Using asset prices to measure the long-run impact of monetary policy

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Abstract: We develop a methodology to measure market expectations of the long-run impact of monetary policy from asset prices using the operator approach of Hansen and Scheinkman (2009). We show that the ratio of long term equity returns to long term bond returns, measured over short FOMC windows, can be used to quantify this long-run impact. Empirically, we find that a 25-basis-point expansionary monetary policy shock and a 25-basis-point central bank information shock—identified following Jarociński and Karadi (2020)—are on average associated with roughly 100-basis-point and 200-basis-point upward revisions in long-run output, respectively.

Keywords: TBA

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1 Introduction

Does monetary policy impact the long-run growth of the economy? We provide an answer to this question by measuring financial markets' expectations about the long-run impact of monetary policy. Let T denote the time of an FOMC announcement and let T^- denote the time right before the announcement. Let Y_t denote aggregate output and $E_t\left[\frac{Y_{\mathcal{T}}}{Y_t}\right]$ denote markets' expectation about the growth rate of the economy from time t to $\mathcal{T}$. Over a short FOMC announcement window, investors update their expectation about economic growth from $E_T^-\left[\frac{Y_{\mathcal{T}}}{Y_T}\right]$ to $E_T\left[\frac{Y_{\mathcal{T}}}{Y_T}\right]$, where E_T incorporates the information about monetary policy announcement, and E_T^- does not. We show that $\lim_{\mathcal{T}\to\infty}\frac{E_T\left[\frac{Y_{\mathcal{T}}}{Y_T}\right]}{E_T^-\left[\frac{Y_{\mathcal{T}}}{Y_T}\right]}$ can be directly inferred from the difference in the return on long term equity and that on long term bond over short FOMC announcement windows. Empirically, we construct a long term equity portfolio and long term bond portfolio and use our methodology to measure the financial market's expectation about long-run growth over short FOMC announcement windows. We find evidence for permanent impacts of monetary policy with a standard deviation of 60 basis points per FOMC announcement. We follow Jarociński and Karadi (2020) to construct monetary policy shock and central bank information shock. We document that a 25-basis-point expansionary monetary policy shock and a 25-basis-point central bank information shock are on average associated with 100-basis-point and 200-basis-point upward revisions in long-run economic growth, respectively.

Presumably, stock market reaction over a short FOMC announcement window incorporates the market's belief about the impact of monetary policy on future economic growth. However, using stock market reaction to measure the market's expectation about long-run economic growth faces two challenges. First, stock market valuation reacts to monetary policy either due to a discount rate effect or a cash flow effect. It is hard to separately identify the cash flow effect. Second, even if we find a cash flow effect, stock market dividend payout is only a small fraction of the aggregate economy. It may be difficult to generalize.

We develop an asset pricing framework with FOMC announcements and use the operator approach of Hansen and Scheinkman (2009) to address both of the above challenges. First, we define long term equity as the claim to dividends paid far into the future. We show by computing the difference in the returns on long term equity and that on long term bond, we can completely eliminate the impact of discount rate on stock prices and the residue

reflects the market expectation about long term cash flow. Second, we show that as far as measuring $E_t\left[\frac{Y_{\mathcal{T}}}{Y_t}\right]$ is concerned, the difference between dividend cash flow and aggregate output is negligible for large values of $\mathcal{T}$, i.e. as $\mathcal{T} \to \infty$, as long as the dividend payout and aggregate GDP are cointegrated.

We present two key theoretical results to link returns on long term equity and long term bonds to expectations about long-run growth. Our first result, Proposition 1, states that if monetary policy does not have a long-run impact on the economy, the returns on long term equity and long term bond will be perfectly correlated over short FOMC announcement windows.

Our second result, Proposition 1, provides conditions under which the difference between the return on long term equity and that on long term bond can be expressed as the sum of announcement premium and an innovation in market expectation of long-run growth. The key assumption for the exact identification is that growth rates and the stochastic discount factor are log linear functions of state variables. Even when this condition fails, it is still possible to use the difference between the return on long term equity and that on long term bonds to approximate expectations about long-run growth. We provide several examples and show that the approximation errors are typically negligible under plausibly calibrated parameter values.

Empirically, we construct long term equity by using a portfolio of non-dividend paying stocks as a proxy.[1] We use futures on the 30-year T-bond as the proxy for long term bond following Alvarez and Jermann (2005). We show that empirical evidence strongly rejects the null hypothesis of zero long-run impact of monetary policy. Quantitatively, the standard deviation of changes in long-run growth expectations is around 60 basis points on an average FOMC announcement day. Our evidence highlights the need to study monetary models with long-run impact. In most models of monetary policy, including the New Keynesian models (Galí (2015) and Woodford (2003)), the economy has a steady state, and monetary policy has a temporary impact on deviations from the steady state but no permanent impact on the level of output.

The unconditional correlation between our measured innovations in long-run growth expectation and Fed funds rates, a conventional measure of monetary policy shock, is low. We show that this is due to the Fed information effect. We follow Jarociński and Karadi

[1] We thank Sang Seo for advising us on non-dividend paying stocks to construct the long-equity portfolio.

(2020) to construct monetary policy shocks and central bank information shocks and show that changes in long-run growth expectations are highly correlated with both components of monetary policy surprises. A 25-basis-point monetary policy shock on average leads to 100-basis-point upward revisions in long-run economic growth. The impact of Fed information shocks is stronger. A 25-basis-point central bank information shock typically leads to 200-basis-point revisions of long-run economic growth.

Literature Our identification methodology builds on the operator approach to long term pricing pioneered by Hansen and Scheinkman (2009) and is closely related to the literature on recovery of probabilities from asset prices (Ross (2015), Borovička, Hansen, and Scheinkman (2016),Qin and Linetsky (2017) and Bakshi, Chabi-Yo, and Gao (2018)). Borovička, Hansen, and Scheinkman (2016) demonstrates that in general it is not possible to recover subjective probabilities from asset prices when there exists a permanent component in the pricing kernel. Cieslak and Khorrami (2025) shows that under the Long-Run Neutrality (LRN) condition, changes in expectations under the subjective belief and expectations under the long-run risk neutral measure are identical, and as a result, changes in all subjective expectations can be identified from asset prices. The LRN is a strong requirement and is not satisfied in most of the models we examine in this paper. Due to the failure of LRN, we cannot recover a full set of expectations. Nevertheless, we demonstrate that the recovery or approximate recovery of long-run growth expectations is possible under much more general conditions.

Much of the empirical work in long term pricing focuses on the permanent and transitory components of the stochastic discount factor (SDF). Bansal and Lehmann (1997) provides a decomposition of the SDF into permanent and transitory components. Alvarez and Jermann (2005) and Bakshi and Chabi-Yo (2012) develop volatility bounds on these two components. Christensen (2017) estimates them with a structural model and a non-parametric approach to the Perron-Frobenius eigenfunction problem. Lustig, Stathopoulos, and Verdelhan (2019) links the carry trade risk premium of long-term bonds to the cross-country differences in the volatility of the permanent components of the SDF. Cieslak and Khorrami (2025) finds evidence that the permanent component of the SDF shifts around the FOMC announcement. Different from the focus of the literature, our paper develops a measure of investors' revisions to long-run output expectation around short announcement windows by characterizing the relationship between the eigenfunctions of the relevant multiplicative functionals.

Our paper connects to the literature on the high-frequency identification of monetary policy surprises over FOMC announcements and their causal effects on asset prices. In early work, Bernanke and Kuttner (2005) and Kuttner (2001) identify the surprises as the daily changes in the current-month or next-month federal funds futures rate and study the responses in stock prices and Treasury yields, respectively. More recent studies exploit narrower windows around announcements to identify monetary policy surprises. Gürkaynak, Sack, and Swanson (2005) constructs a target factor and a path factor from high-frequency responses of several federal funds and Eurodollar futures with one year or less to expiration. Nakamura and Steinsson (2018) provides evidence for the Fed information effect and estimates the non-neutrality from the responses of real rates, expected inflation and expected output growth to the high-frequency surprises. Jarociński and Karadi (2020) decomposes the surprises into a pure monetary policy component and a Fed information component. Bauer and Swanson (2023) highlights the importance of removing predictable components from these surprises. Also related is the literature that aims to explain the responses in asset prices. Kekre and Lenel (2022) and Pflueger and Rinaldi (2022) highlight the risk premium channel in the models with wealth redistribution and habit formation, respectively. Bauer, Pflueger, and Sunderam (2024) shows that perceptions about the Fed's policy rule have shaped the transmission of monetary policy to financial markets. Kekre, Lenel, and Mainardi (2025) explains the responses of long term Treasury yields in a segmented markets model. Nagel and Xu (2025) attributes the responses of stock prices to movements in the yield curve.

To study the impacts of monetary policy on the macroeconomy, several papers, for example, Cochrane and Piazzesi (2002), Gertler and Karadi (2015), Ramey (2016) and Stock and Watson (2018), exploit the exogeneity of high-frequency surprises, using them either as regression variables or instruments in structural VARs or local projection methods. A related paper by Golez and Matthies (2025) also leverages intraday financial market data to measure the impact of monetary policy on cash flows. They provide evidence for the Fed information effect on short-term growth by finding that the announcement returns of a six-month dividend strip constructed from options load positively on the monetary policy surprises. Different from the above paper, our purpose is to identify the long-run component of monetary policy shocks, and more importantly, quantify the long run impact.

Our paper is broadly related to the literature that uses asset market data to identify

macroeconomic risk factors. Giglio, Kelly, and Kozak (2024) employs a large cross-section of equity returns to identify the priced latent factors and then estimate the term structure of dividend yields. Bryzgalova, Huang, and Julliard (2025) adopts a similar approach to study the term structure of the risk premia associated with macroeconomic processes.

The importance of long-run growth in asset pricing is emphasized by the long-run risk asset pricing literature. Bansal and Yaron (2004) demonstrates how low-frequency variations in expected consumption growth and its volatility can account for leading asset pricing puzzles. Bansal, Kiku, and Yaron (2016) provides empirical support for this framework. Hansen, Heaton, and Li (2008) formalizes how long-run growth variation in consumption and asset cash flows contributes to cross-sectional differences in valuation and risk premia. Croce (2014), Kaltenbrunner and Lochstoer (2010) and Kung and Schmid (2015) develop macro asset pricing models where long-run consumption risks arise as an equilibrium outcome in production economies.

The rest of the paper is organized as follows. In Section 2, we develop a no-arbitrage model with FOMC announcements. In Section 3, we prove our main result, that is, the difference between the return on long term equity and that on long term bond includes a predictable macroeconomic announcement premium and an unpredictable component, which represents changes in the market's expectation about the long run growth of the economy. In Section 4, we use non-dividend paying stock and long-maturity Treasury bond as empirical proxies for long term equity and long term bond to estimate the long-run impact of monetary policy. In Section 5, we demonstrate that even in the cases where the sufficient conditions do not hold, our measure can still provide a good approximation. Section 6 concludes.

2 Model setup

A Hidden Markov Environment Let $(\Omega, \mathcal{F}, Pr)$ be a probability space and let $\{\mathcal{F}_t\}_{t\geq 0}$ be a filtration. Let W be a k dimensional Brownian motion adapted to $\{\mathcal{F}_t\}_{t\geq 0}$ and let X_t be a $k-$dimensional continuous-time VAR process defined as

$$dX_t = (\bar{x} - AX_t)\, dt + \Sigma_X dW_{X,t},$$

where $\bar{x} \in R^k$ is a k dimensional vector, A is a $k \times k$ matrix with k strictly positive and semi-simple eigenvalues,[2] Σ_X is a $k \times k$ matrix of local volatility of the VAR process, and W_X is a k dimensional standard Brownian motion. We assume that A has strictly positive eigenvalues so that X_t has a unique stationary distribution that is a multivariate normal distribution. Here, we allow the Brownian motions $W_{X,t}$ to be multi-dimensional. Some elements in $W_{X,t}$ may be aggregate shocks and some may be purely idiosyncratic, depending on whether they are priced by the stochastic discount factor, which we describe later in this section.

We interpret X_t as a vector of state variables for the macroeconomy in a recursive competitive equilibrium (Mehra and Prescott (1985)). We allow some or all of the elements in X_t to be influenced by monetary policy. To incorporate monetary policy announcements, we allow X_t to be partially unobservable, and we model monetary policy announcements as revealing information about X_t. One such example is that monetary policy is set by a Taylor rule with unknown parameters, and FOMC announcements reveal information about these parameters, which we discuss in Example 1 below. Because X_t is not fully observable, our general framework allows agents to update their beliefs about X_t based on observables both on and off FOMC announcement days.

Information and announcements We assume that agents in the economy observe an l dimensional vector of noisy signals of X_t, denoted ξ_t, where

$$d\xi_t = \left(\bar{\xi} + BX_t\right) dt + \Upsilon_X dW_{X,t} + \Upsilon_\xi dW_{\xi t}, \tag{1}$$

where B is a $l \times k$ matrix and Υ_X is a $l \times k$ dimensional matrix, Υ_ξ is a $l \times l$ dimensional matrix, and W_ξ is a l-dimension standard Brownian motion such that elements in W_X and W_ξ are pairwise independent. This setup allows the innovations in the signals to be correlated or independent on the innovations of the Brownian motion $W_{X,t}$. It also allows some elements of X_t to be contained in ξ_t, so that only a subset of X_t is not observable.

In addition to the continuous signal process ξ_t, we assume that there are pre-scheduled FOMC announcements that occur at 0,T, $2T$, $3T$, ... Every announcement provides a j-dimensional noisy signal about X_t in the form of $s_{nT} = CX_{nT} + \epsilon_{nT}$, where C is a $j \times k$

[2] An eigenvalue is semi-simple if its algebraic multiplicity equals its geometric multiplicity. This condition guarantees that A is diagonalizable and we can rotate X into a vector of Ornstein–Uhlenbeck processes.

matrix and ϵ is a j-dimension standard normal distribution.

In our setup, X is the hidden Markov state of the economy including state variables that can be influenced by the Federal reserve bank's decisions. The main assumption we make is that state variables that govern the Federal reserve bank's decision rules are not perfectly observable and FOMC announcements provide information about those state variables. These state variables could be the Fed's private information about its own policy objectives, or its private information about the future prospects of the economy. Whatever the interpretation may be, we allow these state variables to impact future economic growth and asset valuation as we specify below.

Learning and information processing Because X_t is not observable, agents in the economy need to make their decisions based on their beliefs about X_t. Thanks to the conditional Gaussian setup, the posterior belief about X_t can be characterized by its first two moments, $\left(\hat{X}_t, Q_t\right)$, where $\hat{X}_t = E\left[X_t \middle| \xi^t, s^t\right]$ represents the conditional mean of X_t based on all observed information up to time t. Here, we use superscripts to denote history: $\xi^t = \{\xi_s : 0 \leq s \leq t\}$, and $s^t = \{s_s : 0 \leq s \leq t\}$. In addition, $Q_t = Var\left[X_t \middle| \xi^t, s^t\right]$ is the conditional variance-covariance matrix.

We describe investors' learning problem on non-FOMC announcement days and on FOMC announcement days separately. In between FOMC announcements, using the results in Liptser and Shiryaev (2001), if we define a vector of innovation processes as

$$d\hat{W}_t = (\Upsilon \circ \Upsilon)^{-\frac{1}{2}} \left[d\xi_t - \left(\bar{\xi} + B\hat{X}_t\right) dt\right], \tag{2}$$

where $\Upsilon \circ \Upsilon = \Upsilon_X \Upsilon_X' + \Upsilon_\xi \Upsilon_\xi'$, then $\hat{W}_t$ is a vector of standard Brownian motions adapted to investors' information set generated by the history of observations, $\hat{\mathcal{F}}_t = (\xi^t, s^t)$. Using this notation, the law of motion of $\hat{X}_t$ can be written as a diffusion process adapted to $\left\{\hat{\mathcal{F}}_t\right\}$:

$$d\hat{X}_t = \left(\bar{x} - A\hat{X}_t\right) dt + \left(\Sigma_X \Upsilon_X' + Q_t B'\right) (\Upsilon \circ \Upsilon)^{-\frac{1}{2}} d\hat{W}_t, \tag{3}$$

and Q_t evolves deterministically according to

$$dQ_t = \left[\Sigma_X \Sigma_X' - \left(AQ_t + Q_t A'\right) - \left(\Sigma_X \Upsilon_X' + Q_t B'\right) (\Upsilon \circ \Upsilon)^{-1} \left(\Sigma_X \Upsilon_X' + Q_t B'\right)'\right] dt.$$

At a typical FOMC announcement at time nT, investors receive noisy signals $s_{nT} = CX_{nT} + \epsilon_{nT}$. We denote investors' beliefs about X_{nT} before the announcement as $\left(\hat{X}_{nT}^{-}, Q_{nT}^{-}\right)$ and their beliefs after the announcement as $\left(\hat{X}_{nT}, Q_{nT}\right)$. Using formula for the conditional distribution of multi-variate normal distributions, we have

$$\hat{X}_{nT} = \hat{X}_{nT}^{-} + Q_{nT}^{-} C' \left(CQ_{nT}^{-} C' + I\right)^{-1} \left(s_{nT} - C\hat{X}_{nT}^{-}\right),$$

and

$$Q_{nT} = Q_{nT}^{-} - Q_{nT}^{-} C' \left(CQ_{nT}^{-} C' + I\right)^{-1} CQ_{nT}^{-}.$$

Because the Markov state variable X_t is not observable, equilibrium prices and quantities cannot depend on X_t. Thanks to the conditional Gaussian setup, $\left(\hat{X}_t, Q_t\right)$ can instead be used as the Markov state variable to construct equilibrium prices and quantities. Thanks to the assumption of constant volatilities for both the hidden Markov chain and the signals, Q_t will be a deterministic function of time. Because the distance between two FOMC announcements is constant over time, in the learning steady state, Q_t will cycle across announcements, that is $Q_t = Q_{t+T}$ for all t. This property allows us to replace Q_t with $Q_{\tau(t)}$, where $\tau(t) = t \mod T$ as the state variable. In what follows, we will construct equilibrium prices by using the pair of state variables $\left(\hat{X}_\tau, \tau\right)$. Under this formulation, $\tau \in [0, T]$, and $\left(\hat{X}_\tau, \tau\right)$ is jointly Markov. Our convention is that FOMC announcements are made at $\tau = T$ and $\tau = T^-$ is the instant right before an announcement. Because we focus on the learning steady state, Q_t increases monotonically on $[0, T)$. At an announcement time T, the signal s_T reduces uncertainty, and resets Q_T to Q_0.

