The long-run impact of monetary policy

Hengjie Ai and Leitao Fu*

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Abstract: The ratio of long-term equity returns to long-term bond returns over short FOMC announcement windows provides a quantitative measure of the long-run impact of monetary policy. Empirically, we find that a 25-basis-point expansionary monetary policy shock and a 25-basis-point central bank information shock—identified following Jarociński and Karadi (2020)—are on average associated with roughly 100-basis-point and 250-basis-point upward revisions in expected long-run economic growth, respectively. These findings strongly reject the hypothesis of long-run neutrality of monetary policy. To provide guidance for structural models in which monetary policy has a nontrivial impact on long-run economic growth, we discuss conditions under which these responses may be interpreted as revisions in expected long-run growth, in the long-run risk premium, or both.

Keywords: monetary policy, FOMC announcements, long-run growth expectations, Fed information effect, long-run risk-neutral probability.
JEL Classification: E43, E52, G12.

*Hengjie Ai (hengjie.ai@wisc.edu) and Leitao Fu (lfu38@wisc.edu) are both from Wisconsin School of Business, University of Wisconsin-Madison. We thank Lars Peter Hansen, Paymon Khorrami, Ricardo Reis, Sang Seo, Ivan Shaliastovich, and seminar participants of CKGSB, Loyola University, the Macro Finance Society Workshop, Michigan State University, Richmond CORE week, PBC school of finance, and Wuhan University for helpful comments.

1 Introduction

Long-run monetary neutrality is a long-held belief in economics that dates back at least to Friedman (1968) and Lucas (1972). Blanchard (2018) refers to the hypothesis that long-run potential output is independent of monetary policy as the independence hypothesis. But as remarked by Blanchard (2018, p. 118), "[i]f the independence hypothesis fails, adverse shocks are more costly, and stabilization policy more powerful." Whether the independence hypothesis holds is ultimately an empirical question.

The purpose of this paper is to propose an asset-pricing test of the independence hypothesis and a quantitative measure of the long-run impact of monetary policy. Applying our methodology, we find that asset-market-based evidence strongly rejects a version of the independence hypothesis. To quantify the long-run impact of monetary policy, we follow Jarociński and Karadi (2020) in constructing monetary policy shocks and central bank information shocks. In our sample, a 25-basis-point expansionary monetary policy shock and a 25-basis-point central bank information shock are associated, on average, with 100-basis-point and 250-basis-point upward revisions in long-run economic growth (under the long-run risk-neutral measure, as defined in Section 3), respectively.

To test the independence hypothesis and to quantify the failure of the independence hypothesis, we need a measure of the long-run impact of monetary policy. Building on the operator approach of Hansen and Scheinkman (2009), this paper constructs such a measure by using the response of long-term assets over short FOMC announcement windows. The operator approach provides theoretical foundations for our measure. First, it allows us to separate discount rates and cash flow expectations. The long-term equity we construct is a claim to dividends to be paid far in the future, with maturities of 30 years or more. The price of long-term equity therefore reflects market participants' beliefs about long-run discount rates and long-run growth. Similarly, the price of long-term bonds reflects long-run discount rates. We show that computing the ratio of the return on long-term equity to the return on long-term bond over short FOMC announcement windows differences out the common discount rate component and therefore provides a measure of market participants' expectations about the long-run impact of monetary policy. Empirically, we find that financial markets' beliefs about the long-run impact of monetary policy respond significantly to FOMC announcements.

Second, the operator approach provides a theoretical justification for using the present value of long-term dividends to measure expectations about aggregate output: we show that for the purpose of measuring the long-run impact, the difference between dividend cash flows and aggregate output becomes negligible as the horizon of the impact becomes large, provided that dividends and aggregate GDP are cointegrated.

Our main result is that the ratio of returns on long-term equity to long-maturity bonds over a short FOMC announcement window identifies the long-run impact of the announcement, with expectations taken under the long-run risk-neutral probability measure in the sense of Hansen and Scheinkman (2009). This result has two implications. First, imperfect correlation between returns on long-term equity and long-maturity bonds over short FOMC announcement windows rejects the

independence hypothesis. We provide empirical evidence that strongly rejects this hypothesis. Our findings highlight the importance of studying monetary models with long-run effects.

Second, the ratio of returns on long-term equity to long-maturity bonds over short FOMC announcement windows provides a quantitative measure of the impact of monetary policy on expected long-run growth. We find evidence of a substantial long-run impact of monetary policy—the average magnitude of the revision in expected long-run growth is about 60 basis points per FOMC announcement.

Empirically, we construct long-term equity using a portfolio of non-dividend-paying stocks as a proxy.[1] We use the Treasury bond of the longest available maturity as a proxy for long-term bonds, following Alvarez and Jermann (2005). The unconditional correlation between our measured innovations in long-run growth expectations and high-frequency changes in the federal funds futures rate—a conventional measure of monetary policy shocks—is low. We show that this reflects the Fed information effect. Following Jarociński and Karadi (2020), we construct monetary policy shocks and central bank information shocks and show that changes in long-run growth expectations are highly correlated with both components of monetary policy surprises. A 25-basis-point expansionary monetary policy shock leads, on average, to a 100-basis-point upward revision in long-run economic growth, while a 25-basis-point central bank information shock typically leads to a 250-basis-point upward revision.

The long-run impact measured under the long-run risk-neutral probability includes both growth expectations under investors' subjective probability and a risk compensation for long-run economic growth. To study the relationship between this measure and the long-run impact under investors' subjective probability, we provide several results. First, we provide a set of sufficient conditions under which the two measures coincide. Second, under the assumption of rational expectations, we use the methodology of Ai and Fu (2026) to decompose the long-run impact of monetary policy into long-run cash-flow-expectation and long-run risk-premium components. We find that the two components are of comparable importance: each accounts for about half of the variance of the long-run impact.

Literature Our paper is related to the large literature on the long-run impact of monetary policy. In standard models with nominal rigidity, such as the textbook treatments of Galí (2015) and Woodford (2003), monetary policy does generate real effects, but they dissipate in the long run. Empirical work that identifies monetary policy shocks as deviations from a policy rule often finds that output responses to monetary policy shocks are hump-shaped and die out within a few years (Christiano, Eichenbaum, and Evans, 1999; Romer and Romer, 2004; Ramey, 2016). At the same time, a smaller literature emphasizes the long-run impact of monetary policy. Stadler (1990) shows that once technology is endogenous, demand shocks, including monetary policy shocks, have permanent output effects. Moran and Queralto (2018); Garga and Singh (2021); Bianchi, Kung, and Morales (2019); Fornaro and Wolf (2023) build New Keynesian models with endogenous R&D

[1] We thank Sang Seo for advising us on non-dividend paying stocks to construct the long-equity portfolio.

in which policy affects productivity growth. Jordà, Singh, and Taylor (2024) find that identified monetary policy shocks depress output, TFP, and capital for more than a decade.

Our paper connects to the literature on the high-frequency identification of monetary policy surprises around FOMC announcements and their causal effects on asset prices. In early work, Bernanke and Kuttner (2005) and Kuttner (2001) identify the surprises as the daily changes in the current-month or next-month federal funds futures rate and study the responses in stock prices and Treasury yields, respectively. More recent studies exploit narrower windows around announcements to identify monetary policy surprises. Gürkaynak, Sack, and Swanson (2005) construct a target factor and a path factor from high-frequency responses of several federal funds and Eurodollar futures with one year or less to expiration. Nakamura and Steinsson (2018) provide evidence for the Fed information effect and estimate non-neutrality from the responses of real rates, expected inflation, and expected output growth to the high-frequency surprises. Jarociński and Karadi (2020) decompose the surprises into a pure monetary policy component and a Fed information component. Bauer and Chernov (2024) show that option-implied skewness of interest rates can predict these surprises. Bauer and Swanson (2023) highlight the importance of removing predictable components from these surprises. Also related is the literature that aims to explain the responses in asset prices. Kekre and Lenel (2022) and Pflueger and Rinaldi (2022) highlight the risk premium channel in models with wealth redistribution and habit formation, respectively. Bauer, Pflueger, and Sunderam (2024) show that perceptions about the Fed's policy rule have shaped the transmission of monetary policy to financial markets. Kekre, Lenel, and Mainardi (2025) explain the responses of long-term Treasury yields in a segmented markets model. Nagel and Xu (2025) attribute the responses of stock prices to movements in the yield curve. Our identification strategy also builds on the evidence of a macroeconomic announcement premium documented by Savor and Wilson (2013), Lucca and Moench (2015), and Ai and Bansal (2018).

