relatively small set of pre-scheduled macroeconomic announcements that are released at monthly frequencies or lower. Within this category, we select the top five announcements ranked by investor attention from Bloomberg users. This procedure yields, on average, 48 announcement days per year for 1961-2021.

level, the average stock market excess return is 14.05 basis points (bps) on announcement days and 1.28 bps on days without major macroeconomic announcements. As a result, the cumulative excess stock market return on the 30 announcement days averages 6.75% per year, accounting for about 73% of the annual equity premium (9.29%).

The above fact not only highlights the importance of information in determining equity market risk compensation but also motivates a GRS-based asset pricing model to explain the joint dynamics of information and asset returns. As shown in Ai and Bansal (2018), the existence of the macroeconomic announcement premium implies that investors' preferences must satisfy generalized risk sensitivity. As we will demonstrate in this paper, under a condition slightly stronger than GRS, which we call strong GRS, information-driven volatility induces a negative relationship between realized volatility and expected returns.

Table 1: Macroeconomic Announcement Premium

	# of days p.a.	Daily prem.	Prem. p.a.	t-stat
Market	247	3.76 bps	9.29%	2.61
Ann	48	14.05 bps	6.75%	4.06
Non-Ann	199	1.28 bps	2.54%	0.81

This table documents the average excess return of the U.S. stock market during the 1961-2021 period. The column "# days p.a." is the average number of trading days per annum. The second column is the daily market equity premium on all days, announcement days, and days without announcements. The column "premium p.a." is the cumulative excess market returns within a year, computed by multiplying the daily premium by the number of event days and converting it into percentage points.

3 Information-Driven Volatility and Risk Premium

Our theory of information-driven volatility has two main theses. First, more precise information is associated with higher realized stock return volatility upon the informational event and lower volatility afterward. Second, more precise information is associated with a higher realization of the risk premium upon the informational event and a lower expected return afterward. The first thesis is simply an implication of the variance decomposition formula in (1) and does not depend on any assumptions on preferences and technology. The second thesis requires an equilibrium model. In this section, we extend the Ai and Bansal (2018) model to prove a general result between information-driven volatility and the risk premium. We demonstrate that the second thesis holds true under a condition that is slightly stronger than GRS, which we call strong GRS.

We consider a continuous-time endowment economy and allow aggregate consumption to be an arbitrary diffusion process such that the representative agent's utility is well defined. We use discrete time to illustrate the timing of events and the information structure but study its continuous-time limit to provide a sharp theoretical result. To fix ideas, we think of information as macroeconomic announcements, but our result applies to any event that carries information about the macroeconomy.

We assume that at time 0, the representative investor anticipates an announcement at time $t = \Delta$, where $\Delta > 0$. Here, 0 is a notationally convenient normalization and can be interpreted as any calendar time. We compare two economies that differ in the informativeness of the announcement at time t: an economy with a vague announcement, where $s_t = \eta$, and an economy with an informative announcement, where $s_t = (\eta, \varepsilon)$ contains an additional signal ε. We also assume that investors' information at time $T = 2\Delta$ is represented by the state variable θ_T, which contains more information than (η, ε).[2] The above setup implies that the only difference between the vague announcement economy and the informative announcement economy is that the additional ε is announced earlier in the informative announcement economy at time t but later in the vague information economy at time T. By time T, investors in both economies have the same information θ_T. In what follows, we call the return earned from time 0 to t the announcement return and the return earned from t to T the post-announcement return.

The variance decomposition formula (1), together with Jensen's inequality, implies that

$$\mathrm{Var}\left(\mathbb{E}\left[\theta_T \middle| \eta, \varepsilon\right]\right) \geq \mathrm{Var}\left(\mathbb{E}\left[\theta_T \middle| \eta\right]\right), \tag{2}$$

and

$$\mathbb{E}\left[\mathrm{Var}\left(\theta_T \middle| \eta, \varepsilon\right)\right] \leq \mathbb{E}\left[\mathrm{Var}\left(\theta_T \middle| \eta\right)\right]. \tag{3}$$

Because the stock market price is determined by investors' beliefs about fundamentals, Equation (2) can be interpreted as the variance in the announcement return is higher in the informative announcement economy relative to that in the vague announcement economy. Equation (3) can be interpreted as the variance in the post-announcement return is on average lower in the informative announcement economy relative to that in the vague announcement economy. In what follows, we show that under strong GRS, an analogous comparison also holds for expected returns.

Let V_τ denote the representative investors' utility at time τ. As in Ai and Bansal (2018),

[2] Formally, the three information sets, η, (η, ε), and θ_T are ranked by informativeness in the sense of Blackwell (1953). Using Blackwell (1953)'s notation for the "more informative than" relationship, $\theta_T \supset (\eta, \varepsilon) \supset \eta$.

we consider intertemporal preferences that are represented by a pair of aggregators $\{u, \mathcal{I}\}$ so that utility can be computed recursively as:

$$V_\tau = \left(1 - e^{-\rho\Delta}\right) u\left(C_\tau\right) + e^{-\rho\Delta}\mathcal{I}\left[V_{\tau+\Delta}\right], \tag{4}$$

where $\mathcal{I}$ converts continuation utility V into its certainty equivalent, ρ is the subjective discount rate, and C is aggregate consumption. We also assume that $\mathcal{I}$ has the following representation: $\mathcal{I}[V] = \phi^{-1}\{\mathbb{E}[\phi(V)]\}$, where ϕ is a strictly increasing and three times continuously differentiable function and ϕ^{-1} refers to the inverse function of ϕ. As shown in Ai and Bansal (2018), several important classes of non-expected utility can be represented in this form, such as the recursive utility of Kreps and Porteus (1978) and Epstein and Zin (1989), the robust control preference of Hansen and Sargent (2008), and the second order expected utility of Ergin and Gul (2009).

We illustrate the information structure for the vague announcement economy (panel A) and the informative announcement economy (panel B) in Table 2. Under our assumption of preferences, the stochastic discount factor that prices the (t, s_t)-state-contingent payoff into time-0 consumption units is given by

$$SDF_{0,t}\left(0, s_t\right) = e^{-\rho t}\frac{u'\left(C_t\right)}{u'\left(C_0\right)}\frac{\phi'\left[V_t\left(s_t\right)\right]}{\phi' \circ \phi^{-1}\left\{\mathbb{E}\left[\phi \circ V_t\left(s_t\right)\right]\right\}}, \tag{5}$$

where the symbol "$\circ$" stands for the function composition. The above expression allows $s_t = (\eta, \varepsilon)$ for the case of the informative announcement economy and $s_t = \eta$ for the case of the vague announcement economy. Here, we use the notation $V_t(s_t)$ to emphasize that time-t utility depends on investors' information. The stochastic discount factor that prices the (T, s_T)-state-contingent payoff into (t, s_t) consumption units is given by

$$SDF_{t,T}\left(s_t, s_T\right) = e^{-\rho(T-t)}\frac{u'\left(C_T\right)}{u'\left(C_t\right)}\frac{\phi'\left[V_T\left(s_T\right)\right]}{\phi' \circ \phi^{-1}\left\{\mathbb{E}\left[\phi \circ V_T\left(s_T\right)\middle| s_t\right]\right\}}. \tag{6}$$

To provide a sharp theoretical result, we consider the continuous time limit as $\Delta \to 0$. We denote $s^+ = \lim_{\Delta\to 0} s_\Delta$. We denote the stochastic discount factors as $SDF^+\left(0^-, s^+\right) = \lim_{\Delta\to 0} SDF_{0,\Delta}\left(0, s_\Delta\right)$ and $SDF^{++}\left(s^+, \theta_T\right) = \lim_{\Delta\to 0} SDF_{\Delta,2\Delta}\left(s_\Delta, \theta_T\right)$. We compare the risk premium in the vague announcement and informative announcement economies by comparing the entropy of the stochastic discount factors. We define the entropy of a stochastic discount factor as $\mathcal{H}(SDF) = -\mathbb{E}[\ln SDF]$. The term $\mathcal{H}(SDF)$ can be used to compare the variability of stochastic discount factors and expected returns because, as shown in Bansal and Lehmann (1997) and Backus, Chernov, and Zin (2014), no arbitrage implies that

Table 2: Information Structure

Panel A: Vague announcement			
Time	0	t	T
Info	0	$s_t = \eta$	θ_T
Utility	$V_0 = \left(1 - e^{-\rho\Delta}\right) u\left(C_0\right) + e^{-\rho\Delta}\phi^{-1}\left\{\mathbb{E}\left[\phi \circ V_t(\eta)\right]\right\}$	$V_t(\eta) = \left(1 - e^{-\rho\Delta}\right) u\left(C_t\right) + e^{-\rho\Delta}\phi^{-1}\left\{\mathbb{E}\left[\phi \circ V_T(\theta_T) \middle\| \eta\right]\right\}$	$V_T\left(\theta_T\right)$
SDF		$e^{-\rho t}\frac{u'(C_t)}{u'(C_0)}\frac{\phi'[V_t(\eta)]}{\phi'\circ\phi^{-1}\{\mathbb{E}[\phi\circ V_t(\eta)]\}}$	$e^{-\rho(T-t)}\frac{u'(C_T)}{u'(C_t)}\frac{\phi'[V_T(\theta_T)]}{\phi'\circ\phi^{-1}\{\mathbb{E}[\phi\circ V_T(\theta_T)\|\eta]\}}$
Panel B: Informative announcement			
Time	0	t	T
Info	0	$s_t = (\eta, \varepsilon)$	θ_T
Utility	$V_0 = \left(1 - e^{-\rho\Delta}\right) u\left(C_0\right) + e^{-\rho\Delta}\phi^{-1}\left\{\mathbb{E}\left[\phi \circ V_t(\eta, \varepsilon)\right]\right\}$	$V_t(\eta, \varepsilon) = \left(1 - e^{-\rho\Delta}\right) u\left(C_t\right) + e^{-\rho\Delta}\phi^{-1}\left\{\mathbb{E}\left[\phi \circ V_T(\theta_T) \mid \eta, \varepsilon\right]\right\}$	$V_T\left(\theta_T\right)$
SDF		$e^{-\rho t}\frac{u'(C_t)}{u'(C_0)}\frac{\phi'[V_t(\eta,\varepsilon)]}{\phi'\circ\phi^{-1}\{\mathbb{E}[\phi\circ V_t(\eta,\varepsilon)]\}}$	$e^{-\rho(T-t)}\frac{u'(C_T)}{u'(C_t)}\frac{\phi'[V_T(\theta_T)]}{\phi'\circ\phi^{-1}\{\mathbb{E}[\phi\circ V_T(\theta_T)\|\eta,\varepsilon]\}}$

This table illustrates the timing, information set, continuation utility, and SDF in the vague announcement economy (panel A) and informative announcement economy (panel B), respectively.

$\mathcal{H}\left(SDF\right) \geq \mathbb{E}\left[\ln R\right]$ for any risky return R, and the equality can be achieved by the growth optimal portfolio if markets are complete.

To establish our main result, we need a condition that is slightly stronger than GRS. The theorem of generalized risk sensitivity in Ai and Bansal (2018) demonstrates that the announcement premium is positive if and only if the certainty-equivalent functional $\mathcal{I}$ satisfies generalized risk sensitivity. Under the representation $\mathcal{I}\left[V\right] = \phi^{-1} \circ \mathbb{E}\left[\phi\left(V\right)\right]$, GRS is equivalent to the concavity of ϕ.[3] Here, we are interested in the conditions under which more informative announcements are associated with a higher announcement premium, which require a concept we will call strong GRS.

Definition 1. *(Strong GRS)*
An intertemporal preference represented by the aggregator $\{u, \mathcal{I}\}$, where $\mathcal{I}$ has the representation $\mathcal{I}\left[V\right] = \phi^{-1} \circ \mathbb{E}\left[\phi\left(V\right)\right]$ and ϕ is a strictly increasing and three-times continuously

[3]See Observation (ii) in Section 4.3 under the Subsection "Generalized Risk Sensitivity and Uncertainty Aversion" in Ai and Bansal (2018).

differentiable function, is said to satisfy strong generalized risk sensitivity if ϕ satisfies

$$-\frac{\phi''}{\phi'} \geq \sqrt{\frac{1}{2}\frac{\max\{\phi''', 0\}}{\phi'}}. \tag{7}$$

The above condition has an intuitive interpretation. Note that $-\frac{\phi''}{\phi'}$ is the Arrow-Pratt measure of the absolute risk aversion of ϕ. If $\phi''' \leq 0$, then the above condition simply requires ϕ to be concave and is equivalent to GRS. In most applications, $\phi''' > 0$; for example, the constant elasticity of substitution (CES) function $\phi(x) = \frac{1}{1-\gamma}x^{1-\gamma}$. The above condition then imposes a lower bound on the Arrow-Pratt measure of absolute risk aversion for ϕ. In this sense, this condition is stronger than the concavity of ϕ, or equivalently, the requirement of GRS. It is easy to show that the CES example satisfies condition (7) if and only if $\gamma > 1$, and the negative exponential example, $\phi(x) = -e^{-\alpha x}$, satisfies the above condition for all $\alpha > 0$.

The following proposition summarizes our main result on information-driven volatility and the risk premium. We provide details of the proof in Appendix 7.1.

Proposition 1. *(Information-driven volatility and risk premium)*
Suppose $\{u, \mathcal{I}\}$ satisfy strong generalized risk sensitivity. The entropy of the announcement SDF is higher in the informative announcement economy relative to the vague announcement economy; that is,

$$\mathcal{H}\left[SDF^{+}\left(T^{-}, (\eta, \varepsilon)\right)\right] \geq \mathcal{H}\left[SDF^{+}\left(T^{-}, \eta\right)\right], \tag{8}$$

but the entropy of the post-announcement SDF is lower:

$$\mathcal{H}\left[SDF^{++}\left((\eta, \varepsilon), \theta_T\right)\right] \leq \mathcal{H}\left[SDF^{++}\left(\eta, \theta_T\right)\right]. \tag{9}$$

Under additively separable expected utility, (8) and (9) hold with equality. That is, the informativeness of announcements does not affect the announcement and post-announcement SDFs.

The above proposition formalizes the idea that the announcement SDF is more volatile in the informative announcement economy relative to the vague announcement economy. At the same time, the post-announcement SDF is less volatile compared to the vague announcement economy. In other words, more informative announcements are associated with a higher risk premium upon the announcement but a lower risk premium afterward.

In summary, the basic mechanism of our theory of information-driven volatility is the relationship between the informativeness of announcements and volatility and expected returns. When the announcement at time 0^{+} is more informative, it triggers a stronger stock market response upon the announcement and therefore a higher realized volatility. Under strong GRS, this high realized volatility is also associated with a high average return. However, because the informative announcement resolves more uncertainty, the stock market volatility going forward is lower and the expected returns going forward are lower. Strong GRS is important for expected return dynamics because under the expected utility, the informativeness of announcements does not affect the risk premium. In the next section, we build on the above intuition to develop a quantitative model to study the joint dynamics of expected returns and stock market volatility.

4 A Dynamic Model

In this section, we develop a dynamic model with time-varying informativeness of announcements. Our model provides a quantitative benchmark for us to replicate the empirical tests for the information-driven volatility channel that we develop in the next section, and to assess the relevance of this mechanism in affecting stock market return predictability. To provide a realistic model for stock market volatility dynamics, our setup incorporates both low-frequency variations in fundamental volatility and high-frequency movements in information-driven volatility. The presence of fundamental volatility allows our model to capture the well documented positive autocorrelation of realized volatility of stock returns in the data. The presence of information-driven volatility allows our model to explain the puzzling relationship between realized volatility and expected returns in the data and to highlight the important distinction between fundamental-driven volatility and information-driven volatility. We provide details of the model solutions and derivations in Appendix 7.2.

Preferences and endowment We consider an endowment economy where the representative agent has a Duffie and Epstein (1992a) recursive preference with time discount rate ρ, constant risk aversion γ, and constant intertemporal elasticity of substitution (IES) ψ. This preference has the representation $\mathcal{I}[V] = \phi^{-1} \circ \mathbb{E}[\phi(V)]$ with $\phi(x) = \frac{1}{1-\gamma}\left[\left(1-\frac{1}{\psi}\right)x\right]^{\frac{1-\gamma}{1-1/\psi}}$.[4] It is straightforward to show that it satisfies strong GRS if and only if $\gamma > 1 > \frac{1}{\psi}$. This parameterization coincides with standard long-run risk calibrations and will be assumed throughout this section.

[4] See Section S.2 in "Supplement to Risk Preferences and the Macroeconomic Announcement Premium" in Ai and Bansal (2018).

We assume that aggregate endowment follows a diffusion process of the form

$$\frac{dY_t}{Y_t} = \theta_t dt + \sigma_Y dB_{Y,t}, \tag{10}$$

where $B_{Y,t}$ is a standard Brownian motion and $\{\theta_t\}_{t\geq 0}$ is a two-state Markov process with the state space $\Theta = \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L$. The transition probability for θ_t over an infinitesimal interval Δ is given by

$$\begin{bmatrix} e^{-\lambda_H \Delta} & 1 - e^{-\lambda_H \Delta} \\ 1 - e^{-\lambda_L \Delta} & e^{-\lambda_L \Delta} \end{bmatrix}, \tag{11}$$

where the intensity λ_H is the rate of transition from a high state to low state, and λ_L is the transition probability from low to high.

Information quality and announcements We assume that the latent state variable θ_t is unobservable to investors. However, information about θ_t continuously arrives at financial markets. Investors observe two sources of information about θ_t. First, the aggregate endowment (10) itself contains information about θ_t. Second, pre-scheduled macroeconomic announcements are made at time $T, 2T, \ldots, nT$ for $n = 1, 2, \ldots$.[5] We assume that the announcement at time nT carries a noisy signal s_n about θ_{nT}, which reveals the true value of θ_t with probability ν_n. The distribution of the signal is given as follows: if $\theta_{nT} = \theta_H$, that is, the true state is the high growth state,

$$\begin{array}{lll} s_n = \theta_H & \text{with prob.} & \nu_n \\ s_n = \theta_L & \text{with prob.} & 1 - \nu_n \end{array}, \tag{12}$$

and if $\theta = \theta_L$, that is, the true state is the low growth state,

$$\begin{array}{lll} s_n = \theta_H & \text{with prob.} & 1 - \nu_n \\ s_n = \theta_L & \text{with prob.} & \nu_n \end{array}. \tag{13}$$

Here $\nu_n \in \left[\frac{1}{2}, 1\right]$ is the key parameter in our model that measures the *information quality*, or the time-varying *informativeness* of announcements. When $\nu_n = 1$, announcements carry perfectly accurate information because the signal reveals the true state with probability one. And $\nu_n = 0.5$ indicates that announcements are completely uninformative. Namely,

[5]Our benchmark model matches the quantitative results based on FOMC announcements, which are pre-scheduled and periodically announced. However, our information-volatility theory does not rely on the assumption of fixed announcements. Instead, we show that if we allow announcements to be an unexpected Poisson process, our results still hold.

the announcement discloses the true state with half probability, whereas with another half probability, the information is entirely wrong. For simplicity, we assume that $\nu_1, \nu_2, \ldots, \nu_n$ are i.i.d. over time.

Because θ_t is not observable, equilibrium prices and quantities are functions of investors' posterior beliefs about θ_t. Thanks to the assumption of a two-state Markov chain, the posterior distribution can be fully summarized by the posterior probability of $\theta_t = \theta_H$, $\pi_t = P_t(\theta_t = \theta_H)$. Given π_t, we define $\hat{\theta}_t = \mathbb{E}_t[\theta_t]$ to be the posterior mean of θ_t (i.e., $\hat{\theta}_t = \pi_t\theta_H + (1-\pi_t)\theta_L$). Because π_t and $\hat{\theta}_t$ have a one-to-one relationship, we find it more convenient to use $\hat{\theta}_t$ as the state variable that summarizes the posterior distribution. In the interior of $((n-1)T, nT)$ without announcements, investors update their beliefs based on the observed consumption process (10). The posterior mean of θ_t satisfies

$$d\hat{\theta}_t = (\lambda_H + \lambda_L)\left(\bar{\theta} - \hat{\theta}_t\right)dt + \left(\theta_H - \hat{\theta}_t\right)\left(\hat{\theta}_t - \theta_L\right)\frac{1}{\sigma_Y}d\hat{B}_{Y,t}, \tag{14}$$

where $\bar{\theta} = \frac{\lambda_L\theta_H + \lambda_H\theta_L}{\lambda_L + \lambda_H}$ is the steady-state mean of θ_t, and $\hat{B}_{Y,t}$ is the innovation process defined by $d\hat{B}_{Y,t} = \frac{1}{\sigma_Y}\left(\frac{dY_t}{Y_t} - \hat{\theta}_t dt\right)$, which indicates the surprises from the difference between observed and expected consumption growth.

Right after observing the announcement s_n at nT, $n = 1, 2, \ldots$, investors update their beliefs using Bayes' rule. In our model, equilibrium prices are functions of two Markov state variables $\left(\hat{\theta}_t, t \mod T\right)$, where $\hat{\theta}_t$ is investors' posterior mean for θ_t, and $t \mod T$ — the remainder when time t is divided by the length of the announcement cycle T — is the number of time periods since the last announcement. To simplify notation, we focus on one representative announcement cycle, $[0, T]$. We use T for the announcement time and T^- and T^+ for the instant before and after the announcement, respectively. In this paper, 0 should be understood as right after the announcement T^+, and T should be understood as right before the announcement T^-. This convention allows us to use $\left(\hat{\theta}_t, t\right)$ as the Markov state variables in the following sections.