Asset markets We specify the dynamics of GDP of the economy, denote $\{Y_t\}_{t\geq 0}$ by using the state variables $\left(\hat{X}_\tau, \tau\right)$ and the innovation processes $\hat{W}_t$:

$$d \ln Y_t = \left[\mu_Y \left(\hat{X}_t, \tau(t)\right) - b \ln Y_t\right] dt + \sigma_Y' d\hat{W}_t, \tag{4}$$

where $\mu_Y : R^{k+1} \to R$ is a continuous function that models how the state variables $\left(\hat{X}, \tau\right)$ impact the expected growth rate of aggregate output, and σ_Y is a l-dimensional vector. We consider two cases for the parameter b, $b > 0$ and $b = 0$. When $b > 0$, Y_t is stationary and

our model does not allow long-run growth. When $b = 0$, Y_t grows without bound in the long run and information about X_t will potentially impact the expected long-run growth of Y_t. In this setup, Y_t is a multiplicative functional with respect to $\{X_t\}_{t\geq 0}$ in the sense of Hansen and Scheinkman (2009).[3] We specify the dynamics of Y_t in terms of the observable processes, $\left\{\hat{X}_t, \hat{W}_t\right\}$, because in equilibrium models where Y_t is endogenously determined as equilibrium outcomes, it needs to be adapted to the agents' information filtration.[4]

We think of Y_t as a model of the fundamental growth process of the economy. In the short run, other cash flows in the economy, such as aggregate consumption, investment, labor income, dividend payout, etc. may have very different dynamics from Y_t. In the long run, they will need to be cointegrated with Y_t in the sense that the ratios of the above cash flows with respect to Y_t need to be stationary. We interpret the dependence of the growth rate of Y_t on X_t, captured by $\mu_Y\left(\hat{X}, \tau\right)$ as determined by some equilibrium mechanism where production decisions are functions of the Markov state variables $\left(\hat{X}, \tau\right)$. We are particularly interested in the case where state variables that govern the dynamics of monetary policy are included in X_t, and as a result, signals about X_t are potentially informative about monetary policy and its impact on economic growth. The specifications of $b > 0$ and $b = 0$ allow our framework to encompass stationary models, such as the standard textbook New Keynesian models, for example, Woodford (2003) and Galí (2015), and models with long-run economic growth. Over a very short FOMC announcement window, information about monetary policy-relevant state variables is revealed. Investors' belief about the long-run growth of the economy responds to such information, and so do asset prices. The primary focus of the paper is to develop a methodology to infer the long-run impact of monetary policy announcements from asset pricing moments.

In the above environment, we focus on two classes of assets: a long term bond that pays one unit of consumption good at time $\mathcal{T}$, where $\mathcal{T}$ is a distant future date, and a long term equity that pays $\psi\left(\hat{X}_{\mathcal{T}}, \tau\left(\mathcal{T}\right)\right) Y_{\mathcal{T}}$ at time $\mathcal{T}$. We assume that $\psi\left(X, \tau\right) \in (0, 1)$, so that the only restriction we put on dividend is that it has the same growth trend as total output

[3] In the case of $b > 0$, we need to augment our state space to include $\ln Y_t$ to make sure Y_t is a multiplicative functional.

[4] Another special case of interest is that $\mu_Y\left(X\right)$ is a linear function, in which case the above equation can be interpreted as the outcome of learning. That is, the GDP is determined by the state variable X_t: $d\ln Y_t = \left(\mu_Y' X_t - b \ln Y_t\right) dt + \sigma_{Y,X}' dW_{X,t} + \sigma_{Y,\xi}' dW_{\xi,t}$ and $\ln Y_t$ is part of the vector of signal process, ξ_t. Equation (4) can then be interpreted as rewriting the law of motion of Y_t by using the innovation process consistently with (2). The Veronesi (2000) model can be interpreted in this way.

and the dividend-to-output ratio is between 0 and 1. As shown by Hansen and Scheinkman (2009), and as we will make precise in the rest of this paper, if our interest is in the long-run growth rate of the economy, details of the functional form of $\psi\left(\hat{X},\tau\right)$ are unimportant. This implies that empirically, the details of the construction of the long term equity are not important, as long as we believe the associated cash flow is cointegrated with Y_t.

Stochastic discount factor To compute asset prices, we assume a stochastic discount factor that follows the following law of motion on non-announcement days:

$$\frac{dS_t}{S_t} = -r\left(\hat{X}_t,\tau\left(t\right)\right)dt - \sigma_S\left(\hat{X}_t,\tau\left(t\right)\right)d\hat{W}_t, \tag{5}$$

where $r\left(\hat{X},\tau\right)$ is the instantaneous risk-free rate and $\sigma_S\left(\hat{X}_\tau,\tau\right)$ is a vector a market prices of risk associated with the Brownian motions. We allow some elements of $\sigma\left(\hat{X},\tau\right)$ to be zero, in which case, the corresponding Brownian motion represents idiosyncratic risk. On FOMC announcement days, we assume that the stochastic discount factor takes the form of

$$\frac{S_T}{S_T^-} = \frac{e^{-\alpha\left(\hat{X}_T,T\right)}}{E\left[e^{-\alpha\left(\hat{X}_T,T\right)}\middle| X_T^-,T^-\right]}. \tag{6}$$

Equations (5) and (6) allow a very general specification of the SDF as functions of the Markov state $\left(\hat{X}_\tau,\tau\right)$. The specification of the announcement SDF, Equation (6), imposes the restriction that the instantaneous risk-free rate from time T^- to T is zero. This restriction is motivated by the fact that empirically, the return on risk-free assets over a short FOMC announcement is virtually zero. The dependence of $\frac{S_T}{S_T^-}$ on $\hat{X}_T$ allows for an announcement premium, which is motivated by the empirical evidence of the large FOMC announcement premium, for example, Lucca and Moench (2015). The announcement stochastic discount factor in the continuous time model in Ai and Bansal (2018), for example, is a special case of (5) and (6).

The challenge for inferring the impact of monetary policy on economic growth from asset prices is that monetary policy potentially impacts asset prices through both the discount rate channel, which is encoded in the stochastic discount factor, and growth expectations. We are interested in conditions under which, for large values of $\mathcal{T}$, the impact of monetary

policy on the stochastic discount factor can be differenced out by comparing equity and bond returns over short FOMC announcement windows.

3 Theoretical Foundations

We focus on three quantities: the price of a long bond, the price of a long equity, and the expectation of the long-run growth. Given our setup, the price of a long term bond at time τ with maturity date $\mathcal{T}$, denoted $p_{B,t}(\mathcal{T})$, can be computed as

$$p_{B,t}(\mathcal{T}) = E_t\left[\frac{S_{\mathcal{T}}}{S_t}\right].$$

The price of a long term equity with maturity date $\mathcal{T}$ is

$$p_{E,t}(\mathcal{T})\, Y_t = E_t\left[\frac{S_{\mathcal{T}}}{S_t} Y_{\mathcal{T}} \psi\left(\hat{X}_{\mathcal{T}}, \tau(\mathcal{T})\right)\right].$$

Here, we use the notation $p_{E,t}(\mathcal{T})$ for the price of the equity normalized by the growth trend Y_t. The expectation of long-run GDP growth with maturity T is $E_t\left[\frac{Y_{\mathcal{T}}}{Y_t}\right]$. In our setup, $\{S_t\}_{t\geq 0}$, $\{Y_t\}_{t\geq 0}$, and $\{S_t Y_t\}_{t\geq 0}$ are all multiplicative functionals. To investigate how updates of $E_t\left[\frac{Y_{\mathcal{T}}}{Y_t}\right]$ over short FOMC announcement windows can be computed from $p_{B,t}(\mathcal{T})$ and $p_{E,t}(\mathcal{T})$ and thus used as a measure of the impact of monetary policy on growth, we rely on the study of the Perron-Frobenius problems for multiplicative functionals developed by Hansen and Scheinkman (2009).

3.1 Eigenfunctions for Multiplicative Semigroups

General analytical framework We first provide a review of the general analytical framework of Hansen and Scheinkman (2009) to provide a definition of the relevant eigenvalue problems which provides a foundation for our analysis. We start from a vector of continuous time, strong Markov processes $\{Z_t : t \geq 0\}$ defined on a probability space $\{\Omega, F, Pr\}$. We denote $\mathbb{Z}$ as the state space of $\{Z_t\}$. We assume that $\{Z_t : t \geq 0\}$ is stationary and ergodic under measure Pr. Let $\{\mathcal{F}_t : t \geq 0\}$ be the filtration generated by $\{Z_t : t \geq 0\}$. A functional is a real-valued process $\{M_t : t \geq 0\}$ that is adapted to $\{\mathcal{F}_t : t \geq 0\}$. We as-

sume that M_t has a version with sample paths that are continuous from the right with left limits. Because the functional M_t is constructed from Z_t, we can write the full notation $M_t\left(\{Z_s : s \in [0,t]\}\right)$ to indicate that for each t, M_t is a function of the history of Z from time 0 to t. We use θ to denote the shift operator. That is, $M_u\left(\theta_t\right)$ denotes the time-shifted process: $M_u\left(\theta_t\right) = M_u\left(\{Z_s : s \in [t, t+u]\}\right)$. A functional M is multiplicative (with respect to $\{Z_t : t \geq 0\}$) if $M_0 = 1$ and $M_{t+u} = M_u\left(\theta_t\right) M_t$, for all $t, u > 0$.

Given any positive multiplicative functional, M_t, we can construct a semigroup of linear operators, $\{\mathbf{M}_t\}_{t\geq 0}$, indexed by t, in the following way. For any measurable function $\psi : \mathbb{Z} \to R$ such that $E\left[|M_t\psi\left(Z_t\right)|\right] < \infty$ for all t, $\mathbf{M}_t$ is defined as

$$\mathbf{M}_t\psi\left(Z\right) = E\left[M_t\psi\left(Z_t\right)\middle| Z_0 = Z\right]. \tag{7}$$

Let ρ be a real number and $\mathbf{e}\left(Z\right)$ be a measurable function of Z. We say ρ is an eigenvalue of the multiplicative semigroup $\{\mathbf{M}_t\}_{t\geq 0}$ and $\mathbf{e}$ is an associated eigenfunction if for all $t > 0$ and for all $Z \in \mathbb{Z}$,[5]

$$e^{\rho t}\mathbf{e}\left(Z\right) = E\left[M_t\mathbf{e}\left(Z_t\right)\middle| Z_0 = Z\right]. \tag{8}$$

A eigenfunction $\mathbf{e}$ is a principal eigenfunction if $\mathbf{e}\left(Z\right) > 0$ for all $Z \in \mathbb{Z}$.

Given a multiplicative functional M_t and the associated solution to the eigenvalue problem (8), $(\rho, \mathbf{e})$, we define

$$\hat{M}_t = e^{-\rho t} M_t \frac{\mathbf{e}\left(Z_t\right)}{\mathbf{e}\left(Z_0\right)}. \tag{9}$$

It is straightforward to verify that Equation (8) implies $\hat{M}_t$ is a martingale. We can use this martingale to construct a conditional probability system, represented by $\hat{E}$ such that $\hat{E}\left[f\middle|\mathcal{F}_t\right] = E\left[\frac{\hat{M}_{t+s}}{\hat{M}_t} f\middle| X_t\right]$ for any f measurable with respect to $\mathcal{F}_{t+s}$. As in Hansen and Scheinkman (2009), we focus on principal eigenfunctions such that the original process Z_t is stationary under the measure $\hat{P}r$ induced by the martingale defined in (9).

We are primarily interested in the eigenvalue problem for three multiplicative semigroups. The first one is the semigroup induced by the stochastic discount factor S_t, which we denote

[5] Hansen and Scheinkman (2009) defines eigenvalues and eigenfunctions with respect to the generator of the semigroup. Our definition is equivalent to theirs under the conditions of Proposition 6.2 of their paper. We define the eigenvalue problem with respect to the semigroup but not the generator, because the generator may fail to exist in some of our examples with FOMC announcements, in which case, we need to rely on this more general definition.

as $(-\bar{r}, \hat{\mathbf{e}}_S)$. By definition, $(-\bar{r}, \hat{\mathbf{e}}_S)$ satisfies

$$e^{-\bar{r}t}\mathbf{e}_S(Z) = E\left[\frac{S_\mathcal{T}}{S_t}\mathbf{e}_S(Z_\mathcal{T})\middle| Z_t = Z\right].$$

This equation helps us compute the price of a long bond, which is the claim to one unit of consumption goods paid at time $\mathcal{T}$:

$$E\left[\frac{S_\mathcal{T}}{S_t}\middle| Z_t\right] = e^{-\bar{r}(\mathcal{T}-t)}\mathbf{e}_S(Z_t)\, E\left[\underbrace{e^{\bar{r}(\mathcal{T}-t)}\frac{S_\mathcal{T}}{S_t}\frac{\mathbf{e}_S(Z_\mathcal{T})}{\mathbf{e}_S(Z_t)}}_{martingale}\frac{1}{\mathbf{e}_S(Z_\mathcal{T})}\middle| Z_t\right].$$

The term $e^{\bar{r}(\mathcal{T}-t)}\frac{S_\mathcal{T}}{S_t}\frac{\mathbf{e}_S(Z_\mathcal{T})}{\mathbf{e}_S(Z_t)}$ is a martingale by Equation (9). The probability induced by this martingale is termed the long-run risk-neutral probability by BHS (2015). This allows us to write the price of the long term bond as $e^{-\bar{r}(\mathcal{T}-t)}\mathbf{e}_S(Z_t)\,\hat{E}\left[\frac{1}{\mathbf{e}_S(Z_\mathcal{T})}\middle| Z_t\right]$, where $\hat{E}$ is expectation induced by long-run risk-neutral probability. Assuming that the change of measure preserves the ergodicity of the process Z_t, a condition that needs to be verified in applications, $\lim_{\mathcal{T}\to\infty}\hat{E}\left[\frac{1}{\mathbf{e}(Z_\mathcal{T})}\middle| Z_t\right] = \hat{E}^*\left[\frac{1}{\mathbf{e}_S(Z)}\right]$, where $\hat{E}^*$ denotes the stationary distribution of Z_t under $\hat{E}$. This implies

$$\lim_{\mathcal{T}\to\infty} e^{\bar{r}(\mathcal{T}-t)}E\left[\frac{S_\mathcal{T}}{S_t}\middle| Z_t\right] = \mathbf{e}_S(Z_t)\,\hat{E}^*\left[\frac{1}{\mathbf{e}_S(Z_\mathcal{T})}\right]. \tag{10}$$

Because eigenfunctions are unique up to a multiplicative constant, changes in the price of the long term bond, or in its return, depend only on the eigenfunction $\mathbf{e}_S$.