To study the impacts of monetary policy on the macroeconomy, several papers, for example, Cochrane and Piazzesi (2002), Gertler and Karadi (2015), Ramey (2016) and Stock and Watson (2018), exploit the exogeneity of high-frequency surprises, using them either as regression variables or instruments in structural VARs or local projections. A related paper by Golez and Matthies (2025) also leverages intraday financial market data to measure the impact of monetary policy on cash flows. They provide evidence for the Fed information effect on short-term growth by finding that the announcement returns of a six-month dividend strip constructed from options load positively on the monetary policy surprises. In contrast to their work, our goal is to identify the long-run component of monetary policy shocks and, more importantly, to quantify the long-run impact.

On the methodology side, our paper builds on the operator approach to long-term pricing pioneered by Hansen and Scheinkman (2009) and is closely related to the literature on the recovery of probabilities from asset prices (Ross, 2015; Borovička, Hansen, and Scheinkman, 2016; Qin and Linetsky, 2017; Bakshi, Chabi-Yo, and Gao, 2018; Jensen, Lando, and Pedersen, 2019). Borovička, Hansen, and Scheinkman (2016) demonstrate that, in general, it is not possible to recover subjec-

tive probabilities from asset prices alone when the permanent component of the pricing kernel is important. Our benchmark measure, the ratio of long-term equity to long-term bond returns, is constructed solely from asset prices without assumptions about the relationship between subjective and objective probabilities. As a result, this measure generally captures the impact of monetary policy on both long-run growth expectations and long-run risk premia. In a companion paper, Ai and Fu (2026), we develop a general methodology for constructing a time series of long-run growth expectations from asset prices under the assumption of rational expectations. The benchmark measure of the present paper does not require rational expectations—identification relies only on no-arbitrage over short announcement windows—and it delivers a formal test of the long-run neutrality of monetary policy together with a quantification of its magnitude.

Our paper is also related to the literature that studies the decomposition of the stochastic discount factors (SDF) into permanent and transitory components. Bansal and Lehmann (1997) provide a decomposition of the SDF into permanent and transitory components. Alvarez and Jermann (2005) and Bakshi and Chabi-Yo (2012) develop volatility bounds on these two components. Christensen (2017) estimates them with a structural model and a non-parametric approach to the Perron-Frobenius eigenfunction problem. Backus, Chernov, and Zin (2014) highlight the tension between generating a large permanent component of the SDF and low spreads between yields. Lustig, Stathopoulos, and Verdelhan (2019) link the carry trade risk premium of long-term bonds to the cross-country differences in the volatility of the permanent components of the SDF. Cieslak and Khorrami (2025) propose a Long-Run Neutrality (LRN) condition, which tests the impact of monetary policy shocks on the permanent component of stochastic discount factors. Whereas we focus on long-run economic growth, the LRN condition concerns the stochastic discount factor. LRN is neither necessary nor sufficient for our test of long-run impact.

The rest of the paper is organized as follows. In Section 2, we define long-term equity and long-term bonds and explain the construction of our measure of the long-run impact of monetary policy. In Section 3, we present the empirical evidence for the long-run impact of monetary policy. Section 4 concludes.

2 Evidence for the long-run impact of monetary policy

2.1 Overview of main empirical results

We first provide a non-technical summary of our construction of the long-term portfolio and its connection to the long-run impact of monetary policy. We denote the time t price of a zero-coupon inflation-protected bond with maturity T as $b(t,T)$. Let $\{D_t\}$ be the process of aggregate dividends paid by the stock market. We define the time t price of a tail dividend claim with maturity T, denoted $V(t,T)$, as the present value of all dividends paid after time T. That is, $V(t,T)$ is the present value of $\{D_{T+u}\}_{u=0}^{\infty}$, where we will refer to T as the maturity of the tail dividends. For T large, we call $b(t,T)$ the price of the long-term bond and $V(t,T)$ the price of long-term equity.

To focus on FOMC announcements, we assume that a pre-scheduled FOMC announcement is

made at time $t = \tau, 2\tau, 3\tau, \cdots$. We use $E_{t^-}[Y_T]$ to denote the expected long-run level of output at some distance future time T conditioning on pre-announcement time t^- information, and $E_t[Y_T]$ as the revised belief about Y_T right after the FOMC announcement at time t. Under this notation, $\frac{E_t[Y_T]}{E_{t^-}[Y_T]}$ is the update of the expectation about long-run output Y_T upon an FOMC announcement at time t. As T gets large, $E_t[Y_T]$ may not be finite; however, under general conditions, $\lim_{T\to\infty} \frac{E_t[Y_T]}{E_{t^-}[Y_T]}$ is finite. We will refer to $\lim_{T\to\infty} \frac{E_t[Y_T]}{E_{t^-}[Y_T]}$ as the update of expected long-run output upon an FOMC announcement at time t. We are interested in conditions under which $\lim_{T\to\infty} \frac{E_t[Y_T]}{E_{t^-}[Y_T]}$ can be measured from asset prices. We first provide a non-technical summary of our main results. Formal statements and proofs will be given in Section 3.

Claim 1. Suppose the long-run growth rate of Y_t is independent of monetary policy, then

$$\lim_{T\to\infty} \frac{b(t,T)}{b(t^-,T)} = \lim_{T\to\infty} \frac{V(t,T)}{V(t^-,T)}, \tag{1}$$

that is, the return on the long-term bond and that on long-term equity must be perfectly correlated upon FOMC announcements.

Claim 1 above is our version of the independence hypothesis of Blanchard (2018). In Section 3, we will provide a rigorous statement of the hypothesis that the long-run growth rate of Y_t is independent of monetary policy. One such example is the textbook version of the New Keynesian model, for example, in Galí (2015) and Woodford (2003), in which output Y_t has a long-run stationary distribution. Claim 1 implies that under the independence hypothesis, the returns on long-term equity and the long-term bond must be perfectly correlated. As a result, rejection of the perfect correlation between long-term equity and long-term bond over short FOMC announcement windows is a rejection of the independence hypothesis.

Equation (1) motivates the construction of the return on a portfolio that has a long position in long-term equity and a short position in the long-term bond:

$$\ln \frac{V(t,T)}{V(t^-,T)} - \ln \frac{b(t,T)}{b(t^-,T)}.$$

We call this portfolio the long-short long term portfolio, or long-term portfolio for short. Claim 1 implies that a non-trivial return of the long-short long term portfolio over a short FOMC announcement window leads to the rejection of the independence hypothesis.

More generally, the following claim implies that the return on the long-short long-term portfolio over a short FOMC announcement window is a measure of the long-run impact of monetary policy in the sense that it is the update of the belief about long-run growth under the long-run risk neutral measure.

Claim 2. Let E^* denote the long-run risk neutral measure in the sense of Hansen and Scheinkman (2009), then

$$\lim_{T\to\infty} \ln\left(\frac{E_t^*[Y_T]}{E_{t^-}^*[Y_T]}\right) = \ln \frac{V(t,T)}{V(t^-,T)} - \ln \frac{b(t,T)}{b(t^-,T)}. \tag{2}$$

That is, the difference between the returns on long-term equity and the long-term bond identifies the long-run impact of monetary policy under the long-run risk-neutral measure.

In general, $\lim_{T\to\infty} \frac{E_t^*[Y_T]}{E_{t^-}^*[Y_T]} \neq 1$ indicates a non-trivial long-run impact of monetary policy, but this impact may be a combination of risk premium and growth expectations. In most structural models, $E_t^*[Y_T] < E_t[Y_T]$, and the difference between the two reflects investors' demand for compensation for shocks that affect long-run economic growth, or long-term risk. The long-run risk-neutral measure understates the probability of high-growth states and inflates the probability of low-growth states. We can write

$$\ln \frac{E_t^*[Y_T]}{E_{t^-}^*[Y_T]} = \ln \frac{E_t[Y_T]}{E_{t^-}[Y_T]} + \left\{ \ln \frac{E_t^*[Y_T]}{E_t[Y_T]} - \ln \frac{E_{t^-}^*[Y_T]}{E_{t^-}[Y_T]} \right\}, \tag{3}$$

where the first term $\ln \frac{E_t[Y_T]}{E_{t^-}[Y_T]}$ is the impact of monetary policy on expected long-run growth, and the second term represents the change in risk compensation for long-run growth before and after FOMC announcements. In the next section, we will compute this measure of the long-run impact of monetary policy and provide a decomposition into the expected growth component and the long-run risk premium component. In Section 4, we provide conditions under which the long-run impact can come from growth rate expectations and/or discount rate variations to provide guidance for structural models of the long-run impact of monetary policy.