In Appendix 7.2.1, we show that given the prior belief $\hat{\theta}_T^-$ before the announcement, the posterior distribution of $\hat{\theta}_T^+$ after the announcement follows

$$\hat{\theta}_T^+ = \begin{cases} \theta_L + \frac{\left(\hat{\theta}_T^- - \theta_L\right)\nu(\theta_H - \theta_L)}{\left(\hat{\theta}_T^- - \theta_L\right)\nu + \left(\theta_H - \hat{\theta}_T^-\right)(1-\nu)} & \text{w.p.} \quad \frac{\hat{\theta}_T^- - \theta_L}{\theta_H - \theta_L}\nu + \frac{\theta_H - \hat{\theta}_T^-}{\theta_H - \theta_L}(1-\nu) \\ \theta_L + \frac{\left(\hat{\theta}_T^- - \theta_L\right)(1-\nu)(\theta_H - \theta_L)}{\left(\hat{\theta}_T^- - \theta_L\right)(1-\nu) + \left(\theta_H - \hat{\theta}_T^-\right)\nu} & \text{w.p.} \quad \frac{\hat{\theta}_T^- - \theta_L}{\theta_H - \theta_L}(1-\nu) + \frac{\theta_H - \hat{\theta}_T^-}{\theta_H - \theta_L}\nu \end{cases}. \tag{15}$$

The informativeness ν not only influences the posterior mean of investors' beliefs about the latent growth rate but also governs the probability of observing an accurate signal about the underlying state. For example, it is straightforward to verify that if $\nu = 1$, the signal is perfectly informative, and the above becomes $\hat{\theta}^{+}=\begin{cases}\theta_H & \text{w.p. } \frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}\\ \theta_L & \text{w.p. } \frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}\end{cases}$. And if $\nu=0.5$, the signal is absolutely uninformative and $\hat{\theta}^{+}=\hat{\theta}^{-}$ with probability one.

Asset prices We assume that the aggregate stock market is the claim to the following dividend process:

$$\frac{dD_t}{D_t}=\left[\xi\left(\hat{\theta}_t-\bar{\theta}\right)+\bar{\theta}\right]dt+\sigma_Y d\hat{B}_{Y,t}+\omega_t dB_{\omega,t}, \tag{16}$$

where ξ is the leverage on expected consumption growth, and $dB_{\omega,t}$ is a Brownian motion uncorrelated with the consumption shock $dB_{Y,t}$ or θ_t. Here, we allow the idiosyncratic volatility ω_t in the dividend growth rate to be time varying and assume that it follows a two-state Markov chain with state space $\{\omega_H,\omega_L\}$, where $\omega_H>\omega_L$. The transition matrix over a small interval Δ is $\begin{bmatrix} e^{-\kappa_H\Delta} & 1-e^{-\kappa_H\Delta} \\ 1-e^{-\kappa_L\Delta} & e^{-\kappa_L\Delta}\end{bmatrix}$, where κ_H (κ_L) denotes the rate of transition from a high (low) to low (high) state of dividend idiosyncratic volatility. This specification allows our model to capture the low-frequency movements in fundamental volatility and generate a positive autocorrelation in realized stock market volatility, which is an important feature of the data.

The lifetime utility of the representative agent can be written as a function of state variables: $V\left(\hat{\theta}_t,t,Y_t\right)=\frac{1}{1-\gamma}H(\hat{\theta}_t,t)Y_t^{1-\gamma}$. As a result, changes in beliefs about θ_t are immediately reflected through the variations in the continuation utility $H(\hat{\theta}_t,t)$. We show the solutions of the value function in Lemma 1 in Appendix 7.2.1. Given the value function, we can construct the pricing kernel M_t. The law of motion of M_t in the interior of $(0,T)$ without announcements can be written as

$$\frac{dM_t}{M_t} = -r\left(\hat{\theta}_t,t\right)dt-\sigma_M\left(\hat{\theta}_t,t\right)d\hat{B}_{Y,t}, \tag{17}$$

where the risk-free rate $r(\hat{\theta}_t,t)$ and the market price of risk $\sigma_M(\hat{\theta}_t,t)$ are given in Equations (68) and (69) in Appendix 7.2.2.

Denote $p(\hat{\theta}_t,t)$ as the price-to-dividend ratio so that the stock price is given by $p(\hat{\theta}_t,t)D_t$. By definition, the stock price is the discounted future cash flow:

$$p\left(\hat{\theta}_t,t\right)D_t=\mathbb{E}\left[\int_0^\infty \frac{M_{t+s}}{M_t}D_{t+s}ds \mid \hat{\theta}_t,t\right]. \tag{18}$$

We provide the expression for the partial differential equations (PDE) together with the boundary conditions that determine the solution of $p(\hat{\theta}_t,t)$ in Appendix 7.2.2. With the pricing kernel and the price-to-dividend ratio, the market risk premium is given by the following proposition.

Proposition 2. *(Equity premium)*
In the interior of $(0,T)$, the instantaneous risk premium is given by

$$\mathbb{E}_t\left[\frac{d\left[p\left(\hat{\theta}_t,t\right)D_t\right]+D_t dt}{p\left(\hat{\theta}_t,t\right)D_t}\right]-r\left(\hat{\theta}_t,t\right)dt=\sigma_M\left(\hat{\theta}_t,t\right)\left(\frac{p_\theta\left(\hat{\theta}_t,t\right)}{p\left(\hat{\theta}_t,t\right)}\frac{\sigma_\theta\left(\hat{\theta}_t\right)}{\sigma_Y}+\sigma_Y\right), \quad (19)$$

where $\sigma_\theta\left(\hat{\theta}_t\right)=\left(\theta_H-\hat{\theta}_t\right)\left(\hat{\theta}_t-\theta_L\right)\frac{1}{\sigma_Y}$ and $\frac{p_\theta}{p}=\frac{\partial p(\hat{\theta}_t,t)/\partial\hat{\theta}_t}{p(\hat{\theta}_t,t)}$. At announcement T, the announcement premium is given by

$$\mathbb{E}_{T^-}\left[\frac{p\left(\hat{\theta}_T^+,T^+\right)}{p\left(\hat{\theta}_T^-,T^-\right)}\right]-1=\frac{\left(\mathbb{E}_{T^-}\left[H\left(\hat{\theta}_T^+,T^+\right)\right]\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\mathbb{E}_{T^-}\left[p\left(\hat{\theta}_T^+,T^+\right)\right]}{\mathbb{E}_{T^-}\left[H\left(\hat{\theta}_T^+,T^+\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}p\left(\hat{\theta}_T^+,T^+\right)\right]}-1. \quad (20)$$

where $\hat{\theta}_T^+$ is drawn from the distribution in Equation (15).

In Appendix 7.2.2, we show that $\frac{H(\hat{\theta}_T^+,T^+)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}{\left\{\mathbb{E}_{T^-}\left[H(\hat{\theta}_T^+,T^+)\right]\right\}^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}$ is the announcement stochastic discount factor. Note that under the parameter restriction $\gamma>1$, the value function $H(\hat{\theta}_t,t)$ is decreasing in $\hat{\theta}_t$ while the price-to-dividend ratio $p(\hat{\theta}_t,t)$ is an increasing function of $\hat{\theta}_t$. Under the assumption $\gamma>1/\psi$, the preference satisfies GRS, the term $H(\hat{\theta}_T^+,T^+)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}$ will be negatively correlated with $p(\hat{\theta}_T^+,T^+)$. As a result, $\text{Cov}\left[H(\hat{\theta}_T^+,T^+)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}},p(\hat{\theta}_T^+,T^+)\right]<0$ and the announcement requires positive risk compensation. This gives rise to the macroeconomic announcement premium we observe in the data (Savor and Wilson, 2013). Mathematically, upon announcements, investors' beliefs jump from $\hat{\theta}_T^-$ to $\hat{\theta}_T^+$ according to Equation (15). The value function and price-to-dividend ratio are simultaneously affected as they are all functions of $\hat{\theta}_t$. Intuitively, investors require compensation for risk because the arrival of information over a short time interval creates significant variations in the continuation utility. The risk premium realizes upon the resolution of uncertainty. If $\gamma=1/\psi$, investors' utility function becomes an additively separable CRRA utility. The announcement stochastic discount factor equals one; therefore, the information contained in the announcement will not be priced.

Finally, we define the implied variance of the stock return at time t with maturity $\tau-t$ as the expected variance of the log return from t to τ, where the expiration date $\tau\geq t$ and

$h = \tau - t$ is the day to maturity:

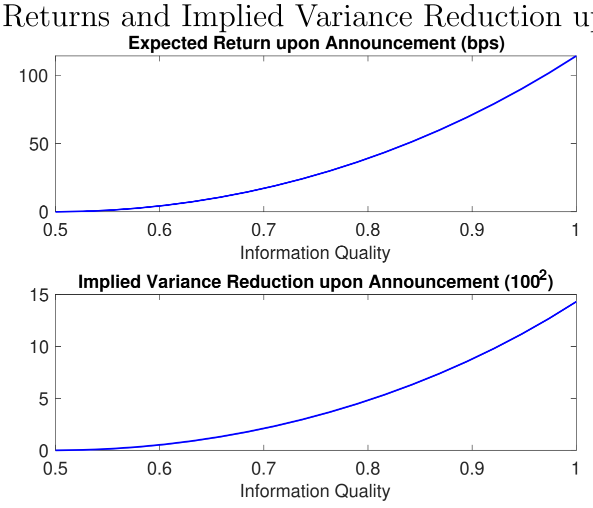
$$IV_{t,\tau}\left(\hat{\theta}_t, t\right) = \text{Var}_t\left[\ln\left[p\left(\hat{\theta}_\tau, \tau\right) D_\tau\right] - \ln\left[p\left(\hat{\theta}_t, t\right) D_t\right]\right]. \tag{21}$$

Because the log price $\ln\left[p\left(\hat{\theta}_t, t\right) D_t\right]$ is known at time t, we can simply write $IV_{t,\tau}\left(\hat{\theta}_t, t\right) = \text{Var}_t\left[\ln\left[p\left(\hat{\theta}_\tau, \tau\right) D_\tau\right]\right]$. We present solutions to our model-implied variance in Lemmas 4 and 5 in Appendix 7.2.3.

Comparative statics with respect to informativeness of announcements In this section, we use the policy functions from the above model to illustrate the two main theses of our theory of information-driven volatility, as highlighted in Section 3. We provide the formulas for the policy functions in Appendix 7.2.4.

Our parameterization satisfies $\gamma > 1 > 1/\psi$, which implies strong GRS. This condition guarantees that the announcement premium in (20) is not only positive but also increasing in the informativeness of announcements, as we demonstrated in Proposition 1. In Figure 2, we plot the announcement premium in the top panel and the implied variance reduction in the bottom panel as functions of the information quality ν. Instead of plotting the variance of announcement return, we plot the implied variance reduction as the latter is a market-price-based measure of the variance of announcement return, which we also use in our empirical exercises.

Figure 2: Expected Returns and Implied Variance Reduction upon Announcements



The top panel is the expected announcement return as a function of the information quality ν, where expected returns are measured in basis points. The bottom panel is the option-implied variance reduction upon announcements (in percentage squared) implied by our model as a function of informativeness ν.

The implied variance reduction in Figure 2 is defined as follows. Using Equation (21), the implied variance at time T^+ with expiration τ right after an announcement is $IV^+_{T,\tau}\left(\hat{\theta}^+_T, T^+\right) =$

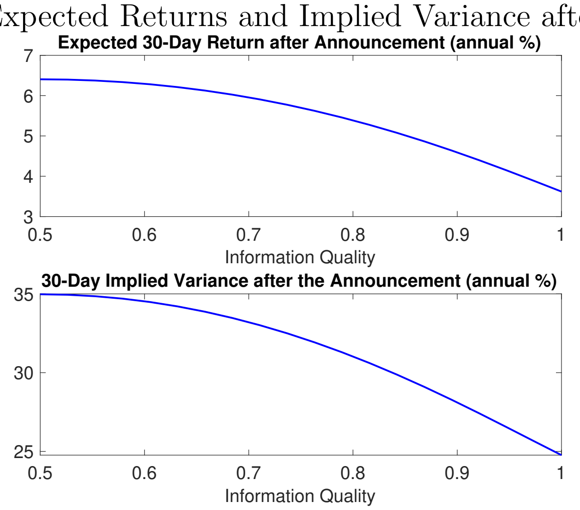
$\mathrm{Var}\left[\ln\left[p\left(\hat{\theta}_\tau,\tau\right)D_\tau\right]\middle|\hat{\theta}_T^+\right]$, and the implied variance before the announcement and after the value of ν is revealed is $IV_{T,\tau}^-\left(\hat{\theta}_T^-,T^-,\nu\right) = \mathrm{Var}\left[\ln\left[p\left(\hat{\theta}_\tau,\tau\right)D_\tau\right]\middle|\hat{\theta}_T^-,\nu\right]$. We define the implied variance reduction upon the announcement as the difference between the implied variance before and after the announcement,

$$\Delta IV_T = IV_{T,\tau}^-\left(\hat{\theta}_T^-,T^-,\nu\right) - IV_{T,\tau}^+\left(\hat{\theta}_T^+,T^+\right). \tag{22}$$

In Appendix 7.2.3, we show that ΔIV_T can be used to compute the variance of the announcement return: $\mathbb{E}\left[\Delta IV_T|\nu,\hat{\theta}_T^-,T^-\right] = \mathrm{Var}\left(\ln p\left(\hat{\theta}_T^+,T^+\right) - \ln p\left(\nu,\hat{\theta}_T^-,T^-\right)|\nu,\hat{\theta}_T^-,T^-\right)$.

As shown in Figure 2, at $\nu = 0.5$, the announcement is completely uninformative. The expected announcement return and the implied variance reduction upon the announcement are both zero. As ν increases from 0.5 to 1, the expected return and the implied variance reduction rise monotonically. Consistent with the intuition in Proposition 1, the expected return and expected variance on announcement days are both increasing functions of the informativeness of the announcement. Because of the monotonic relationship between the informativeness ν and the implied variance reduction ΔIV_T, the latter provides a market-price-based measure of the informativeness of announcements, which we use to exploit in the empirical exercises in the following section.

Figure 3: Future Expected Returns and Implied Variance after Announcements



The top panel is the expected 30-day return (annual percentage) during the post-announcement period as a function of the information quality ν implied by our model. The bottom panel is the 30-day implied variance (annual percentage) after the announcement implied by our model.

The second implication of our theory of information-driven volatility is that more informative announcements are associated with a lower expected return and a lower volatility during the period after the announcement. In Figure 3, we plot the model-implied expected return during the 30-day period after the announcement (top panel), and the 30-day implied variance during the same period (bottom panel) as a function of the information quality ν. Clearly, as the informativeness of announcements increases, the expected return after the announcement in the future is reduced as is the expected variance of the market return during the same period. More informative announcements resolve a larger fraction of uncertainty about future consumption growth and are associated with lower expected returns and lower expected variance after the announcement going forward.

5 Quantitative Results

In this section, we first calibrate our model and demonstrate that it can quantitatively match a broad set of moments — especially announcement-related moments — of the aggregate market. We then develop empirical tests for the information-driven volatility channel and replicate these tests in our model. Finally, we show that our model provides an explanation for the variance risk premium return predictability without assuming high-frequency variations in the volatility of macroeconomic fundamentals.

Parameter values We choose a discount rate $\rho = 1.5\%$, a risk aversion $\gamma = 20$, an IES $\psi = 2$, and a leverage parameter $\xi = 3$, in line with the standard long-run risk literature. We set the volatility of consumption growth $\sigma_Y = 3\%$ to match the volatility of annual consumption growth in the U.S. in our sample period from 1929.01-2019.12. We calibrate the value of the two Markov states $\theta_H = 4.4\%$, $\theta_L = -1.7\%$ and the transition probabilities to match the mean, standard deviation, and autocorrelation of the aggregate consumption data. For simplicity, we assume $\lambda_H = \lambda_L$. We set the dividend volatility parameters ω_H and ω_L and the transition probabilities $\kappa_H = 0.06$ and $\kappa_L = 0.025$ by estimating a regime-switching model of dividend growth rates. We provide details for the data sources and estimation procedures in Appendix 7.3 and 7.4.

Most of our quantitative and empirical results target FOMC announcements in the data for two reasons. First, as documented by Lucca and Moench (2015), FOMC announcements require significant risk compensation. Second, compared to other macroeconomic announcements, the informativeness of FOMC announcements is more likely to vary over time because these announcements typically reflect the Federal Reserve's contingent response to macroeconomic conditions. In contrast, other macro announcements, such as the unemployment report or the publication of the producer price index, typically provide a given statistic of the macroeconomy, and their informativeness is likely to be relatively constant over time. In our calibration, the parameters ν_H and ν_L govern the informativeness of the announcements. We set $\nu_H = 0.999$ and $\nu_L = 0.60$ so that our model matches the mean and standard deviation

Table 3: Calibrated Parameters

Panel A. Preferences					
ρ	Time discount rate	0.015	γ	Relative risk aversion	20
ψ	IES	2			
Panel B. Consumption and dividend dynamics					
σ_Y	Endowment growth volatility	0.03	ξ	Leverage	3
θ_H	High endowment growth state	0.044	θ_L	Low endowment growth state	−0.017
λ_H	θ transition prob. (high to low)	0.8	λ_L	θ transition prob. (low to high)	0.8
ω_H	High dividend idiosyncratic vol.	0.14	ω_L	Low dividend idiosyncratic vol.	0.04
κ_H	ω transition prob. (high to low)	0.06	κ_L	ω transition prob. (low to high)	0.025
Panel C: Information					
ν_H	High informativeness of ann.	0.999	ν_L	Low informativeness of ann.	0.6
$\frac{1}{T}$	Frequency of announcements	8			

This table displays the calibrated annual parameters in our model.

of the implied variance reduction on FOMC announcement days. We choose $T = \frac{1}{8}$ so that there are eight announcements per year in our model, matching the frequency of FOMC announcements in the data. All calibrated parameters are listed in Table 3.

Basic statistics of model-implied unconditional moments We list the asset pricing moments in the data and the corresponding statistics for our calibrated model in Table 4. We simulate our continuous-time model at daily frequencies and aggregate it quantitively to appropriate frequencies to compare with the data.

Our model matches well with both the return and volatility moments in the data. The average equity market premium in the model is 5.69% per year, and the volatility of annual market return is 12%. Our model generates an average level of 0.42% for the risk-free rate, with a standard deviation of 0.92% per year. Both moments are close to their data counterparts in the data. Our model also captures the dynamics of consumption and dividend growth in the data. The first two moments and autocorrelation of consumption growth in the data (model) are 1.74% (1.33%), 2.72% (3.36%), and 0.38 (0.36). The model-based mean, volatility, and autocorrelation of dividend growth are 1.24%, 9.12%, and 0.35, which are fairly close to 1.55%, 10.66%, and 0.17 in the data. Our model produces a significant announcement premium. The average announcement-day return is 29 bps, and the average non-announcement day return is 1 bps, which are very close to the same moments (26 bps and 2 bps) reported in the data.

Several key moments in the data are particularly important in assessing the quantitative importance of the information-driven volatility channel. First, as we explain earlier, changes

Table 4: Asset Pricing Moments

Panel A: Aggregate market moments		Data	Model
$\mathbb{E}[R] - r_f$	Equity premium	7.46%	5.69%
$\text{Std}[R]$	Vol of market return	18.55%	12.03%
$\mathbb{E}[r_f]$	Average risk-free rate	0.26%	0.42%
$\text{Std}[r_f]$	Vol of risk-free rate	1.08%	0.92%
$\mathbb{E}[dY/Y]$	Consumption growth rate	1.74%	1.33%
$\text{Std}[dY/Y]$	Vol of consumption growth rate	2.72%	3.36%
$\text{AC}[dY/Y]$	AC(1) of consumption growth rate	0.38	0.36
$\mathbb{E}[dD/D]$	Dividend growth rate	1.55%	1.24%
$\text{Std}[dD/D]$	Vol of dividend growth rate	10.66%	9.12%
$\text{AC}[dD/D]$	AC(1) of dividend growth rate	0.17	0.35
$\text{Corr}(dY/Y, dD/D)$	Corr b.t. consumption and dividend	0.43	0.64
$\text{AC}[RV]$	AC(1) of RV	0.65	0.79
Panel B: Announcement moments		Data	Model
$\mathbb{E}\left[R^A\right]$	A-day average return	26 bps	29 bps
$\mathbb{E}\left[R^N\right]$	NA-day average return	2 bps	1 bps
$\mathbb{E}[\Delta IV_T]$	Av. IV reduction on A-days	2.14	4.18
$\text{Std}[\Delta IV_T]$	Std. of IV reduction on A-days	8.86	4.87

This table displays the asset pricing moments in the data and those implied by the model. The data in panel A include the period 1929.01-2019.12. The data in panel B include the period 1994.09-2019.12.

in fundamental-driven volatility and information-driven volatility both affect the dynamics of stock return volatility. Fundamental-driven volatility induces a positive autocorrelation of return volatility, and information-driven volatility induces a negative autocorrelation. The autocorrelation of stock market return volatility in the data therefore provides an upper bound on how strong the information-driven volatility channel can be: a large magnitude of information-driven volatility may result in a counterfactually negative auto-correlation of return volatility. Thanks to the stochastic volatility in dividend growth, our calibration matches the autocorrelation of stock market return volatility quite well. The first-order autocorrelation of realized volatility of annual stock market returns is 0.65 in the data, and the same moment is 0.79 in our model.

Second, our model successfully matches moments of the implied variance reduction on announcement days. As we remarked earlier, ΔIV_T provides a market-based measure of the informativeness of announcements. In Table 4, the implied variance reduction is significantly positive on announcement days and averages 2.14 (monthly percentage squared) in the data. Our model features a similar magnitude for the implied variance reduction and a standard deviation similar to that as in the data.