The second multiplicative semigroup is the one induced by S_tY_t, which helps us compute the price of long term equity. Let $(\rho_{SY}, \hat{\mathbf{e}}_{SY})$ be the solution to the eigenvalue problem (8) with $M_t = S_tY_t$. We can write the price of long term equity as the present value of $Y_\mathcal{T}\psi(Z_\mathcal{T})$:

$$E_t\left[\frac{S_\mathcal{T}Y_\mathcal{T}}{S_t}\psi(Z_\mathcal{T})\right] = e^{\rho(\mathcal{T}-t)}\mathbf{e}_{SY}(Z_t)\,Y_t E\left[\frac{\hat{M}_\mathcal{T}}{\hat{M}_t}\frac{\psi(Z_\mathcal{T})}{\mathbf{e}_{SY}(Z_\mathcal{T})}\middle| Z_t\right], \tag{11}$$

where $\hat{M}_t = e^{-\rho(\mathcal{T}-t)}\frac{S_\mathcal{T}Y_\mathcal{T}}{S_tY_t}\frac{\mathbf{e}_{SY}(Z_\mathcal{T})}{\mathbf{e}_{SY}(Z_t)}$ is the martingale constructed from S_tY_t by using (9). If we assume the distribution of Z_t under the measure induced by $\hat{M}_t$ is ergodic, then for large

$\mathcal{T}$, $\lim_{T\to\infty} E\left[\frac{\hat{M}_{\mathcal{T}}}{\hat{M}_t}\frac{1}{\mathbf{e}_{SY}(Z_{\mathcal{T}})}\middle| Z_t\right] = \hat{E}\left[\frac{\psi(Z_{\mathcal{T}})}{\mathbf{e}_{SY}(Z_{\mathcal{T}})}\middle| Z_t\right]$, where $\hat{E}$ denotes expectation under the measure induced by $\hat{M}_t = e^{-\rho(\mathcal{T}-t)}\frac{S_{\mathcal{T}}Y_{\mathcal{T}}}{S_tY_t}\frac{\mathbf{e}_{SY}(Z_{\mathcal{T}})}{\mathbf{e}_{SY}(Z_t)}$.

The final multiplicative semigroup is the one induced by Y_t, which we use to compute expectations of future economic growth. Let $(\rho_Y, \hat{\mathbf{e}}_Y)$ be the solution to the eigenvalue problem (8) with $M_t = Y_t$ and use the same calculation as above to compute the expectation of future growth:

$$\begin{aligned} E_t\left[\frac{Y_{\mathcal{T}}}{Y_t}\right] =& e^{\rho_Y(\mathcal{T}-t)}\mathbf{e}_Y(Z_t)\, E\left[\underbrace{e^{-\rho_Y(\mathcal{T}-t)}\frac{S_{\mathcal{T}}}{S_t}\frac{\mathbf{e}_Y(Z_{\mathcal{T}})}{\mathbf{e}_Y(Z_t)}}_{martingale}\frac{1}{\mathbf{e}_Y(Z_{\mathcal{T}})}\middle| Z_t\right] \\ =& e^{\rho_Y(\mathcal{T}-t)}\mathbf{e}_Y(Z_t)\,\hat{E}\left[\frac{1}{\mathbf{e}_Y(Z_{\mathcal{T}})}\middle| Z_t\right], \end{aligned} \tag{12}$$

where $\hat{E}$ denotes the measure induced by the martingale $e^{-\rho_Y(\mathcal{T}-t)}\frac{S_{\mathcal{T}}}{S_t}\frac{\mathbf{e}_Y(Z_{\mathcal{T}})}{\mathbf{e}_Y(Z_t)}$. Under the ergodicity condition, $\hat{E}\left[\frac{1}{\mathbf{e}_Y(Z_{\mathcal{T}})}\middle| Z_t\right]$ converges to a constant and the expectation about future growth can be computed from the eigenfunction $\mathbf{e}_Y(Z_t)$.

Relationship between eigenfunctions In the context of our model, the relevant state variable is $\left(\hat{X}, \tau\right)$. We are primarily interested in investors' update of $E_t\left[\frac{Y_{\mathcal{T}}}{Y_t}\right]$ over short FOMC announcement windows. We denote the update of this expectation over an FOMC announcement window as

$$\Delta E^{\infty}(T) \equiv \lim_{\mathcal{T}\to\infty}\frac{E\left[\frac{Y_{\mathcal{T}}}{Y_T}\middle|\hat{X}_T, T\right]}{E\left[\frac{Y_{\mathcal{T}}}{Y_T}\middle|\hat{X}_T^-, T^-\right]} = \frac{\mathbf{e}_Y\left(\hat{X}_T, T\right)}{\mathbf{e}_Y\left(\hat{X}_T^-, T^-\right)}, \tag{13}$$

where the second equality is true if $\left(\hat{X}_\tau, \tau\right)$ is ergodic under the probability measure induced by the relevant martingales. We also define the announcement return on long term bond as

$$R_B^{\infty}(T) \equiv \lim_{\mathcal{T}\to\infty}\frac{E\left[\frac{S_{\mathcal{T}}}{S_T}\middle|\hat{X}_T, T\right]}{E\left[\frac{S_{\mathcal{T}}}{S_T^-}\middle|\hat{X}_T^-, T^-\right]} = \frac{\mathbf{e}_S\left(\hat{X}_T, T\right)}{\mathbf{e}_S\left(\hat{X}_T^-, T^-\right)}, \tag{14}$$

and the announcement return on long term equity as

$$R_E^{\infty}(T) \equiv \lim_{\mathcal{T}\to\infty} \frac{E\left[\frac{S_{\mathcal{T}}}{S_T} Y_{\mathcal{T}} \psi(Z_{\mathcal{T}}) \middle| \hat{X}_T, T\right]}{E\left[\frac{S_{\mathcal{T}}}{S_T^-} Y_{\mathcal{T}} \psi(Z_{\mathcal{T}}) \middle| \hat{X}_T^-, T^-\right]} = \frac{\mathbf{e}_{SY}\left(\hat{X}_T, T\right)}{\mathbf{e}_{SY}\left(\hat{X}_T^-, T^-\right)}, \tag{15}$$

provided that the distribution of $\left(\hat{X}_\tau, \tau\right)$ in (14) and (15) are ergodicity under the relevant probability measures.

We are primarily interested in investors' updates of expectations about future growth, $\Delta E^{\infty}(T)$ over short FOMC announcement windows. Equation (13) implies that updates of expectations of long-run growth can be computed from the eigenfunction $\mathbf{e}_Y$. However, neither expectations nor eigenfunctions are directly observable, while asset returns over a short FOMC announcement window can be accurately measured. The main result of the paper is to provide conditions under which the eigenfunction $\mathbf{e}_Y$ can be recovered from the eigenfunctions $\mathbf{e}_S$ and $\mathbf{e}_{SY}$. In general, if the three eigenfunctions satisfy

$$\mathbf{e}_{SY}\left(\hat{X}, \tau\right) f(\tau) = \mathbf{e}_S\left(\hat{X}, \tau\right) \mathbf{e}_Y\left(\hat{X}, \tau\right), \tag{16}$$

for some function f that depends only on τ, then

$$\Delta E^{\infty}(T) = \frac{\mathbf{e}_Y\left(\hat{X}_T, T\right)}{\mathbf{e}_Y\left(\hat{X}_T^-, T^-\right)} = \frac{\frac{\mathbf{e}_{SY}\left(\hat{X}_T, T\right)}{\mathbf{e}_{SY}\left(\hat{X}_T^-, T^-\right)}}{\frac{\mathbf{e}_S\left(\hat{X}_T, T\right)}{\mathbf{e}_S\left(\hat{X}_T^-, T^-\right)}} \frac{f(T)}{f(T^-)} \tag{17}$$

can be computed from the ratio of the return on long term equity and the return on long term bond over short FOMC announcement windows by using Equations (14) and (15). In the rest of the paper, we provide sufficient conditions under which (16) holds so that inference about the long-run impact of monetary policy from asset prices using (17) is exact. We also demonstrate through examples that even if Equation (16) fails, the inference equation remains a good approximation.

The setup of FOMC announcements in our measurement exercise is important for two reasons. First, we are primarily interested in measuring the impact of monetary policy and the short FOMC announcement window allows us to identify monetary policy shocks. Second, using the return on long term equity to identify the eigenfunction $\mathbf{e}_{SY}$ is valid only if

we focus on short FOMC announcement windows where changes in GDP Y_t are negligible. To see this, note that the return on long term equity, (15) from t to T is $\frac{\mathbf{e}_{SY}\left(\hat{X}_T,T\right)Y_T}{\mathbf{e}_{SY}\left(\hat{X}_t,t\right)Y_t}$ in general depends both on the eigenfunction $\mathbf{e}_{SY}$ as well as shocks to the growth rate of Y_t. By focusing on short FOMC announcement windows, we set $t = T^-$, and $\frac{\mathbf{e}_{SY}\left(\hat{X}_T,T\right)Y_T}{\mathbf{e}_{SY}\left(\hat{X}_t,t\right)Y_t} = \frac{\mathbf{e}_{SY}\left(\hat{X}_T,T\right)}{\mathbf{e}_{SY}\left(\hat{X}_{T^-},T^-\right)}$ because Y_t is a continuous process. The assumption that Y_t is continuous is a reasonable one because the changes in the level of aggregate output over short FOMC announcement windows are quantitatively negligible.

3.2 The stationary case

We first focus on the case where Y_t is stationary, that is, $b > 0$. Because $b > 0$, the growth rate of Y_t depends on $y_t = \ln Y_t$. To make Y_t a multiplicative functional, we need to augment the state space to include y_t: $\left(\hat{X}, \tau, y\right)$. In this setup, the eigenfunctions in general take the form of $\mathbf{e}\left(\hat{X}, \tau, y\right)$. Under our assumptions, $\left(\hat{X}_t, \tau_t\right)$ clearly has a unique ergodic distribution. We also assume that $\mu_Y\left(\hat{X}, \tau\right)$ is such that y_t also has a unique ergodic distribution. Under these assumptions, we obtain two main results. First, Equation (16) holds with $f(\tau) = 1$ and, therefore, by (17), changes in growth expectations can be directly computed from the ratio of the return on long term equity and that on long term bond:

$$\Delta E^{\infty}(T) = \frac{R_E^{\infty}(T)}{R_B^{\infty}(T)}. \tag{18}$$

Our second result is that over a short FOMC announcement window, the above ratio is always equal to 1. This is not surprising, because aggregate GDP is a stationary process, it converges to a steady state in the long run. Monetary policy or any policy might have a temporary impact on the level of output, but does not have a long-run impact on the economy. The textbook version of the New Keynesian model has this feature. We summarize our main result in the following proposition.

Proposition 1. *Suppose* $\left(-\bar{r}, \mathbf{e}_S\left(\hat{X}, \tau\right)\right)$ *solve the eigenvalue problem (8) for the stochastic discount factor. Suppose also that under the measure induced by the associated martingale, the process* $\left(\hat{X}_t, \tau_t\right)$ *is stationary and ergodic. Then* $\left(-\bar{r}, \mathbf{e}_S\left(\hat{X}, \tau\right)e^{-y}\right)$ *solves the eigenvalue problem (8) for the multiplicative functional* S_tY_t*, and* $(1, e^{-y})$ *solves the same problem*

for the multiplicative functional Y_t. In addition, the distributions of $\left(\hat{X}_t, \tau_t, y_t\right)$ are stationary under the respective measures constructed from the martingale (9).

Proof. See Section 7.1 in the appendix. □

Clearly, Proposition 1 implies that $\mathbf{e}_Y = \frac{\mathbf{e}_{SY}}{\mathbf{e}_S}$. By Equation (18), we can compute the announcement return implied expectation update by normalizing the announcement return on long term equity by the announcement return on long bond. Because $\mathbf{e}_Y\left(\hat{X}, \tau, y\right) = e^{-y}$, under the assumptions of Proposition 1,

$$\Delta E^{\infty}(T) = \frac{R_E^{\infty}(T)}{R_B^{\infty}(T)} = \frac{e^{y_T}}{e^{y_T^-}} = 1, \tag{19}$$

because y_t is a continuous process. That is, monetary policy cannot have any long-run impact. This is intuitive, because the economy converges to a stationary distribution in the long run and does not have long-run growth. Monetary policy in this economy cannot have an impact on the long-run growth rate.

An immediate implication of our result is that if we observe that the ratio of the return on long term equity and that on long term bond are not identical over short FOMC announcement windows, we should reject the hypothesis that monetary policy does not have a permanent impact on aggregate economic growth. We next provide an example of a New Keynesian model where Equation (18) holds.

Example 1. A New Keynesian model of FOMC announcement

We slightly modify the textbook New Keynesian model to fit our general analytical framework.

3.3 Measuring long run impact on growth

As we will see in Section 4 below, the data strongly reject the hypothesis of zero long run impact of monetary policy. To provide a framework by which we can quantitatively measure the long-run impact of monetary policy, we set $b = 0$ so that our model allows for long-run growth. While the relationship between updates of expectation and returns on long term assets (18) holds generally in stationary economies, a similar relationship in economies with long-run growth requires more assumptions. In this section, we derive a version of Equation

(18) by assuming log linear dynamics of the SDF and the cash flow process. We make the following assumptions.

Assumption 1: $r(X,\tau) = r_0(\tau) + r_1' X$, $\sigma(X,\tau) = \sigma(\tau)$. That is, the risk-free rate is linear in X and the market price of risk is only a function of τ.

Assumption 2: The SDF that prices announcement return is exponentially linear in X:

$$\frac{S_T}{S_T^-} = \frac{e^{-\alpha \hat{X}_T}}{E\left[e^{-\alpha \hat{X}_T} \middle| X_T^-, T^-\right]}. \tag{20}$$

Assumption 3: $b = 0$ and $\mu_Y\left(\hat{X}_t\right) = \mu_Y' \hat{X}$ is also linear in $\hat{X}$.

Proposition 2. *Under Assumptions 1-3, the eigenfunctions of the eigenvalue problem for the multiplicative functionals, S_t, Y_t and $S_t Y_t$ all take the following form:*

$$\mathbf{e}_i\left(\hat{X},\tau\right) = e^{\beta_i' X + g_i(\tau)}, \tag{21}$$

for $i = S, SY, Y$ with $\beta_{SY} = \beta_S + \beta_Y$. In addition

$$\frac{R_E^\infty(T)}{R_B^\infty(T)} = \Delta E^\infty(T)\, e^{\beta_Y Q_T^- \alpha}, \tag{22}$$

where $e^{\beta_Y Q_T^- \alpha}$ is the announcement risk premium on long term equity over long term bond.

Proof. See Appendix. □

Equation (22) generalizes the stationary case, (18). The update of expectation of long-run growth can be computed from the difference in the return on long term equity and the return on long bond with an adjustment term, $e^{\beta_Y Q_T^- \alpha}$ which can be interpreted as the announcement premium of long term equity over long bond. By construction, $E[\Delta E^\infty(T)] = 1$, because it is an update of expectation. On the left side, the expected return on long term equity, $E[R_E^\infty(T)]$ in general is different from the expected return on long term bond, $E[R_B^\infty(T)]$, because our stochastic discount factor, (20) allows for an announcement premium. The above proposition implies that after adjusting for a risk premium, the return on long term equity normalized by the return on long term bond equals the update of belief about long-run growth. In other words, deviations of $\frac{R_E^\infty(T)}{R_B^\infty(T)}$ from its expectation can be used to infer

the long-run impact of monetary policy during short announcement windows.

In general, monetary policy may affect asset valuations through both a discount rate channel and a cash flow expectations channel, and it is hard to separately identify the two. The above proposition implies that if we are interested in long-run cash flow expectations, then under Assumptions 1-3, the discount rate effect can be completely eliminated by computing the ratio of the return on long term equity and that on long term bond. The residue, after subtracting an announcement risk premium, reflects the market expectation about long-run growth. The key assumption for the above result is the linearity of risk-free rate and the linearity of expected growth in state variables. This can be interpreted as coming from the log-linearized solution of equilibrium models. In the rest of this section, we will discuss several general equilibrium models where Assumptions 1-3 hold and the ratio of the long term equity return and long term bond return identifies exactly growth rate expectations.

Example 2. Ai and Bansal (2018)

Section 5 of Ai and Bansal (2018) presented a continuous-time model with macroeconomic announcements and recursive preferences. The unit IES version of the model satisfies the assumptions of Proposition 2, which we briefly describe below. The aggregate consumption follows

$$\frac{dC_t}{C_t} = x_t dt + \sigma dB_{C,t}, \tag{23}$$

where x_t is an Ornstein–Uhlenbeck processes of the form:

$$dx_t = a\left(\bar{x} - x_t\right) dt + \sigma_x dB_{x,t}, \tag{24}$$

and $B_{C,t}$ and $B_{x,t}$ are independent standard Brownian motions. The true value of x_t is not observable but investors can learn about x_t through two sources. First, investors can update their beliefs about x_t through their observations of C_t. Second, upon pre-scheduled FOMC announcements at $t = 0, T, 2T, \cdots$, investors receive additional signals of the form $s_{nT} = x_{nT} + \epsilon_{nT}$, where $\epsilon_{nT} \sim N\left(0, \sigma_S^2\right)$. In this setting, the posterior distribution of x_t is a Gaussian distribution with mean $\hat{x}$ and variance q. For $t \in (nT, (n+1)T)$,

$$d\hat{x}_t = a\left(\bar{x} - \hat{x}_t\right) dt + \frac{q_t}{\sigma} d\hat{B}_t, \tag{25}$$

where the Brownian motion $\hat{B}_t$ is defined as $d\hat{B}_t = \frac{1}{\sigma}\left[\frac{dC_t}{C_t} - \hat{x}_t dt\right]$. The posterior variance q_t satisfies a Riccati equation in the form

$$dq_t = \left[\sigma_x^2 - 2aq_t - \frac{1}{\sigma^2}q_t^2\right]dt. \tag{26}$$

On the boundary, $\hat{x}_T = \frac{1}{q_{nT}}\left[\frac{1}{\sigma_S^2}s_{nT} + \frac{1}{q_{nT}^-}\hat{x}_{nT}^-\right]$. In addition, q_t cycles deterministically across announcements and satisfies $q_t = q_{\tau(t)}$ with $\frac{1}{q_{nT}} = \frac{1}{\sigma_S^2} + \frac{1}{q_{nT}^-}$.