2.2 Proxies for long-term equity and long-term bond

Before our empirical analysis, we first summarize our construction of long-term portfolios. More details on the data are provided in the data appendix. For notational convenience, we denote the announcement returns on long-term equity and the long-term bond as

$$\ln R_E^\infty = \lim_{T\to\infty} \ln \frac{V(t,T)}{V(t^-,T)} \qquad \text{and} \qquad \ln R_B^\infty = \lim_{T\to\infty} \ln \frac{b(t,T)}{b(t^-,T)}$$

Long-term bond Following Alvarez and Jermann (2005), we proxy the long-term bond with the Treasury bond of the longest available maturity. Given our focus on the high-frequency price responses, we use futures on the 30-year Treasury bond[2], as in Cieslak and Khorrami (2025). Although this benchmark bond portfolio is nominal, it covers our entire sample period from Feb 1994 to Dec 2023 and we observe its high-frequency price changes. For robustness, we regress high-frequency changes in the on-the-run 30Y TIPS yield on changes in the on-the-run 30Y nominal Treasury bond yield around FOMC announcements[3]. High-frequency TIPS yield changes are available only from Feb 2005, so this exercise uses a shorter sample from Feb 2005 to Dec 2023. As shown in table 7.1 in the Appendix, we find a slope coefficient of 0.98 and an R^2 of 0.88.

[2]The underlying bond is the cheapest-to-deliver Treasury bond with a maturity of more than 15 years

[3]These yield changes are from U.S. Monetary Policy Event-Study Database of the Federal Reserve Bank of San Francisco, which is based on the work in Acosta, Ajello, Bauer, Loria, and Miranda-Agrippino (2025)

The non-dividend-paying stock portfolio We proxy long-term equity with a portfolio of non-dividend-paying stocks. We construct our benchmark portfolio in each year with firms that have never paid any dividend from listing through the end of the previous year. In doing so, we form this long term equity portfolio with only information available at the formation date. We assume this portfolio has ultra-long duration and is a good proxy for the long term equity. Indeed, from 1926 to 2024, 82.1% of the firm-year observations in this portfolio correspond to firms that have never paid any dividend at any point in the sample. For robustness, we construct a second portfolio of firms that explicitly state in their annual 10-K filings that they will not pay dividends. Such statements are highly standardized: we find that firms typically report that they do not anticipate paying dividends in the foreseeable future. We use GPT-4o-mini to classify each firm-year based on the dividend-policy discussion in Item 5 of the 10-K. To be conservative, we require that the statement contain the phrase "foreseeable future". Among the firm-year observations in this portfolio, 76.4% additionally state the intention to retain earnings to fund growth. As above, we lag the classification in our empirical analysis by one year to avoid using look-ahead information.

Figure 2.1 plots four series over 1994-2024: the total number of firms in CRSP (solid line), the number of firms that had paid no dividend from listing through the end of that year (dotted line), the number of firms that pay no dividend at any point in 1926-2024 (dashed line), and the number of firms whose 10-K states that they do not anticipate paying dividends in the foreseeable future (dash-dotted line). Our benchmark non-dividend-paying portfolio consists of the second group—firms with no dividend history as of the formation date—which accounts for more than half of all CRSP firms in the sample.

To further validate our portfolio as a proxy for long term equity, we compare it with an alternative construction using dividend futures data. Dividend futures allow us to compute the time t value of the dividend strip of the S&P 500 index with maturity n as $P_t^n = B_t^n G_t^n$, where B_t^n is the time t price of a zero-coupon bond maturing at $t+n$ and G_t^n is the time t price of dividend futures maturing at $t+n$ (see, for example, Van Binsbergen and Koijen (2017), Bansal, Miller, Song, and Yaron (2021), and Nagel and Xu (2025)). Given that a stock is a claim to all future dividends, we can construct the price of a synthetic portfolio of long-term dividends of the S&P 500 index after year M as

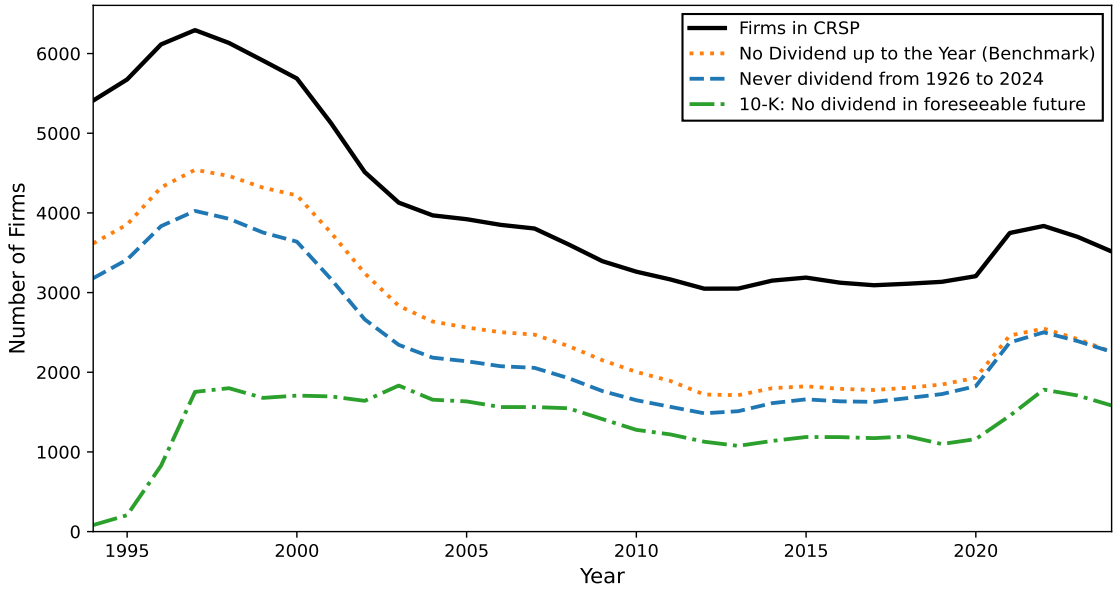
$$P_t^{M+} = P_t^{S\&P500} - \sum_{n=1}^{M} P_t^n. \tag{4}$$

In what follows, we will call P_t^{M+} the price of the tail dividends with maturity M.

Table 2.1 reports the correlations between the daily log return on our non-dividend paying stock portfolio $\ln R_E^\infty$ and the return on the synthetic tail dividend claim of the S&P 500 index with maturities $M=$ 5, 6 and 7 and that with the S&P 500 itself[4]. Following the literature, we exclude dividend futures with maturities beyond seven years, as they are less liquid. Consistent with our theory, the correlation between the return on our non-dividend paying stock portfolio and the return

[4]The availability of individual stock price data in daily CRSP is different from that in TAQ so the set of firms in the portfolio can be different.

Figure 2.1: Number of Firms



Notes: The figure plots four annual series over 1994-2024. The solid line is the total number of firms in CRSP. The dotted line is the number of firms that had paid no dividend from listing through the end of that year. The dashed line is the number of firms that pay no dividend at any point in 1926-2024. The dash-dotted line is the number of firms whose 10-K for that year states that they do not anticipate paying dividends in the foreseeable future. For clarity, all classifications in this figure are contemporaneous. For empirical analysis, we lag these classification by one year to avoid using look-ahead information in forming the portfolios.

on the tail-dividend claim is higher than its correlation with the S&P 500 index itself.

Table 2.1: Correlation with an alternative proxy

	$\ln R^{5+}$		$\ln R^{6+}$		$\ln R^{7+}$		$S\&P500$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FOMC	0.93		0.93		0.92		0.90	
All days		0.90		0.90		0.90		0.87
N	101	3154	92	2893	87	2723	238	8816

Note: The table reports the correlations of the log daily return of the non-dividend paying stock portfolio $\ln R_E^\infty$ with the log daily return of an alternative proxy constructed using the S&P500 and dividend futures. The last row of the table reports the number of observations available in the sample to compute the correlations.