Table 5: Implied Variance Reduction

	Q5	Q25	Q50	Q75	Q95
Data	-5.54	-0.26	1.14	3.88	10.00
Model	-0.29	0.24	1.82	7.07	14.63

This table displays the quantile of the 30-day implied variance drop on announcement days in the data and in the model. The data include the period 1994.09-2019.12. Implied variance reductions are in monthly percentage squared units.

Although our calibration is fairly simple, it matches well a key feature of the implied variance reduction in the data: the implied variance typically drops right after announcements but occasionally *increases*. In Table 5, we tabulate the histogram of ΔIV_T in the data and in our model. Consistent with the data, our model shows a similar pattern of a positive reduction on average but an occasional increase in the fifth percentile. In our model, the implied variance is mainly determined by investors' posterior variance about the latent variable θ_t. Below, we illustrate the above pattern of the implied variance reduction using the variance decomposition identity.

First, an average positive reduction in the implied variance after announcements is a general property of Bayesian updating. Because $\mathrm{Var}\left[\mathbb{E}\left(\theta_T|s_t\right)\right] \geq 0$, Equation (1) implies that $\mathrm{Var}\left[\theta_T\right] \geq \mathbb{E}\left[\mathrm{Var}\left(\theta_T|s_t\right)\right]$. That is, *on average*, the conditional variance must decline after the arrival of information. Second, the possibility of increases in the implied variance after announcements requires a deviation from the Gaussian distribution. Under the Kalman filter with Gaussian distribution, $\mathrm{Var}\left(\theta_T|s_t\right)$ is deterministic and does not depend on the realization of s_t. In this case, we must have $\mathrm{Var}\left[\theta_T\right] \geq \mathrm{Var}\left(\theta_T|s_t\right)$; that is, the implied variance can only reduce after the announcement s_t. In general, however, $\mathrm{Var}\left[\theta_T\right] \geq \mathbb{E}\left[\mathrm{Var}\left(\theta_T|s_t\right)\right]$ allows the possibility of $\mathrm{Var}\left[\theta_T\right] < \mathrm{Var}\left(\theta_T|s_t\right)$ for some realizations of s_t. In our setup of a two-state Markov chain, signals are wrong with probability $1-\nu$ (see Equations (12) and (13)). A surprise signal that is very far from investors' prior belief about θ_t may trigger an increase in the posterior variance and therefore an increase in the implied variance of stock returns.

Testing the information-driven volatility channel This section conducts several statistical tests for the two main implications of our theory of information-driven volatility. We first present these tests using U.S. stock market return data and then replicate them with the simulated data from our model.

Our first set of tests are on the relationships between the informativeness of announcements and the mean and volatility of announcement returns as illustrated in Figure 2. The test requires a return predictability regression using some measure of the informativeness of FOMC announcements. We cannot use the implied variance reduction directly, as it requires knowledge of the implied variance after the announcement.[6] An ideal measure of informativeness should only use ex-ante information before announcements and be a good predictor for the implied variance reduction after announcements.

Our construction of the informativeness measure is based on the following intuition. Suppose the implied variance reflects realized return variances induced by the arrival of information, and suppose FOMC announcement days have more information arrivals relative to non-announcement days. Denote the variance of information on the upcoming announcement day as $Info$ and the variance of information on non-announcement days as $InfoN$. Before the announcement at time $T-1$, suppose we observe an implied variance with 9 days to maturity ($IV_{T-1,T+8}$) and another implied variance with 30 days to maturity ($IV_{T-1,T+29}$). The two market-implied variances then allow us to back out the two unknowns, $Info$ and $InfoN$:

$$IV_{T-1,T+8} = Info + 8 \times InfoN, \tag{23}$$

$$IV_{T-1,T+29} = Info + 29 \times InfoN. \tag{24}$$

Empirically, we make two modifications to the above simple construction. First, we find that normalizing the implied variance on the left-hand side of (23) and (24) by the realized variance with the same maturity yields a more effective measure of informativeness. This is intuitive because a high implied variance may result from either a period of heightened fundamental volatility or the anticipation of more informative events in the future. Normalizing by realized variance allows us to control for the fundamental volatility. Given the nature of the stochastic volatility in dividend growth, this correction also yields a better measure of informativeness in our model. Second, whenever we have more than two maturities for the implied variance, we use all available maturities by finding an $Info$ and $InfoN$ that minimize the mean squared error of equations similar to (23) and (24) constructed from all maturities between 7 and 180 calendar days.

In Table 6, we present the results of the following two regressions on the informativeness measure constructed above:

$$R_t^A = \alpha + \beta \times Info_{t-1} + \varepsilon_t, \tag{25}$$

[6] In fact, it is well-known that the implied variance and stock returns are strongly negatively correlated. This ex post negative correlation may simply be a result of the "leverage" effect and may have nothing to do with the relationship between informativeness and the announcement premium.

and

$$\Delta IV_t = \alpha + \beta \times Info_{t-1} + \varepsilon_t, \tag{26}$$

where R_t^A is the announcement-day return, earned from the beginning of the announcement day to the end of the announcement day; $\Delta IV_t = IV_{t-1,t+29} - IV_{t,t+30}$ is the difference between the 30-day option-implied variance (VIX index squared) on the day before the announcement and that on the announcement day; and $Info_{t-1}$ is the informativeness measure we construct using option prices on the day before the announcement at time $t-1$.

Table 6: Announcement return and IV reduction predictability by informativeness

		R_t^A	ΔIV_t
$Info_{t-1}$	Data	65.04	6.32
		(4.70)	(3.25)
R^2 (%)		15.44	22.20
ν_t	Model	22.22	3.98

This table presents the results of the return predictability regressions defined in (25) and the implied variance reduction predictability regression defined in (26). The data include the period 1996.01-2019.12. The bottom panel is the model-implied regression coefficients of the informativeness of announcements ν_t. Informativeness in both the data and the model, $Info_{t-1}$ and ν_t, is normalized by the mean and standard deviation. Returns are in daily basis points; implied variance reductions are in monthly percentage squared units. Newey-West t-statistics are in parentheses.

First, our measure of informativeness has strong predictive power for the implied variance reduction on announcement days. The coefficient is significantly positive with a t-statistic of 3.25 and an R^2 of 22.20%. This result indicates that $Info_{t-1}$ is an effective measure of the informativeness of the upcoming announcement perceived by the market.

Second, as shown in Table 6, in the return predictability regression, the regression coefficient on $Info_{t-1}$ is positive and significant with a t-statistic of 4.70. The above regression has an impressive R^2 of 15.44%, especially given that daily returns are notoriously hard to be predicted in the data. This evidence shows that more informative announcements are associated with a higher risk premium upon the announcement and provides an empirical support for Proposition 1.

In the model, we run exactly the same regressions as in the data. Our model has a precise measurement of informativeness: ν_t, which captures the information quality of the announcements. The announcement return and implied variance reduction are defined in Equations (20) and (22), respectively. Our model produces significantly positive regression coefficients of 22.22 and 3.98 in predicting the announcement returns and implied variance reduction upon the announcements, which are close to the counterparts in the data. These results are consistent with what we illustrated using policy functions in Figure 2 that higher informativeness is associated with higher expected returns and higher expected variance upon the arrival of information.

Our second set of tests are predictability regressions for post-announcement returns and the post-announcement realized variance by the informativeness of the announcement. Because we are interested in predicting the returns and realized variance *after* the announcement, we can directly use the announcement-day drops in the implied variance ΔIV_t instead of the ex-ante measure of $Info_{t-1}$ as the measure of informativeness. We consider the following regression specifications:

$$RV_{t,t+h} = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \varepsilon_{t,t+h} \tag{27}$$

for the realized variance predictability and

$$R_{t,t+h} - r_f = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \varepsilon_{t,t+h} \tag{28}$$

for the realized excess return predictability. In the above regressions, $RV_{t,t+h}$ and $R_{t,t+h}$ are the realized variance and realized returns, respectively, from announcement day t (not including the announcement day itself) to h days after the announcement, for various choices of h: $h = 1, 2, 3, 4, 5, 30, 60$ up to two months.[7] We control for $RV_{t-2,t-1}$, the realized variance on the day before announcement days, as a measure of the level of uncertainty before the announcement.[8] Our main interest is the regression coefficients on the measure of the informativeness of the announcement, ΔIV_t. We conduct the same regressions in the model and report our regression results in Table 7.[9]

Consistent with the policy functions we plot in Figure 3, in our model, more informative announcements are associated with a lower realized variance and lower expected returns after the announcements going forward. As a result, the betas in both regressions, (27) and (28), are negative in the model-simulated data. Consistent with the model, these regressions show a similar pattern in the data. The drops in the implied variance on announcement days negatively predict post-announcement day variances at horizons of 1-5 days. This pattern extends to the one-month horizon but dissipates over time and becomes insignificant over the two-month horizon. The announcement-day drop in implied variance can also negatively predict post-announcement day returns up to the two-month horizon. The decreasing patterns

[7]Note that h indicates the number of calendar days. In the data, we use $[1, 2, 3, 4, 5, 21, 42]$ trading days.

[8]In the Online Appendix, we show that our predictability results remain true if we also include the implied variance on the announcement day, IV_t, in regressions (27) and (28).

[9]We provide expressions for $R_{t,t+h}$ and $RV_{t,t+h}$ in our model in the Online Appendix.

Table 7: Model-Implied Return and Variance Predictability by IV Reduction

Number of days		1	2	3	4	5	30	60
$RV_{t,t+h}$	Data	-0.12	-0.08	-0.05	-0.05	-0.05	-0.07	-0.04
		(-3.83)	(-3.43)	(-2.41)	(-2.57)	(-2.70)	(-4.15)	(-2.54)
R^2 (%)		65.30	58.80	44.94	43.63	43.70	33.16	23.61
$RV_{t,t+h}$	Model	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.04
$R_{t,t+h} - r_f$	Data	-2.55	-2.02	-1.08	-1.26	-0.61	-0.19	-0.30
		(-1.03)	(-1.62)	(-1.92)	(-2.08)	(-1.61)	(-1.25)	(-2.03)
R^2 (%)		4.55	3.83	3.13	7.83	1.66	1.09	1.73
$R_{t,t+h} - r_f$	Model	-0.56	-0.48	-0.34	-0.25	-0.31	-0.16	-0.23

This table presents the results of the realized variance predictability regression (27) and the excess return predictability regression (28) in the data and the model. The columns 3-9 represent the horizon of returns and variances on the left-hand side of Equations (27) and (28), respectively, with $h = 1, 2, 3, 4, 5, 30, 60$ calendar days. The data include the period 1994.09-2019.12.We normalized the right-hand side variables in the model. Returns and realized variances are in daily basis points. Implied variance reductions are in monthly percentage squared units. Newey-West t-statistics are in parentheses.

of predictabilities over time further establish that information-driven volatility is crucial in explaining the expected return and variance, especially in the short run over higher frequencies. All of the above evidence confirms the basic mechanism of the information-driven volatility we illustrate in Figures 2 and 3.

Additional tests for the information-driven volatility channel In searching for major events with large public information releases about the economy, we identify informational days on which the S&P 500 Index daily returns (close to close) are above 2.5% or below -2.5%. According to Baker, Bloom, Davis, and Sammon (2021), all these informational days are well matched with at least one type of news.[10] In our 30-year sample, we have 295 informational days in total. We then classify a month as "informational" if at least one of the trading days within this month is an informational day. This procedure identifies 95 months as informational months and allows us to provide an additional test for the model implication that informational events are followed by lower future expected returns. We run the following regression:

$$R_{t,t+h} - r_f = \alpha + \beta RV_{t-h,t} + \varepsilon_{t,t+h}, \tag{29}$$

[10] In Baker, Bloom, Davis, and Sammon (2021), the categories for news causes are Macroeconomic News & Outlook, Corporate Earnings & Outlook, Sovereign Military & Security Actions, Government Spending, Commodities, and so on.

where $h = 30$ so that $RV_{t-h,t}$ is the realized variance in the current month and $R_{t,t+h}$ is the return in the future month.[11]

In the model, because the announcement is pre-scheduled eight times per year, the informational month is naturally the month with one announcement. We again run the same regressions as in the data and report the regression results in Table 8.

Table 8: Monthly Return Predictability by Monthly Realized Variance

		Full Sample	Informational Month t	The Rest
$R_{t,t+h} - r_f$	Data	-0.16	-0.25	-0.07
		(-2.11)	(-4.12)	(-0.22)
R^2 (%)		1.48	4.91	0.01
$R_{t,t+h} - r_f$	Model	-0.04	-0.03	0.05

This table presents the results of the monthly return predictability regression (29). The columns 1-3 represent the monthly returns (in percentages) and realized variances on the left-hand side of Equation (29) for different samples with $h = 30$ within a month. The data include the period 1990.01-2019.12. Returns are in annual percentage points and realized variances are in monthly percentage squared units. Newey-West t-statistics are in parentheses.

The first column in Table 8 uses all of the months available in our sample. We find a significantly negative relationship between the current month's realized variance and the next month's return. This finding is consistent with that of Nelson (1991), Glosten, Jagannathan, and Runkle (1993), and Moreira and Muir (2017). The second and the third column split our sample into two subsamples. The second column reports a version of regression (29) that uses only the realized variance in informational months to predict the returns in the next month. The negative relationship is much stronger and has a much higher degree of significance, with a t-statistic of -4.12 and an R^2 of 4.91%. The third column reports the same regression where realized variance is computed only in the subsample of non-informational months. The regression coefficient β is not statistically significant. The above results provide additional support for the information-driven volatility mechanism emphasized by our model. Because informational months are months with large information releases, they drive the negative relationship between past realized variance and future expected returns.

We report the same regression results using model-simulated data in the last row of Table 8. In our model, we interpret informational months as those with FOMC announcements. Our model exhibits a similar pattern as the data. The informational months are associated with a significant negative relationship between realized variance and future expected returns

[11] Unlike pre-scheduled FOMC announcements, these informational days are typically not anticipated by the market. We therefore cannot use these days to test the relationship between informativeness and the expected returns on the informational days.

whereas the non-informational months are not.

We next present a more rigorous test by repeating the realized variance predictability regression in Equation (27) and the return predictability regression in Equation (28) using the above identified informational days. In this analysis, the $RV_{t,t+h}$ and $R_{t,t+h}$ are the realized variance and realized returns, respectively, from informational day t (not including the informational day itself) to h days after the informational day, and ΔIV_t is the implied variance reduction on informational days. We report our regression results in Table 9.

Table 9: Expected Return and Variance Predictability by IV Reduction around Jump Days

Number of days	1	2	3	4	5	21	42
$RV_{t,t+h}$	−0.06	−0.06	−0.05	−0.05	−0.04	−0.03	−0.02
	(−3.59)	(−4.41)	(−5.05)	(−4.53)	(−4.20)	(−2.63)	(−2.35)
R^2 (%)	37.12	41.56	41.08	40.32	39.45	37.49	34.26
$R_{t,t+h} - r_f$	−1.05	−1.07	−0.46	−0.51	−0.58	−0.09	−0.00
	(−3.93)	(−6.09)	(−3.14)	(−3.62)	(−4.71)	(−1.69)	(−0.05)
R^2 (%)	3.88	7.20	2.19	3.62	6.07	3.27	4.65

This table presents the results of the realized variance predictability regression (27) and the return predictability regression (28) in the data. The columns 3-9 represent the horizon of returns (in percentages) and variances on the left-hand side of Equations (27) and (28), respectively, with $h = 1, 2, 3, 4, 5, 21, 42$ trading days. The data include the period 1990.01-2019.12. Returns and realized variances are in daily basis points. Implied variance reductions are in monthly percentage squared units. Newey-West t-statistics are in parentheses.

The predictability regression results in Table 9 again provide consistent support for our information-driven volatility mechanism. The drops in the implied variance on informational days significantly negatively predict post-informational day variances and returns at horizons of 1-5 days. Overall, our analysis on informational days echoes the analysis on FOMC announcement days. Through the information-driven volatility channel, the informational event days of higher informativeness usually predict lower risk as a result of the variance decomposition formula and a lower risk premium on subsequent days because investors' preferences satisfy strong GRS in our model.

Variance risk premium predictability In this section, we report our model's implications on variance risk premium (VRP) return predictability. Previous literature has documented robust empirical evidence on return predictability by the difference between the option-implied variance and the realized variance up to six-month horizons. The stochastic volatility models developed to address this empirical phenomenon (e.g., Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011)) typically relied on large high-frequency variations in the volatility of aggregate consumption.

Our model of information driven volatility provides an alternative explanation for the above facts without assuming high-frequency variations in consumption volatility. The information-driven volatility in our model creates a wedge between the implied and realized variance because the implied variance before the announcement includes investors' expectations about the upcoming announcements, whereas the past realized variance does not. Therefore, the difference between the implied and realized variance in our model immediately reflects the informativeness of the upcoming announcement. Because the implied variance is a forward-looking measure of variance, it increases when the upcoming announcement is expected to be informative. The difference between the implied and realized variance predicts returns because more informative announcements are associated with higher realizations of announcement premiums.

Table 10: Return Predictability by Variance Risk Premium

Number of calendar days		1	30	60	90	120	150	180
$IV_{t,t+30} - RV_{t-30,t}$	Data	30.21	4.14	5.64	4.27	5.71	3.60	2.93
		(2.68)	(1.44)	(3.42)	(2.83)	(3.05)	(2.68)	(2.35)
R^2 (%)		7.58	0.72	2.55	2.11	4.36	2.32	1.67
$IV_{t,t+30} - RV_{t-30,t}$	Model	12.02	1.77	1.06	0.65	0.51	0.43	0.38

This table shows the results of the return predictability regression (30) using U.S. stock market return data and those using data simulated from the model. Columns 2 to 7 represent returns on the left-hand side of (30) with $h = [1, 30, 60, 90, 120, 150, 180]$ calendar days ($[1, 21, 42, 63, 84, 105, 126]$ trading days in the data) including the upcoming announcement day. The regression includes returns and VRPs for every announcement day during the period 1994.09-2019.12. Newey-West t-statistics with 1-6 lags are in parentheses. We normalize the VRP in both the data and the model. Returns are in daily basis points for one-day-ahead forecasts, are in annualized percentages for 30 or more days, and VRPs are in monthly percentage squared units.

In Table 10, we report the results of the return predictability regression:

$$R_{t,t+h} - r_f = \alpha + \beta \left[IV_{t,t+30} - RV_{t-30,t} \right] + \varepsilon_{t,t+h}, \tag{30}$$

where $R_{t,t+h}$ is the cumulative market return from time t to time $t+h$. Here, t denotes one day before the announcement so that $R_{t,t+1}$ indicates the announcement-day return. We use $IV_{t,t+30}$ to denote the forward-looking 30-day implied variance (VIX index squared) at time t, and $RV_{t-30,t}$ is the past 30-day realized variance. The regression coefficients are statistically significant and increase up to six months. As in the data, in our model, returns are predictable by the differences between IV and RV. The regression coefficients of returns on VRP are significant up to the six-month horizon in our model as well. Note that the return predictability is strongest for the announcement-day return because the announcement premium immediately reflects the risk compensation required by the strong GRS through our information-driven volatility channel.

6 Conclusion

In this paper, we present a model of information-driven volatility. Traditional asset pricing models of stochastic volatility typically rely on high-frequency variations in the volatility of macroeconomic fundamentals, which lack strong empirical support. We believe that information is more likely to be responsible for high-frequency variations in financial market volatility than the volatility of macroeconomic fundamentals. We extend the theoretical analysis of Ai and Bansal (2018) and develop the concept of strong generalized risk sensitivity. We show that under this condition, the informativeness of macroeconomic news not only affects financial market volatility but also expected returns. Based on the above insights, we present a quantitative asset pricing model and demonstrate that the information-driven volatility channel can resolve several prominent volatility-related asset pricing puzzles, including the negative relationship between past realized volatility and future expected returns.