The representative investor has a recursive preference with unit IES. For $t \in (nT, (n+1)T)$, the SDF takes the form of

$$dS_t = S_t\left[-r\left(\hat{x}_t, \tau\left(t\right)\right)dt - \sigma_S\left(\tau\left(t\right)\right)d\hat{B}_t\right],$$

where $r\left(\hat{x}, \tau\right) = \beta + \hat{x} - \gamma\sigma^2 - \frac{\gamma-1}{a+\beta}q\left(\tau\right)$, and $\sigma_S\left(\tau\right) = \left(\gamma + \frac{\gamma-1}{a+\beta}\frac{q(\tau)}{\sigma^2}\right)\sigma$. On the boundary,

$$\frac{S_{nT}}{S_{nT}^-} = e^{-\frac{1}{2}\left(\frac{\gamma-1}{a+\beta}\right)^2 q\left(\tau\left(nT^-\right)\right) - \frac{\gamma-1}{a+\beta}\left(\hat{x}_{nT} - \hat{x}_{nT}^-\right)}.$$

In the above setup, we consider a slightly more general dynamics of aggregate output than (23):

$$\frac{dY_t}{Y_t} = b\hat{x}_t dt + \sigma_Y d\hat{B}_t. \tag{27}$$

The interpretation is that we allow the consumption of marginal investors to be potentially different from aggregate output. The two are equal if we set $b = 1$ and $\sigma_Y = \sigma_C$. The dynamics of the aggregate output (27) and the equilibrium stochastic discount factor in this example clearly satisfy the assumptions of Proposition 2. As we show in the appendix, the three eigenfunctions take the form of (21) with

$$\beta_S = -\frac{1}{a_x}, \beta_{SY} = \frac{b-1}{a_x}, \ \beta_Y = \frac{b}{a_x}.$$

Clearly, Equation (22) holds with $\alpha = \frac{\gamma-1}{a+\beta}$. In this case, innovations in growth rate expectations can be directly recovered from the ratio of long term equity returns and long term bond returns after adjusting for an announcement premium.

We make several observations on the above example. First, in the above example, the

eigenvalue problem (8) is well-defined for any t. However, the limit

$$\lim_{\Delta\to 0}\frac{1}{\Delta}E_t\left[\frac{S_{t+\Delta}}{S_t}\psi\left(\hat{x}_{t+\Delta},\tau\left(t+\Delta\right)\right)-\psi\left(x_t,\tau\left(t\right)\right)\right]$$

is not well-defined for every ψ at $t=T^-$. As a result, the extended generator in the sense of Hansen and Scheinkman (2009) is not well-defined at announcement times, nT^-, even though the eigenvalue problem (8) is. This is why we use (8) to formulate the eigenvalue problem instead of using the generator definition as in Hansen and Scheinkman (2009).

Second, the above example does not satisfy the long-run neutrality (LRN) condition of Cieslak and Khorrami (2025). The LRN condition requires the monetary policy news to be independent of the permanent component of the pricing kernel. In the above example, let $\hat{M}_t = e^{-\rho t}S_t\frac{\mathbf{e}_S(\hat{x}_t,\tau(t))}{\mathbf{e}_S(\hat{x}_0,\tau(0))}$ be the permanent component of the SDF, i.e. the martingale constructed form the multiplicative functional S_t using formula (9), then

$$\frac{\hat{M}_{nT}}{\hat{M}_{nT}^-}=e^{-\frac{1}{2}\left(\frac{1}{a}+\frac{\gamma-1}{a+\beta}\right)^2 q_{\tau(nT^-)}}e^{-\left(\frac{1}{a}+\frac{\gamma-1}{a+\beta}\right)\left(\hat{x}_{nT}-\hat{x}_{nT}^-\right)},$$

where $\hat{x}_{nT}-\hat{x}_{nT}^-$ is the monetary policy shock. That is, the monetary policy shock in the above example is perfectly correlated with the innovation in the permanent component of the pricing kernel at announcements. Despite the failure of the LRN condition, our formula perfectly recovers the innovation in the subjective belief about long-run growth. The reason for this is that LRN is a condition where changes in all expectations under the physical probability can be recovered from changes in long-run risk-neutral expectations. This puts a very strong requirement on the relationship between monetary policy news and the pricing kernel. Our objective is not to recover all expectations, but just the expectation of long-run growth. The latter is possible under much more general conditions.

Our next example is adapted from the Veronesi (2000) model.

Example 3. Veronesi (2000) with Gaussian shocks

Veronesi (2000) also considers a model where the consumption process takes the form of (23) and x_t is not observable. Veronesi (2000) assumes that x_t is a finite state Markov chain, in which case the dynamics of the posterior mean of x_t do not satisfy the linear stochastic differential equation in (3). We will study the case of a finite-state Markov Chain in Example

6 below. Here, we modify the Veronesi (2000) model and assume that x_t is an Ornstein–Uhlenbeck process of the form (24) and therefore the posterior beliefs follow (25) and (26). The Veronesi (2000) model assumes a representative agent with CRRA utility and constant risk aversion γ. Under expected utility, the stochastic discount factor is of the form

$$dS_t = S_t \left[-r\left(\hat{x}_t\right) dt - \gamma \sigma dB_t \right], \tag{28}$$

where the risk-free $r\left(x\right)$ takes the form of

$$r\left(x\right) = \beta + \gamma x - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma^2.$$

We allow aggregate output to be different from the consumption of marginal investors. Again, Proposition holds with

$$\beta_S = -\frac{\gamma}{a_x}, \beta_{SY} = \frac{b-\gamma}{a_x}, \ \beta_Y = \frac{b}{a_x}.$$

Due to expected utility, in this example, there is no need to adjust for announcement risk premium.

The last example is the Brennan and Xia (2001) model.

Example 4. Brennan and Xia (2001)

Brennan and Xia (2001) is a model where the consumption of the marginal investor is a geometric Brownian motion with constant drift. As a result, the pricing kernel takes the form of

$$dS_t = S_t \left[-rdt - \sigma_S dB_t \right],$$

for some constant r and σ_S. However, the cash flow takes the form of (23). In this case, Equation holds with

$$\beta_S = 0, \beta_{SY} = \frac{1}{a_x}, \ \beta_Y = \frac{1}{a_x}.$$

In this example, monetary policy announcements do not affect the permanent component of the SDF and the LRN condition in Cieslak and Khorrami (2025) holds. Our formula (22) recovers expectations about long-run growth. In fact, due to the LRN condition, it is possible to recover innovations of all expectations upon announcements from asset prices.

4 Evidence for the long-run impact of monetary policy

Our empirical exercise is guided by Equation (22). Taking log on both sides of this equation, we have:

$$\ln \Delta E^{\infty}(T) = \beta_Y Q_T^- \alpha + \ln R_E^{\infty}(T) - \ln R_B^{\infty}(T). \tag{29}$$

In the above equation, $\ln \Delta E^{\infty}(T)$ is the market's update of long-run growth expectations. $\ln R_E^{\infty}(T)$ and $\ln R_B^{\infty}(T)$ are announcement returns of long term equity and long term bonds. All three quantities are measurable functions of X_T, that is, they are functions of the content of announcements. The term $\beta_Y Q_T^- \alpha$ is the announcement premium and is measurable with respect to pre-announcement information. The above suggest the following measurement of update of long-run growth expectation upon FOMC announcements based on linear regressions:

$$\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T) = \xi_0 + \xi' Z_T^- + \epsilon_T, \tag{30}$$

where Z_T^- is a vector of macro variables that are know pre-FOMC announcements and $\ln R_E^{\infty}(T)$ and $\ln R_B^{\infty}(T)$ are long term equity return and long term bond return over short FOMC announcement windows. The residue of the above regression, ϵ_T, is our measurement of markets' expectation of the long-run impact of monetary policy.

Using the above methodology, our empirical exercise has two key findings. First, the hypothesis that surprises contained in the FOMC announcements do not have a permanent impact on output is strongly rejected by the data. Second, we find that a 25 basis point surprise cut in nominal rates from a conventional monetary policy shock (Fed information shock) is, on average, associated with a 1% upward revision (2% downward revision) in long-run economic growth expectation. In what follows, we first introduce our empirical proxies of long term equity and long term bond and present the empirical findings.

4.1 Proxies for long-term equity and long-term bond

Following the literature on high-frequency identification, for example, Gürkaynak, Sack, and Swanson (2005), we examine asset returns in a short window spanning from 10 minutes before to 20 minutes after each FOMC announcement. The sample covers the period from

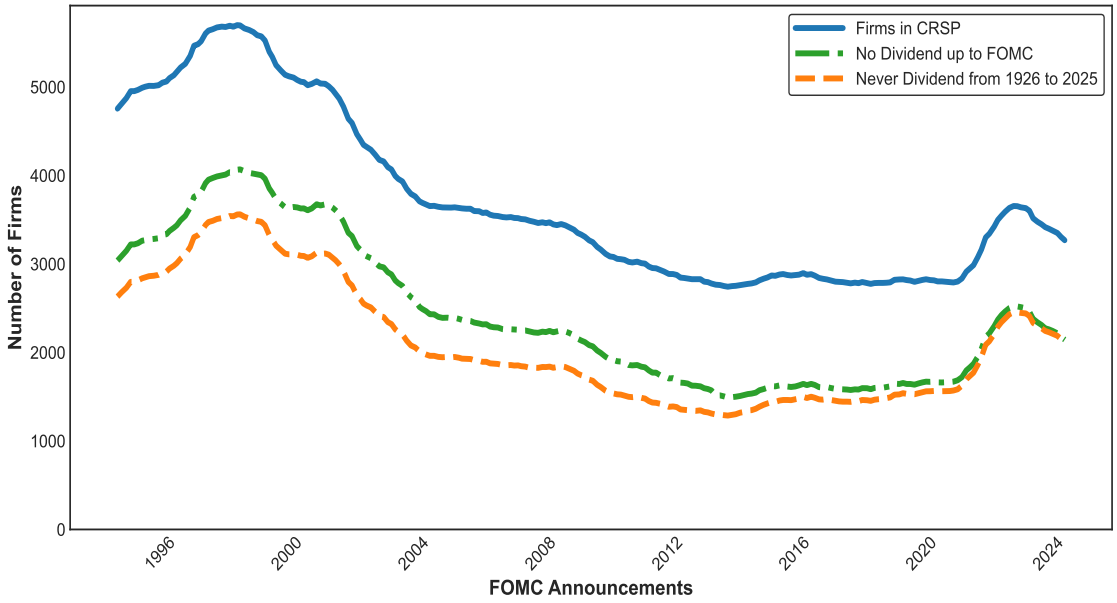
February 4, 1994, when the FOMC began publishing its federal funds rate target decisions after each meeting, through December 13, 2023.

Consistent with the literature, for example, Alvarez and Jermann (2005), we proxy the long term bond with the Treasury bond of the longest maturity available. Given our focus on the high-frequency price responses, we use futures on the 30-year Treasury bond, as in Cieslak and Khorrami (2025). We proxy the long term equity with a portfolio of non-dividend paying stocks. We obtain the intraday price data for individual stocks from TAQ and for 30-year T-Bond futures from Tick Data. The tick-by-tick price data are aggregated to the minute level by taking the median of all prices within each minute. We define the closest available minute-level prices 10 minutes before and 20 minutes after the announcements on FOMC days as the pre- and post- announcement prices. Then, we compute the announcement returns as the log price difference for the 30-year T-Bond futures and each stock. To mitigate concerns about microstructure noise, we exclude observations with stock prices below \$5, although our results remain robust when they are included.

The non-dividend paying stock portfolio To construct a portfolio of non-dividend-paying stocks as a proxy for long term equity, we obtain stock-level variables from monthly CRSP and classify a stock as non-dividend-paying in month t if its total return and ex-dividend return are identical. Figure 1 plots three series at each FOMC announcement between 1994 and 2025: the total number of firms in CRSP (solid line), the number of firms that had never paid any dividend since their listing up to the announcement date (dash-solid line), and the number of firms that did not pay any dividend at any point during the sample period (dashed line). Our non-dividend-paying portfolio consists of the last group—firms that never paid dividends during the entire sample period—which accounts for roughly 50% of all CRSP firms in the sample.

To validate our portfolio as a proxy for long term equity, we compare it against an alternative construction using dividend futures data. Dividend futures allows us to compute time t value of the dividend strip of the the S&P 500 index with maturity n as $P_t^n = B_t^n G_t^n$ where B_t^n is the price of a zero-coupon bond maturing in $t+n$ and G_t^n is the price of dividend futures maturing in $t+n$ (See, for example,Van Binsbergen and Koijen (2017), Bansal, Miller, Song, and Yaron (2021) and Nagel and Xu (2025)). Given that a stock is a claim to all future dividends, we can construct the price of a synthetic portfolio of long-term dividends of the

Figure 1: Number of Firms



Notes: The figure plots the total number of firms in the CRSP database at each FOMC announcement from 1994 to 2025. The solid line is the total number of firms in the CRSP database at each FOMC announcement. The dash-solid line is the number of firms that had never paid any dividend since their listing up to the announcement date. The dashed line indicates the number of firms that did not pay any dividend at any point during the sample period of 1926-2025.

S&P 500 index after year M as

$$P_t^{M+} = P_t^{S\&P500} - \sum_{n=1}^{M} P_t^n. \tag{31}$$

In what follows, we will call P_t^{M+} the price of the tail dividends with maturity M.

We obtain daily prices of dividend futures on the S&P 500 index from January 2007 to August 2014 from Bansal, Miller, Song, and Yaron (2021) and from November 2015 onwards from Bloomberg. The dividend futures expire on the third Friday of December. As is common in the literature, we linearly interpolate futures prices between two adjacent maturities to obtain the price for yearly maturities up to seven years for each trading day. The first date with valid data for the nearest five years in our sample is December 2009. We obtain the daily nominal yields data from Gürkaynak, Sack, and Wright (2007) and the daily

S&P 500 index from CRSP. Table 1 reports the correlations between the daily log return of our non-dividend paying stock portfolio $\ln R_E^\infty$ and the return on the synthetic tail dividend claim of the S&P 500 index with maturities $M=$ 5, 6 and 7 and that with the S&P 500 itself[6]. Following the literature, we exclude dividend futures with maturities beyond seven years, as they are less liquid. Consistent with our theory, the correlation between the return on our non-dividend paying stock portfolio and the tail dividend claim is higher than its correlation with the S&P 500 index itself, and substantially so on FOMC announcement days.

Table 1: Correlation with an alternative proxy

	$\ln R^{5+}$		$\ln R^{6+}$		$\ln R^{7+}$		$S\&P500$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FOMC	0.938		0.936		0.935		0.867	
All days		0.897		0.895		0.895		0.854
N	101	3154	92	2893	87	2723	238	7803

Note: The table reports the correlations of the log daily return of the non-dividend paying stock portfolio $\ln R_E^\infty$ with the log daily return of an alternative proxy constructed using the S&P500 and dividend futures. The last row of the table reports the number of observations available in the sample to compute the correlations.