We prefer the non-dividend-paying portfolio to the synthetic tail-dividend claim for several reasons. First, stocks are far more liquid than dividend futures, and their prices are available at high frequency, which improves data quality and is crucial for high-frequency identification. Second, our theory requires long-maturity dividends. The duration of the non-dividend-paying stocks is likely to be much longer than the synthetic tail-dividend portfolio. Finally, as shown in Table 2.1, data coverage is also substantially better for the non-dividend-paying portfolio.

2.3 Testing the independence hypothesis

Following the literature on high-frequency identification (see, for example, Gürkaynak, Sack, and Swanson (2005)), we examine asset returns in a short window spanning from 10 minutes before to 20 minutes after each FOMC announcement. Our sample covers the period from February 4, 1994, when the FOMC began publishing its federal funds rate target decisions after each meeting, through December 13, 2023. It comprises 239 scheduled meetings over this period. We obtain the intraday price data for individual stocks from TAQ and for 30-year T-Bond futures from Tick Data. We aggregate the tick-by-tick price data to the minute level by taking the median of all prices within each minute. We define the closest available minute-level prices 10 minutes before and 20 minutes after the announcements on FOMC days as the pre- and post-announcement prices. Then, we compute the announcement returns as the log price difference for the 30-year T-Bond futures and each stock. To compute the value-weighted announcement return on the non-dividend-paying portfolio ($\ln R_E^\infty$), we use the market value of each stock in the portfolio at the end of the last trading date as the weights.

A necessary condition for the absence of a long-run impact of monetary policy, as suggested by Claim 1, is that the coefficient β_1 in the regression of long-term equity return on the long-term bond return is one. Table (2.2) formally test this necessary condition for the independence hypothesis, using our constructed series of $\ln R_E^\infty$ and $\ln R_B^\infty$. The point estimate of β_1 is 0.24 and the 95% confidence interval is $[0.11, 0.38]$ with an adjusted R^2 of 0.04. A formal t-test of the null hypothesis $H_0 : \beta_1 = 1$ produces a test statistic of -11.20. This evidence suggests the realized announcement returns on long-term equity and the long-term bond are far from being perfectly correlated. Therefore, the hypothesis that monetary policy has no long-run impact is strongly rejected by the data. In what follows, we discuss the quantitative magnitude of this long-run impact.

Table 2.2: Testing the Independence Hypothesis

	$\ln R_E^\infty$	
	(1) Benchmark	(2) 10-K
$\ln R_B^\infty$	0.24	0.22
	$[0.11, 0.38]$	$[0.09, 0.35]$
Adj. R^2	0.04	0.03
	$H_0 : \beta_1 = 1$	
t-test	-11.20	-11.65
N	239	231

Notes: The table reports OLS estimates of the regression $\ln R_E^\infty = \beta_0 + \beta_1 \ln R_B^\infty + u$ on returns measured over the 30-minute window (-10 to +20 minutes) around FOMC announcements. Column (1) uses the benchmark long-term equity portfolio, formed from firms with no dividend history as of the formation date. Column (2) uses the portfolio of firms whose 10-K states that no dividend is anticipated in the foreseeable future. Under the independence hypothesis, the long-term equity and long-term bond portfolios earn the same announcement return, for which $\beta_1 = 1$ is a necessary condition. The row labeled t-test reports the statistic for $H_0 : \beta_1 = 1$. 95% confidence intervals are reported in brackets. The sample is scheduled FOMC announcements from February 1994 to December 2023. Standard errors are Heteroskedasticity-robust.

2.4 Expected long-run growth under the long-run risk-neutral measure

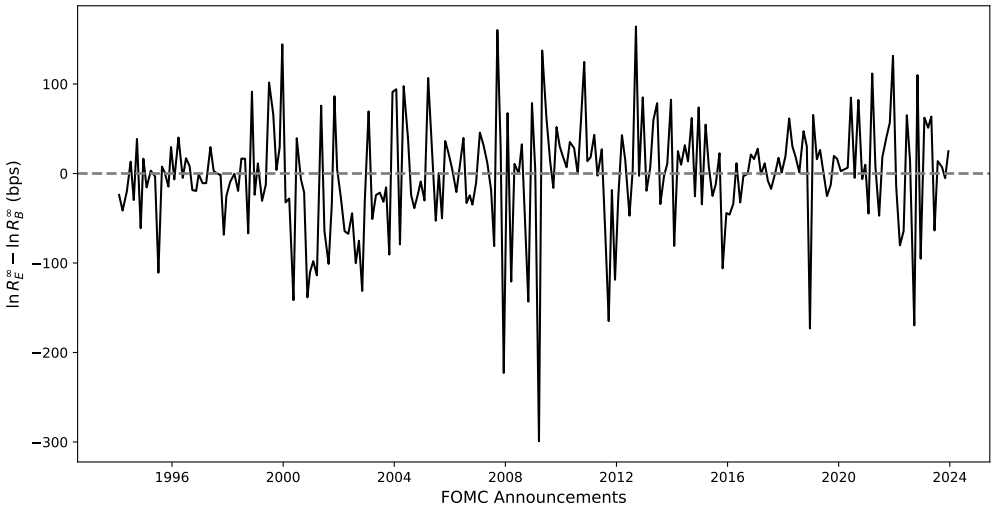
Figure 2.2 displays the constructed series of $\ln R_E^\infty - \ln R_B^\infty$. As shown in Table (2.3), the mean of $\ln R_E^\infty - \ln R_B^\infty$ is close to zero, consistent with its interpretation as a measure of innovations in expectations. The standard deviation of innovations in growth-rate expectations under long-run risk-neutral measure in our sample period is about 60 basis points, slightly higher than that of the S&P 500 futures return in the same window, 51 basis points, indicating a sizable permanent impact of monetary policy announcements.

Table 2.3: Summary Statistics of key variables

	Panel A			Panel B	
	(1)	(2)	(3)	(4)	(5)
	$\ln R_E^\infty - \ln R_B^\infty$	$\ln R_E^\infty$	$\ln R_B^\infty$	MPS	CBI
Mean	-2.17	-1.67	0.50	-0.19	-0.38
SD	62.19	52.75	45.94	4.72	2.72
25th	-27.75	-28.42	-19.87	-2.32	-1.79
75th	29.60	26.61	16.23	2.12	1.19
AC	-0.05	-0.04	-0.07	-0.18	0.12
Skew	-0.72	-0.10	2.66	-0.59	-0.80
N	239	239	239	239	239

Notes: The table reports the summary statistics of key variables in the 30-minute window around the scheduled FOMC announcements from 1994/01/01 to 2023/12/13. Panel A presents summary statistics for the log returns of the long term equity portfolio ($\ln R_E^\infty$), 30-year Treasury bond futures ($\ln R_B^\infty$) and the difference of them ($\ln R_E^\infty - \ln R_B^\infty$). All variables are quoted in basis points. Panel B presents summary statistics for the pure monetary policy surprise and the Fed information surprise from Jarociński and Karadi (2020), reported in basis points.

Figure 2.2: Time series of $\ln R_E^\infty - \ln R_B^\infty$ in 30-minute window around announcements



Notes: The figure plots our measure of the growth expectation revisions under long-run risk neutral measure around the 30-minute window (-10 to +20 minutes) at each scheduled FOMC meeting. The sample spans 1994/01/01 to 2023/12/13, covering 239 meetings. Growth expectation revisions are quoted in basis points. The horizontal dashed line represents the case of no revision in growth expectation.

2.5 Relationship with conventional measures of monetary policy shocks

To further quantify the permanent impact of monetary policy, we compare our time series of $\ln R_E^\infty - \ln R_B^\infty$ with other measures of monetary policy shocks, including the monetary policy shocks of Bauer and Swanson (2023) (BS), the orthogonalized monetary policy shocks of Bauer and Swanson (2023) ($\text{BS}^\perp$), and the monetary policy shock (MPS) and the central bank information shock (CBI) constructed in Jarociński and Karadi (2020). Table 2.4 reports the correlations between the measured long run impact of monetary policy, $\ln R_E^\infty - \ln R_B^\infty$, and the above measures of monetary policy shocks over the same FOMC announcement window. The correlations between our measure of long-run impact of monetary policy and BS, $\text{BS}^\perp$, and MPS are all negative, indicating that surprise interest-rate reductions typically lead to upward revisions of long-run growth expectations. The CBI shocks of Jarociński and Karadi (2020) are identified as interest-rate increases associated with simultaneous stock-market rises. These shocks typically lead to positive revisions of long-run growth rates as well.