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7 Appendix

7.1 Information-Driven Volatility and Risk Premium

Proof. We denote $V^{+}(s^{+}) = \lim_{\Delta\to 0} V_{\Delta}(s_{\Delta})$, and $V^{++}(\theta_T) = \lim_{\Delta\to 0} V_{2\Delta}(\theta_T)$. Taking the continuous-time limit of the utility recursion (4), $V^{+}(s^{+})$ and $V^{++}(\theta_T)$ are related by:

$$V^{+}\left(s^{+}\right) = \phi^{-1}\left\{\mathbb{E}\left[\phi \circ V^{++}(\theta_T)\middle|\, s^{+}\right]\right\}. \tag{31}$$

Because C_t is a continuous function of time, we obtain the continuous-time limit of the SDF in (5) and (6) as:

$$SDF^{+}\left(0^{-}, s^{+}\right) = \frac{\phi'\left(V^{+}(s^{+})\right)}{\phi' \circ \phi^{-1}\left(\mathbb{E}\left[\phi \circ V^{+}(s^{+})\right]\right)}, \tag{32}$$

and

$$SDF^{++}\left(s^{+}, \theta_T\right) = \frac{\phi'\left\{V^{++}(\theta_T)\right\}}{\phi' \circ \phi^{-1}\mathbb{E}\left[\phi \circ V^{++}(\theta_T)\middle|\, s^{+}\right]}. \tag{33}$$

We first prove Eq. (8). To save notation, we denote $\Phi = \phi' \circ \phi^{-1}$ and use (32) to write the SDF in the informative announcement economy and that in the vague announcement economy as:

$$SDF^{+}\left(0^{-}, (\eta, \varepsilon)\right) = \frac{\Phi\left(\phi \circ V^{+}(\eta, \varepsilon)\right)}{\Phi\left(\mathbb{E}\left[\phi \circ V^{+}(\eta, \varepsilon)\right]\right)}; \quad SDF^{+}(0, \eta) = \frac{\Phi\left(\phi \circ V^{+}(\eta)\right)}{\Phi\left(\mathbb{E}\left[\phi \circ V^{+}(\eta)\right]\right)}. \tag{34}$$

Eq. (31) implies that $\phi \circ V^{+}(s^{+}) = \mathbb{E}\left[\phi \circ V^{++}(\theta_T)\middle|\, s^{+}\right]$. Because $V^{++}(\theta_T)$ are the same in the vague announcement economy and in the informative announcement economy, by law of iterated expectations, $\mathbb{E}\left[\phi \circ V^{+}(\eta)\right] = \mathbb{E}\left[\phi \circ V^{+}(\eta, \varepsilon)\right]$ and the denominators in (34) equal each other: $\Phi\left(\mathbb{E}\left[\phi \circ V^{+}(\eta, \varepsilon)\right]\right) = \Phi(\mathbb{E}\left[\phi \circ V^{+}(\eta)\right])$. To prove inequality (8), that is, $-\mathbb{E}\left[\ln SDF^{+}(0, (\eta, \varepsilon))\right] \geq -\mathbb{E}\left[\ln SDF^{+}(0, \eta)\right]$, we only need to show $-\mathbb{E}\left[\ln \Phi\left(\phi \circ V^{+}(\eta, \varepsilon)\right)\right] \geq -\mathbb{E}\left[\ln \Phi\left(\phi \circ V^{+}(\eta)\right)\right]$. To apply Jensen's inequality, it is enough to establish the concavity of $\ln \Phi$. Taking the first and the second order derivatives of $\ln \Phi$, we have:

$$\frac{\partial}{\partial x} \ln \Phi(x) = \frac{\partial}{\partial x} \ln \phi' \circ \phi^{-1}(x) = \frac{\phi''\left[\phi^{-1}(x)\right]}{\left\{\phi'\left[\phi^{-1}(x)\right]\right\}^{2}},$$

and

$$\frac{\partial^2}{\partial x^2} \ln \Phi(x) = \frac{1}{\left\{\phi'\left[\phi^{-1}(x)\right]\right\}^{2}} \left\{\phi'''\left[\phi^{-1}(x)\right] \phi'\left[\phi^{-1}(x)\right] - 2\left(\phi''\left[\phi^{-1}(x)\right]\right)^{2}\right\}.$$

It is straightforward to show that condition (7) is equivalent to $\frac{d}{dx}\left[\frac{\phi''}{(\phi'(x))^2}\right] \leq 0$, which is equivalent to $\phi'''\phi' - 2(\phi'')^2 \leq 0$. This establish the concavity of $\ln \Phi$.

We next prove Eq. (9). Given the utility recursion $\phi \circ V^+(s^+) = \mathbb{E}\left[\phi \circ V^{++}(\theta_T) \middle| s^+\right]$, it convenient to the post-announcement SDF as:

$$SDF^{++}((\eta, \varepsilon), \theta_T) = \frac{\phi'\{V^{++}(\theta_T)\}}{\Phi[\phi \circ V^+(\eta, \varepsilon)]}, \quad , SDF^{++}(\eta, \theta_T) = \frac{\phi'\{V^{++}(\theta_T)\}}{\Phi[\phi \circ V^+(\eta)]}. \tag{35}$$

Because $V^{++}(\theta_T)$ are the same in the vague announcement economy and the informative announcement economy, it is enough to show

$$\mathbb{E}\left[\ln \Phi\left\{\phi \circ V^+(\eta, \varepsilon)\right\}\right] \leq \mathbb{E}\left[\ln \Phi\left\{\phi \circ V^+(\eta)\right\}\right]. \tag{36}$$

Given the concavity of $\ln \Phi$, Jensen's equality implies that for all η,

$$\mathbb{E}\left[\ln \Phi\left\{\phi \circ V^+(\eta, \varepsilon)\right\} \middle| \eta\right] \leq \ln \Phi\left\{\mathbb{E}\left[\phi \circ V^+(\eta, \varepsilon) \middle| \eta\right]\right\} = \ln \Phi\left(\phi \circ V^+(\eta)\right), \tag{37}$$

where the second equality uses the utility recursion (31). Taking unconditional expectation on both sides of (37), we obtain (36). □

7.2 The Infinite-horizon Model Solutions

7.2.1 Preferences and beliefs

In this subsection, we start by deriving the posterior belief of a representative agent with two sources of information. We first solve for learning from the observable consumption using optimal filtering. Then we derive belief updating upon the announcements. Finally, we present solutions to the value functions and the associated boundary conditions at the announcement.

Learning in the interior: optimal filtering The two-state Markov chain process can be conveniently represented as an integration with respect to a Poisson process. In particular, let $\{N_{j,t}\}_{t \geq 0}$ be a Poisson process with intensity λ_j, for $j = H, L$. Let $I_{\{x\}}$ be an indicator function, that is,

$$I_{\{\theta_j\}}(\theta_t) = \begin{cases} 1 & \text{if } \theta_t = \theta_j \\ 0 & \text{if } \theta_t \neq \theta_j \end{cases}, \tag{38}$$

Then $\{\theta_t\}$ can be represented as the following compound Poisson process:

$$d\theta_t = (\theta_H - \theta_L)\, \eta\left(\theta_t^-\right) dN_t \tag{39}$$

and $\eta(\theta)$, and $N(t)$ are vectors:

$$\eta(\theta_t) = \left[-I_{\{\theta_H\}}(\theta_t), I_{\{\theta_L\}}(\theta_t)\right], \text{and } N(t) = \left[N_{H,t}, N_{L,t}\right]^{\top}. \tag{40}$$

Here we use the convention that $\{\theta_t\}$ is right-continuous with left limit, and use the notation $\theta_t^- = \lim_{s\to t, s<t} \theta_s$. And $N_{j,t}$ is the counting processes with intensity λ_j. That is,

$$dN_{j,t} = \begin{cases} 1 & \text{with prob. } \lambda_j dt \\ 0 & \text{with prob. } 1 - \lambda_j dt \end{cases}. \tag{41}$$

For example, if the true state is the high growth state $\theta_t^- = \theta_H$, and the transition from high state to low state happens, i.e., $dN_{H,t} = 1$. Then $\eta(\theta_t) = [-1, 1]$ gives rise to $d\theta_t = (\theta_H - \theta_L)\left(-I_{\{\theta_H\}}(\theta_t)\, dN_{H,t} + I_{\{\theta_L\}}(\theta_t)\, dN_{L,t}\right) = \theta_L - \theta_H$. Therefore, $\theta_t^+ = \theta_t^- + d\theta_t = \theta_L$.

Applying optimal filtering (see Chapter 9 of Liptser and Shiryaev (2001)), we could obtain

$$d\pi_t = \left[\lambda_L - (\lambda_H + \lambda_L)\pi_t\right] dt + \pi_t (1 - \pi_t)(\theta_H - \theta_L) \frac{1}{\sigma_Y} d\hat{B}_{Y,t}, \tag{42}$$

where $\hat{B}_{Y,t}$ is the innovation process defined in the main text. Note that the mapping between $\hat{\theta}_t$ and π_t is one-to-one, so we can equivalently use $\hat{\theta}_t$ as the state variable. By definition, $\hat{\theta}_t = \pi_t \theta_H + (1 - \pi_t)\theta_L$. This recovers π_t from $\hat{\theta}_t$

$$\pi_t = \frac{\hat{\theta}_t - \theta_L}{\theta_H - \theta_L}. \tag{43}$$

Applying Ito's lemma, we get Eq.(14) in the main text.

Learning upon the announcements At the announcement, investors observe a noisy signal s_n where the distribution is given as

$$\begin{aligned} P(s_H|\theta_H) &= \nu, \; P(s_L|\theta_H) = 1 - \nu \\ P(s_H|\theta_L) &= 1 - \nu, \; P(s_L|\theta_L) = \nu. \end{aligned}$$

Here we denote s_j as investors receive a signal of θ_j upon the announcement. Given the prior distribution $P^-(\theta_H) = \pi^-$, we need to compute the posterior distribution of π^+. Applying Bayes' rule,

$$P^+(\theta_i|s_j) = \frac{P(s_j|\theta_i) P^-(\theta_i)}{\sum_{\theta_i \in \Theta} P(s_j|\theta_i) P^-(\theta_i)}.$$

That is, given that $P^{-}(\theta_H)=\pi^{-}$,

$$P^{+}(\theta_H|s_H)=\frac{\pi^{-}\nu}{\pi^{-}\nu+(1-\pi^{-})(1-\nu)};\qquad P^{+}(\theta_L|s_H)=1-P^{+}(\theta_H|s_H)=\frac{(1-\pi^{-})(1-\nu)}{\pi^{-}\nu+(1-\pi^{-})(1-\nu)} \tag{44}$$

$$P^{+}(\theta_H|s_L)=\frac{\pi^{-}(1-\nu)}{\pi^{-}(1-\nu)+(1-\pi^{-})\nu};\qquad P^{+}(\theta_L|s_L)=1-P^{+}(\theta_H|s_L)=\frac{(1-\pi^{-})\nu}{\pi^{-}(1-\nu)+(1-\pi^{-})\nu} \tag{45}$$

This is equivalent to say, if we see s_H, then, $\pi^{+}=\frac{\pi^{-}\nu}{\pi^{-}\nu+(1-\pi^{-})(1-\nu)}$, and if we see s_L, $\pi^{+}=\frac{\pi^{-}(1-\nu)}{\pi^{-}(1-\nu)+(1-\pi^{-})\nu}$. So π^{+} only has two possible realizations: the probability of seeing s_H and the probability of seeing s_L. Denote h_{s_H} and h_{s_L} as the probability of seeing s_H and s_L *after* the announcement, respectively,

$$\begin{aligned} h_{s_H} &= P(s_H|\theta_H)P^{-}(\theta_H)+P(s_H|\theta_L)P^{-}(\theta_L)=\pi^{-}\nu+\left(1-\pi^{-}\right)(1-\nu) && (46)\\ h_{s_L} &= P(s_L|\theta_H)P^{-}(\theta_H)+P(s_L|\theta_L)P^{-}(\theta_L)=\pi^{-}(1-\nu)+\left(1-\pi^{-}\right)\nu. && (47)\end{aligned}$$

For notational convenience, we denote

$$\pi^{+}_{s_H}=\frac{\pi^{-}\nu}{\pi^{-}\nu+(1-\pi^{-})(1-\nu)}\text{ and }\pi^{+}_{s_L}=\frac{\pi^{-}(1-\nu)}{\pi^{-}(1-\nu)+(1-\pi^{-})\nu} \tag{48}$$

as the probability of $\theta^{+}=\theta_H$ *after* the announcement if we see s_H and s_L, respectively.

Note that our signal generates the "correct" result with probability ν and produces a "wrong" signal with probability $1-\nu$. That is to say, the conditional distribution of π^{+} is given as follows. If the true state is θ_H, then $\pi^{+}=\begin{cases}\pi^{+}_{s_H} & \text{w.p. } \nu\\ \pi^{+}_{s_L} & \text{w.p. } 1-\nu\end{cases}$, and if the true state is θ_L, $\pi^{+}=\begin{cases}\pi^{+}_{s_L} & \text{w.p. } \nu\\ \pi^{+}_{s_H} & \text{w.p. } 1-\nu\end{cases}$.

Since the mapping between $\hat{\theta}_t$ and π_t is one-to-one, we could instead use $\hat{\theta}$ as the state variable. Substituting Eq.(43) back, we can derive the probability of seeing s_H and s_L after the announcement respectively as

$$h_{s_H}=\frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}\nu+\frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}(1-\nu), \tag{49}$$

$$h_{s_L}=\frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}(1-\nu)+\frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}\nu. \tag{50}$$

Also, given $\pi^{+}=\left\{\pi^{+}_{s_H},\pi^{+}_{s_L}\right\}$, we can compute $\hat{\theta}^{+}=\theta_L+\pi^{+}(\theta_H-\theta_L)$ using Eq.(48). Therefore, with probability h_{s_H} and h_{s_L}, the posterior mean becomes

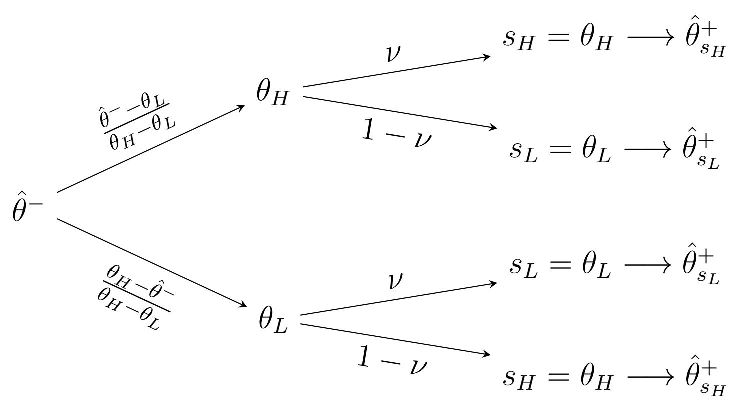
$$\hat{\theta}^+_{s_H} = \theta_L + \frac{\left(\hat{\theta}^- - \theta_L\right)\nu\left(\theta_H - \theta_L\right)}{\left(\hat{\theta}^- - \theta_L\right)\nu + \left(\theta_H - \hat{\theta}^-\right)(1-\nu)}, \tag{51}$$

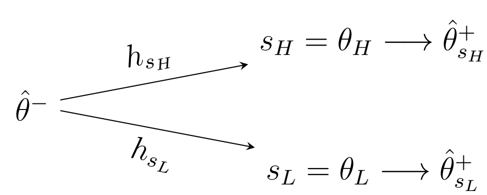
$$\hat{\theta}^+_{s_L} = \theta_L + \frac{\left(\hat{\theta}^- - \theta_L\right)(1-\nu)\left(\theta_H - \theta_L\right)}{\left(\hat{\theta}^- - \theta_L\right)(1-\nu) + \left(\theta_H - \hat{\theta}^-\right)\nu}. \tag{52}$$

Here $\hat{\theta}^+_{s_H}$ and $\hat{\theta}^+_{s_L}$ indicate the posterior mean of $\hat{\theta}^+$ after the announcement if we observe s_H and s_L, respectively.

In summary, the *conditional* distribution of $\hat{\theta}^+$ is: if the underlying state is θ_H, $\hat{\theta}^+ = \begin{cases} \hat{\theta}^+_{s_H} & \text{w.p. } \nu \\ \hat{\theta}^+_{s_L} & \text{w.p. } 1-\nu \end{cases}$. If the true state is θ_L, then $\hat{\theta}^+ = \begin{cases} \hat{\theta}^+_{s_L} & \text{w.p. } \nu \\ \hat{\theta}^+_{s_H} & \text{w.p. } 1-\nu \end{cases}$. The *unconditional* distribution of $\hat{\theta}^+$ after the announcement follows: $\hat{\theta}^+ = \begin{cases} \hat{\theta}^+_{s_H} & \text{w.p.} & h_{s_H} \\ \hat{\theta}^+_{s_L} & \text{w.p.} & h_{s_L} \end{cases}$.

To summarize the evolution of $\hat{\theta}$ (from prior mean $\hat{\theta}^-$to posterior mean $\hat{\theta}^+$) , we provide the following two graphs. In both graphs, we can observe the prior mean $\hat{\theta}^-$and the noisy signal s. The first graph highlights the intermediate step where we list the two cases of true unobservable state θ with the associated conditional distributions, while the equivalent second graph omits the intermediate step and uses unconditional distributions instead.





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As a matter of notation, for any function f, we denote

$$f\left(\nu,\hat{\theta}^{-},T\right)=\mathbb{E}\left[f\left(\hat{\theta}^{+},0\right)|\nu,\hat{\theta}^{-},T\right]=h_{s_H}f\left(\hat{\theta}^{+}_{s_H},0\right)+h_{s_L}f\left(\hat{\theta}^{+}_{s_L},0\right) \tag{53}$$

$$\begin{aligned}&=\left[\frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}\nu+\frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}(1-\nu)\right]f\left(\theta_L+\frac{\left(\hat{\theta}^{-}-\theta_L\right)\nu\left(\theta_H-\theta_L\right)}{\left(\hat{\theta}^{-}-\theta_L\right)\nu+\left(\theta_H-\hat{\theta}^{-}\right)(1-\nu)},0\right)\\&+\left[\frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}(1-\nu)+\frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}\nu\right]f\left(\theta_L+\frac{\left(\hat{\theta}^{-}-\theta_L\right)(1-\nu)\left(\theta_H-\theta_L\right)}{\left(\hat{\theta}^{-}-\theta_L\right)(1-\nu)+\left(\theta_H-\hat{\theta}^{-}\right)\nu},0\right).\end{aligned} \tag{54}$$

The value function of the representative agent The representative consumer's preference is specified by a pair of aggregators $(f,\mathcal{A})$ such that:

$$dV_t=[-f(Y_t,V_t)-\frac{1}{2}\mathcal{A}(V_t)||\sigma_V(t)||^2]dt+\sigma_V(t)dB_t \tag{55}$$

for some square-integrable process $\sigma_V(t)$. We adopt the convenient normalization $\mathcal{A}(V_t)=0$ (Duffie and Epstein, 1992b), and denote $\bar{f}$ as the normalized aggregator. Under this normalization, for $\psi\neq 1$, $\bar{f}(Y_t,V_t)$ is,

$$\bar{f}(Y_t,V_t)=\frac{\rho}{1-1/\psi}\frac{Y_t^{1-1/\psi}-((1-\gamma)V_t)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)V_t)^{\frac{1-1/\psi}{1-\gamma}-1}}. \tag{56}$$

The Hamilton-Jacobi-Bellman (HJB) equation for the recursive utility is

$$\bar{f}\left(Y_t,V\left(\hat{\theta},t,Y\right)\right)+\mathcal{L}\left[V\left(\hat{\theta},t,Y\right)\right]=0, \tag{57}$$

where $\mathcal{L}$ is the infinitesimal generator defined as

$$\mathcal{L}\left(V_t\right)=\lim_{\Delta\to 0}\frac{1}{\Delta}\mathbb{E}_t\left[V_{t+\Delta}-V_t\right]. \tag{58}$$

Consider the following homogeneous form of

$$V\left(\hat{\theta}_t,t,Y_t\right)=\frac{1}{1-\gamma}H\left(\hat{\theta}_t,t\right)Y_t^{1-\gamma} \tag{59}$$

where

$$\frac{dY_t}{Y_t} = \hat{\theta}_t dt + \sigma_Y d\hat{B}_{Y,t}, \tag{60}$$

$$d\hat{\theta}_t = \mu_{\theta,t} dt + \frac{\sigma_{\theta,t}}{\sigma_Y} d\hat{B}_{Y,t}, \tag{61}$$

where $\mu_{\theta,t} = (\lambda_H + \lambda_L)\left(\bar{\theta} - \hat{\theta}_t\right)$, $\sigma_{\theta,t} = \left(\theta_H - \hat{\theta}_t\right)\left(\hat{\theta}_t - \theta_L\right)$. The following lemma summarizes the solution to the value function. We provide details for numerical solutions in the Online Appendix.

Lemma 1. *In the interior* $(0,T)$, $H\left(\hat{\theta}_t, t\right)$ *satisfies the following HJB equation*

$$\begin{aligned} 0 = &\frac{1}{(1-\gamma)H}\left\{H_t + H_\theta\left[\mu_{\theta,t} + (1-\gamma)\sigma_{\theta,t}\right] + \frac{\sigma_{\theta,t}^2}{2\sigma_Y^2} H_{\theta\theta}\right\} \\ &+ \frac{\rho}{1-\frac{1}{\psi}}\left(H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} - 1\right) + \left(\hat{\theta}_t - \frac{1}{2}\gamma\sigma_Y^2\right). \end{aligned} \tag{62}$$

where we use notations $H_t = \frac{\partial H(\hat{\theta}_t,t)}{\partial t}$, $H_\theta = \frac{\partial H(\hat{\theta}_t,t)}{\partial \hat{\theta}_t}$, *and* $H_{\theta\theta} = \frac{\partial^2 H(\hat{\theta}_t,t)}{\partial \hat{\theta}_t^2}$.

Assume the informativeness of the announcement ν *takes* m *values, i.e.,* $\nu_1, \nu_2, \nu_3, \ldots, \nu_m$ *with the associated probabilities* $q_1, q_2, q_3, \ldots, q_m$. *Then the boundary condition at the announcement satisfies*

$$\tilde{H}\left(\nu, \hat{\theta}^-, T\right) = \mathbb{E}\left[H\left(\hat{\theta}^+, 0\right) \mid \nu, \hat{\theta}^-, T\right] = h_{s_H} H\left(\hat{\theta}^+_{s_H}, 0\right) + h_{s_L} H\left(\hat{\theta}^+_{s_L}, 0\right), \tag{63}$$

$$\text{and } H\left(\hat{\theta}^-, T\right) = \mathbb{E}\left[\tilde{H}\left(\nu, \hat{\theta}^-, T\right) | \hat{\theta}^-, T\right] = \sum_{j=1}^{m} q_j \tilde{H}\left(\nu_j, \hat{\theta}^-, T\right), \tag{64}$$

where h_{s_H}, h_{s_L}, $\hat{\theta}^+_{s_H}$, *and* $\hat{\theta}^+_{s_L}$ *are defined in Eqs. (49) to (52).*

Proof. The form of value function implies: $\bar{f}(Y,V) = \frac{\rho}{1-\frac{1}{\psi}} Y^{1-\gamma}\left(H^{1-\frac{1-\frac{1}{\psi}}{1-\gamma}} - H\right)$. Using Ito's lemma, we have

$$\begin{aligned} \frac{\mathcal{L}\left[V\left(\hat{\theta}_t, t, Y_t\right)\right]}{Y_t^{1-\gamma}} &= \frac{\mathcal{L}\left[H\left(\hat{\theta}_t, t\right) Y_t^{1-\gamma}\right]}{(1-\gamma) Y_t^{1-\gamma}} \\ &= H\left(\hat{\theta}_t - \frac{1}{2}\gamma\sigma_Y^2\right) + \frac{1}{1-\gamma}\left(H_t + H_\theta\mu_{\theta,t} + \frac{1}{2\sigma_Y^2} H_{\theta\theta}\sigma_{\theta,t}^2\right) + H_\theta\sigma_{\theta,t}. \end{aligned} \tag{65}$$

Therefore, HJB equation is written as Eq.(62).