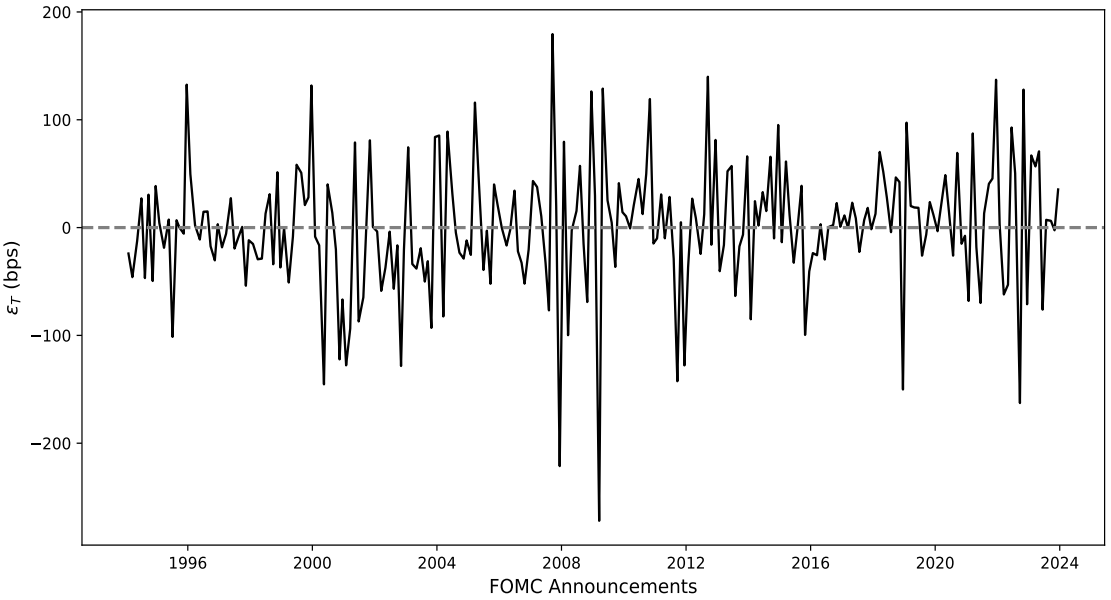
We prefer the non-dividend-paying portfolio to the synthetic tail-dividend claim for several reasons. First, stocks are far more liquid than dividend futures, and their prices are available at high frequency, which improves data quality and is crucial for high-frequency identification. Second, our theory requires long-maturity dividends. The synthetic tail-dividend portfolio has a maturity of seven years or longer, whereas the maturity of the non-dividend-paying stocks is likely much higher, since these firms have never paid dividends since their inclusion in the CRSP database. Finally, as shown in Table 1, data coverage is also substantially better for the non-dividend-paying portfolio.

Constructing long-run growth expectations Applying Equation (30), we first compute the value-weighted announcement return of the non-dividend paying portfolio ($\ln R_E^\infty(T)$)and then normalize the returns by the return of the 30-year T-Bond futures to obtain the

[6]The availability of individual stock price data in CRSP daily is different from that in TAQ so that the set of firms in the portfolio can be different.

infinite-maturity excess return $(\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T))$. To remove the predictive component, we run the predictive regression (30) using the set of macro variables known prior to the announcements from Bauer and Swanson (2023) to construct the orthogonalized series ϵ_T. The adjusted R^2 of this regression is 0.048, indicating very minimal predictability of $\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T)$. The regression results are reported in the Appendix 8. Figure (2) displays the constructed series of ϵ_T.

Figure 2: Time series of ϵ_T in 30-minute window around announcements



Notes: The figure plots our measure of the growth expectation revisions around the 30-minute window (-10 to +20 minutes) at each scheduled FOMC meeting. The sample spans 1994/01/01 to 2023/12/13, covering 239 meetings. Growth expectation revisions are quoted in basis points. The horizontal dashed line represents the case of no revision in growth expectation.

The lack of predictability in regression (30) is consistent with the finding that most of the FOMC announcement premiums are realized before, rather than within, the 30-minute window surrounding the announcements (Lucca and Moench (2015)). This implies that both $\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T)$ and ϵ_T are valid measures of changes in long-run growth expectations during short FOMC windows. As shown in Table (2), the two series exhibit similar summary statistics. The mean of $\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T)$ is close to zero, consistent with its interpretation as a measure of innovations in expectations. The standard deviation of innovations

of growth rate expectations in our sample period is about 60 bps, slightly higher than that of the S&P 500 futures return in the same window, 51 bps, indicating a sizable permanent impact of monetary policy announcements.

Table 2: Summary Statistics of key variables

	Panel A		Panel B	
	(1)	(2)	(3)	(4)
	ϵ_T(bps)	$\ln R_E^\infty - \ln R_B^\infty$ (bps)	MPS(bps)	CBI(bps)
Mean	0	-1.13	-0.2	-0.4
SD	59.46	61.72	4.7	2.7
25th	-26.02	-25.61	-2.3	-1.8
75th	30.62	28.02	2.1	1.2
AC	-0.13	-0.04	-0.183	0.124
Skew	-0.54	-0.79	-0.585	-0.805
N	239	239	239	239

Notes: The table reports the summary statistics of key variables in the 30-minute window around the scheduled FOMC announcements from 1994/01/01 to 2023/12/13. Panel A presents summary statistics for the difference in log returns between the long term equity portfolio and 30-year Treasury bond futures ($\ln R_E^\infty - \ln R_B^\infty$) and for our measure of revisions in long-run growth expectations (ϵ_T). Both variables are quoted in basis points. Panel B presents summary statistics for the pure monetary policy surprise and the Fed information surprise from Jarociński and Karadi (2020), reported in basis points.

4.2 Testing the absence of long-run impact

A necessary condition for the absence of long-run impact of monetary policy, as shown in Equation (19), is the coefficient β_1 of the regression of long term equity return, $\ln R_E^\infty(T)$ on that of long term bond return, $\ln R_B^\infty(T)$,

$$\ln R_E^\infty(T) = \beta_0 + \beta_1 \ln R_B^\infty(T) + u_T$$

is one. Using our constructed series of $\ln R_E^\infty(T)$ and $\ln R_B^\infty(T)$ the point estimate of β_1 is 0.235 and the 95% confidence interval is $[0.109, 0.361]$ with an adjusted R^2 of 0.04. A formal t-test of the null hypothesis $H_0 : \beta_1 = 1$ produces a test statistic of -11.9. Therefore, the hypothesis that monetary policy has no long-run impact is strongly rejected by the data. In what follows, we discuss the quantitative magnitude of this long-run impact.

4.3 Relationship with conventional measures of monetary policy shocks

To further quantify the permanent impact of monetary policies, we compare our time series of $\ln R_E^\infty(T) - \ln R_B^\infty(T)$ and ϵ_T with other measures of monetary policy shocks, including the monetary policy shocks of Bauer and Swanson (2023) (BS), the orthogonalized monetary policy shocks of Bauer and Swanson (2023) ($BS^\perp$), and monetary policy shock (MPS) and the central bank information shock (CBI) constructed in Jarociński and Karadi (2020). Table 3 reports the correlation between the measured long run impact of monetary policy, $\ln R_E^\infty(T) - \ln R_B^\infty(T)$ and ϵ_T, with the above measures of monetary policy shocks over the same FOMC announcement window. The correlations between our measure of long run impact of monetary policy and BS, $BS^\perp$, and MPS are all negative, indicating surprise interest rate reductions typically lead to upward revisions of long-run growth expectations. The CBI shock of Jarociński and Karadi (2020) is identified as interest rate increases that are associated with simultaneous stock market rises. These shocks typically lead to positive revisions of long-run growth rates as well.

Table 3: Correlations with high frequency monetary policy shocks

	MPS	CBI	BS	$BS^\perp$
	(1)	(2)	(3)	(4)
ϵ_T(bps)	-0.38	0.43	-0.11	-0.12
$\ln R^\infty$(bps)	-0.35	0.49	-0.06	-0.12

Notes: The table reports the correlations of ϵ_T and $\ln R^\infty$ with four measures of monetary policy shocks constructed in the literature. MPS and CBI represent the pure monetary policy shocks and Fed information shocks from Jarociński and Karadi (2020). BS and $BS^\perp$ are two monetary policy shocks from Bauer and Swanson (2023). BS is the first principal component of the price changes of the Eurodollar futures expiring in the nearest four quarters. $BS^\perp$is the residual from regressing BS on six macroeconomic variables known prior to the announcement. All variables are measured in the 30-minute window around the announcements.

To quantify the impact of different components of monetary policy shock on long run growth rates, we estimate the following regression

$$Long-run\ Impact_T = \beta_0 + \beta_1 MPS_T + \beta_2 CBI_T + u_T, \tag{32}$$

where $Long-run\ Impact_T$ can be either $\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T)$ or ϵ_T. All shocks are measure within the same 30-minutes window around FOMC announcements. In the above equation, MPS_T and CBI_T are the monetary policy shock and the central bank information shock, respectively, constructed by Jarociński and Karadi (2020). The regression coefficient β_1 and β_2 can be interpreted as the impact of MPS_T and CBI_T on long-run growth expectations, respectively. The regression results are reported in Table (4).

Table 4: High Frequency revisions in long-run growth to MPS and CBI

	Panel (A): ϵ_T(bps)			Panel (B): $\ln R^{\infty}$(bps)		
	(1)	(2)	(3)	(4)	(5)	(6)
MPS	-4.18^{***}			-3.86^{***}		
	(1.27)			(1.31)		
CBI	8.60^{***}			10.35^{***}		
	(2.09)			(2.15)		
BS		-1.25			-0.67	
		(1.05)			(0.99)	
$BS^{\perp}$			-1.42			-1.47
			(1.12)			(1.09)
Const.	2.43	0.50	0.04	2.02	-0.86	-1.50
	(3.50)	(3.90)	(3.93)	(3.63)	(4.02)	(4.04)
N	239	239	233	239	239	233
Adj. R^2	0.29	0.01	0.01	0.32	-0.001	0.01

Notes: The table reports OLS estimates of (A) the revisions in long-run growth expectations (B) log returns of the long term equity portfolio normalized by the long term bond. Heteroskedasticity-robust standard errors are reported in brackets.$^{*}p < 0.1$,$^{**}p < 0.05$,$^{***}p < 0.01$.

We find that a 25 basis points cut in short rates due to a pure monetary policy shock, on average, leads to a $417.81 * 0.25 \approx 104.45$ basis points upward revision in long-run output and the same magnitude of cut due to the Fed information effect is associated with $859.73 * 0.25 \approx 214.93$ downward revision (see the column 1 in the Panel A of Table (4)). The slope coefficients are highly significant and the adjusted R^2 is about 0.29, which is in contrast to the insignificant coefficients and near-zero adjusted R^2 obtained from regressions that directly use raw high-frequency changes in short rates (see columns (2) and (3)). Our results indicate that the pure monetary policy is not neutral in the long run and the Fed

announcements contain information about the long-run output. Panel B of Table (4) reports the same regressions using the $\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T)$ series. The results are similar overall.

5 Approximations

Our main identification Equation (22) relies on Assumptions 1-3. In this section, we analyze two examples where Assumptions 1-3 do not satisfy and demonstrate the robustness of Equation (22) as approximations. We obtain global solutions for the eigenfunctions and show that the errors of log-linear approximations are typically small under plausible parameter values. Our first example is built on the Wright-Fisher process where the Markov state variable $x \in [0,1]$ is highly nonlinear. This example allows us to obtain closed-form solutions for the three eigenfunctions in (13), (14), and (15) and illustrate why Equation (16) is a good approximation despite the failure of Assumptions 1-3. Using the closed-form solutions, we demonstrate that (16) holds under a first-order approximation of the eigenfunctions. To simplify exposition, we abstract from learning and announcements, and focus on the difference between $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ and $\mathbf{e}_Y(x)$ in Equation (16).

Example 5. An equilibrium model with Wright-Fisher dynamics

We assume that the underlying Markov process is a Wright-Fisher process:

$$dx_t = (-ax_t + b(1-x_t))\,dt + \sqrt{x_t(1-x_t)}\sigma dB_t, \tag{33}$$

where $a, b > 0$ and $x_t \in [0,1]$. As shown in Karlin and Taylor (1981) (Example 8, Page 239), the above process has a unique ergodic distribution under the following parameter restrictions, which we will maintain throughout this example.[7]

ASSUMPTION 4: $\frac{2a}{\sigma^2} > 1$ and $\frac{2b}{\sigma^2} > 1$.

We assume that the dynamics of cash flow takes the form of

$$dG_t = G_t\left[\nu(x_t)\,dt + \sigma_G(x_t)\,dB_t\right], \tag{34}$$

[7]See also Durrett (2008) for related discussions on the Wright-Fisher process.

where the drift and diffusion coefficients are:

$$\nu(x) = \nu_0 + \nu_1 \frac{x}{1-x} + \nu_2 \frac{1-x}{x}, \ \sigma_G(x_t) = \sigma_{G,2}\sqrt{\frac{1-x}{x}} + \sigma_{G,1}\sqrt{\frac{x}{1-x}}. \tag{35}$$

with $\nu_1 > 0$, and $\nu_2 > 0$. We are primarily interested in the case where $\nu_2 = 0$, and $\sigma_{G,1} = 0$ so that $\nu(x) \to \nu_0$ as $x \to 0$, and $\nu(x) \to \infty$ as $x \to 1$. However, we keep the formulation general. When $\sigma_{G,1} = 0$, the term $\sigma_G(x_t) = \sigma_{G,2}\sqrt{\frac{1-x}{x}}$ models counter cyclical volatility in the cash flow process. We assume that the SDF takes the form of

$$dS_t = S_t\left[-r(x_t)\,dt - \sigma_S(x_t)\,dB_t\right],$$

where the risk-free rate and the market price of risk takes the form of:

$$r(x) = r_0 + r_1 \frac{x}{1-x} - r_2 \frac{1-x}{x}, \ \sigma_S(x) = \sigma_{S,2}\sqrt{\frac{1-x}{x}} + \sigma_{S,1}\sqrt{\frac{x}{1-x}}. \tag{36}$$

We assume the parameters are such that $r_1, r_2 > 0$ and $\sigma_{S,1}, \sigma_{S,2} > 0$. That is, the risk-free rate is increasing in x. In Appendix 7.3, we show that the above dynamics of the pricing kernel can arise in a general equilibrium model where the representative agent has a recursive preference and consumption process of the form (34) and (35). Under the general equilibrium interpretation of the SDF, the parameters r_0, r_1, r_2 and $\sigma_{S,1}, \sigma_{S,2}$ will be functions of preference and technology parameters. The proposition below derives the eigenfunctions under the general specification without imposing general equilibrium.

Proposition 3. *The eigenfunctions associated with the multiplicative functional, S_t, $\mathbf{e}_S(x)$ is given by:*

$$\mathbf{e}_S(x) = x^{\beta_S}(1-x)^{-\alpha_S}, \tag{37}$$

where the parameters, α_S and β_S are given by:

$$\alpha_S = \left(\frac{a + \sigma\sigma_{S,1}}{\sigma^2} - \frac{1}{2}\right) - \sqrt{\left(\frac{a + \sigma\sigma_{S,1}}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r_1}{\sigma^2}}, \tag{38}$$

$$\beta_S = \left(\frac{1}{2} - \frac{b - \sigma\sigma_{S,2}}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{b - \sigma\sigma_{S,2}}{\sigma^2}\right)^2 - \frac{2r_2}{\sigma^2}}. \tag{39}$$

In addition, the eigenfunctions associated with S_tY_t, and Y_t are also of the form $\mathbf{e}_i(x) = x^{\beta_i}(1-x)^{\alpha_i}$, for $i = SY$ and $i = Y$ with

$$\alpha_{SY} = \left(\frac{a+\sigma(\sigma_{S,1}-\sigma_{G,1})}{\sigma^2} - \frac{1}{2}\right) - \sqrt{\left(\frac{a+\sigma(\sigma_{S,1}-\sigma_{G,1})}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r_1-\nu_1+\sigma_{G,1}\sigma_{S,1})}{\sigma^2}}, \tag{40}$$

$$\beta_{SY} = \left(\frac{1}{2} - \frac{b+\sigma(\sigma_{G,2}-\sigma_{S,2})}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{b+\sigma(\sigma_{G,2}-\sigma_{S,2})}{\sigma^2}\right)^2 - \frac{2(r_2+\nu_2-\sigma_{G,2}\sigma_{S,2})}{\sigma^2}}, \tag{41}$$

$$\alpha_Y = \left(\frac{a-\sigma\sigma_{G,1}}{\sigma^2} - \frac{1}{2}\right) - \sqrt{\left(\frac{a-\sigma\sigma_{G,1}}{\sigma^2} - \frac{1}{2}\right)^2 - \frac{2\nu_1}{\sigma^2}}, \tag{42}$$

$$\beta_Y = \left(\frac{1}{2} - \frac{b+\sigma\sigma_{G,2}}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{b+\sigma\sigma_{G,2}}{\sigma^2}\right)^2 + \frac{2\nu_2}{\sigma^2}}. \tag{43}$$

We make several observations on the above example. First, in the above model, the valuation of the ratio of long term equity and long term bond can be written as $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)} = x^{(\beta_{SY}-\beta_S)}(1-x)^{(\alpha_S-\alpha_{SY})}$. Clearly, in general,

$$\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)} = x^{(\beta_{SY}-\beta_S)}(1-x)^{(\alpha_S-\alpha_{SY})} \neq \mathbf{e}_Y(x).$$

Given the functional form of the eigenfunctions in (37), the equation $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)} = \mathbf{e}_Y(x)$ is not true unless $\alpha_S + \alpha_Y = \alpha_{SY}$ and $\beta_S + \beta_Y = \beta_{SY}$, which in general will not be the case.