Table 2.4: Correlations with high frequency monetary policy shocks

	MPS	CBI	BS	$BS^{\perp}$
	(1)	(2)	(3)	(4)
$\ln R_E^\infty - \ln R_B^\infty$	−0.38	0.52	−0.07	−0.13
$\ln R_E^\infty$	−0.80	0.57	−0.47	−0.49
$\ln R_B^\infty$	−0.41	−0.05	−0.44	−0.38

Notes: The table reports the correlations of $\ln R_E^\infty$, $\ln R_B^\infty$ and $\ln R_E^\infty - \ln R_B^\infty$ with four measures of monetary policy shocks constructed in the literature. MPS and CBI represent the pure monetary policy shocks and Fed information shocks from Jarociński and Karadi (2020). BS and $BS^{\perp}$ are two monetary policy shocks from Bauer and Swanson (2023). BS is the first principal component of the price changes of the Eurodollar futures expiring in the nearest four quarters. $BS^{\perp}$is the residual from regressing BS on six macroeconomic variables known prior to the announcement. All variables are measured in the 30-minute window around the announcements.

To quantify the impact of different components of the monetary policy shock on long-run growth rates, we estimate the following regression

$$\ln R_E^\infty - \ln R_B^\infty = \beta_0 + \beta_1 MPS + \beta_2 CBI + u, \tag{5}$$

where $\ln R_E^\infty - \ln R_B^\infty$ is our measure of the long-run growth rate under the long-run risk-neutral measure $\lim_{T\to\infty} \ln\left(\frac{E_t^*[Y_T]}{E_{t-}^*[Y_T]}\right)$. All shocks are measured within the same 30-minute window around FOMC announcements. In the above equation, MPS and CBI are the monetary policy shock and the central bank information shock, constructed by Jarociński and Karadi (2020), respectively. The regression coefficients β_1 and β_2 can be interpreted as the impact of MPS and CBI on long-run growth expectations, respectively. The regression results are reported in Table (2.5).

Table 2.5: High Frequency revisions in long-run growth to MPS and CBI

	Panel (A): $\ln R_E^\infty - \ln R_B^\infty$			Panel (B): $\ln R_E^\infty$			Panel (C): $\ln R_B^\infty$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
MPS	-4.24^{***}			-8.31^{***}			-4.08^{***}		
	(1.31)			(0.35)			(1.12)		
CBI	10.93^{***}			9.27^{***}			-1.65		
	(2.18)			(0.62)			(1.87)		
BS		-0.86			-4.64^{***}			-3.78^{***}	
		(1.04)			(0.67)			(0.67)	
$BS^\perp$			-1.67			-5.17^{***}			-3.51^{***}
			(1.14)			(0.78)			(0.63)
Const.	1.12	-1.83	-2.49	0.21	0.18	-0.65	-0.91	2.01	1.84
	(3.57)	(4.07)	(4.08)	(1.27)	(3.05)	(3.06)	(2.98)	(2.74)	(2.83)
N	239	239	233	239	239	233	239	239	233
Adj. R^2	0.36	0.00	0.01	0.87	0.22	0.24	0.17	0.19	0.14

Notes: The table reports OLS estimates from regressions of (A) the revisions in long-run growth expectations under long-run risk neutral measure $\ln R_E^\infty - \ln R_B^\infty$ (B) the log returns on the long-term equity portfolio ($\ln R_E^\infty$) and (C) the log returns on the long-term bond portfolio ($\ln R_B^\infty$), on the monetary policy surprises. Heteroskedasticity-robust standard errors are reported in brackets.$^{*}p < 0.1,^{**}p < 0.05,^{***}p < 0.01$.

We find that a 25-basis-points cut in short rates due to a pure monetary policy shock, on average, leads to a $4.237 * 25 \approx 106$ basis points upward revision in long-run output under long-run risk neutral measure, and the same magnitude of cut due to the Fed information effect is associated with $10.926 * 25 \approx 273$ basis points downward revision (see column 1 in Panel A of Table (2.5)). The slope coefficients are highly significant and the adjusted R^2 is about 0.36, which is in contrast to the insignificant coefficients and near-zero adjusted R^2 obtained from regressions that directly use raw high-frequency changes in short rates (see columns (2) and (3)). Our results indicate that pure monetary policy is not neutral in the long run and Fed announcements contain information about long-run output. Table 7.2 in the Appendix provides a robustness check using the 10-K portfolio.

2.6 Decomposition of the long-run impact

Having documented the sizable long-run impact of monetary policy, we further decompose this long-run impact into news about long-run consumption growth and the long-run risk premium. As shown in Ai and Fu (2026), news about long-run consumption growth can be approximated by

$$\lim_{T\to\infty} \frac{E\left[Y_T \middle| X_{t+\Delta}\right]}{E\left[Y_T \middle| X_t\right]} \approx e^{-\rho\Delta} \frac{R^\infty_{E,t+\Delta}}{\exp\left(\sum_{j=1}^m \omega_j \left(n^j_{t+\Delta} - n^j_t\right)\right)}$$

where $R^\infty_{E,t+\Delta} = \lim_{T\to\infty} \ln \frac{V(t+\Delta,T)}{V(t,T)}$ is the gross return on long-term equity over the time interval $[t, t+\Delta]$, $n_t = \left\{n^j\left(X_t\right) : j = 1, \ldots m\right\}$ is a set of predictors of the long-term equity return and $\omega = \{\omega_j : j = 1, \ldots, m\}$ is the set of weights associated with each predictor. Under rational expectations,

the update of the belief on the long-run growth $\lim_{T\to\infty} \frac{E[Y_T|X_{t+\Delta}]}{E[Y_T|X_t]}$ has to be a multiplicative martingale.

$$E_t\left[\frac{R^{\infty}_{E,t+\Delta}}{\exp\left(\sum_{j=1}^{m}\omega_j\left(n^j_{t+\Delta}-n^j_t\right)\right)}\right]=e^{\rho\Delta}$$

Motivated by this, we estimate the set of parameters $\{\rho,\omega\}$ using GMM based on the following moment conditions

$$E\left[\left(\frac{R^{\infty}_{E,t+\Delta}}{\exp\left(\sum_{j=1}^{m}\omega_j\left(n^j_{t+\Delta}-n^j_t\right)\right)}-e^{\rho\Delta}\right)Z_t\right]=0$$

where $Z_t \in \mathcal{F}_t$ is a set of instruments measurable at time t.

Given our focus on high-frequency announcements, the predictors should have high-frequency data available. As a result, we pick the log VIX and the credit spread (Baa - Aaa) as the predictors in n_t. We obtain their daily series from FRED and construct monthly series using the last daily observation of the month. We estimate the weights ω at monthly frequency since identification requires predictability. Table 2.6 shows the estimation results from the two-step GMM. The negative values of the both weights reflect that a positive unexpected increase in either VIX or credit spread is associated with positive discount rate news for long-term equity. The J-test has a p-value of 0.22, so the overidentifying restrictions are not rejected.

Table 2.6: Two-step GMM

ω_{VIX}	ω_{CS}	ρ
-0.066	-14.06	0.0068^{***}
(0.05)	(9.10)	(0.0026)
	J-test	
p-value	0.22	
N	419	

Notes: The table reports two-step GMM estimates of ρ and $\omega=(\omega_{VIX},\omega_{CS})$. The predictor n_t are the log VIX and the credit spread (Baa-Aaa). The instrument vector is $Z_t=\left(1,\log VIX_t, CS_t, R^{\infty}_{E,t}, R_{f,t}\right)$, so the model is overidentified with two degrees of freedom. The first-step weighting matrix is $\left(Z^TZ/T\right)^{-1}$ and the second-step weighting matrix is the Newey-West HAC estimator with a bandwidth of five months. The sample is monthly from Feb 1990 to Dec 2024. $^{*}p<0.1,^{**}p<0.05,^{***}p<0.01$.