The boundary condition upon the announcement satisfies

$$H\left(\hat{\theta}^{-},T\right)=\mathbb{E}\left[\mathbb{E}\left[H\left(\hat{\theta}^{+},0\right)\mid\nu,\hat{\theta}^{-},T\right]|\hat{\theta}^{-},T\right].\tag{66}$$

To understand the above boundary condition, we provide the timeline and the two steps in the table below.

Table 11: Continuation Utility

Time	T (or T^{-})		0 (or T^{+})
	don't know ν	draw ν, know ν	after announcement
Information set	$\left\{\hat{\theta}^{-},T\right\}$	$\left\{\nu,\hat{\theta}^{-},T\right\}$	$\left\{\nu,\hat{\theta}^{+},0\right\}$
Continuation utility	$H\left(\hat{\theta}^{-},T\right)$	$\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)$	$H\left(\hat{\theta}^{+},0\right)$
Step1: $\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)=\mathbb{E}\left[H\left(\hat{\theta}^{+},0\right)\mid\nu,\hat{\theta}^{-},T\right]$			
Step2: $H\left(\hat{\theta}^{-},T\right)=\mathbb{E}\left[\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)|\hat{\theta}^{-},T\right]$			

Step 1, we draw the i.i.d. random variable ν. Because investors know the distributions of ν so that they could update their beliefs about the associated realized values and probabilities of $\hat{\theta}^{+}$ according to (15) conditioning on each ν. It is useful to denote this intermediate step as $\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)=\mathbb{E}\left[H\left(\hat{\theta}^{+},0\right)\mid\nu,\hat{\theta}^{-},T\right]$, where $\mathbb{E}\left[H\left(\hat{\theta}^{+},0\right)\mid\nu,\hat{\theta}^{-},T\right]$ is defined in (54). In this step, we calculate the expected value of the continuation utility right after the announcement conditional on the information set $\left\{\nu,\hat{\theta}^{-},T\right\}$, as defined in Eq. (63).

Step 2, we compute the unconditional expectation by integrating over all possible realizations of ν to get Eq. (64). This step allows us to derive the expected value function based on the information set $\left\{\hat{\theta}^{-},T\right\}$ right before the announcement. If, for example, ν only takes only two values, ν_H and ν_L with probability q and $1-q$. Then,

$$H\left(\hat{\theta}^{-},T\right)=\mathbb{E}\left[\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)|\hat{\theta}^{-},T\right]=q\tilde{H}\left(\nu_H,\hat{\theta}^{-},T\right)+(1-q)\,\tilde{H}\left(\nu_L,\hat{\theta}^{-},T\right).\tag{67}$$

□

7.2.2 Asset Prices

In this subsection, we first derive the pricing kernel for the representative investor. We then derive the risk-free rate and the partial differential equation (PDE) for the price-to-dividend ratio with the boundary condition at the announcement. Finally, we calculate the cumulative return and the risk premium.

Pricing kernel and the risk-free rate We first provide proof for the law of motion of the pricing kernel M_t, which satisfies the stochastic differential equation (SDE) of Eq. (17), and

$$r\left(\hat{\theta}_t, t\right) = \rho + \frac{1}{\psi}\hat{\theta}_t - \frac{1+\frac{1}{\psi}}{2}\gamma\sigma_Y^2 + \frac{(\frac{1}{\psi}-\gamma)(1-\frac{1}{\psi})}{2\left(1-\gamma\right)^2}\left(\frac{H_\theta}{H}\right)^2\frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_\theta}{H}\sigma_{\theta,t}, \tag{68}$$

$$\sigma_M\left(\hat{\theta}_t, t\right) = \gamma\sigma_Y - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_\theta}{H}\frac{\sigma_{\theta,t}}{\sigma_Y}, \tag{69}$$

where $\sigma_{\theta,t} = (\theta_H - \hat{\theta}_t)(\hat{\theta}_t - \theta_L)$, and we use notations $\frac{H_\theta}{H} = \frac{\partial H(\hat{\theta}_t,t)/\partial\hat{\theta}_t}{H(\hat{\theta}_t,t)}$ and $\frac{H_{\theta\theta}}{H} = \frac{\partial^2 H(\hat{\theta}_t,t)/\partial\hat{\theta}_t^2}{H(\hat{\theta}_t,t)}$.

Proof. The pricing kernel is defined as

$$\frac{dM_t}{M_t} = \frac{d\bar{f}_Y\left(Y,V\right)}{\bar{f}_Y\left(Y,V\right)} + \bar{f}_V\left(Y,V\right)dt, \tag{70}$$

where $\bar{f}_Y\left(Y,V\right) = \rho H^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}Y^{-\gamma}$, and $\bar{f}_V\left(Y,V\right) = \rho\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} - \rho\frac{1-\gamma}{1-\frac{1}{\psi}}$. Applying Ito's lemma,

$$\begin{aligned}\frac{d\bar{f}_Y\left(Y,V\right)}{\bar{f}_Y\left(Y,V\right)} &= \frac{d[H^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}Y_t^{-\gamma}]}{H^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}Y_t^{-\gamma}} = \Bigg\{-\gamma\hat{\theta}_t + \frac{1}{2}\gamma\left(\gamma+1\right)\sigma_Y^2 + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left(\frac{H_t}{H} + \frac{H_\theta}{H}\mu_{\theta,t}\right)\\ &\quad + \frac{1}{2}\left[\frac{\left(\frac{1}{\psi}-\gamma\right)\left(\frac{1}{\psi}-1\right)}{\left(1-\gamma\right)^2}\left(\frac{H_\theta}{H}\right)^2 + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_{\theta\theta}}{H}\right]\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}\\ &\quad -\gamma\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_\theta}{H}\sigma_{\theta,t}\Bigg\}dt + \left(-\gamma\sigma_Y + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_\theta}{H}\frac{\sigma_{\theta,t}}{\sigma_Y}\right)d\hat{B}_{Y,t}.\end{aligned} \tag{71}$$

Matching the drift and diffusion of Eq.(17), we can get (69) and the risk-free rate

$$\begin{aligned}r_t &= -\frac{\frac{1}{\psi}-\gamma}{\left(1-\gamma\right)H}\left[H_t + H_\theta\mu_{\theta,t} + \frac{1}{2}\left(\frac{\frac{1}{\psi}-1}{1-\gamma}\frac{H_\theta^2}{H} + H_{\theta\theta}\right)\frac{\sigma_{\theta,t}^2}{\sigma_Y^2} - \gamma H_\theta\sigma_{\theta,t}\right]\\ &\quad +\gamma\hat{\theta}_t - \frac{1}{2}\gamma\left(\gamma+1\right)\sigma_Y^2 - \rho\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}} + \rho\frac{1-\gamma}{1-\frac{1}{\psi}}.\end{aligned} \tag{72}$$

Using the HJB equation to simplify r_t by multiplying $\frac{1}{\psi}-\gamma$ on both sides of (62),

$$\begin{aligned} 0 &= \frac{\frac{1}{\psi}-\gamma}{(1-\gamma)H}\left\{H_t + H_\theta\left[\mu_{\theta,t}+(1-\gamma)\sigma_{\theta,t}\right]+\frac{1}{2}H_{\theta\theta}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}\right\} \\ &\quad + \frac{\rho\left(\frac{1}{\psi}-\gamma\right)}{1-\frac{1}{\psi}}\left(H^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}-1\right)+\left(\hat{\theta}_t-\frac{1}{2}\gamma\sigma_Y^2\right)\left(\frac{1}{\psi}-\gamma\right), \end{aligned} \tag{73}$$

and adding up with (72), we get the instantaneous risk-free rate in the main text. □

Price-to-dividend ratio We show the solution for $p(\hat{\theta}_t,t)$ in the following lemma. We provide details for numerical solutions in the Online Appendix.

Lemma 2. *In the interior $(0,T)$, the price-to-dividend ratio $p(\hat{\theta}_t,t)$ satisfies the PDE of*

$$\varpi\left(\hat{\theta}_t,t\right)p = p_t + p_\theta\varrho\left(\hat{\theta}_t,t\right)+\frac{1}{2}p_{\theta\theta}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}+1, \tag{74}$$

where we use notations $p_t=\frac{\partial p(\hat{\theta}_t,t)}{\partial t}$, $p_\theta=\frac{\partial p(\hat{\theta}_t,t)}{\partial\hat{\theta}_t}$, $p_{\theta\theta}=\frac{\partial^2 p(\hat{\theta}_t,t)}{\partial\hat{\theta}_t^2}$, and

$$\begin{aligned} \varpi\left(\hat{\theta}_t,t\right) &= (\xi-1)\bar{\theta}+\rho-\frac{1}{2}\gamma\sigma_Y^2\left(\frac{1}{\psi}+1\right)+\gamma\sigma_Y^2-\left(\xi-\frac{1}{\psi}\right)\hat{\theta}_t+\frac{\left(\frac{1}{\psi}-\gamma\right)\left(1-\frac{1}{\psi}\right)}{2(1-\gamma)^2}\left(\frac{H_\theta}{H}\right)^2\frac{\sigma_{\theta,t}^2}{\sigma_Y^2} \\ \varrho\left(\hat{\theta}_t,t\right) &= \mu_{\theta,t}+(1-\gamma)\sigma_{\theta,t}+\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_\theta}{H}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \end{aligned} \tag{75}$$

with the boundary condition at the announcement satisfying

$$\begin{aligned} \tilde{p}\left(\nu,\hat{\theta}_T^-,T\right) &= \mathbb{E}\left[\left.\frac{H\left(\hat{\theta}_T^+,0\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}p\left(\hat{\theta}_T^+,0\right)}{\mathbb{E}\left[H\left(\hat{\theta}_T^+,0\right)\middle|\nu,\hat{\theta}_T^-,T\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}\right|\nu,\hat{\theta}_T^-,T\right] \\ &= \frac{h_{s_H}H\left(\hat{\theta}_{s_H}^+,0\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}p\left(\hat{\theta}_{s_H}^+,0\right)+h_{s_L}H\left(\hat{\theta}_{s_L}^+,0\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}p\left(\hat{\theta}_{s_L}^+,0\right)}{\left[h_{s_H}H\left(\hat{\theta}_{s_H}^+,0\right)+h_{s_L}H\left(\hat{\theta}_{s_L}^+,0\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}, \end{aligned} \tag{76}$$

$$\textit{and } p\left(\hat{\theta}_T^-,T\right) = \mathbb{E}\left[\left.\frac{\tilde{H}\left(\nu,\hat{\theta}_T^-,T\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\tilde{p}\left(\nu,\hat{\theta}_T^-,T\right)}{\mathbb{E}\left[\tilde{H}\left(\nu,\hat{\theta}_T^-,T\right)|\hat{\theta}_T^-,T\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}\right|\hat{\theta}_T^-,T\right] = \frac{\sum_{j=1}^m q_j\tilde{H}\left(\nu_j,\hat{\theta}_T^-,T\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\tilde{p}\left(\nu_j,\hat{\theta}_T^-,T\right)}{\left[\sum_{j=1}^m q_j\tilde{H}\left(\nu_j,\hat{\theta}_T^-,T\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}, \tag{77}$$

where h_{s_H}, h_{s_L}, $\hat{\theta}^+_{s_H}$, and $\hat{\theta}^+_{s_L}$ are defined in Eqs.(49) to (52).

Proof. The present value relationship (18) implies

$$M_t D_t dt + \mathcal{L}\left[M_t p(\hat{\theta}_t, t) D_t\right] = 0. \tag{78}$$

This gives $\frac{\mathcal{L}\left[M_t p(\hat{\theta}_t,t)D_t\right]}{M_t p(\hat{\theta}_t,t)D_t} + \frac{1}{p(\hat{\theta}_t,t)} = 0$. Applying Ito's lemma and using Eqs.(16) and (17),

$$\begin{aligned}\frac{\mathcal{L}\left[M_t p(\hat{\theta}_t, t) D_t\right]}{M_t p(\hat{\theta}_t, t) D_t} &= -r_t + \frac{1}{p}\left(p_t + p_\theta \mu_{\theta,t} + \frac{1}{2} p_{\theta\theta} \frac{\sigma^2_{\theta,t}}{\sigma^2_Y}\right) \\ &\quad + \xi\left(\hat{\theta}_t - \bar{\theta}\right) + \bar{\theta} + \frac{p_\theta}{p}\sigma_{\theta,t} - \sigma_{M,t}\left(\sigma_Y + \frac{p_\theta}{p}\frac{\sigma_{\theta,t}}{\sigma_Y}\right).\end{aligned} \tag{79}$$

Put in r_t from Eq. (68) would give the PDE for $p(\hat{\theta}_t, t)$.

We next solve the boundary condition. Another way to write Eq. (70) is: $M_t = f_Y(Y_t, V_t)\, e^{\int_0^t f_V(Y_s,V_s)ds}$. From this formula, we could derive the announcement SDF as $\frac{H(\hat{\theta}^+_T,0)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}{\mathbb{E}\left[H(\hat{\theta}^+_T,0)\middle|\hat{\theta}^-_T,T\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}$. The intuition is as follows. Upon the announcement, Y_t is continuous while the continuation utility $H(\hat{\theta}_t, t)$ jumps when new information about $\hat{\theta}_t$ arrives because of generalized risk sensitivity in preferences (Ai and Bansal, 2018). Therefore, using the announcement SDF, the boundary condition for $p(\hat{\theta}_t, t)$ is

$$p\left(\hat{\theta}^-_T, T\right) = \mathbb{E}\left[\frac{H(\hat{\theta}^+_T,0)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} p(\hat{\theta}^+_T,0)}{\mathbb{E}\left[H(\hat{\theta}^+_T,0)\middle|\hat{\theta}^-_T,T\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}\middle|\hat{\theta}^-_T, T\right]. \tag{80}$$

We again understand the above boundary condition in two steps. First, conditioning on a given ν, the distribution of $\hat{\theta}^+_T$ is given by Eqs.(49)-(52). This intermediate step $\tilde{p}\left(\nu, \hat{\theta}^-_T, T\right)$ can be computed in Eq.(76). Next, we compute the unconditional expectation by averaging over all possible realizations of ν, as shown in Eq.(77). $\square$

Risk premium Here we provide proof for Proposition 2. For notational convenience, denote $\mu_{R,t} = \mathbb{E}\left[\frac{d\left(p\left(\hat{\theta}_t,t\right)D_t\right)+D_t dt}{p\left(\hat{\theta}_t,t\right)D_t}\right]$ as the expected return and $\sigma_{R,t}$ as the instantaneous return volatility. We show the following lemma holds.

Lemma 3. *The cumulated return takes the following form*

$$\frac{dR_t}{R_t} = \mu_{R,t}dt + \sigma_{R,t}d\hat{B}_{Y,t} + \omega_t dB_{i,t}, \tag{81}$$

where

$$\mu_{R,t} = \frac{1}{p}\left(1 + p_t + p_\theta \mu_{\theta,t} + \frac{1}{2}p_{\theta\theta}\frac{\sigma^2_{\theta,t}}{\sigma^2_Y}\right) + \xi\left(\hat{\theta}_t - \bar{\theta}\right) + \bar{\theta} + \frac{p_\theta}{p}\sigma_{\theta,t}, \tag{82}$$

$$\sigma_{R,t} = \frac{p_\theta}{p}\frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y. \tag{83}$$

Proof. The cumulative return can be computed as

$$\frac{dR_t}{R_t} = \frac{d\left(p\left(\hat{\theta}_t, t\right)D_t\right) + D_t dt}{p\left(\hat{\theta}_t, t\right)D_t} = \frac{1}{p}dt + \frac{d\left(pD_t\right)}{pD_t}. \tag{84}$$

Applying Ito's lemma and using (16),

$$\begin{aligned}\frac{d\left(p\left(\hat{\theta}_t, t\right)D_t\right)}{p\left(\hat{\theta}_t, t\right)D_t} &= \left[\frac{1}{p}\left(p_t + p_\theta\mu_{\theta,t} + \frac{1}{2}p_{\theta\theta}\frac{\sigma^2_{\theta,t}}{\sigma^2_Y}\right) + \xi\left(\hat{\theta}_t - \bar{\theta}\right) + \bar{\theta} + \frac{p_\theta}{p}\sigma_{\theta,t}\right]dt \\ &\quad + \left(\frac{p_\theta}{p}\frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y\right)d\hat{B}_{Y,t} + \omega_t dB_{i,t}.\end{aligned} \tag{85}$$

Together with the expression of the pricing kernel (17), the risk premium is therefore

$$\begin{aligned}\mu_{R,t} - r_t &= -\mathrm{Cov}_t\left[\frac{dM_t}{M_t}, \frac{dR_t}{R_t}\right] \\ &= \sigma_{M,t}\sigma_{R,t} = \left(\gamma\sigma_Y - \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\frac{H_\theta}{H}\frac{\sigma_{\theta,t}}{\sigma_Y}\right)\left(\frac{p_\theta}{p}\frac{\sigma_{\theta,t}}{\sigma_Y} + \sigma_Y\right),\end{aligned} \tag{86}$$

which corresponds to Eq.(19) in Proposition 2.

Upon the announcement, since dividend flow is continuous, the announcement premium is given by $\left(\mathbb{E}\left[p\left(\hat{\theta}^+_T, 0\right)/p\left(\hat{\theta}^-_T, T\right) \mid \hat{\theta}^-_T, T\right] - 1\right)$. Using the boundary condition of the price-to-dividend ratio (80), we can therefore obtain Eq.(20). □

7.2.3 Forward Looking Measure of Implied Variance

In this session, we compute the model implied variance defined in Eq.(21). We first simplify the log return. The law of motion of dividend from Eq.(16) implies

$$\ln D_{\tau} = \ln D_t + \int_t^{\tau} \left(\xi \hat{\theta}_s + \bar{\theta}\left(1-\xi\right) - \frac{1}{2}\sigma_Y^2 - \frac{1}{2}\omega_s^2 \right) ds + \int_t^{\tau} \left(\sigma_Y d\hat{B}_{Y,s} + \omega_s dB_{\omega,s} \right) \quad (87)$$

For simplicity, we define

$$\delta\left(t\right) = \int_0^{t} \left(\xi \hat{\theta}_s + \bar{\theta}\left(1-\xi\right) - \frac{1}{2}\sigma_Y^2 \right) ds + \int_0^{t} \sigma_Y d\hat{B}_{Y,s}, \quad (88)$$

or equivalently,

$$d\delta\left(t\right) = \left[\xi \hat{\theta}_t + \bar{\theta}\left(1-\xi\right) - \frac{1}{2}\sigma_Y^2 \right] dt + \sigma_Y d\hat{B}_{Y,t}. \quad (89)$$

Therefore, the log return could be simplified as

$$\ln\left\{p\left(\hat{\theta}_{\tau},\tau\right) D_{\tau}\right\} - \ln\left\{p\left(\hat{\theta}_{t},t\right) D_{t}\right\} = \ln p\left(\hat{\theta}_{\tau},\tau\right) - \ln p\left(\hat{\theta}_{t},t\right) + \delta\left(\tau\right) - \delta\left(t\right) + \int_t^{\tau}\left(-\frac{1}{2}\omega_s^2 ds + \omega_s dB_{\omega,s}\right). \quad (90)$$

Then the implied variance could be decomposed as

$$\begin{aligned} IV_{t,\tau} &= \mathrm{Var}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right)\right] + 2\mathrm{Cov}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right), \delta\left(\tau\right)-\delta\left(t\right)\right] \\ &\quad + \mathrm{Var}_t\left[\delta\left(\tau\right)-\delta\left(t\right)\right] + \mathrm{Var}_t\left[-\int_t^{\tau}\frac{1}{2}\omega_s^2 ds + \int_t^{\tau}\omega_s dB_{\omega,s}\right], \end{aligned} \quad (91)$$

where we use the fact that $\ln p\left(\hat{\theta}_t,t\right)$ is known at time t, and $dB_{\omega,t}$ is uncorrelated with others. The four terms in Eq.(91) can be further decomposed into:

$$\mathrm{Var}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right)\right] = \mathbb{E}_t\left[\ln^2 p\left(\hat{\theta}_{\tau},\tau\right)\right] - \left(\mathbb{E}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right)\right]\right)^2, \quad (92)$$

$$\begin{aligned} \mathrm{Cov}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right), \delta\left(\tau\right)-\delta\left(t\right)\right] &= \mathbb{E}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right)\left(\delta\left(\tau\right)-\delta\left(t\right)\right)\right] \\ &\quad - \mathbb{E}_t\left[\ln p\left(\hat{\theta}_{\tau},\tau\right)\right]\mathbb{E}_t\left[\left(\delta\left(\tau\right)-\delta\left(t\right)\right)\right], \end{aligned} \quad (93)$$

$$\mathrm{Var}_t\left[\delta\left(\tau\right)-\delta\left(t\right)\right] = \mathbb{E}_t\left[\left\{\delta\left(\tau\right)-\delta\left(t\right)\right\}^2\right] - \mathbb{E}_t\left[\delta\left(\tau\right)-\delta\left(t\right)\right]^2, \quad (94)$$

$$\begin{aligned} \mathrm{Var}_t\left[-\int_t^{\tau}\frac{1}{2}\omega_s^2 ds + \int_t^{\tau}\omega_s dB_{\omega,s}\right] &= \mathbb{E}_t\left[\left(-\int_t^{\tau}\frac{1}{2}\omega_s^2 ds + \int_t^{\tau}\omega_s dB_{\omega,s}\right)^2\right] \\ &\quad - \left(\mathbb{E}_t\left[-\int_t^{\tau}\frac{1}{2}\omega_s^2 ds + \int_t^{\tau}\omega_s dB_{\omega,s}\right]\right)^2. \end{aligned} \quad (95)$$

The first term on the right hand side of the last equation above is equivalent to

$$\begin{aligned}
&\frac{1}{4}\mathbb{E}_t\left[\left(\int_t^\tau \omega_s^2 ds\right)^2\right]+\mathbb{E}_t\left[\left(\int_t^\tau \omega_s dB_{\omega,s}\right)^2\right]-\mathbb{E}_t\left[\left(\int_t^\tau \omega_s^2 ds\right)\left(\int_t^\tau \omega_s dB_{\omega,s}\right)\right]\\
=\ &\frac{1}{4}\mathbb{E}_t\left[\left(\int_t^\tau \omega_s^2 ds\right)^2\right]+\mathbb{E}_t\left[\int_t^\tau \omega_s^2 ds\right],
\end{aligned}$$

where we applied Ito's isometry for $\mathbb{E}_t\left[\left(\int_t^\tau \omega_s dB_{\omega,s}\right)^2\right]=\mathbb{E}_t\left[\int_t^\tau \omega_s^2 ds\right]$. As a result, Eq. (95) can be simplified to

$$\mathrm{Var}_t\left[-\frac{1}{2}\int_t^\tau \omega_s^2 ds+\int_t^\tau \omega_s dB_{\omega,s}\right]=\frac{1}{4}y\left(\omega_t,t\right)+g\left(\omega_t,t\right)-\frac{1}{4}g^2\left(\omega_t,t\right). \tag{96}$$

where $g\left(\omega_t,t\right)=\mathbb{E}_t\left[\int_t^\tau \omega_s^2 ds\right]$ and $y\left(\omega_t,t\right)=\mathbb{E}_t\left[\left(\int_t^\tau \omega_s^2 ds\right)^2\right]$. Using simplified notations defined above, we show the following lemma holds. We provide details for numerical solutions in the Online Appendix.