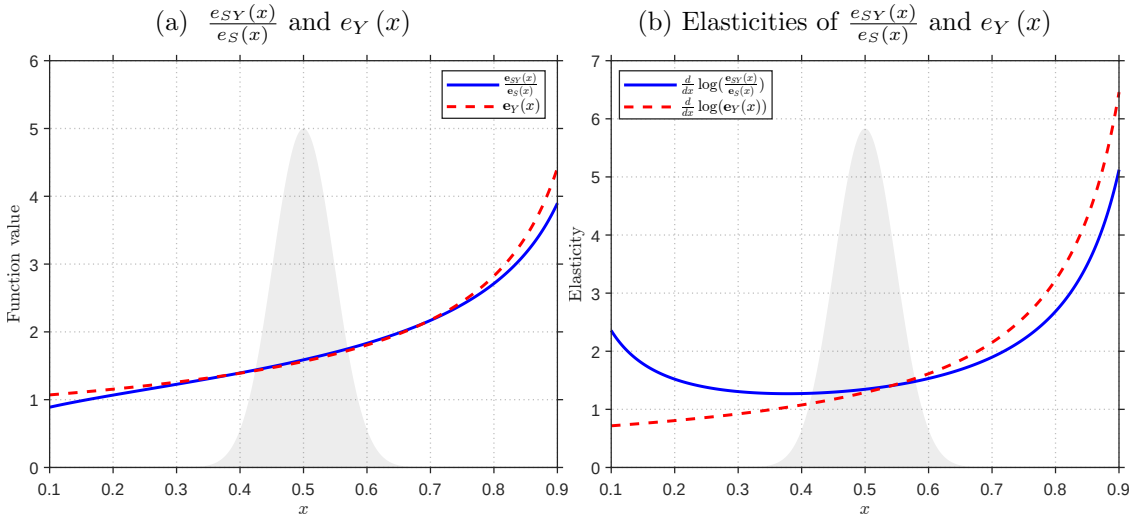
Second, it is straightforward to verify that under the condition that news about the state variable x does not affect the distribution of long-run cash flow growth, that is, $\nu_1 = \nu_2 = \sigma_{G,1} = \sigma_{G,2} = 0$,

$$\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)} = \mathbf{e}_Y(x) = 1,$$

which is consistent with the result in Proposition 1.

In Figure 3 we compare $\mathbf{e}_Y(x)$ (dashed line) with $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ (solid line) in a calibrated version of the model. We calibrate the parameters of the x process so that its steady-state distribution is symmetric. We calibrate the parameters of the cash flow process to match aggregate consumption data, as we list in Table 6. Instead of choosing parameters of the

Figure 3: Comparison of Eigenfunctions (Wright-Fisher)



Notes: Figure 3a plots the numerical solutions of $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ and $\mathbf{e}_Y(x)$ for the Wright-Fisher and Figure 3b plots their elasticities. Given that the eigenfunctions are unique up to a scale, we rescale them in Figure 3a so that they are comparable in the plots. The shaded grey area represents the stationary distribution of the state variable x, scaled for visualization.

SDF directly, we derive the SDF dynamics from a representative agent model where G_t is the aggregate consumption processes and the agent's preference is of the Epstein-Zin with parameters listed in Table 5. In Appendix, we provide detailed derivations of the Equilibrium SDF. On the left panel, 3a, we compare the level of $\mathbf{e}_Y(x)$ and $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$. Because eigenfunctions are unique up to a multiplicative constant, we align the levels of $\mathbf{e}_Y(x)$ and $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ at the steady state $x = 0.5$. The shaded area indicates the density of the steady-state distribution of x. The eigenfunction $\mathbf{e}_Y(x)$ and its approximation, $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ track each quite closely, especially in regions where the density of steady-state distribution of x focuses on.

Empirically, when using the return on the long-equity and the long-bond portfolio to measure the long-run impact of monetary policy as in (29), we are effectively using $\frac{d}{dx}\ln\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ to approximate $\frac{d}{dx}\ln\mathbf{e}_Y(x)$. In 3b, we compare the two elasticities, $\frac{d}{dx}\ln\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ (solid line) and $\frac{d}{dx}\ln\mathbf{e}_Y(x)$ (dashed line). This comparison is more informative than the comparison of levels, because the distance between $\frac{d}{dx}\ln\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ and $\frac{d}{dx}\ln\mathbf{e}_Y(x)$ is the approximation error of our empirical implementation (29). In addition, even though the level of eigenfunctions depends multiplicative constants, the elasticities eliminates the dependence on multiplicative

Table 5: Calibrated Parameters (Wright-Fisher)

Panel A. Wright-Fisher process		
$a = b$	Drift parameters	0.053
σ	Vol of x	0.044
Panel B. Preference		
β	Time discount rate	0.06
γ	Relative risk aversion	6
ψ	IES	3.3
Panel C. Consumption process		
$\sigma_{C,2}$	Vol of consumption	0.013

constants and can be compared directly.

To understand the approximation error in 3b, using Proposition 3,

$$\frac{d}{dx}\ln \mathbf{e}_Y(x) = \beta_Y \frac{1}{x} - \alpha_Y \frac{1}{1-x},$$

and

$$\frac{d}{dx}\ln \frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)} = (\beta_{SY} - \beta_S)\frac{1}{x} - (\alpha_{SY} - \alpha_S)\frac{1}{1-x}.$$

As a result, the approximation error depends on the difference between β_Y and $\beta_{SY} - \beta_S$ and that between α_Y and $\alpha_{SY} - \alpha_S$. In Appendix 7.3, we show that $\alpha_S + \alpha_Y \approx \alpha_{SY}$, and $\beta_S + \beta_Y \approx \beta_{SY}$, where the approximations are computed by using a first-order Taylor expansion around the deterministic steady-state of the model.

Our next example is adapted from the model of Veronesi (2000) and Ai (2010). The Veronesi (2000) features a non-linear filtering problem, which violates the assumption of the linear dynamics of the Markov state variable. The related Ai (2010) model assumes general recursive utility, which violates Assumption 1, that is, the log linear form of the stochastic discount factor in state variables. We consider an environment that combines both features and demonstrate that quantitatively, condition (22) still provides a very accurate approximation under plausible parameter values.

Example 6. A Veronesi-Ai Economy

We consider an endowment economy with consumption following (23). Instead of as-

Table 6: Model Moments (Wright-Fisher)

Panel A. Consumption		
ρ_Y	Long-run growth rate	2.5%
$E\left(\frac{dY}{Y}\right)$	Mean growth rate	2.4%
$Std\left(\frac{dY}{Y}\right)$	Vol of consumption growth rate	1.3%
Panel B. Asset pricing		
$-\rho_{SY}+\rho_Y+\rho_S$	Long-run equity premium	3.5%
$-\rho_S$	Long bond yield	1%
$\sqrt{E\left(\sigma_S^2\right)}$	Max. Sharpe ratio	0.8
$E\left(r_f\right)$	Average risk-free rate	2.5%
$Std\left(r_f\right)$	Vol of risk-free rate	0.3%

suming that x_t follows an an Ornstein–Uhlenbeck process, we assume that it is a two-state Markov chain with state space $\{x_H, x_L\}$ and infinitesimal generating matrix:

$$\begin{bmatrix} -\lambda_H & \lambda_H \\ \lambda_L & -\lambda_L \end{bmatrix}.$$

In addition, instead of assuming that the representative consumption has expected utility, we assume a general recursive preference with risk aversion γ and IES ψ. For simplicity, we assume that the true value of x_t is fully revealed upon FOMC announcements. The rest of the setup is the same as in Example 2.

As shown in Veronesi (2000), the posterior distribution of x_t can be characterized by its conditional mean, $\hat{x}_t$, thanks to the assumption to a two-state Markov chain.

$$d\hat{x}_t = \left(\lambda_H + \lambda_L\right)\left(\bar{x} - \hat{x}_t\right)dt + \left(x_H - \hat{x}_t\right)\left(\hat{x}_t - x_L\right)\frac{1}{\sigma}d\hat{B}_t, \tag{44}$$

where $\bar{\theta}$ is the steady-state mean of θ:

$$\bar{\theta} = \frac{\lambda_L\theta_H + \lambda_H\theta_L}{\lambda_L + \lambda_H}, \tag{45}$$

and $d\hat{B}_t$ is defined in the same way as in Example 2. Veronesi (2000) considers CRRA

expected utility. Here, we allow for recursive utility with constant relative risk aversion and constant IES as in Ai (2010). As we show below, recursive utility introduces another source of non-linearity is the pricing kernel, which allows us to evaluate the robustness of Equation (22). Under the above assumption, the life-time utility of the representative consumer takes the form of $V(\hat{x},\tau,C)=\frac{1}{1-\gamma}H(\hat{x},\tau)C^{1-\gamma}$, where $H(\hat{x},\tau)$ is determined by Equations (72) and (73) in the appendix.

In the above setup, the SDF takes the form of

$$\frac{dS(t)}{S(t)} = -r(\hat{x}_t,\tau(t))\,dt-\sigma_S(\hat{x}_t,\tau(t))\,d\hat{B}_t \tag{46}$$

where the instantaneous risk free rate is

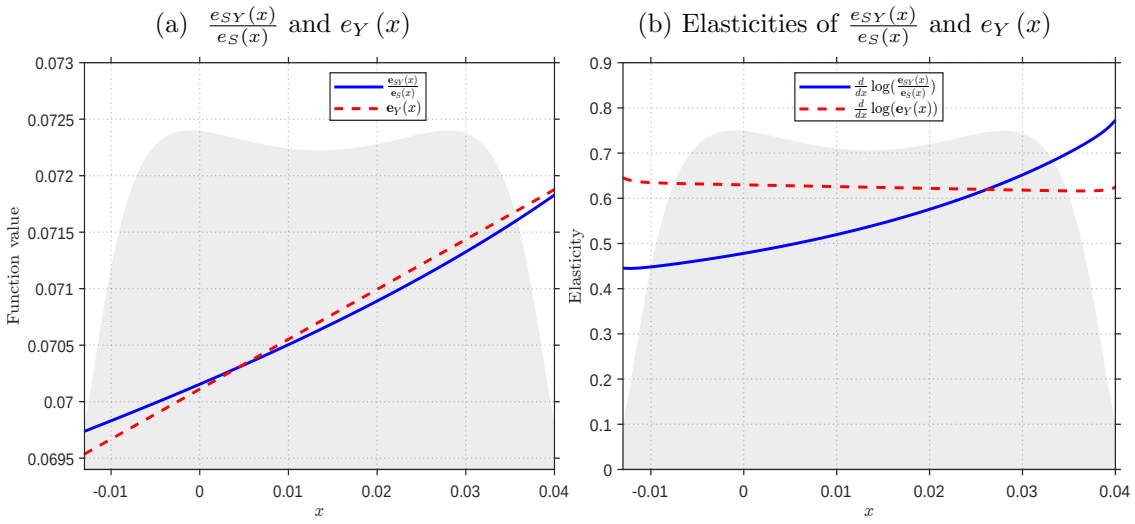
$$r(\hat{x},\tau)=\rho+\frac{1}{\psi}\hat{x}-\frac{1}{2}\gamma\sigma^2\left(\frac{1}{\psi}+1\right)+\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_x(\hat{x},\tau)}{H(\hat{x},\tau)}(x_H-\hat{x})(\hat{x}-x_L)\frac{1}{\sigma}, \tag{47}$$

$$+\frac{\left(\frac{1}{\psi}-\gamma\right)\left(1-\frac{1}{\psi}\right)}{2(1-\gamma)^2}\left(\frac{H_x(\hat{x},\tau)}{H(\hat{x},\tau)}\right)^2(x_H-\hat{x})^2(\hat{x}-x_L)^2\frac{1}{\sigma^2}, \tag{48}$$

$\sigma_S(\hat{x},\tau)=\gamma\sigma-\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\frac{H_x(\hat{x},\tau)}{H(\hat{x},\tau)}(x_H-\hat{x})(\hat{x}-x_L)\frac{1}{\sigma}$. As in Example 2, we consider the output process of the form (27).

The assumptions of Proposition 2 fail for two reasons. First, the law of motion of the state variable (44) is nonlinear due to the non-normality of the Markov state variable x. Second, whenever $\gamma\neq\frac{1}{\psi}$, the nonlinearity of $\frac{H_x(\hat{x},\tau)}{H(\hat{x},\tau)}$ introduces a nonlinear term in the risk-free rate and the market price of risk. In Figure (4), we compare the level of eigenfunctions f $\mathbf{e}_Y(\hat{x})$ and $\frac{\mathbf{e}_{SY}(\hat{x})}{\mathbf{e}_S(\hat{x})}$ on the left panel and the elasticities, $\frac{d}{dx}\ln\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ and $\frac{d}{dx}\ln\mathbf{e}_Y(x)$, on the right panel. Parameter values are taken from the calibration of Ai, Han, and Xu (2025). The shaded areas are the density of the stationary distribution of the state variable $\hat{x}$. As shown in the right panel, the approximation errors are in general within 15% in most of the domain of the stationary distribution of $\hat{x}$.

Figure 4: Comparison of Eigenfunctions (Veronesi-Ai)



Notes: Figure 4a plots the numerical solutions of $\frac{\mathbf{e}_{SY}(x)}{\mathbf{e}_S(x)}$ and $\mathbf{e}_Y(x)$ for the Veronesi-Ai and Figure 4b plots their elasticities. Given that the eigenfunctions are unique up to a scale, we rescale them in Figure 4a so that they are comparable in the plots. The shaded grey area represents the stationary distribution of the state variable x, scaled for visualization.

In summary, our measurement equation (29) should be viewed as an approximation. In the above examples, this approximation is quite accurate quantitative. It is also possible to construct examples where the approximation errors can be large. Here we want to emphasize that our rejection of the absence of the long-run impact of monetary policy holds under much more general conditions and does not reply on any approximation. One may want to be conservative and interpret our approximation with caution. In that case, the moments of the long equity and long bond portfolio should still be used as a quantitative discipline for structural models of monetary policy, where the spread between the long equity and long bond portfolio in Equation (29) can potentially come from a combination of news about long-run impact and news about risk premiums.

6 Conclusion

Using the operator approach of Hansen and Scheinkman (2009), we provide a formula that measures the permanent impact of monetary policy, Equation (22). We provide conditions

under which Equation (22) holds exactly, and we demonstrate that even in situations where the sufficient conditions do not hold, Equation (22) provides a good approximation. Using this formula, we identify a quantitatively large impact of monetary policy. A 25-basis-point expansionary monetary policy shock and a 25-basis-point central bank information shock—identified following Jarociński and Karadi (2020)—are on average associated with roughly 100-basis-point and 200-basis-point upward revisions in long-run output, respectively.

7 Appendix

7.1 Proof for Proposition 1

Let $\left(-\bar{r}, \mathbf{e}_S\left(\hat{X}, \tau\right)\right)$ solve the eigenvalue problem (8) for the stochastic discount discount factor. In addition, we assume that the distribution of $\left(\hat{X}_t, \tau_t\right)$ is ergodic under the change of measure induced by martingale (9) constructed from this eigenvalue and eigenfunction. By definition (8),

$$e^{-\bar{r}t}\mathbf{e}_S\left(\hat{X}, \tau\right) = E\left[\frac{S_t}{S_0}\mathbf{e}_S\left(\hat{X}_t, \tau\left(t\right)\right)\middle| \hat{X}_0 = \hat{X}, \tau\left(0\right) = \tau\right] \tag{49}$$

for all $\left(\hat{X}, \tau\right)$ and all $t > 0$. We need to verify that

1. The pair $\left(-\bar{r}, \mathbf{e}_S\left(\hat{X}, \tau\right)e^{-y}\right)$ is the eigenvalue and eigenfunction for the multiplicative functional S_tY_t. In addition, $\left(\hat{X}, \tau, y_t\right)$ is jointly ergodic under the change of measure induced by the martingale as constructed in (9).

2. The pair $(0, e^{-y})$ is the eigenvalue-eigenfunction for the multiplicative functional Y_t. In addition, the distribution of $\left(\hat{X}, \tau, y_t\right)$ is jointly ergodic under the relevant change of measure.

To verify that $\left(-\bar{r}, \mathbf{e}_S\left(\hat{X}, \tau\right)e^{-y}\right)$ is the eigenvalue and eigenfunction for the multiplicative functional S_tY_t,

$$E\left[\frac{S_tY_t}{S_0Y_0}\mathbf{e}_S\left(\hat{X}_t, \tau\left(t\right)\right)e^{-y_t}\middle| \hat{X}_0 = \hat{X}, \tau\left(0\right) = \tau\right] = e^{-\bar{r}t}\mathbf{e}_S\left(\hat{X}, \tau\right)e^{-y_0}.$$

by (49). In addition, $(0, e^{-y})$ is the eigenvalue-eigenfunction for the multiplicative functional Y_t because

$$E\left[\frac{Y_t}{Y_0}e^{-y_t}\middle| \hat{X}_0 = \hat{X}, \tau\left(0\right) = \tau\right] = e^{-y_0}.$$

It is straightforward to verify that the $\left(\hat{X}, \tau, y_t\right)$ is jointly ergodic under the relevant change of measures.