Over the short announcement window, we can measure the changes in the long-run growth expectation (η^{∞}_{CF}) and long-run risk premium (η^{∞}_{RP}) respectively as

$$\eta^{\infty}_{CF}=\lim_{T\to\infty}\ln\frac{E_t\left[Y_T\right]}{E_{t^-}\left[Y_T\right]}=\ln R^{\infty}_E-\sum_{j=1}^{m}\omega_j\left(n^j_t-n^j_{t-}\right)$$

$$-\eta^{\infty}_{RP}=\lim_{T\to\infty}\left(\ln\frac{E^*_t\left[Y_T\right]}{E_t\left[Y_T\right]}-\ln\frac{E^*_{t^-}\left[Y_T\right]}{E_{t^-}\left[Y_T\right]}\right)=\sum_{j=1}^{m}\omega_j\left(n^j_t-n^j_{t-}\right)-\ln R^{\infty}_B$$

As shown in equation (3), our measure of the long-run impact of the announcements can be additively decomposed into this long-run growth expectation and the long–run risk premium component $\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]} = \eta^{\infty}_{CF} - \eta^{\infty}_{RP}$. As a result, its variance can be decomposed into two covariance components

$$Var\left(\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]}\right) = Cov\left(\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]}, \eta^{\infty}_{CF}\right) + Cov\left(\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]}, -\eta^{\infty}_{RP}\right)$$

Table (2.7) reports the decomposition. Long-run cash flow news covaries positively with the long-run impact and long-run risk premium news covaries negatively, which is consistent with the theory. We find that about half of the variance comes from long-term cash flow news and the other half from changes in the long-run risk premium. These two components are constructed from our measure of the long–run impact over the announcement window, the weights estimated in Table (2.6), and the daily changes in the VIX and the credit spread around each announcement.

Table 2.7: Variance decomposition of the long-run impact of the announcements

	$Var\left(\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]}\right)$	$Cov\left(\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]}, \eta^{\infty}_{CF}\right)$	$Cov\left(\ln \frac{E^*_{t-}[Y_T]}{E_{t-}[Y_T]}, -\eta^{\infty}_{RP}\right)$
	(1)	(2)	(3)
Size	0.387	0.193	0.194
Fraction	1.0	0.5	0.5

Notes: The table decomposes the variance of the long-run impact of an announcement into its cash-flow and risk-premium components. Sizes are reported in percent-squared. The Fraction row expresses each covariance as a share of variance of the measure of long-run impact and sums to one by construction. The sample is 239 FOMC announcements from February 1994 to December 2023.

3 Interpreting long-run impact

In this section, we provide a theoretical foundation for Claim 1 and Claim 2 in Section 2 and discuss conditions under which the return on the long-short long-term portfolio in 3 can be interpreted as a subjective belief about long-run growth or the risk premium about long-run cash flow, or both. We first describe an analytical framework for our analysis based on the setup in Hansen and Scheinkman (2009).

3.1 An analytical framework

Model setup In this section, we describe a general no-arbitrage asset pricing setup where we can provide a theoretical foundation for the three empirical exercises in Equations (1), (2), and (3). Our general setup follows Hansen and Scheinkman (2009) and Qin and Linetsky (2017). Time is continuous and infinite and prescheduled FOMC announcements are made at time $\tau, 2\tau, 3\tau, \cdots$. Let X_t be a $J-$dimensional strong Markov process defined on the probability space $(\Omega, \mathcal{F}, Pr)$ with an open and connected state space $\mathbb{X} \subseteq R^J$, and let B_t be a $K-$dimensional Brownian motion defined

on $(\Omega, \mathcal{F}, Pr)$. We assume that between announcements, for $t \in (n\tau, (n+1)\tau)$, X_t is driven by a diffusion process of the form: $dX_{j,t} = \mu_j(X_t, t)\,dt + \sigma_j^T(X_t, t)\,dB_t$ for $j = 1, 2, \cdots J$, where for each j, $\sigma_j(X_t, t)$ is a $K \times 1$ vector of risk exposure with respect to the vector Brownian motion B_t, and the superscript T denotes the matrix transpose. In vector notation, we write

$$dX_t = \mu_X(X_t, t)\,dt + \Sigma_X^T(X_t, t)\,dB_t, \tag{6}$$

where $\mu_X(X_t, t) = [\mu_1(X_t, t), \mu_2(X_t, t), \cdots \mu_J(X_t, t)]^T$ is a $J \times 1$ vector of drift parameters, and $\Sigma_X(X_t, t) = [\sigma_1(X_t, t), \sigma_2(X_t, t), \cdots \sigma_J(X_t, t)]$ is a $K \times J$ matrix of diffusion coefficients. We allow part of the X_t vector to jump at announcement times, $\tau, 2\tau, 3\tau, \cdots$, and adopt the convention that X_t is right-continuous with left limits. At each announcement time $n\tau$, for $n = 1, 2, \cdots$, the transition probability of $X_{n\tau}$ given $X_{n\tau^-} = y$ is given by a Markov kernel $Q(y, dx)$ that does not depend on n.

We interpret X_t as a vector of state variables in an appropriately defined recursive competitive equilibrium (Mehra and Prescott (1985)). Some elements of X_t may be exogenous state variables such as TFP, or exogenous monetary policy shocks. Part of X_t may be endogenous state variables such as the capital stock of the economy or financial intermediary net worth. We allow some or all of the elements in X_t to be influenced by monetary policy, so that a monetary policy announcement may result in a discrete jump of X_t at announcement points. Let $\{\mathcal{F}_t : t \geq 0\}$ denote the filtration generated by the history of X_t, that is, $\forall t > 0$, $\mathcal{F}_t = \sigma\left(\{X_s\}_{s=0}^t\right)$. We assume the state variables are rich enough so that the Brownian motion B_t is $\mathcal{F}_t$ adapted. Because we allow jumps in X_t, the history of $\{X_s\}_{s=0}^t$ potentially contains more information than the filtration generated by the Brownian motion.

To model the periodicity of announcements, we assume that X_t is τ periodic in the sense that $\mu_X(x, t) = \mu_X(x, s)$ and $\Sigma_X(x, t) = \Sigma_X(x, s)$ if $t = s \mod \tau$. We also assume that $\mu_X(x, t)$ and $\Sigma_X(x, t)$ are locally Lipschitz on $\mathbb{X} \times [0, \infty)$ with Lipschitz constants uniform in t. In addition, $\Sigma_X^T(x, t)\,\Sigma_X(x, t)$ is positive definite for all $x \in \mathbb{X}$ and all t. Most of our results will extend to the case where $\mu_X(x, t)$ and $\Sigma_X(x, t)$ are asymptotically periodic but the extension involves significantly heavier notation.

We assume that all asset prices can be computed by using an $\mathcal{F}_t$ adapted real stochastic discount factor (SDF) process, S_t, where for $t \in (n\tau, (n+1)\tau)$,

$$\frac{dS_t}{S_t} = -r(X_t, t)\,dt - \sigma_S^T(X_t, t)\,dB_t, \tag{7}$$

where $r(X_t, t)$ is the real risk-free interest rate and $\sigma_S^T(X_t, t)$ is the market price of risk, both of which are assumed to be $\tau-$periodic. We allow S_t to jump at announcement times and denote the joint distribution of $\left(\frac{S_t}{S_{t^-}}, X_{t^-}\right)$ given X_{t^-} as $Q(y, d(z, x))$. Under this formulation, let f_T be a payoff at time T measured in time-T consumption goods. The present value of f_T as of time

t, evaluated in time t consumption units, is $E\left[\frac{S_T}{S_t}f_T\middle| X_t\right]$.[5] The possibility of jumps of S_t at pre-scheduled announcement times allows us to model the FOMC announcement premium (Ai and Bansal (2018)). Let P_t be the price of one unit of real consumption goods at time t. We assume that the dynamics of price level are given by the following diffusion process with $\tau-$periodic coefficients

$$\frac{dP_t}{P_t} = \pi\left(X_t, t\right)dt + \sigma_\pi^T\left(X_t, t\right)dB_t. \tag{8}$$

Under the above setup, $\left\{\frac{S_t}{P_t}\right\}$ is the nominal SDF. That is, if f_T is a time-T payoff measured in dollar terms, then $E\left[\frac{S_T/P_T}{S_t/P_t}f_T\middle| X_t\right]$ is the (nominal) price of the payoff at time t.

We assume that real aggregate output is also $\mathcal{F}_t$ adapted, $\tau-$periodic, and given by

$$\frac{dY_t}{Y_t} = \mu_Y\left(X_t, t\right)dt - \sigma_Y^T\left(X_t, t\right)dB_t. \tag{9}$$

The aggregate stock market is a claim to the following dividend process, $D_t = \Psi\left(X_t\right)Y_t$ measured in real consumption terms, where $\Psi\left(X_t\right)$ is bounded. Because we are interested in the long-run behavior of dividends and output, the functional form, $D_t = \Psi\left(X_t\right)Y_t$, effectively imposes that aggregate dividends and aggregate output are co-integrated. We maintain this assumption throughout.