Lemma 4. *For $0\le t\le\tau$, the implied variance is given by*

$$\begin{aligned}
IV_{t,\tau}\left(\hat{\theta}_t,\omega_t,t\right)\ =\ &w_2\left(\hat{\theta}_t,t\right)-w_1\left(\hat{\theta}_t,t\right)^2+2\left[a_3\left(\hat{\theta}_t,t\right)-w_1\left(\hat{\theta}_t,t\right)a_1\left(\hat{\theta}_t,t\right)\right]\\
&+a_2\left(\hat{\theta}_t,t\right)-a_1\left(\hat{\theta}_t,t\right)^2+\frac{1}{4}y\left(\omega_t,t\right)+g\left(\omega_t,t\right)-\frac{1}{4}g^2\left(\omega_t,t\right),
\end{aligned} \tag{97}$$

where

$$w_1\left(\hat{\theta}_t,t\right)=\mathbb{E}_t\left[\ln p\left(\hat{\theta}_\tau,\tau\right)\right], \tag{98}$$

$$w_2\left(\hat{\theta}_t,t\right)=\mathbb{E}_t\left[\ln p^2\left(\hat{\theta}_\tau,\tau\right)\right], \tag{99}$$

$$a_1\left(\hat{\theta}_t,t\right)=\mathbb{E}_t\left[\delta\left(\tau\right)-\delta\left(t\right)\right], \tag{100}$$

$$a_2\left(\hat{\theta}_t,t\right)=\mathbb{E}_t\left[\left\{\delta\left(\tau\right)-\delta\left(t\right)\right\}^2\right], \tag{101}$$

$$a_3\left(\hat{\theta}_t,t\right)=\mathbb{E}_t\left[\ln p\left(\hat{\theta}_\tau,\tau\right)\left\{\delta\left(\tau\right)-\delta\left(t\right)\right\}\right], \tag{102}$$

and $g\left(\omega_t,t\right)=\mathbb{E}_t\left[\int_t^\tau \omega_s^2 ds\right]$ and $y\left(\omega_t,t\right)=\mathbb{E}_t\left[\left(\int_t^\tau \omega_s^2 ds\right)^2\right]$ have the following closed form solutions

$$g\left(\omega_H,t\right)=\frac{\kappa_H\omega_L^2+\kappa_L\omega_H^2}{\kappa_H+\kappa_L}\left(\tau-t\right)+\kappa_H\frac{\omega_H^2-\omega_L^2}{\left(\kappa_H+\kappa_L\right)^2}\left[1-e^{-\left(\kappa_H+\kappa_L\right)\left(\tau-t\right)}\right], \tag{103}$$

$$g\left(\omega_L,t\right)=\frac{\kappa_H\omega_L^2+\kappa_L\omega_H^2}{\kappa_H+\kappa_L}\left(\tau-t\right)-\kappa_L\frac{\omega_H^2-\omega_L^2}{\left(\kappa_H+\kappa_L\right)^2}\left[1-e^{-\left(\kappa_H+\kappa_L\right)\left(\tau-t\right)}\right]. \tag{104}$$

$$
\begin{aligned}
y(\omega_H,t) &= \frac{\left(\omega_H^2\kappa_L+\kappa_H\omega_L^2\right)^2}{\left(\kappa_H+\kappa_L\right)^2}(\tau-t)^2+\frac{2\kappa_H\left(\omega_H^2-\omega_L^2\right)\left(1-e^{-(\kappa_H+\kappa_L)(\tau-t)}\right)\left(\kappa_H\omega_H^2-2\omega_H^2\kappa_L-\kappa_H\omega_L^2\right)}{\left(\kappa_H+\kappa_L\right)^4} \\
&\quad+\frac{2\kappa_H\left(\omega_H^2-\omega_L^2\right)\left[-e^{-(\kappa_H+\kappa_L)(\tau-t)}\left(\kappa_H\omega_H^2-\kappa_L\omega_L^2\right)+2\omega_H^2\kappa_L+\kappa_H\omega_L^2-\kappa_L\omega_L^2\right]}{\left(\kappa_H+\kappa_L\right)^3}(\tau-t), \quad (105) \\
y(\omega_L,t) &= \frac{\left(\omega_H^2\kappa_L+\kappa_H\omega_L^2\right)^2}{\left(\kappa_H+\kappa_L\right)^2}(\tau-t)^2+\frac{2\kappa_L\left(\omega_H^2-\omega_L^2\right)\left(1-e^{-\Delta(\kappa_H+\kappa_L)}\right)\left(-2\kappa_H\omega_H^2+\omega_H^2\kappa_L-\kappa_L\omega_L^2\right)}{\left(\kappa_H+\kappa_L\right)^4} \\
&\quad+\frac{2\kappa_L\left(\omega_H^2-\omega_L^2\right)\left[\kappa_H\omega_H^2+e^{-(\kappa_H+\kappa_L)(\tau-t)}\left(\kappa_H\omega_H^2-\kappa_L\omega_L^2\right)-\omega_H^2\kappa_L-2\kappa_H\omega_L^2\right]}{\left(\kappa_H+\kappa_L\right)^3}(\tau-t). \quad (106)
\end{aligned}
$$

Note that all of them also depend on the expiration date τ, but we dropped τ to save notations. We first solve for $g(\omega_t,t)$ and $y(\omega_t,t)$, where $\omega_t\in\{\omega_H,\omega_L\}$ is a two-state Markov chain. We construct the martingale

$$
\mathbb{E}_t\left[\int_0^\tau \omega_s^2 ds\right]=\mathbb{E}_t\left[\int_t^\tau \omega_s^2 ds\right]+\mathbb{E}_t\left[\int_0^t \omega_s^2 ds\right]=g(\omega_t,t)+\int_0^t \omega_s^2 ds. \quad (107)
$$

Because $\mathbb{E}_t\left[\mathbb{E}_t\left[\int_0^\tau \omega_s^2 ds\right]\right]=\mathbb{E}_t\left[\int_0^\tau \omega_s^2 ds\right]$, $\mathbb{E}_t\left[\int_0^\tau \omega_s^2 ds\right]$ is a martingale, we have

$$
\mathcal{L}\left[g(\omega_t,t)\right]+\omega_t^2=0, \quad (108)
$$

with the boundary condition $g(\omega_\tau,\tau)=0$. The above can be written as

$$
\begin{aligned}
\omega_H^2+\frac{\partial}{\partial t}g(\omega_H,t)+\kappa_H\left[g(\omega_L,t)-g(\omega_H,t)\right] &= 0, \quad (109) \\
\omega_L^2+\frac{\partial}{\partial t}g(\omega_H,t)+\kappa_L\left[g(\omega_H,t)-g(\omega_L,t)\right] &= 0, \quad (110)
\end{aligned}
$$

which further gives Eqs.(103) and (104) in Lemma 4.

We use the same methodology to solve for $y(\omega_t,t)$. Denote $z(t)=\int_0^t \omega_s^2 ds$,

$$
\begin{aligned}
\mathbb{E}_t\left[\left(\int_0^\tau \omega_s^2 ds\right)^2\right] &= \mathbb{E}_t\left[\left(\int_0^t \omega_s^2 ds\right)^2+\left(\int_t^\tau \omega_s^2 ds\right)^2+2\left(\int_0^t \omega_s^2 ds\right)\left(\int_t^\tau \omega_s^2 ds\right)\right] \\
&= z(t)^2+y(\omega_t,t)+2z(t)\,g(\omega_t,t). \quad (111)
\end{aligned}
$$

Because $\mathbb{E}_t\left[\left(\int_0^\tau \omega_s^2 ds\right)^2\right]$ is a martingale, we have

$$
\begin{aligned}
0 &= \mathcal{L}\left\{z(t)^2+y(\omega_t,t)+2z(t)\,g(\omega_t,t)\right\} \\
&= 2z(t)\,z'(t)+\mathcal{L}y(\omega_t,t)+2z(t)\,\mathcal{L}g(\omega_t,t)+2z'(t)\,g(\omega_t,t). \quad (112)
\end{aligned}
$$

Note that $z'(t) = \omega_t^2$, the above condition can be written as

$$2z(t)\left[\omega_t^2 + \mathcal{L}g(\omega_t, t)\right] + \mathcal{L}y(\omega_t, t) + 2\omega_t^2 g(\omega_t, t) = 0. \tag{113}$$

Because $z(t)$ is a random variable and can take any values, for the above to hold, we must have (108) and

$$\mathcal{L}y(\omega_t, t) + 2\omega_t^2 g(\omega_t, t) = 0, \tag{114}$$

together with the boundary condition $y(\omega_\tau, \tau) = 0$. Clearly, the above implies the following ODEs of $y(\omega_t, t)$ function

$$\begin{aligned} \frac{\partial}{\partial t}y(\omega_H, t) + \kappa_H\left[y(\omega_L, t) - y(\omega_H, t)\right] + 2\omega_H^2 g(\omega_H, t) &= 0 \qquad (115)\\ \frac{\partial}{\partial t}y(\omega_L, t) + \kappa_L\left[y(\omega_H, t) - y(\omega_L, t)\right] + 2\omega_L^2 g(\omega_L, t) &= 0. \qquad (116) \end{aligned}$$

which further gives Eqs.(105) and (106) in Lemma 4.

Next, we solve for $w_j\left(\hat{\theta}_t, t\right)$, $j = 1, 2$, and $a_j\left(\hat{\theta}_t, t\right)$ where $j = 1, 2, 3$ separately. In general, there are two cases. In the first case, $\tau < T$ so that there will be no announcement before expiration τ. We construct martingales and use martingale property to determine the PDE and the associated boundary condition for each variable. In the second case, $\tau \geq T$, and the forward-looking implied variance will cover one announcement before expiration. If so, we solve $w_j\left(\hat{\theta}_t, t\right)$ and $a_j\left(\hat{\theta}_t, t\right)$ using backward induction in three steps: after, upon, and before the announcement. We summarize the results in the following lemma.

Lemma 5. *If $0 \leq t \leq \tau < T$, there is no announcement before expiration. The PDEs for $w_j\left(\hat{\theta}_t, t\right)$, $j = 1, 2$ and $a_j\left(\hat{\theta}_t, t\right)$, $j = 1, 2, 3$ are*

$$\begin{aligned} 0 &= \frac{\partial}{\partial t}w_j\left(\hat{\theta}_t, t\right) + \frac{\partial}{\partial \hat{\theta}}w_j\left(\hat{\theta}_t, t\right)\mu_{\theta,t} + \frac{1}{2}\frac{\partial^2}{\partial \hat{\theta}^2}w_j\left(\hat{\theta}_t, t\right)\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \qquad (117)\\ 0 &= \frac{\partial}{\partial t}a_1\left(\hat{\theta}_t, t\right) + \frac{\partial}{\partial \hat{\theta}}a_1\left(\hat{\theta}_t, t\right)\mu_{\theta,t} + \frac{1}{2}\frac{\partial^2}{\partial \hat{\theta}^2}a_1\left(\hat{\theta}_t, t\right)\frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + \xi\hat{\theta}_t + \bar{\theta}(1-\xi) - \frac{1}{2}\sigma_Y^2 \qquad (118)\\ 0 &= 2a_1\left(\hat{\theta}_t, t\right)\left(\xi\hat{\theta}_t + \bar{\theta}(1-\xi) - \frac{1}{2}\sigma_Y^2\right) + 2\frac{\partial}{\partial \hat{\theta}}a_1\left(\hat{\theta}_t, t\right)\sigma_{\theta,t} + \sigma_Y^2\\ &\quad + \frac{\partial}{\partial t}a_2\left(\hat{\theta}_t, t\right) + \frac{\partial}{\partial \hat{\theta}}a_2\left(\hat{\theta}_t, t\right)\mu_{\theta,t} + \frac{1}{2}\frac{\partial^2}{\partial \hat{\theta}^2}a_2\left(\hat{\theta}_t, t\right)\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \qquad (119)\\ 0 &= w_1\left(\hat{\theta}_t, t\right)\left(\xi\hat{\theta}_t + \bar{\theta}(1-\xi) - \frac{1}{2}\sigma_Y^2\right) + \frac{\partial}{\partial \hat{\theta}}w_1\left(\hat{\theta}_t, t\right)\sigma_{\theta,t} + \frac{\partial}{\partial t}a_3\left(\hat{\theta}_t, t\right)\\ &\quad + \frac{\partial}{\partial \hat{\theta}}a_3\left(\hat{\theta}_t, t\right)\mu_{\theta,t} + \frac{1}{2}\frac{\partial^2}{\partial \hat{\theta}^2}a_3\left(\hat{\theta}_t, t\right)\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \qquad (120) \end{aligned}$$

with the associated boundary conditions of

$$w_1\left(\hat{\theta}_\tau,\tau\right) = \ln p\left(\hat{\theta}_\tau,\tau\right) \text{ and } w_2\left(\hat{\theta}_\tau,\tau\right) = \ln p^2\left(\hat{\theta}_\tau,\tau\right), \tag{121}$$

$$a_1\left(\hat{\theta}_\tau,\tau\right) = 0,\ a_2\left(\hat{\theta}_\tau,\tau\right) = 0 \text{ and } a_3\left(\hat{\theta}_\tau,\tau\right) = 0. \tag{122}$$

If $0 \le t < T \le \tau$, there will be an announcement before expiration. For $t \in [T^+,\tau]$, we use the above PDEs and boundary conditions (121) and (122) to compute $w_j\left(\hat{\theta}_{T^+},T^+\right)$ and $a_j\left(\hat{\theta}_{T^+},T^+\right)$. For $t \in [T^-,T^+]$, we update the boundary condition according to

$$w_j(\hat{\theta}_T^-,T^-) = \sum_{s=1}^m q_s\tilde{w}_j(\nu_s,\hat{\theta}_T^-,T^-),\ \tilde{w}_j(\nu,\hat{\theta}_T^-,T^-) = h_{s_H}w_j(\hat{\theta}_{s_H}^+,T^+) + h_{s_L}w_j(\hat{\theta}_{s_L}^+,T^+), \tag{123}$$

$$a_j(\hat{\theta}_T^-,T^-) = \sum_{s=1}^m q_s\tilde{a}_j(\nu_s,\hat{\theta}_T^-,T^-),\ \tilde{a}_j(\nu,\hat{\theta}_T^-,T^-) = h_{s_H}a_j(\hat{\theta}_{s_H}^+,T^+) + h_{s_L}a_j(\hat{\theta}_{s_H}^+,T^+), \tag{124}$$

For $t \in [t,T^-]$, we use the PDEs again with the above boundary conditions (123) and (124).

Proof. **Case 1: IV for $0 \le t \le \tau < T$: no announcement before expiration.**

$w_j\left(\hat{\theta}_t,t\right)$ is a martingale because $\mathbb{E}_t\left[w_j\left(\hat{\theta}_t,t\right)\right] = \mathbb{E}_t\left(\mathbb{E}_t\left[\ln p^j\left(\hat{\theta}_\tau,\tau\right)\right]\right) = w_j\left(\hat{\theta}_t,t\right)$ for $t \le \tau$, where $j = 1,2$, using law of iterated expectations. Therefore, $w_j\left(\hat{\theta}_t,t\right)$ is determined by the PDE

$$\mathcal{L}w_1\left(\hat{\theta}_t,t\right) = 0 \text{ and } \mathcal{L}w_2\left(\hat{\theta}_t,t\right) = 0, \tag{125}$$

which further determines Eq.(117) with the boundary condition at time τ described in (121).

Similarly, $\mathbb{E}_t[\delta(\tau)] = a_1\left(\hat{\theta}_t,t\right) + \delta(t)$ is a martingale. Therefore we must have the PDE

$$\mathcal{L}\left[a_1\left(\hat{\theta}_t,t\right) + \delta(t)\right] = 0. \tag{126}$$

together with the boundary condition $a_1\left(\hat{\theta}_\tau,\tau\right) = \mathbb{E}_\tau[\delta(\tau) - \delta(\tau)] = 0$. Using the law of motion of $\hat{\theta}_t$ from (61), δ_t from (89) and $a_j\left(\hat{\theta}_t,t\right)$, $j = 1,2,3$ from below

$$da_j\left(\hat{\theta}_t,t\right) = \left(\frac{\partial a_j}{\partial t} + \frac{\partial a_j}{\partial\hat{\theta}}\mu_{\theta,t} + \frac{1}{2}\frac{\partial^2 a_j}{\partial\hat{\theta}^2}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}\right)dt + \frac{\partial a_j}{\partial\hat{\theta}}\frac{\sigma_{\theta,t}}{\sigma_Y}d\hat{B}_{Y,t}, \tag{127}$$

Eq. (126) immediately gives (118).

Next, consider $a_2\left(\hat{\theta}_t, t\right)$. Note that

$$\begin{aligned}\mathbb{E}_t\left[\delta^2(\tau)\right] &= \mathbb{E}_t\left[(\delta(\tau)-\delta(t)+\delta(t))^2\right] \\ &= \delta(t)^2+2a_1\left(\hat{\theta}_t, t\right)\delta(t)+a_2\left(\hat{\theta}_t, t\right). \end{aligned} \tag{128}$$

The fact that $\mathbb{E}_t\left[\delta^2(\tau)\right]$ is a martingale implies that

$$\mathcal{L}\left[\delta(t)^2+2a_1\left(\hat{\theta}_t, t\right)\delta(t)+a_2\left(\hat{\theta}_t, t\right)\right]=0, \tag{129}$$

together with the boundary condition $a_2\left(\hat{\theta}_\tau, \tau\right)=\mathbb{E}_\tau\left[\{\delta(\tau)-\delta(\tau)\}^2\right]=0$. This gives

$$\begin{aligned} 2\delta_t\left(\xi\hat{\theta}_t+\bar{\theta}(1-\xi)-\frac{1}{2}\sigma_Y^2\right)+2\delta_t\left(\frac{\partial a_1}{\partial t}+\frac{\partial a_1}{\partial\hat{\theta}}\mu_{\theta,t}+\frac{1}{2}\frac{\partial a_1^2}{\partial\hat{\theta}^2}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}\right)+\sigma_Y^2 & \\ +2a_1\left(\xi\hat{\theta}_t+\bar{\theta}(1-\xi)-\frac{1}{2}\eta^2\sigma_Y^2\right)+2\frac{\partial a_1}{\partial\hat{\theta}}\sigma_{\theta,t}+\left(\frac{\partial a_2}{\partial t}+\frac{\partial a_2}{\partial\hat{\theta}}\mu_{\theta,t}+\frac{1}{2}\frac{\partial a_2^2}{\partial\hat{\theta}^2}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}\right) &= 0. \end{aligned} \tag{130}$$

Because the above equation must hold for all δ_t, matching the coefficient on δ_t and the rest, we obtain Eq.(118) and (119).