7.2 Proof for Proposition 2

We first focus on the eigenvalue problem for S_tY_t. The other two problems can be solved similarly. In the interior, $(\rho_{SY}, \mathbf{e}_{SY})$ will solve the following PDE:

$$\rho_{SY}\mathbf{e}\left(\hat{X},\tau\right)=\frac{\mathcal{L}\left[S_tY_t\mathbf{e}_{SY}\left(\hat{X}_t,\tau_t\right)\right]}{S_tY_t}, \tag{50}$$

where, by Ito lemma,

$$\begin{aligned}\frac{\mathcal{L}\left[S_tY_t\mathbf{e}_{SY}\left(\hat{X}_t,\tau_t\right)\right]}{S_tY_t}&=\mathbf{e}_{SY}\left(\hat{X},\tau\right)\left[\mu_Y\left(\hat{X}\right)-r\left(\hat{X},\tau\right)-\sigma_S'\left(\tau\right)\sigma_Y\left(\tau\right)\right]\\&+D_\tau\mathbf{e}_{SY}\left(\hat{X},\tau\right)+D_X\mathbf{e}\left(\hat{X},\tau\right)\left(\bar{a}-A\hat{X}\right)+\frac{1}{2}tr\left[D_{XX}\mathbf{e}_{SY}\left(\hat{X},\tau\right)\Gamma\left(\tau\right)\Gamma'\left(\tau\right)\right]\\&+D_X\mathbf{e}_{SY}\left(\hat{X},\tau\right)\Gamma\left(\tau\right)\left[\sigma_Y\left(\tau\right)-\sigma_S\left(\tau\right)\right]'.\end{aligned}$$

On the boundary, the eigenfunction must satisfy:

$$\mathbf{e}_{SY}\left(X_T^-,T^-\right)=\frac{E\left[e^{-\alpha\left(\hat{X}_T,T\right)}\mathbf{e}\left(\hat{X}_T,T\right)\middle|X_T^-,T^-\right]}{E\left[e^{-\alpha\left(\hat{X}_T,T\right)}\middle|X_T^-,T^-\right]}.$$

Under Assumptions 1-3, Equation (50) can be written as:

$$\begin{aligned}\left[\rho_{SG}+r_0\left(\tau\right)+r_X'\hat{X}-\mu_Y'\hat{X}+\sigma_S'\left(\tau\right)\sigma_Y\left(\tau\right)\right]\mathbf{e}_{SY}\left(\hat{X},\tau\right)&=D_X\mathbf{e}_{SY}\left(\hat{X},\tau\right)\left[\left(\bar{a}-A\hat{X}\right)+\Gamma\left(\tau\right)\left[\sigma_Y\left(\tau\right)-\sigma_S\right.\right.\right.\\&+D_\tau\mathbf{e}_{SY}\left(\hat{X},\tau\right)+\frac{1}{2}tr\left[D_{XX}\mathbf{e}_{SY}\left(\hat{X},\tau\right)\Gamma\left(\tau\right)\Gamma'\left(\tau\right)\right.\end{aligned}$$

We guess $\mathbf{e}_{SY}\left(\hat{X},\tau\right)=e^{\beta_{SY}'X+g_{SY}(\tau)}$ for some β and some undetermined function $g_{SY}\left(\tau\right)$ and use method of undetermined coefficients. The above equation becomes

$$\begin{aligned}\rho_{SG}+r_0\left(\tau\right)+r_X'\hat{X}-\mu_Y'\hat{X}+\sigma_S'\left(\tau\right)\sigma_Y\left(\tau\right)&=\beta_{SY}'\left[\left(\bar{a}-A\hat{X}\right)+\Gamma\left(\tau\right)\left[\sigma_Y\left(\tau\right)-\sigma_S\left(\tau\right)\right]'\right]\\&+g_{SY}'\left(\tau\right)+\frac{1}{2}\sum_i\sum_j\beta_{SY}\left(i\right)\beta_{SY}\left(j\right)\gamma_i'\left(\tau\right)\gamma_j'\left(\tau\right).\end{aligned} \tag{51}$$

Matching coefficient on X, we have:

$$\beta_{SY} = A^{-1}\left(\mu_Y - r_X\right).$$

Using this expression for β_{SY}, Equation (51) becomes an ODE for $g'(\tau)$:

$$g_{SY}\left(T^-\right) - g_{SY}(0) = \int_0^T \left[\rho_{SG} + r_0(s) + \sigma_S'(s)\,\sigma_Y(s) - \beta'\left[\bar{a} + \Gamma(s)\left[\sigma_Y(s) - \sigma_S(s)\right]'\right] - \frac{1}{2}\sum_i \sum_j \beta_{SY}(i)\,\beta_{SY}(j)\,\gamma_i'(s)\,\gamma_j'(s)\right] ds. \tag{52}$$

Using the above functional form for the SDF, we write the boundary condition as:

$$e^{\beta_{SY}'\hat{X}_T^- + g_{SY}(T)} = \frac{E\left[e^{-\alpha'\hat{X}_T + \beta_{SY}'\hat{X}_T + g_{SY}(0)}\middle| X_T^-, T^-\right]}{E\left[e^{-\alpha'\hat{X}_T}\middle| X_T^-, T^-\right]},$$

where we impose the stationarity condition $g_{SY}(0) = g_{SY}(T)$. This implies that

$$g_{SY}\left(T^-\right) - g_{SY}(0) = \frac{1}{2}\beta_{SY}'Q_T^-\beta_{SY} - \beta_{SY}'Q_T^-\alpha. \tag{53}$$

Equations (52) and (53) together determines eigenvalue ρ_{SG}. It is useful to note that the announcement premium on the long-term equity can be written as expected payoff divided by present value:

$$\frac{E\left[e^{\beta_{SY}'\hat{X}_T + g_{SY}(0)}\middle| X_T^-, T^-\right] E\left[e^{-\alpha'\hat{X}_T}\middle| X_T^-, T^-\right]}{E\left[e^{-\alpha'\hat{X}_T + \beta_{SY}'\hat{X}_T + g_{SY}(0)}\middle| X_T^-, T^-\right]} = e^{\beta_{SY}'Q_T^-\alpha}.$$

Similarly, the eigenfunction for S_t takes the form $\mathbf{e}_S\left(\hat{X}, \tau\right) = e^{\beta_S'X + g_S(\tau)}$, where $\beta_S = -A^{-1}r_X$, and $g(T) - g(0) = \frac{1}{2}\beta_S'Q_T^-\beta_S - \beta_S'Q_T^-\alpha$. In addition, the eigenfunction for Y_t takes the form $\mathbf{e}_Y\left(\hat{X}, \tau\right) = e^{\beta_Y'X + g_Y(\tau)}$, where $\beta_Y = A^{-1}\mu_Y$, and $g_Y(T) - g_Y(0) = \frac{1}{2}\beta_Y'Q_T^-\beta_Y - \beta_Y'Q_T^-\alpha$. Clearly,

$$\beta_{SY} = \beta_S + \beta_Y. \tag{54}$$

Using the above, we can write the announcement-day return $\frac{R_E^\infty(T)}{R_B^\infty(T)}$ as:

$$\begin{aligned}\frac{R_E^\infty(T)}{R_B^\infty(T)} &= \frac{\frac{e^{\beta_{SY}'\hat{X}_T+g_{SY}(0)}}{E\left[e^{-\alpha'\hat{X}_T+\beta_{SY}'\hat{X}_T+g_{SY}(0)}\middle|X_T^-,T^-\right]}}{\frac{e^{\beta_S'\hat{X}_T+g_S(0)}}{E\left[e^{-\alpha'\hat{X}_T+\beta_S'\hat{X}_T+g_S(0)}\middle|X_T^-,T^-\right]}} \\ &= e^{(\beta_{SY}-\beta_S)'\hat{X}_T-\frac{1}{2}(\beta_{SY}-\alpha)'Q_T^-(\beta_{SY}-\alpha)+\frac{1}{2}(\beta_Y-\alpha)'Q_T^-(\beta_Y-\alpha)} \\ &= e^{\beta_Y'\left(\hat{X}_T-X_T^-\right)+\beta_YQ_T^-\alpha-\frac{1}{2}\beta_YQ_T^-\beta_Y}, \end{aligned} \tag{55}$$

where the last line uses the property $\beta_{SY}=\beta_S+\beta_Y$.

We can solve for the expectation update during announcement window using Equation (13):

$$\Delta E^\infty(T)=\frac{\mathbf{e}_Y\left(\hat{X}_T,T\right)}{\mathbf{e}_Y\left(\hat{X}_T^-,T^-\right)}=e^{\beta_Y\left(\hat{X}_T-\hat{X}_T^-\right)-\frac{1}{2}\beta_Y'Q_T^-\beta_Y}. \tag{56}$$

Comparing (55) and (56), we obtain Equation (22).

7.3 Details of Example 5

The eigenvalue problems We first solve the eigenvalue problem for the multiplicative functionals. We start with the multiplicative functional S_tG_t. Given the functional form of the SDF in (36) and the growth process, (35), we can write the eigenvalue problem for S_tG_t as:

$$\begin{aligned}&\left[\rho+\left(r_0+r_1\frac{x}{1-x}-r_2\frac{1-x}{x}\right)-\left(\nu_0+\nu_1\frac{x}{1-x}+\nu_2\frac{1-x}{x}\right)\right. \\ &\left.+\sigma_{G,2}\sigma_{S,2}\frac{1-x}{x}+\sigma_{G,2}\sigma_{S,1}+\sigma_{G,1}\sigma_{S,2}+\sigma_{G,1}\sigma_{S,1}\frac{x}{1-x}\right]\mathbf{e}_{SG}(x) \\ =&\mathbf{e}_{SG}'(x)\left[-ax+b(1-x)+\sigma\left(\sigma_{G,2}-\sigma_{S,2}\right)(1-x)+\sigma\left(\sigma_{G,1}-\sigma_{S,1}\right)x\right] \\ &+\frac{1}{2}\sigma^2x(1-x)\mathbf{e}_{SG}''(x).\end{aligned}$$

Collecting terms, we can write the above as:

$$\left[\underbrace{(\rho + r_0 - \nu_0 + \sigma_{G,2}\sigma_{S,1} + \sigma_{G,1}\sigma_{S,2})}_{\bar{\rho}} + \underbrace{(r_1 - \nu_1 + \sigma_{G,1}\sigma_{S,1})}_{\bar{r}_1}\frac{x}{1-x} - \underbrace{(r_2 + \nu_2 - \sigma_{G,2}\sigma_{S,2})}_{\bar{r}_2}\frac{1-x}{x}\right]\mathbf{e}_{SG}(x)$$
$$= \left[-\underbrace{(a + \sigma(\sigma_{S,1} - \sigma_{G,1}))}_{\bar{a}}x + \underbrace{(b + (\sigma_{G,2} - \sigma_{S,2})\sigma)}_{\bar{b}}(1-x)\right]\mathbf{e}'_{SG}(x) + \frac{1}{2}\sigma^2 x(1-x)\mathbf{e}''_{SG}(x). \tag{57}$$

To simplify notation, we define $\bar{\rho} = \rho + r_0 - \nu_0 + \sigma_{G,2}\sigma_{S,1} + \sigma_{G,1}\sigma_{S,2}$, $\bar{r}_1 = r_1 - \nu_1 + \sigma_{G,1}\sigma_{S,1}$, $\bar{r}_2 = r_2 + \nu_2 - \sigma_{G,2}\sigma_{S,2}$, $\bar{a} = a + \sigma(\sigma_{S,1} - \sigma_{G,1})$, and $\bar{b} = b + (\sigma_{G,2} - \sigma_{S,2})\sigma$. We conjecture that the eigenfunction take the form of $e_{SG}(x) = x^{\beta_{SG}}(1-x)^{-\alpha_{SG}}$. Using the conjectured functional form of $e_{SG}(x)$, we write Equation (57) as:

$$\left(\bar{\rho} + \bar{r_1}\frac{x}{1-x} - \bar{r}_2\frac{1-x}{x}\right) = \left[-\bar{a}x + \bar{b}(1-x)\right]\left[\beta x^{-1} + \alpha(1-x)^{-1}\right]$$
$$+ \frac{1}{2}\sigma^2\left[\beta(\beta-1)\frac{1-x}{x} + 2\alpha\beta + \alpha(1+\alpha)\frac{x}{1-x}\right],$$

which leads to:

$$\bar{\rho} + \bar{r_1}\frac{x}{1-x} - \bar{r}_2\frac{1-x}{x} = \left(\alpha\bar{b} - \beta\bar{a}\right) - \bar{a}\alpha\frac{x}{1-x} + \bar{b}\beta\frac{1-x}{x} + \frac{1}{2}\sigma^2\left[\beta(\beta-1)\frac{1-x}{x} + 2\alpha\beta + \alpha(1+\alpha)\frac{x}{1-x}\right].$$

We match the coefficients on the constant to get:

$$\bar{\rho} = \alpha\bar{b} - \bar{a}\beta + \alpha\beta\sigma^2, \tag{58}$$

those on $\frac{x}{1-x}$ to get:

$$\bar{r_1} = -\bar{a}\alpha + \frac{1}{2}\sigma^2\alpha(1+\alpha), \tag{59}$$

and those on $\frac{1-x}{x}$ to get:

$$-\bar{r}_2 = \bar{b}\beta + \frac{1}{2}\sigma^2\beta(\beta-1). \tag{60}$$

Equation (60) has two solutions for β:

$$\beta_{\pm} = \left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right) \pm \sqrt{\left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right)^2 - \frac{2\bar{r}_2}{\sigma^2}}. \tag{61}$$

Equation (59) also has two solutions:

$$\alpha_{\pm} = \left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right) \pm \sqrt{\left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\bar{r}_1}{\sigma^2}}. \tag{62}$$

In order for real solutions to exist, we need $\left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right)^2 - \frac{2\bar{r}_2}{\sigma^2} \geq 0$, and $\left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\bar{r}_1}{\sigma^2} \geq 0$.

As we will show below, ergodicity requires us to pick the solution α_- and β_+. Given α and β, we can solve the eigenvalue ρ_{SY} as:

$$\rho_{SG} = -r_0 + \nu_0 - \sigma_{G,2}\sigma_{S,1} - \sigma_{G,1}\sigma_{S,2} + \alpha\left(b + \left(\sigma_{G,2} - \sigma_{S,2}\right)\sigma\right) - \beta\left(a + \sigma\left(\sigma_{S,1} - \sigma_{G,1}\right)\right) + \alpha\beta\sigma^2.$$

Similarly, the eigenvalue problem for the stochastic discount factor can be formulated in the form of (57) with $\bar{\rho} = \rho + r_0$, $\bar{r}_1 = r_1$, $\bar{r}_2 = r_2$, $\bar{a} = a + \sigma\sigma_{S,1}$ and $\bar{b} = b - \sigma\sigma_{S,2}$. The eigenfunctions for the SDF take the form of $e_S(x) = x^{\beta_S}(1-x)^{-\alpha_S}$, where α_S and β_S take the form of (61) and (62) with the new definitions of $\bar{a}$, $\bar{b}$, $\bar{r}_1$, and $\bar{r}_2$. The eigenvalue problem for the growth functional G_t is a special of (57) with $\bar{\rho} = \rho - \nu_0$, $\bar{r}_1 = -\nu_1$, $\bar{r}_2 = \nu_2$, $\bar{a} = a - \sigma\sigma_{G,1}$ and $\bar{b} = b + \sigma\sigma_{G,2}$. The eigenfunction takes the form of $e_G(x) = x^{\beta_G}(1-x)^{-\alpha_G}$ where α_G and β_G given in (61) and (62) with the new definitions of $\bar{a}$, $\bar{b}$, $\bar{r}_1$, and $\bar{r}_2$.

Imposing ergodicity The quadratic equations (61) and (62) have two solutions. To pick the right solution, we need to impose ergodicity of the x_t process under the relevant transformed measures. We first state a sufficient condition for the ergodicity of the Wright-Fisher process defined in (33).

Lemma. *(Stationarity of WF process)*

Suppose $\frac{2b}{\sigma^2} \geq 1$ and $\frac{2a}{\sigma^2} \geq 1$, then the x_t process in (33) has a unique ergodic distribution

on $[0,1]$, which is a beta distribution with parameter $\left(\frac{2b}{\sigma^2}, \frac{2a}{\sigma^2}\right)$:

$$\psi(x) = \frac{\Gamma\left(\frac{2(a+b)}{\sigma^2}\right)}{\Gamma\left(\frac{2b}{\sigma^2}\right)\Gamma\left(\frac{2a}{\sigma^2}\right)} x^{\frac{2b}{\sigma^2}-1}(1-x)^{\frac{2a}{\sigma^2}-1}. \tag{63}$$

The proof of the above lemma can be found in Karlin and Taylor (1981) (Example 8, Page 239).