We think of the dynamics of the SDF in (7) and price level in (8) and those of output in (9) as equilibrium outcomes. The impact of monetary policy is modeled through the dependence of asset prices, $r\left(X_t, t\right)$, $\sigma_S\left(X_t, t\right)$, inflation, $\pi\left(X_t, t\right)$, $\sigma_\pi\left(X_t, t\right)$, and output dynamics, $\mu_Y\left(X_t, t\right)$ and $\sigma_Y\left(X_t, t\right)$ on the state variable X_t. To emphasize the dependence of prices on the state variable X_t, we augment our notation for the price of a zero-coupon inflation-protected bond with maturity T at time t as $b\left(t, T\middle| X_t\right)$, that is,

$$b\left(t, T\middle| X_t\right) = E\left[\frac{S_T/P_T}{S_t/P_t}P_T\middle| X_t\right],$$

and the price of tail dividends with maturity T as $V\left(t, T\middle| X_t\right)$:

$$V\left(t, T\middle| X_t\right) = E\left[\int_{u=0}^{\infty}\frac{S_{T+u}/P_{T+u}}{S_t/P_t}P_{T+u}D_{T+u}du\middle| X_t\right].$$

In our formulation, although the underlying state variable X_t may contain jumps at $\tau, 2\tau, 3\tau, \cdots$, we have assumed that, P_t, and Y_t both have continuous paths. The continuity of P_t and Y_t are reasonable because inflation and changes in aggregate output over a short FOMC announcement window are negligible. The continuity assumption on Y_t and P_t allows us to capture the contrast between the fact that over a short FOMC announcement window, macroeconomic quantities change very little, while expectations about the future and forward-looking asset prices can respond instantaneously, which is the basis for some of the sharp identification results we derive below.

[5] If t is an announcement time, then the pre-announcement present value is $E\left[\frac{S_T}{S_{t-}}f_T\middle| X_{t-}\right]$ and post-announcement present value is $E\left[\frac{S_T}{S_t}f_T\middle| X_t\right]$.

Martingale constructions Our setup takes the SDF process (7), the inflation process (8), and the output process (9) as primitives. In this section, we make some assumptions on these processes that allow us to use the results from Hansen and Scheinkman (2009) and Qin and Linetsky (2017) to compute returns on long-term portfolios. To allow for periodically scheduled announcements, we find it convenient to extend the analysis of Hansen and Scheinkman (2009) to the time-inhomogeneous case, where the drift and diffusion coefficients in (6), (7), (8), and (9) are allowed to be functions of t.

We start with a diffusion process M_t of the form

$$dM_t = M_t \left[\mu_M\left(X_t, t\right) dt + \sigma_M^T\left(X_t, t\right) dB_t\right] \tag{10}$$

with $\tau-$periodic coefficients. Suppose there exist a real number ρ and a strictly positive and twice continuously differentiable function $\mathbf{e}\left(X, t\right)$ such that $\hat{M}_t = e^{-\rho t}\mathbf{e}\left(X_t, t\right) M_t$ is a martingale. Because both $\mu_M\left(X_t, t\right)$ and $\sigma_M\left(X_t, t\right)$ are periodic, $\mathbf{e}\left(X_t, t\right)$ must be periodic as well. Using $\hat{M}_t$, we can construct a new probability on $(\Omega, \mathcal{F})$ such that expectation under the new probability, denoted $\hat{E}^M$, is defined as $\hat{E}\left[f_T | X_t\right] = E\left[\frac{\hat{M}_T}{\hat{M}_t} f_T \middle| X_t\right]$ for all X_t and all $\mathcal{F}_T$ measurable functions f_T. In the time-homogeneous case of Hansen and Scheinkman (2009), M_t is formulated as a multiplicative functional of X_t and $\{\rho, \mathbf{e}\left(X, t\right)\}$ are constructed as the principal eigenvalue-eigenfunction pair from the solution to an appropriately defined eigenvalue problem. With a slight abuse of terminology, we will still call ρ and $\mathbf{e}\left(X, t\right)$ the eigenvalue and the eigenfunction, as they must jointly satisfy a partial differential equation similar to that in Hansen and Scheinkman (2009). Formally, we introduce a definition of long-term factorization (see also Qin and Linetsky (2017)).

Definition 1. (Long-term factorization)

A diffusion process M_t of the form 10 is said to have a long-term factorization if there exists $\{\rho, \mathbf{e}\left(X, t\right)\}$ such that

1. $\mathbf{e}\left(X, t\right)$ is strictly positive and twice continuously differentiable and $\hat{M}_t = e^{-\rho t}\mathbf{e}\left(X_t, t\right) M_t$ is a martingale;

2. the inverse of the eigenfunction $\frac{1}{\mathbf{e}(X_t, t)}$ is dominated by a Lyapunov function under the measure $\hat{E}^M$.

The setup of Hansen and Scheinkman (2009) requires X_t to have a stationary and ergodic distribution under the probability induced by $\left\{\hat{M}_t\right\}$. In the time-inhomogeneous case, the ergodic distribution may not exist. Our condition 2 above adapts the Lyapunov function approach of Hansen and Scheinkman (2009). As we show in the appendix, it will allow us to compute the price of long-term assets by using the eigenfunctions in the long-term factorization.

Whenever M_t admits a long-term factorization, we can write

$$M_t = e^{\rho t}\frac{1}{\mathbf{e}\left(X_t, t\right)}\hat{M}_t.$$

Here, ρ is a constant growth rate, and $\hat{M}_t$ is a martingale (the permanent component of M_t). In Hansen and Scheinkman (2009), the eigenfunction $\mathbf{e}(X_t, t)$ depends only on the time-homogeneous Markov state variable X_t. It is the stationary component of M_t in the sense that X_t has a stationary distribution under the probability induced by $\hat{M}_t$. We will keep this terminology and call $\frac{1}{\mathbf{e}(X_t,t)}$ the stationary component. In our setup, X_t does not converge to a unique stationary distribution, but to a family of τ-periodic distributions. In what follows, we assume that both S_t and Y_t have long-term factorizations, and denote the corresponding principal eigenvalue-eigenfunction pairs by $(-\bar{r}, \mathbf{e}_S(X,t))$ and $(\rho_Y, \mathbf{e}_Y(X,t))$.

The long-term factorization can be used to compute changes in expectations about long-run growth over short FOMC announcement windows. Using the long-term factorization of Y_t, we can construct the martingale $\hat{Y}_t = e^{-\rho_Y t}\mathbf{e}_Y(X_t,t)\,Y_t$, which allows us to write expected future output growth as

$$\begin{aligned} E_t\left[\frac{Y_T}{Y_t}\right] &= e^{\rho_Y(T-t)}\mathbf{e}_Y(X_t,t)\,E\left[\underbrace{\frac{e^{-\rho_Y T}\mathbf{e}_Y(X_T,T)\,Y_T}{e^{-\rho_Y t}\mathbf{e}_Y(X_t,t)\,Y_t}}_{martingale}\frac{1}{\mathbf{e}_Y(X_T,T)}\middle| X_t\right] \\ &= e^{\rho_Y(T-t)}\mathbf{e}_Y(X_t,t)\,\hat{E}^Y\left[\frac{1}{\mathbf{e}_Y(X_T,T)}\middle| X_t\right]. \end{aligned} \tag{11}$$

The assumption of long-term factorization allows us to conclude the convergence of the term $\hat{E}^Y\left[\frac{1}{\mathbf{e}_Y(X_T,T)}\middle| X_t\right] \to \int \frac{1}{\mathbf{e}_Y(y,T)}\pi_{T \bmod \tau}(dy)$, where the limit does not depend on X_t. Here, $\{\pi_t\}_{t\in[0,\tau)}$ is a family of stationary distributions to which X_t converges under the probability $\hat{E}^Y$. As a result, beliefs about long-run output can in general be written as $e^{-\rho_Y(T-t)}E_t\left[\frac{Y_T}{Y_t}\right] \to \mathbf{e}_Y(X_t,t)\,Y_t\int \frac{1}{\mathbf{e}_Y(y,T)}\pi_{T \bmod \tau}(dy)$, and the update of belief about the long-run level of GDP at time T, before and after an FOMC announcement, can be written as:

$$\lim_{T\to\infty}\frac{E_t[Y_T]}{E_{t^-}[Y_T]} = \frac{\mathbf{e}_Y(X_t,t)}{\mathbf{e}_Y(X_{t^-},t)}. \tag{12}$$

The continuity of Y_t, that is, the fact that $Y_{t^-} = Y_t$ implies that computing the update of belief about long-run growth is equivalent to computing the eigenfunction $\mathbf{e}_Y(X_t,t)$.