Last, we compute $a_3\left(\hat{\theta}_t, t\right)$. By definition, $\mathbb{E}_t\left[\ln p\left(\hat{\theta}_\tau, \tau\right)\delta(\tau)\right]=a_3\left(\hat{\theta}_t, t\right)+\mathbb{E}_t\left[\ln p\left(\hat{\theta}_\tau, \tau\right)\delta(t)\right]$ is also a martingale. Therefore,

$$\mathcal{L}\left[w_1\left(\hat{\theta}_t, t\right)\delta(t)+a_3\left(\hat{\theta}_t, t\right)\right]=0, \tag{131}$$

with the boundary condition $a_3\left(\hat{\theta}_\tau, \tau\right)=\mathbb{E}_\tau\left[\ln p\left(\hat{\theta}_\tau, \tau\right)\{\delta(\tau)-\delta(\tau)\}\right]=0$. Further,

$$\begin{aligned} 0 &= \delta_t\left(\frac{\partial w_1}{\partial t}+\frac{\partial w_1}{\partial\hat{\theta}}\mu_{\theta,t}+\frac{1}{2}\frac{\partial w_1^2}{\partial\hat{\theta}^2}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}\right)+w_1\left(\xi\hat{\theta}_t+\bar{\theta}(1-\xi)-\frac{1}{2}\sigma_Y^2\right) \\ &\quad +\frac{\partial w_1}{\partial\hat{\theta}}\sigma_{\theta,t}+\frac{\partial a_3}{\partial t}+\frac{\partial a_3}{\partial\theta}\mu_{\theta,t}+\frac{1}{2}\frac{\partial a_3^2}{\partial\theta^2}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2}, \end{aligned}$$

we finally obtain Eq.(120).

Case 2: IV for $\tau \geq T$: one announcement before expiration.

Case 1 computes the expectations only for $\tau < T$. If $\tau \geq T$, which is often the case if we want to talk about implied variance across an announcement, we need to deal with the *announcement boundary* separately, and we do it in three steps backward induction: we start from T^+, go back to (T^-, ν), and go back to T^- without knowing ν. Note that the output in each step will be the input for the next step.

Step 1: from $T^+ \to \tau$, we use the PDEs and boundary conditions (121) in Lemma 5 to calculate $w_j\left(\hat{\theta}_{T^+}, T^+\right)$, $j = 1, 2$ and $a_j\left(\hat{\theta}_{T^+}, T^+\right)$, $j = 1, 2, 3$.

Step 2: from $T^- \to T^+$, we calculate the boundary conditions. Because $\delta(t)$ is a stochastic integral, it must be continuous. We have $\delta(T^-) = \mathbb{E}_{T^-}[\delta(T^+)] = \delta(T^+)$. Using Eqs.(125), (126), (129) and (131), the martingale property implies

$$w_1\left(\hat{\theta}_T^-, T^-\right) = \mathbb{E}_{T^-}\left[w_1\left(\hat{\theta}_T^+, T^+\right)\right] \text{ and } w_2\left(\hat{\theta}_T^-, T^-\right) = \mathbb{E}_{T^-}\left[w_2\left(\hat{\theta}_T^+, T^+\right)\right]$$

$$\begin{aligned} a_1\left(\hat{\theta}_{T^-}, T^-\right) + \delta_{T^-} &= \mathbb{E}_{T^-}\left[a_1\left(\hat{\theta}_{T^+}, T^+\right) + \delta_{T^+}\right] \\ \Rightarrow a_1\left(\hat{\theta}_T^-, T^-\right) &= \mathbb{E}_{T^-}\left[a_1\left(\hat{\theta}_T^+, T^+\right)\right], \end{aligned} \tag{132}$$

$$\begin{aligned} \mathbb{E}_{T^-}\left[\delta_{T^+}^2 + 2a_1\left(\hat{\theta}_T^+, T^+\right)\delta_{T^+} + a_2\left(\hat{\theta}_T^+, T^+\right)\right] &= \delta_{T^-}^2 + 2\mathbb{E}_{T^-}\left[a_1\left(\hat{\theta}_T^+, T^+\right)\right]\delta_{T^-} + \mathbb{E}_{T^-}\left[a_2\left(\hat{\theta}_T^+, T^+\right)\right] \\ \Rightarrow a_2\left(\hat{\theta}_T^-, T^-\right) &= \mathbb{E}_{T^-}\left[a_2\left(\hat{\theta}_T^+, T^+\right)\right], \end{aligned} \tag{133}$$

$$\begin{aligned} \mathbb{E}_{T^-}\left[w_1\left(\hat{\theta}_T^+, T^+\right)\delta_{T^+} + a_3\left(\hat{\theta}_T^+, T^+\right)\right] &= w_1\left(\hat{\theta}_T^-, T^-\right)\delta_{T^-} + a_3\left(\hat{\theta}_T^-, T^-\right) \\ \Rightarrow a_3\left(\hat{\theta}_T^-, T^-\right) &= \mathbb{E}_{T^-}\left[a_3\left(\hat{\theta}_T^+, T^+\right)\right]. \end{aligned} \tag{134}$$

Then we follow the procedures in the proof of Lemma 1 to compute the boundary condition

$$w_j\left(\hat{\theta}_T^-, T^-\right) = \mathbb{E}\left[\mathbb{E}\left[w_j\left(\hat{\theta}_T^+, T^+\right)|\nu, \hat{\theta}_T^-, T^-\right]|\hat{\theta}_T^-, T^-\right], \; j = 1, 2; \tag{135}$$

$$a_j\left(\hat{\theta}_T^-, T^-\right) = \mathbb{E}\left[\mathbb{E}\left[a_j\left(\hat{\theta}_T^+, T^+\right)|\nu, \hat{\theta}_T^-, T^-\right]|\hat{\theta}_T^-, T^-\right] \; j = 1, 2, 3, \tag{136}$$

which correspond to Eqs.(123) and (124) in Lemma 5.

Step 3: from $t \to T^-$, we use the PDEs in Step 1 to solve the entire paths with the boundary conditions $w_j\left(\hat{\theta}_T^-, T^-\right)$ and $a_j\left(\hat{\theta}_T^-, T^-\right)$ obtained from Step 2. □

Implied variance reduction upon the announcement With the implied variance from Lemma 4, we could calculate the implied variance reduction upon the announcement. Right after the announcement, the IV is

$$\begin{aligned} IV_{T^+,\tau}\left(\hat{\theta}_T^+, T^+\right) &= w_2\left(\hat{\theta}_T^+, T^+\right) - w_1\left(\hat{\theta}_T^+, T^+\right)^2 + a_2\left(\hat{\theta}_T^+, T^+\right) - a_1\left(\hat{\theta}_T^+, T^+\right)^2 \\ &\quad + 2\left[a_3\left(\hat{\theta}_T^+, T^+\right) - w_1\left(\hat{\theta}_T^+, T^+\right)a_1\left(\hat{\theta}_T^+, T^+\right)\right] + g\left(\omega_T^+, T^+\right), \end{aligned} \tag{137}$$

Right before the announcement, conditional on a given informativeness ν, the IV is

$$\begin{aligned}\tilde{IV}_{T^-,\tau}\left(\nu,\hat{\theta}_T^-,T^-\right) &= \mathbb{E}\left[IV_{T^+,\tau}\left(\hat{\theta}_T^+,T^+\right)|\nu,\hat{\theta}_T^-,T^-\right] \\ &= \tilde{w}_2\left(\nu,\hat{\theta}_T^-,T^-\right)-\tilde{w}_1\left(\nu,\hat{\theta}_T^-,T^-\right)^2+\tilde{a}_2\left(\nu,\hat{\theta}_T^-,T^-\right)-\tilde{a}_1\left(\nu,\hat{\theta}_T^-,T^-\right)^2 \\ &\quad +2\left[\tilde{a}_3(\nu,\hat{\theta}_T^-,T^-)-\tilde{w}_1(\nu,\hat{\theta}_T^-,T^-)\tilde{a}_1(\nu,\hat{\theta}_T^-,T^-)\right]+g\left(\omega_T^-,T^-\right). \quad (138)\end{aligned}$$

Therefore, the implied variance reduction is defined as

$$\Delta IV_T=\tilde{IV}_{T^-,\tau}\left(\nu,\hat{\theta}_T^-,T^-\right)-IV_{T^+,\tau}\left(\hat{\theta}_T^+,T^+\right), \quad (139)$$

which corresponds to Eq.(22) in the main text. Note that the first term denotes the IV before the announcement. It is captured by the conditional expectation of IV after the announcement, conditional on information right before the announcement at T^- *and* the distribution of ν. Instead of using unconditional expectation before the announcement, we use conditional expectation because it immediately reflects investors' belief updating conditional on the informativeness ν. The conditional expectation better measures information-induced changes in implied variance.

7.2.4 Policy functions

In all of the computations below, we plot policy functions for different choices of $\nu\in[0.5,1]$. We assume that $\hat{\theta}_T^-=\bar{\theta}=\frac{\lambda_L\theta_H+\lambda_H\theta_L}{\lambda_L+\lambda_H}$. Given $\hat{\theta}_T^-=\bar{\theta}$ and given a ν, the distribution of $\hat{\theta}_T^+$ is given by Eq.(15), or equivalently, Eqs.(49) to (52).

Announcement premium Define the return earned upon the announcement as $R^A=\frac{p\left(\hat{\theta}_T^+,0\right)}{\tilde{p}\left(\nu,\hat{\theta}_T^-,0\right)}$. For a given ν, the announcement premium in the model is

$$\mathbb{E}\left[R^A\middle|\,\nu,\hat{\theta}_T^-,T^-\right]-1=\frac{\mathbb{E}\left[p\left(\hat{\theta}_T^+,T^+\right)\middle|\,\nu,\hat{\theta}_T^-,T^-\right]}{\tilde{p}\left(\nu,\hat{\theta}_T^-,T^-\right)}-1=\frac{h_{s_H}p\left(\hat{\theta}_{s_H}^+,T^+\right)+h_{s_L}p\left(\hat{\theta}_{s_L}^+,T^+\right)}{\tilde{p}\left(\nu,\hat{\theta}_T^-,T^-\right)}-1, \quad (140)$$

where $\tilde{p}\left(\nu,\hat{\theta}_T^-,T^-\right)$ is defined in Eq.(76).

Implied variance reduction upon the announcement Because the dividend is continuous upon the announcement, the IV reduction can be computed as

$$\begin{aligned}\Delta IV_T &= \mathrm{Var}\left[\ln p\left(\hat{\theta}_T^+, T^+\right) - \ln\tilde{p}\left(\nu, \hat{\theta}_T^-, T\right)\middle|\, \nu, \hat{\theta}_T^-, T^-\right] = \mathrm{Var}\left[\ln p\left(\hat{\theta}_T^+, T^+\right)\middle|\, \nu, \hat{\theta}_T^-, T^-\right] \quad (141)\\ &= \mathbb{E}\left[\ln p\left(\hat{\theta}_T^+, T^+\right)^2\middle|\, \nu, \hat{\theta}_T^-, T^-\right] - \left(\mathbb{E}\left[\ln p\left(\hat{\theta}_T^+, T^+\right)\middle|\, \nu, \hat{\theta}_T^-, T^-\right]\right)^2\\ &= \left[h_{s_H}\ln p\left(\hat{\theta}_{s_H}^+, T^+\right)^2 + h_{s_L}\ln p\left(\hat{\theta}_{s_L}^+, T^+\right)^2\right] - \left[h_{s_H}\ln p\left(\hat{\theta}_{s_H}^+, T^+\right) + h_{s_L}\ln p\left(\hat{\theta}_{s_L}^+, T^+\right)\right]^2 \quad (142)\end{aligned}$$

30-day expected variance after the announcement going forward We first compute the 30-day implied variance after the announcement:

$$IV_{T^+,T+\Delta}\left(\hat{\theta}_T^+, T^+\right) = \mathrm{Var}\left[\ln p\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) + \ln D_{T+\Delta}\middle|\, \hat{\theta}_T^+, T^+\right]. \tag{143}$$

where $\Delta = \frac{1}{12}$ for monthly return. Note that $IV_{T^+,\tau}\left(\hat{\theta}_T^+, T^+\right)$ is a random variable that depends on the realization of $\hat{\theta}_T^+$. This is the same as the implied variance defined in (137). To compute expected value, we compute

$$\mathbb{E}\left[IV_{T^+,T+\Delta}\left(\hat{\theta}_T^+, T^+\right)\middle|\, \nu, \hat{\theta}_T^-, T^-\right] = h_{s_H} IV_{T^+,T+\Delta}\left(\hat{\theta}_{s_H}^+, T^+\right) + h_{s_L} IV_{T^+,T+\Delta}\left(\hat{\theta}_{s_L}^+, T^+\right). \tag{144}$$

Expected 30-day return after the announcement going forward Using the expression (90), the expected log return is given by:

$$\begin{aligned}ER_{t,T+\Delta}\left(\hat{\theta}_T^+, T^+\right) &= \mathbb{E}\left[\ln\left[\frac{p\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) D_{T+\Delta} + \int_{T^+}^{T+\Delta} D_s ds}{p\left(\hat{\theta}_T^+, T^+\right) D_{T^+}}\right]\middle|\, \hat{\theta}_T^+, T^+\right]\\ &\approx \mathbb{E}\left[\ln\left[\frac{p\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) D_{T+\Delta} + D_{T+\Delta}\Delta}{p\left(\hat{\theta}_T^+, T^+\right) D_{T^+}}\right]\middle|\, \hat{\theta}_T^+, T^+\right]\\ &= \mathbb{E}\left[\ln\left[p\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) + \Delta\right]\middle|\, \hat{\theta}_T^+, T^+\right]\\ &\quad -\ln p\left(\hat{\theta}_T^+, T^+\right) + a_1\left(\hat{\theta}_T^+, T^+\right) + g\left(\omega_T^+, T^+\right),\end{aligned} \tag{145}$$

where $\Delta = \frac{1}{12}$ for monthly return. The only unknown in the above equation is $w_3\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) = \mathbb{E}\left[\ln\left[p\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) + \Delta\right]\middle|\, \hat{\theta}_T^+, T^+\right]$. We apply the same procedure in computing $w_1\left(\hat{\theta}_T^+, T^+\right)$ with the only difference of the boundary condition $w_3\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) = \ln p\left(\hat{\theta}_{T+\Delta}, T+\Delta\right) +$

Δ.

Then we compute the average expected return after announcement as

$$\mathbb{E}\left[ER_{t,\tau}\left(\hat{\theta}_T^+, T^+\right)\middle|\, \nu, \hat{\theta}_T^-, T^-\right] = h_{s_H} ER_{t,\tau}\left(\hat{\theta}_{s_H}^+, T^+\right) + h_{s_L} ER_{t,\tau}\left(\hat{\theta}_{s_L}^+, T^+\right). \tag{146}$$

7.3 Data

For model calibration purposes, we use consumption growth, dividend growth, total market excess returns, and the risk-free rate (all in real terms) for the period 1929-2019. More specifically, we use BEA data on real per capita annual consumption growth of nondurables and services. We also use annual dividends data from the CRSP value-weighted portfolios of all stocks traded on the NYSE, AMEX, and NASDAQ. Stock market excess returns and risk-free rates are from Kenneth French's data library. All nominal quantities are deflated using the annual CPI. To illustrate and compare the volatility of consumption growth and stock market excess returns in Figure 1, we switch to a monthly frequency for the period 1960.02-2019.12.

The dates for FOMC meetings are from the website of the Board of Governors of the Federal Reserve System. Following Savor and Wilson (2014), we only include the pre-scheduled FOMC meetings during our data period (1994.09-2019.12). About eight regularly pre-scheduled FOMC meetings occur each year. When the meeting lasts for two days, we consider the second day the FOMC announcement day. In total, our data period contains 203 FOMC announcement days.

To determine the ex post measure of informativeness, we use the difference between the squared option-implied volatility index, VIX^2 before and after announcement days. We obtain data on VIX from the Chicago Board Options Exchange (CBOE) for the period 1990-2019. The CBOE's VIX is a model-free measure of the implied variance computed from the S&P 500 Index option prices.

To determine the ex ante measure of informativeness, we construct the informativeness measure by exploring the S&P 500 Index option panels before the FOMC meetings. We use equity-options data from OptionMetrics for the period January 1, 1996, to December 31, 2019. We exclude options with missing or negative bid-ask spreads, zero bids, or zero open interest. We restrict the sample to out-of-the-money (OTM) options to estimate the model-free implied variance (Bakshi, Kapadia, and Madan (2003)). To ensure that our results are not driven by misleading prices, we follow Conrad, Dittmar, and Ghysels (2013) and exclude options that do not satisfy the standard option price bounds. We further remove options with maturities less than 7 days or more than 180 days.

We define $IV_{t,\tau}$ as the time-t price of the τ-maturity quadratic payoff on the underlying stock, $IV_{t,\tau} \equiv e^{-r_f \tau}\mathbb{E}_t^{\mathbb{Q}}\left[r_{t,t+\tau}^2\right]$, where r_f is the continuously compounded interest rate and $r_{t,t+\tau}$ is the logarithm return over time τ periods of time. Bakshi, Kapadia, and Madan (2003) show that $IV_{t,\tau}$ can be recovered from the prices of OTM call and put options as follows:

$$IV_{t,\tau} = \int_{S_t}^{\infty} \frac{1 - \ln\left(K/S_t\right)}{K^2/2} C_t\left(\tau; K\right) dK + \int_0^{S_t} \frac{1 + \ln\left(S_t/K\right)}{K^2/2} P_t\left(\tau; K\right) dK, \tag{147}$$

where S_t is the price of the underlying stock and $C_t\left(\tau; K\right)$ and $P_t\left(\tau; K\right)$ are call and put prices with maturity τ and strike K, respectively.

We compute $IV_{t,\tau}$ on each day and each day to maturity. In theory, computing $IV_{t,\tau}$ requires a continuum of strike prices, whereas in practice, we only observe a discrete and finite set of them. Following Jiang and Tian (2005) and others, we discretize the integrals in Eq.(147) by setting up a total of 1,001 grid points in the moneyness (K/S_t) ranging from 1/3 to 3. First, we use cubic splines to interpolate the implied volatility inside the available moneyness range. Second, we extrapolate the implied volatility using the boundary values to fill in the rest of the grid points. Third, we calculate option prices from these 1,001 implied volatilities using the formula of Scholes and Black (1973). Lastly, we compute $IV_{t,\tau}$ if the number of OTM options is more than four (e.g., Conrad, Dittmar, and Ghysels (2013) and others). This process yields a daily panel of the risk-neutral expected quadratic payoff with various maturities.

In the empirical section, we use the S&P 500 Index return at the daily frequency to conduct return predictability regressions. We also use 5-min intra-daily data on the S&P 500 Index return to compute the realized variance for one day, two days, ..., one month, two months,..., several months. Over a small interval dt, where dividend is negligible, we have

$$r_{t,t+dt} = \ln R_{t+dt} - \ln R_t. \tag{148}$$

To conserve space, we illustrate the construction method for the daily logarithm return $r_{t,t+\Delta}$ and the daily realized variance $RV_{t,t,t+\Delta}$. Both are defined by aggregating $r_{t,t+dt}$ and $r_{t,t+dt}^2$, respectively:

$$r_{t,t+\Delta} = \sum_{j=1}^{\Delta/dt} r_{t+(j-1)dt,t+jdt} + r_{t,O}, RV_{t,t+\Delta}\left(dt\right) = \sum_{j=1}^{\Delta/dt} r_{t+(j-1)dt,t+jdt}^2 + r_{t,O}^2, \tag{149}$$

where Δ/dt is the number of high-frequency returns in a day (e.g., $dt = 5$ for 5-min intra-daily returns), $r_{t,O}$ denotes the overnight return between the previous day at close and day t at open, and $r_{t+(j-1)dt,t+jdt}$ denotes the j^{th} high-frequency return of the daily period during day t.

7.4 Regime Switching Model

In this section, we apply a regime switching model to the idiosyncratic shock of the dividend growth rate in Eq. (16). To measure the idiosyncratic shock ϵ_t, we regress the dividend growth rate d_t on the consumption growth rate c_t. The residuals from this regression capture the innovations in the dividend growth rate that are uncorrelated with the consumption growth rate:

$$d_t = \alpha + \beta c_t + \epsilon_t.$$

Next, we assume that the idiosyncratic shock ϵ_t follows a two-state Markov switching model. For each state, ϵ_t depends on an intercept $\mu = [\mu_H, \mu_L]$ and a volatility $\omega = [\omega_H, \omega_L]$ as follows:

$$\begin{array}{cc} \epsilon_t = \mu_H + \omega_H e_t & \text{high volatility state} \\ \epsilon_t = \mu_L + \omega_L e_t & \text{low volatility state} \end{array},$$

where $\omega_H > \omega_L$ and $e_t \sim iid\, \mathcal{N}(0,1)$. The process governing the dynamics of the underlying regime is specified as a homogeneous first-order Markov chain,

$$\begin{bmatrix} 1-\kappa_H & \kappa_H \\ \kappa_L & 1-\kappa_L \end{bmatrix},$$

where $\kappa = [\kappa_H, \kappa_L]$ and $\kappa_H(\kappa_L)$ denotes the probability of transition from a high (low) to low (high) state of dividend idiosyncratic volatility.

Finally, we take this model to the data for consumption growth and dividend growth for the annual sample period 1929-2019. The estimated intercept $\mu = [0.006, -0.004]$, estimated volatility $\omega = [0.1378, 0.0418]$, and the estimated transition probability is $\kappa = [0.0603, 0.0246]$.

Online Appendix

In this Online Appendix, we provide a robustness check for the empirical analysis and we describe the numerical solutions we use to solve for our dynamic model.

Robustness check In the main text Table 7, we show the regression coefficients on the measure of the informativeness of the announcement are significantly negative. Here we test whether our predictability results in Table 7 still hold if we also include the implied variance on the announcement day, IV_t, that is known to predict the realized variance. Specifically, we consider the alternative regressions:

$$RV_{t,t+h} = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \beta_3 IV_t + \varepsilon_{t,t+h}, \tag{150}$$

for the realized variance predictability and

$$R_{t,t+h} - r_f = \alpha + \beta_1 \Delta IV_t + \beta_2 RV_{t-2,t-1} + \beta_3 IV_t + \varepsilon_{t,t+h}, \tag{151}$$

for the realized excess return predictability.