To avoid repetition, we show how to use the ergodicity condition to select the solutions of the quadratic equation for the multiplicative functional of the stochastic discount factor as an example. Other multiplication functionals can be dealt in the same way. In the case of the eigenvalue problem for the SDF, the α and β are given by:

$$\beta_{\pm} = \left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right) \pm \sqrt{\left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right)^2 - \frac{2r_2}{\sigma^2}}. \tag{64}$$

The two solutions of α are:

$$\alpha_{\pm} = \left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right) \pm \sqrt{\left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r_1}{\sigma^2}}. \tag{65}$$

To ensure that α and β have real solutions, we need to impose

$$\left(\frac{1}{2} - \frac{b - \sigma\sigma_{S,2}}{\sigma^2}\right)^2 - \frac{2r_2}{\sigma^2} > 0; \ \left(\frac{a + \sigma\sigma_{S,1}}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r_1}{\sigma^2} > 0. \tag{66}$$

We make the following observation.

Remark. Under condition (66), $\alpha_+ > \left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right)$, $\alpha_- < \left(\frac{\bar{a}}{\sigma^2} - \frac{1}{2}\right)$. In addition, $\beta_+ > \left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right)$ and $\beta_- < \left(\frac{1}{2} - \frac{\bar{b}}{\sigma^2}\right)$.

As in Hansen and Scheinkman (2009), we use the eigenfunctions to construct an MG via the following relationship:

$$S_t = e^{\rho_S t}\hat{M}_t \frac{\mathbf{e}_S(x_0)}{\mathbf{e}_S(x_t)} \Rightarrow \hat{M}_t = e^{-\rho_S t}\frac{\mathbf{e}_S(x_t)}{\mathbf{e}_S(x_0)} S_t = e^{-\rho_S t}\frac{x_t^{\beta}(1-x_t)^{-\alpha}}{x_0^{\beta}(1-x_0)^{-\alpha}} S_t. \tag{67}$$

The results from Hansen and Scheinkman (2009) implies that $\hat{M}_t$ is a martingale. To derive

an expression for $\hat{M}_t$, we use Ito lemma to get:

$$dx_t^{\beta}(1-x_t)^{-\alpha} = \left[\beta x^{\beta-1}(1-x)^{-\alpha} + \alpha x^{\beta}(1-x)^{-(\alpha+1)}\right] dx_t$$
$$+ \frac{1}{2}\left[\beta(\beta-1)x^{\beta-2}(1-x)^{-\alpha} + 2\alpha\beta x^{\beta-1}(1-x)^{-(\alpha+1)} + \alpha(1+\alpha)x^{\beta}(1-x)^{-(\alpha+2)}\right] x_t(1-$$

We know that the dt part of (67) has to be zero, because that is how we constructed the solution. We focus on the BM part:

$$\frac{dx_t^{\beta}(1-x_t)^{-\alpha}}{x_t^{\beta}(1-x_t)^{-\alpha}} = ...dt + \left[\beta\frac{1}{x} + \alpha\frac{1}{1-x}\right]\sqrt{x_t(1-x_t)}\sigma dB_t.$$

We combine this with S_t to get:

$$\frac{d\hat{M}_t}{\hat{M}_t} = 0dt + \left[\left(\beta\frac{1}{x} + \alpha\frac{1}{1-x}\right)\sqrt{x_t(1-x_t)}\sigma - \sigma_{S,2}\sqrt{\frac{1-x}{x}} - \sigma_{S,1}\sqrt{\frac{x}{1-x}}\right] dB_t$$
$$= \left[(\beta\sigma - \sigma_{S,2})\sqrt{\frac{1-x}{x}} + (\alpha\sigma - \sigma_{S,1})\sqrt{\frac{x}{1-x}}\right] dB_t. \tag{68}$$

We require the x_t process to be ergodic under the transformed measure induced by the above martingale. To derive the law of motion of x_t under the transformed measure, we note that by the Girsanov theorem, under the $\hat{M}-$ induced measure,

$$d\hat{B}_t = dB_t - \left[(\beta\sigma - \sigma_{S,2})\sqrt{\frac{1-x}{x}} + (\alpha\sigma - \sigma_{S,1})\sqrt{\frac{x}{1-x}}\right] dt$$

is a standard Brownian motion. Under this distorted measure, we write the LOM of x_t as:

$$dx_t = (-ax_t + b(1-x_t))dt + \sqrt{x_t(1-x_t)}\sigma\left[d\hat{B}_t + \left[(\beta\sigma - \sigma_{S,2})\sqrt{\frac{1-x}{x}} + (\alpha\sigma - \sigma_{S,1})\sqrt{\frac{x}{1-x}}\right] dt\right]$$
$$= [-ax_t + b(1-x_t) + \sigma(\beta\sigma - \sigma_{S,2})(1-x_t) + \sigma(\alpha\sigma - \sigma_{S,1})x_t]dt + \sqrt{x_t(1-x_t)}\sigma d\hat{B}_t$$
$$= [-(a - \sigma(\alpha\sigma - \sigma_{S,1}))x_t + (b + \sigma(\beta\sigma - \sigma_{S,2}))(1-x_t)]dt + \sqrt{x_t(1-x_t)}\sigma d\hat{B}_t.$$
$$= \left[-\left(\bar{a} - \alpha\sigma^2\right)x_t + \left(\bar{b} + \beta\sigma^2\right)(1-x_t)\right]dt + \sqrt{x_t(1-x_t)}\sigma d\hat{B}_t$$

By Lemma 7.3, ergodicity requires:

$$\bar{a} - \alpha\sigma^2 > \frac{1}{2}\sigma^2;\ \bar{b} + \beta\sigma^2 > \frac{1}{2}\sigma^2.$$

That is,

$$\alpha < \frac{\bar{a}}{\sigma^2} - \frac{1}{2};\ \beta > \frac{1}{2} - \frac{\bar{b}}{\sigma^2}.$$

Clearly, by Remark 7.3, we should pick α_- and β_+. These solutions guarantee the ergodicity of x under the transformed measure. This completes the proof of Proposition 3.

Approximation of the eigenfunctions Here, we show that $\alpha_S + \alpha_Y \approx \alpha_{SY}$, and $\beta_S + \beta_Y \approx \beta_{SY}$, where the approximation is obtain by a linear expansion of the solutions (38)-(43) around the deterministic steady state. We keep the stochastic process of x_t unchanged but consider the deterministic steady state where $\sigma_{S,1} = \sigma_{S,2} = \sigma_{G1} = \sigma_{G,2} = 0$. Focusing on α_S in Equation (38), we write

$$\alpha_S = \left(\frac{a + \sigma\sigma_{S,1}}{\sigma^2} - \frac{1}{2}\right) - \sqrt{\left(\frac{a}{\sigma^2} - \frac{1}{2}\right)^2 + \left(\frac{\sigma_{S,1}}{\sigma}\right)^2 + 2\frac{\sigma_{S,1}}{\sigma}\left(\frac{a}{\sigma^2} - \frac{1}{2}\right) + \frac{2r_1}{\sigma^2}}.$$

Consider a first order Taylor approximation of the square root term in the above expression around $\left(\frac{a}{\sigma^2} - \frac{1}{2}\right)^2$ (noting that $\frac{a}{\sigma^2} > \frac{1}{2}$ under our parameter assumptions)

$$\begin{aligned}\alpha_S &\approx \left(\frac{a + \sigma\sigma_{S,1}}{\sigma^2} - \frac{1}{2}\right) - \left(\frac{a}{\sigma^2} - \frac{1}{2}\right) - \frac{1}{2}\frac{1}{\left(\frac{a}{\sigma^2} - \frac{1}{2}\right)}\left\{\left(\frac{\sigma_{S,1}}{\sigma}\right)^2 + 2\frac{\sigma_{S,1}}{\sigma}\left(\frac{a}{\sigma^2} - \frac{1}{2}\right) + \frac{2r_1}{\sigma^2}\right\} \\ &= -\frac{1}{2}\frac{\left(\frac{\sigma_{S,1}}{\sigma}\right)^2 + \frac{2r_1}{\sigma^2}}{\left(\frac{a}{\sigma^2} - \frac{1}{2}\right)} = -\frac{\frac{1}{2}\sigma_{S,1}^2 + r_1}{a - \frac{1}{2}\sigma^2}. \qquad (69)\end{aligned}$$

Similarly, we can write α_Y is Equation (42) as:

$$\begin{aligned}
\alpha_Y &= \left(\frac{a-\sigma\sigma_{G,1}}{\sigma^2}-\frac{1}{2}\right)-\sqrt{\left(\frac{a-\sigma\sigma_{G,1}}{\sigma^2}-\frac{1}{2}\right)^2+\left(\frac{\sigma_{G,1}}{\sigma}\right)^2-2\frac{\sigma_{G,1}}{\sigma}\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)-\frac{2\nu_1}{\sigma^2}}, \\
&\approx \left(\frac{a+\sigma\sigma_{S,1}}{\sigma^2}-\frac{1}{2}\right)-\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)-\frac{1}{2}\frac{1}{\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)}\left\{\left(\frac{\sigma_{G,1}}{\sigma}\right)^2-2\frac{\sigma_{G,1}}{\sigma}\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)-\frac{2\nu_1}{\sigma^2}\right\} \\
&= -\frac{1}{2}\frac{\left(\frac{\sigma_{G,1}}{\sigma}\right)^2-\frac{2\nu_1}{\sigma^2}}{\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)}=-\frac{\frac{1}{2}\sigma_{G,1}^2-\nu_1}{a-\frac{1}{2}\sigma^2}. \qquad (70)
\end{aligned}$$

Finally,

$$\begin{aligned}
\alpha_{SY} &= \left(\frac{a+\sigma\left(\sigma_{S,1}-\sigma_{G,1}\right)}{\sigma^2}-\frac{1}{2}\right)-\sqrt{\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)^2+\left(\frac{\sigma_{S,1}-\sigma_{G,1}}{\sigma}\right)^2+2\frac{\sigma_{S,1}-\sigma_{G,1}}{\sigma}\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)+\frac{2\left(r_1-\nu\right.}{}} \\
&= \left(\frac{a+\sigma\left(\sigma_{S,1}-\sigma_{G,1}\right)}{\sigma^2}-\frac{1}{2}\right)-\sqrt{\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)^2+\left(\frac{\sigma_{S,1}}{\sigma}\right)^2+\left(\frac{\sigma_{G,1}}{\sigma}\right)^2+2\frac{\sigma_{S,1}-\sigma_{G,1}}{\sigma}\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)+\frac{2\left(r_1-\right.}{\sigma^2}} \\
&\approx \left(\frac{a+\sigma\left(\sigma_{S,1}-\sigma_{G,1}\right)}{\sigma^2}-\frac{1}{2}\right)-\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)-\frac{1}{2}\frac{1}{\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)}\left\{\left(\frac{\sigma_{S,1}}{\sigma}\right)^2+\left(\frac{\sigma_{G,1}}{\sigma}\right)^2+2\frac{\sigma_{S,1}-\sigma_{G,1}}{\sigma}\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)\right. \\
&= -\frac{1}{2}\frac{\left(\frac{\sigma_{S,1}}{\sigma}\right)^2+\left(\frac{\sigma_{G,1}}{\sigma}\right)^2+\frac{2(r_1-\nu_1)}{\sigma^2}}{\left(\frac{a}{\sigma^2}-\frac{1}{2}\right)}=-\frac{\frac{1}{2}\left(\frac{\sigma_{S,1}}{\sigma}\right)^2+\frac{1}{2}\left(\frac{\sigma_{G,1}}{\sigma}\right)^2+\frac{r_1-\nu_1}{\sigma^2}}{a-\frac{1}{2}\sigma^2}. \qquad (71)
\end{aligned}$$

Clearly, the above approximations, (69), and (70), and (71) satisfy $\alpha_S+\alpha_Y=\alpha_{SY}$. We can establish the approximation result for $\beta_S+\beta_Y\approx\beta_{SY}$ by following the same arguement.

7.4 The Veronesi-Ai economy

Using the definition of recursive utility, we can show that for $\tau\in(0,T)$, the $H(\hat{x},\tau)$ must satisfy the following PDE:

$$\begin{aligned}
0 \;=\; & \frac{H_x(\hat{x},\tau)}{H(\hat{x},\tau)}\left[\frac{(\lambda_H+\lambda_L)}{1-\gamma}(\bar{x}-\hat{x})+(x_H-\hat{x})(\hat{x}-x_L)\frac{1}{\sigma}\right]+\frac{1}{2}\frac{H_{xx}(\hat{x},\tau)}{(1-\gamma)H(\hat{x},\tau)}(x_H-\hat{x})^2(\hat{x}-x_L)^2\frac{1}{\sigma^2} \\
& +\frac{\beta}{1-\frac{1}{\psi}}\left(H(\hat{x},\tau)^{-\frac{1-\frac{1}{\psi}}{1-\gamma}}-1\right)+\frac{H_\tau(\hat{x},\tau)}{(1-\gamma)H(\hat{x},\tau)}+\left(\hat{x}-\frac{1}{2}\gamma\sigma^2\right). \qquad (72)
\end{aligned}$$

Because the true value of x_{nT} is revealed upon announcement at nT, on the boundaries, we have:

$$H\left(\hat{x}, T^{-}\right) = \frac{\hat{x} - x_L}{x_H - x_L} H\left(x_H, 0\right) + \frac{x_H - \hat{x}}{x_H - x_L} H\left(x_L, 0\right), \tag{73}$$

where $\frac{\hat{x}-x_L}{x_H-x_L} = Prob\left(x = x_H | \hat{x}\right)$ due to the property of the two-state Markov chain.

8 Data appendix

8.1 Data

We obtain monetary policy shocks and Fed information shocks from Jarociński and Karadi (2020). By exploiting the negative and positive comovement between short rates and SP500 prices, they decompose the first principal component of the surprises in interest rate derivatives with maturity from one month to one year into pure monetary policy shocks and fed information shocks. The set of derivatives includes MP1, FF4, ED2, ED3 and ED4. Their decomposition allows both types of shocks to be present in each announcement. Besides, we obtain both the unadjusted and orthogonalized monetary policy shocks from Bauer and Swanson (2023), constructed as the first principal component of the surprises in ED1, ED2, ED3 and ED4.

To classify the stock as long term equity or not, we obtain stock-related variables from monthly CRSP. Specifically, we obtain the price per share (prc), number of common shares outstanding (shrout), the holding period returns including dividends (ret) and excluding dividends (retx). A firm is defined as not paying dividend in month t if ret = retx. We assign a firm to the non-dividend paying portfolio if it has never paid any dividend over the entire CRSP coverage period. Our sample is restricted to U.S. common stocks listed on the NYSE, AMEX and NASDAQ. We exclude utilities (SIC codes 4900-4999) and financial firms (SIC codes 6000-6999) .

8.2 Predictive regression

To remove the predictable component of the infinity-maturity excess return ($\ln R_E^{\infty}(T) - \ln R_B^{\infty}(T)$), we use a set of macro variables known prior to FOMC announcements from Bauer and Swanson (2023). In their study, they use them to remove the predictive component of

Table 7: Predictive regression using pre-announcement macro variables

	$\ln R_E^\infty - \ln R_B^\infty$(bps)
Nonfarm payrolls	0.007*
	(0.004)
Empl. growth (12m)	0.60
	(2.15)
Δlog SP500 (3m)	177.38**
	(71.98)
ΔSlope (3m)	12.66
	(10.06)
Δlog Comm. price (3m)	18.00
	(58.53)
Treasury skewness	14.45
	(17.85)
Const.	−6.82
	(5.92)
N	239
Adj. R^2	0.048

Notes: The table reports OLS estimates from the predictive regression $\ln R_E^\infty - \ln R_B^\infty = \xi_0 + \xi' Z_T^- + \epsilon_T$. The dependent variable $\ln R_E^\infty - \ln R_B^\infty$is the high-frequency log return of long-term equity portfolio normalized by the long-term bond in the 30-minute window around FOMC announcements. We take the set of macro variables known before each FOMC announcement Z_T^- from Bauer and Swanson (2023). The residual ϵ_T is our measure of the revisions in the long-run growth expectations. We report heteroskedasticity-robust standard errors in the brackets.$^*p < 0.1$,$^{**}p < 0.05$,$^{***}p < 0.01$.

the monetary policy shock. The set includes six macro variables: the surprise in the most recent nonfarm payrolls release, last year employment growth, the log change in the S&P500 from 3 months before to the day before the FOMC announcement, the change in the yield curve slope over the same period, the log change in a commodity price index over the same period, and the option-implied skewness of the 10-year Treasury yield. The adjusted R^2 is 0.048 and only two of the six predictors are significant, indicating that the predictable component is small.

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