Table 12: Model-Implied Return and Variance Predictability by IV Reduction

Number of days		1	2	3	4	5	30	60
$RV_{t,t+h}$	Data	-0.07	-0.04	-0.02	-0.02	-0.02	-0.03	-0.01
		(-4.21)	(-3.52)	(-1.53)	(-1.81)	(-1.93)	(-3.32)	(-0.98)
R^2 (%)		81.39	75.86	61.70	64.48	67.30	52.38	41.66
$R_{t,t+h} - r_f$	Data	-2.00	-2.17	-0.96	-1.21	-0.68	-0.26	-0.23
		(-0.77)	(-1.57)	(-1.44)	(-1.80)	(-1.52)	(-1.61)	(-2.65)
R^2 (%)		5.23	3.96	3.27	7.87	1.75	1.52	1.90

This table presents the results of the realized variance predictability regression (150) and the excess return predictability regression (151) in the data and the model. The columns 3-9 represent the horizon of returns and variances on the left-hand side of (150) and (151), respectively, with $h = 1, 2, 3, 4, 5, 30, 60$ calendar days. The data include the period 1994.09-2019.12. Returns and realized variances are in daily basis points. Implied variance reductions are in monthly percentage squared units. Newey-West t-statistics are in parentheses.

Table 12 shows that more informative announcements are associated with a lower realized variance and lower expected return after the announcements going forwards. Therefore, our predictability results remain true if we also include the implied variance on the announcement day, IV_t.

Now we describe the numerical solutions we used to solve for our dynamic model. We first describe how we solve the daily returns, realized variance using the simulated sample paths. Then we demonstrate the finite difference method to solve for the value function, price-to-dividend ratio, as well as the implied variance.

Computing returns The return earned from t to $t+\Delta$ is defined as

$$R_{t,t+\Delta} = \frac{p\left(\hat{\theta}_{t+\Delta}, t+\Delta\right) D_{t+\Delta} + \int_t^{t+\Delta} D_s ds}{p\left(\hat{\theta}_t, t\right) D_t}. \tag{152}$$

Numerically, $\Delta = 1/360$ for daily return, we approximate

$$\begin{aligned} R_{t,t+\Delta} &= \frac{p\left(\hat{\theta}_{t+\Delta}, t+\Delta\right) \frac{D_{t+\Delta}}{D_t} + \int_t^{t+\Delta} \frac{D_s}{D_t} ds}{p\left(\hat{\theta}_t, t\right)} = \frac{p\left(\hat{\theta}_{t+\Delta}, t+\Delta\right)\left(1+\frac{dD_t}{D_t}\right) + \int_t^{t+\Delta}\left(1+\frac{dD_{s-t}}{D_t}\right) ds}{p\left(\hat{\theta}_t, t\right)} \\ &= \frac{p\left(\hat{\theta}_{t+\Delta}, t+\Delta\right)\left(1+\frac{dD_t}{D_t}\right) + \left(1+1+\frac{dD_t}{D_t}\right)\frac{\Delta}{2}}{p\left(\hat{\theta}_t, t\right)} = \frac{p\left(\hat{\theta}_{t+\Delta}, t+\Delta\right)\left(1+\frac{dD_t}{D_t}\right) + \left(\Delta+\frac{dD_t}{D_t}\frac{\Delta}{2}\right)}{p\left(\hat{\theta}_t, t\right)} \end{aligned} \tag{153}$$

Computing realized variance Over a small interval dt, dividend is negligible, we have

$$r_{t,t+dt} = \ln R_{t+dt} - \ln R_t, \tag{154}$$

where the law of motion of R_t in the interior follows Lemma 3. In the data, we compute

$$RV_{t,t+\Delta}(dt) = \sum_{j=1}^{\Delta/dt} r^2_{t+(j-1)dt,t+jdt}. \tag{155}$$

Over a 5-minute interval subtracting a mean is quantitatively irrelevant. It can be shown as the $dt \to 0$, the above quantitative result converges to the quadratic variation of the return process:

$$\lim_{\Delta\to 0} RV_{t,t+\Delta}(dt) = \int_t^{t+dt}\left(\sigma^2_{R,s}+\omega^2_s\right) ds + \sum_{t\le s\le t+\Delta} I_{\{s\}}\left[\ln p\left(\hat{\theta}^+_s, s^+\right) - \ln p\left(\hat{\theta}^-_s, s^-\right)\right]^2 \tag{156}$$

where $I_{\{t\}}$ is the announcement indicator, $I = 1$ if (t mod $T = 0$). Therefore, numerically, over a day, that is, $\Delta = \frac{1}{360}$, we simply compute

$$\begin{aligned} RV_{t,t+\Delta} &\approx \frac{1}{2}\left[\sigma_R^2\left(\hat{\theta}_t, t\right) + \sigma_R^2\left(\hat{\theta}_{t+\Delta}, t+\Delta\right) + \sigma_i^2\left(\hat{\theta}_t, t\right) + \sigma_i^2\left(\hat{\theta}_{t+\Delta}, t+\Delta\right)\right]\Delta \\ &\quad + \sum_{t\le s\le t+\Delta} I_{\{s\}}\left[\ln p\left(\hat{\theta}_s^+, s^+\right) - \ln p\left(\hat{\theta}_s^-, s^-\right)\right]^2. \end{aligned} \tag{157}$$

Calculating the risk-free rate Take the expression of the risk-free rate in (68), the cumulative risk-free return from t to $t+\Delta$ is:

$$e^{\int_t^{t+\Delta} r\left(\hat{\theta}_s, s\right) ds} \approx \exp\left\{\sum_{j=1}^{\Delta/dt} r\left(\hat{\theta}_{t+(j-1)dt}, t + (j-1)\,dt\right) dt\right\}. \tag{158}$$

This way we can accumulate daily risk-free return to compute annual risk-free rate for $\Delta = 1$.

Solve for H function We use finite difference method to solve for the value function. The HJB equation in Eq.(62) can be rewritten as:

$$(1-\gamma)\left(\frac{\rho}{1-\frac{1}{\psi}} - \hat{\theta}_t + \frac{1}{2}\gamma\sigma_Y^2\right) H = H_t + H_\theta\left[\mu_{\theta,t} + (1-\gamma)\,\sigma_{\theta,t}\right] + \frac{1}{2}H_{\theta\theta}\frac{\sigma_{\theta,t}^2}{\sigma_Y^2} + \frac{\rho\,(1-\gamma)}{1-\frac{1}{\psi}} H^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}.$$

Use finite difference method and approximate the functions $H\left(\hat{\theta}_t, t\right)$ at I discrete points in the space dimensions, $\hat{\theta}_i$, $i = 1, 2, ..., I$. Denote $H_i^n = H\left(\hat{\theta}_i, t^n\right)$, where time dimension $n = 0, 1, 2, ..., N$. Denote

$$\beta_i = (1-\gamma)\left(\frac{\rho}{1-\frac{1}{\psi}} - \hat{\theta}_i + \frac{1}{2}\gamma\sigma_Y^2\right), \tag{159}$$

$$u_i^{n+1} = \frac{\rho\,(1-\gamma)}{1-\frac{1}{\psi}}\left(H_i^{n+1}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}. \tag{160}$$

Use implicit method to update the value function,

$$\begin{aligned} \beta_i H_i^n &= \frac{H_i^{n+1} - H_i^n}{\Delta t} + u_i^{n+1} + \frac{1}{2}\partial_{\theta\theta} H_i^n \frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \\ &\quad + \partial_{\theta,F} H_i^n\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^+ + \partial_{\theta,B} H_i^n\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^-. \end{aligned} \tag{161}$$

Use upwind scheme to approximate the derivatives $\partial_\theta H_i^n$ and $\partial_{\theta\theta} H_i^n$,

$$\begin{aligned}
\beta_i H_i^n &= \frac{H_i^{n+1}-H_i^n}{\Delta t} + u_i^{n+1} + \frac{1}{2}\frac{H_{i+1}^n - 2H_i^n + H_{i-1}^n}{\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \\
&\quad + \frac{H_{i+1}^n - H_i^n}{\Delta\hat{\theta}}\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^+ + \frac{H_i^n - H_{i-1}^n}{\Delta\hat{\theta}}\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^- . \quad (162)
\end{aligned}$$

Collecting terms and rewrite HJB equation,

$$\beta_i H_i^n = \frac{H_i^{n+1}-H_i^n}{\Delta t} + u_i^{n+1} + H_{i-1}^n x_i + H_i^n y_i + H_{i+1}^n z_i \quad (163)$$

where

$$x_i = -\frac{\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^-}{\Delta\hat{\theta}} + \frac{1}{2\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (164)$$

$$y_i = -\frac{\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^+}{\Delta\hat{\theta}} + \frac{\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^-}{\Delta\hat{\theta}} - \frac{1}{\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (165)$$

$$z_i = \frac{\left[\mu_{\theta,i} + (1-\gamma)\,\sigma_{\theta,i}\right]^+}{\Delta\hat{\theta}} + \frac{1}{2\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \quad (166)$$

Rewrite in the matrix notation,

$$\beta H^n = u^{n+1} + \mathbf{A}^{n+1} H^n + \frac{H^{n+1}-H^n}{\Delta t}, \quad (167)$$

where

$$\mathbf{A}^{n+1} = \begin{bmatrix} y_1 & z_1 & 0 & \cdots & 0 \\ x_2 & y_2 & z_2 & 0 & \vdots \\ 0 & x_3 & y_3 & z_3 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & x_I & y_I \end{bmatrix}, H^n = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \vdots \\ H_I \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 & 0 & & & \cdots \\ 0 & \beta_2 & & & \\ & & \beta_3 & & \\ & & & \ddots & 0 \\ \vdots & & & & \beta_I \end{bmatrix}, u^{n+1} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_I \end{bmatrix}.$$

The system can be written as

$$\mathbf{B}^{n+1} H^n = b^{n+1},\ \mathbf{B}^{n+1} = \left(\frac{1}{\Delta t} + \beta\right) - \mathbf{A}^{n+1},\ b^{n+1} = u^{n+1} + \frac{1}{\Delta t} H^{n+1}. \quad (168)$$

with the boundary condition

$$H\left(\hat{\theta}^{-},T\right)=\mathbb{E}\left[\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)|\hat{\theta}^{-},T\right];\ \tilde{H}\left(\nu,\hat{\theta}^{-},T\right)=\mathbb{E}\left[H\left(\hat{\theta}^{+},0\right)\mid\nu,\hat{\theta}^{-},T\right],$$

where from (64) and (63),

$$\begin{aligned}\tilde{H}\left(\nu,\hat{\theta}^{-},T\right) &= h_{s_H}H\left(\hat{\theta}^{+}_{s_H},0\right)+h_{s_L}H\left(\hat{\theta}^{+}_{s_L},0\right) \qquad (169)\\ &= \left[\frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}\nu+\frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}\left(1-\nu\right)\right]H\left(\theta_L+\frac{\left(\hat{\theta}^{-}-\theta_L\right)\nu\left(\theta_H-\theta_L\right)}{\left(\hat{\theta}^{-}-\theta_L\right)\nu+\left(\theta_H-\hat{\theta}^{-}\right)\left(1-\nu\right)},0\right)\\ &+\left[\frac{\hat{\theta}^{-}-\theta_L}{\theta_H-\theta_L}\left(1-\nu\right)+\frac{\theta_H-\hat{\theta}^{-}}{\theta_H-\theta_L}\nu\right]H\left(\theta_L+\frac{\left(\hat{\theta}^{-}-\theta_L\right)\left(1-\nu\right)\left(\theta_H-\theta_L\right)}{\left(\hat{\theta}^{-}-\theta_L\right)\left(1-\nu\right)+\left(\theta_H-\hat{\theta}^{-}\right)\nu},0\right).\end{aligned}$$

$$H\left(\hat{\theta}^{-},T\right)=\mathbb{E}\left[\tilde{H}\left(\nu,\hat{\theta}^{-},T\right)|\hat{\theta}^{-},T\right]=\sum_{j=1}^{m}q_j\tilde{H}\left(\nu_j,\hat{\theta}^{-},T\right). \qquad (170)$$

Solve for price-to-dividend ratio Similarly, the PDE for $p\left(\hat{\theta}_t,t\right)$ is defined in Lemma 2. Use finite difference method and approximate the functions $p\left(\hat{\theta}_t,t\right)$ at I discrete points in the space dimensions, $\hat{\theta}_i$, $i=1,2,...,I$. Denote $p_i^n=p\left(\hat{\theta}_i,t^n\right)$, where time dimension $n=0,1,2,...,N$. Denote

$$\begin{aligned}\varpi_i^{n+1} &= \left(\xi-1\right)\bar{\theta}+\rho-\frac{1}{2}\gamma\sigma_Y^2\left(\frac{1}{\psi}+1\right)+\gamma\sigma_Y^2-\left(\xi-\frac{1}{\psi}\right)\hat{\theta}_i+\frac{\left(\frac{1}{\psi}-\gamma\right)\left(1-\frac{1}{\psi}\right)}{2(1-\gamma)^2}\left(\frac{H_{\theta,i}^{n+1}}{H_i^{n+1}}\right)^2\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \qquad (171)\\ \varrho_i^{n+1} &= \mu_{\theta,i}+\left(1-\gamma\right)\sigma_{\theta,i}+\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_{\theta,i}^{n+1}}{H_i^{n+1}}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2}. \qquad (172)\end{aligned}$$

Use implicit method to update the price-to-dividend ratio (74),

$$\varpi_i^{n+1}p_i^n = \frac{p_i^{n+1}-p_i^n}{\Delta t}+1+\frac{1}{2}\partial_{\theta\theta}p_i^n\frac{\sigma_{\theta,i}^2}{\sigma_Y^2}+\partial_{\theta,F}p_i^n\left(\varrho_i^{n+1}\right)^{+}+\partial_{\theta,B}p_i^n\left(\varrho_i^{n+1}\right)^{-}. \qquad (173)$$

Use upwind scheme to approximate the derivatives $\partial_\theta p_{i,j}^n$ and $\partial_{\theta\theta}p_{i,j}^n$,

$$\begin{aligned}\varpi_i^{n+1}p_i^n &= \frac{p_i^{n+1}-p_i^n}{\Delta t}+1+\frac{1}{2}\frac{p_{i+1}^n-2p_i^n+p_{i-1}^n}{\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2}\\ &+\frac{p_{i+1}^n-p_i^n}{\Delta\hat{\theta}}\left(\varrho_i^{n+1}\right)^{+}+\frac{p_i^n-p_{i-1}^n}{\Delta\hat{\theta}}\left(\varrho_i^{n+1}\right)^{-}. \qquad (174)\end{aligned}$$

Collecting terms and rewrite the PDE,

$$\varpi_i^{n+1} p_i^n = \frac{p_i^{n+1} - p_i^n}{\Delta t} + 1 + p_{i-1}^n x_i^{n+1} + p_i^n y_i^{n+1} + p_{i+1}^n z_i^{n+1} \tag{175}$$

where

$$x_i^{n+1} = -\frac{\left(\varrho_i^{n+1}\right)^-}{\Delta\hat{\theta}} + \frac{1}{2\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \tag{176}$$

$$y_i^{n+1} = -\frac{\left(\varrho_i^{n+1}\right)^+}{\Delta\hat{\theta}} + \frac{\left(\varrho_i^{n+1}\right)^-}{\Delta\hat{\theta}} - \frac{1}{\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \tag{177}$$

$$z_i^{n+1} = \frac{\left(\varrho_i^{n+1}\right)^+}{\Delta\hat{\theta}} + \frac{1}{2\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \tag{178}$$

Rewrite in the matrix notation,

$$\varpi^{n+1} p^n = 1 + \mathbf{A}^{n+1} p^n + \frac{p^{n+1} - p^n}{\Delta t}, \tag{179}$$

The system can be written as

$$\mathbf{B}^{n+1} p^n = b^{n+1},\ \mathbf{B}^{n+1} = \left(\frac{1}{\Delta t} + \varpi^{n+1}\right) - \mathbf{A}^{n+1},\ b^{n+1} = 1 + \frac{1}{\Delta t} p^{n+1}. \tag{180}$$

where

$$p^n = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_I \end{bmatrix},\ \varpi^{n+1} = \begin{bmatrix} \varpi_1 & 0 & & & \cdots \\ 0 & \varpi_2 & & & \\ & & \varpi_3 & & \\ & & & \ddots & 0 \\ \vdots & & & & \varpi_I \end{bmatrix}.$$

At the boundary,

$$\tilde{p}\left(\nu,\hat{\theta}_T^-,T\right) = \frac{h_{s_H} H\left(\hat{\theta}_{s_H}^+,0\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} p\left(\hat{\theta}_{s_H}^+,0\right) + h_{s_L} H\left(\hat{\theta}_{s_L}^+,0\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} p\left(\hat{\theta}_{s_L}^+,0\right)}{\left[h_{s_H} H\left(\hat{\theta}_{s_H}^+,0\right) + h_{s_L} H\left(\hat{\theta}_{s_L}^+,0\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}} \tag{181}$$

$$p\left(\hat{\theta}^-,T\right) = = \frac{\sum_{j=1}^m q_j \tilde{H}\left(\nu_j,\hat{\theta}^-,T\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} \tilde{p}\left(\nu_j,\hat{\theta}^-,T\right)}{\left[\sum_{j=1}^m q_j \tilde{H}\left(\nu_j,\hat{\theta}^-,T\right)\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}. \tag{182}$$

Solve for implied variance The PDEs are in general as the form of

$$0 = \frac{w_i^{n+1}-w_i^n}{\Delta t} + u_i^{n+1} + \frac{1}{2}\frac{w_{i+1}^n - 2w_i^n + w_{i-1}^n}{\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} + \frac{w_{i+1}^n - w_i^n}{\Delta\hat{\theta}}\mu_{\theta,i}^+ + \frac{w_{i,j}^n - w_{i-1}^n}{\Delta\hat{\theta}}\mu_{\theta,i}^-. \tag{183}$$

where $u_i^{n+1}=0$ for $w_1 = \mathbb{E}_t\left[\ln p\left(\hat{\theta}_\tau,\tau\right)\right]$ and $w_2 = \mathbb{E}_t\left[\ln^2 p\left(\hat{\theta}_\tau,\tau\right)\right]$, whereas for a_1, a_0 and a_3,

$$u_i^{n+1} = \xi\hat{\theta}_t + \bar{\theta}(1-\xi) - \frac{1}{2}\sigma_Y^2 \tag{184}$$

$$u_i^{n+1} = 2a_{1,i}^{n+1}\left(\xi\hat{\theta}_i + \bar{\theta}(1-\xi) - \frac{1}{2}\sigma_Y^2\right) + 2\frac{a_{1,i+1}^{n+1} - a_{1,i}^{n+1}}{\Delta\hat{\theta}}\sigma_{\theta,t} + \sigma_Y^2 \tag{185}$$

$$u_i^{n+1} = w_{1,i}^{n+1}\left(\xi\hat{\theta}_t + \bar{\theta}(1-\xi) - \frac{1}{2}\sigma_Y^2\right) + \frac{w_{1,i+1}^{n+1} - w_{1,i}^{n+1}}{\Delta\hat{\theta}}\sigma_{\theta,t}. \tag{186}$$

Therefore, rewrite the PDE as

$$0 = \frac{w_i^{n+1}-w_i^n}{\Delta t} + u_i^{n+1} + w_{i-1}^n x_i + w_i^n y_i + w_{i+1}^n z_i, \tag{187}$$

where

$$x_i = -\frac{\mu_{\theta,i}^-}{\Delta\hat{\theta}} + \frac{1}{2\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \tag{188}$$

$$y_i = -\frac{\mu_{\theta,i}^+}{\Delta\hat{\theta}} + \frac{\mu_{\theta,i}^-}{\Delta\hat{\theta}} - \frac{1}{\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2} \tag{189}$$

$$z_i = \frac{\mu_{\theta,i}^+}{\Delta\hat{\theta}} + \frac{1}{2\left(\Delta\hat{\theta}\right)^2}\frac{\sigma_{\theta,i}^2}{\sigma_Y^2}. \tag{190}$$

Rewrite in the matrix notation,

$$0 = u^{n+1} + \mathbf{A}^{n+1} w^n + \frac{w^{n+1} - w^n}{\Delta t}. \tag{191}$$

The system can be written as

$$\mathbf{B}^{n+1} w^n = b^{n+1},\ \mathbf{B}^{n+1} = \frac{1}{\Delta t}\mathbf{I} - \mathbf{A}^{n+1},\ b^{n+1} = u^{n+1} + \frac{1}{\Delta t} w^{n+1}. \tag{192}$$

with the boundary condition for $\mathbb{E}_t \left[\ln p \left(\hat{\theta}_\tau, \tau \right) \right]$ is:

$$w_1 \left(\hat{\theta}, \tau \right) = \ln p \left(\hat{\theta}_\tau, \tau \right), \tag{193}$$

and boundary condition for $\mathbb{E}_t \left[\ln^2 p \left(\hat{\theta}_\tau, \tau \right) \right]$ is:

$$w_2 \left(\hat{\theta}, \tau \right) = \ln^2 p \left(\hat{\theta}_\tau, \tau \right), \tag{194}$$

and the rest boundary conditions

$$a_1 \left(\hat{\theta}_\tau, \tau \right) = 0,\ a_0 \left(\hat{\theta}_\tau, \tau \right) = 0 \text{ and } a_3 \left(\hat{\theta}_\tau, \tau \right) = 0. \tag{195}$$