We will use a similar decomposition of the SDF to relate the update of expectations to asset returns. Given an SDF S_t that admits a long-term factorization, we can write

$$S_t = e^{-\bar{r}t}\mathbf{e}_S(X_t,t)\,\hat{S}_t, \tag{13}$$

which decomposes the SDF into a constant discount rate $\bar{r}$, a transitory component $\mathbf{e}_S(X_t,t)$, and a permanent component $\hat{S}_t$. Following Borovička, Hansen, and Scheinkman (2016), we will call the probability measure induced by $\hat{S}_t$ the long-run risk-neutral measure. Using this construction, we

can compute the price of a zero-coupon inflation-indexed bond with maturity T as:

$$b\left(t,T\middle|\,X_t\right)=P_tE\left[\frac{S_T}{S_t}\middle|\,X_t\right]=e^{-\bar{r}(T-t)}\mathbf{e}_S\left(X_t,t\right)P_tE\left[\underbrace{\frac{e^{\bar{r}T}S_T\mathbf{e}_S\left(X_T,T\right)}{e^{\bar{r}t}S_t\mathbf{e}_S\left(X_t,t\right)}}_{martingale}\frac{1}{\mathbf{e}_S\left(X_T,T\right)}\middle|\,X_t\right].$$

The term $\frac{e^{\bar{r}T}S_T\mathbf{e}_S(X_T)}{e^{\bar{r}t}S_t\mathbf{e}_S(X_t)}$ is a martingale by Equation (13). This allows us to write the price of the long-term bond as $e^{-\bar{r}(T-t)}\mathbf{e}_S\left(X_t,t\right)P_t\hat{E}^S\left[\frac{1}{\mathbf{e}_S(X_T,T)}\middle|\,X_t\right]$, where $\hat{E}^S$ is the expectation induced by long-run risk-neutral probability. Condition 2 in the definition of long-term factorization allows us to compute the bond price for large T as:

$$\lim_{T\to\infty}e^{\bar{r}(T-t)}b\left(t,T\middle|\,X_t\right)=\mathbf{e}_S\left(X_t,t\right)P_t\lim_{T\to\infty}\hat{E}^S\left[\frac{1}{\mathbf{e}_S\left(X_T,T\right)}\middle|\,X_t\right]=\mathbf{e}_S\left(X_t,t\right)P_t\int\frac{1}{\mathbf{e}_S\left(y,T\right)}\pi_{T\mod\tau}\left(dx\right). \tag{14}$$

The above construction allows us to write the return on a long-term inflation-protected bond from time t^- to t as:

$$\lim_{T\to\infty}\frac{b\left(t,T\middle|\,X_t\right)}{b\left(t^-,T\middle|\,X_{t^-}\right)}=\frac{\mathbf{e}_S\left(X_t,t\right)}{\mathbf{e}_S\left(X_{t^-},t\right)}. \tag{15}$$

This result is due to Alvarez and Jermann (2005): the return on long-maturity bonds is determined by the transitory component of the SDF.

If we assume that the product S_tY_t also admits a long-term factorization with an eigenvalue-eigenfunction pair $(\rho_{SY},\hat{\mathbf{e}}_{SY})$, we can compute the present value of a zero-coupon equity paid at time T, that is, the time-t price of a one-time aggregate dividend payment of $D_T=Y_T\psi\left(X_T\right)$ is

$$P_tE_t\left[\frac{S_TY_T}{S_t}\psi\left(X_T\right)\middle|\,X_t\right]=e^{\rho_{SY}(T-t)}\mathbf{e}_{SY}\left(X_t,t\right)P_tY_tE\left[\frac{\hat{M}_T}{\hat{M}_t}\frac{\psi\left(X_T\right)}{\mathbf{e}_{SY}\left(X_T,T\right)}\middle|\,X_t\right].$$

Under the assumption of long-run factorization,

$$e^{-\rho_{SY}(T-t)}P_tE_t\left[\frac{S_TY_T}{S_t}\psi\left(X_T\right)\middle|\,X_t\right]\to\mathbf{e}_{SY}\left(X_t,t\right)P_tY_t\int\frac{\psi\left(y\right)}{\mathbf{e}_{SY}\left(y,T\right)}\pi_{T\mod\tau}\left(dx\right) \tag{16}$$

where $\hat{M}_t=e^{-\rho_{SY}t}S_tY_t\mathbf{e}_{SY}\left(X_t,t\right)$ is the martingale constructed from S_tY_t by using (13), and $\pi_{T\mod\tau}$ is the corresponding measure constructed from Condition 2 of the definition of long-term factorization. In the appendix, we show that a similar calculation also holds for the tail dividend claim, and, as a result,

$$\lim_{T\to\infty}\frac{V\left(t,T\middle|\,X_t\right)}{V\left(t^-,T\middle|\,X_{t^-}\right)}=\frac{\mathbf{e}_{SY}\left(X_t,t\right)}{\mathbf{e}_{SY}\left(X_{t^-},t\right)}. \tag{17}$$

3.2 The independence hypothesis

In this section, we provide a rigorous statement for the independence hypothesis in Claim 1 and provide a formal proof. We start with an SDF process S_t and an output process Y_t, both of which

are assumed to have long-term factorizations. Because Y_t admits a long-term factorization, we can always decompose it into a transitory component and a stochastic growth component. Our independence hypothesis states that there exists one such decomposition such that the stochastic growth component is independent of the SDF.

Definition 2. (The independence hypothesis) There exists $\mathbf{y}(X_t, t)$ such that $\frac{1}{\mathbf{y}(X_t,t)}Y_t$ is independent of S_t.

Whenever the independence hypothesis holds, we can write Y_t as $Y_t = \mathbf{y}(X_t, t)\tilde{Y}_t$, where $\tilde{Y}_t$ represents the stochastic growth component of Y_t and is independent of the stochastic discount factor S_t. In the textbook versions of the New Keynesian models, for example, Galí (2015) and Woodford (2003), the independence hypothesis holds because Y_t itself is stationary. Another example where the independence hypothesis holds is that after detrending by some constant growth rate g, $e^{-gt}Y_t$ is stationary. That is, the long-run growth rate of Y_t is a constant g. In general, by varying the functional form of $\mathbf{y}(X_t, t)$, we can eliminate a transitory component of Y_t as an arbitrary function of (X_t, t), and the residue, $\frac{1}{\mathbf{y}(X_t,t)}Y_t$ contain stochastic growth that cannot be eliminated by using stationary processes in the form of $\mathbf{y}(X_t, t)$. We think of the state variable X_t as capturing the impact of monetary policy, which may impact the real stochastic discount factor S_t. The independence hypothesis formulated above allows the state variable X_t to drive the comovement between the transitory component $\mathbf{y}(X_t, t)$ and the stochastic discount factor S_t but requires the stochastic growth component $\tilde{Y}_t$ to be independent.

Our proposition below states that whenever the independence hypothesis holds, the return on the long-term bond and that on long-term equity must be perfectly correlated over short FOMC announcement windows. As before, we assume that both S_t and Y_t admit long-term factorizations. We denote $\mathbf{e}_S(X_t, t)$ and $\mathbf{y}(X_t, t)$ as the associated eigenfunctions in their long-term factorizations, $\hat{E}^S$ as the measure induced by the martingale $\hat{S}_t$ and $\hat{E}^Y$ as the measure induced by the martingale $\hat{Y}_t$.

Proposition 1. *(The independence hypothesis) Under the independence hypothesis, assume that* $\frac{\mathbf{y}(X_t,t)}{\mathbf{e}_S(X_t,t)}$ *is dominated by a Lyapunov function under the measure* $\hat{E}^S$ *and* $\frac{1}{\mathbf{y}(X_t,t)\mathbf{e}_Y(X_t,t)}$ *is dominated by a Lyapunov function under the probability induced by the* $\hat{Y}$ *martingale, then*

$$\lim_{T\to\infty}\left\{\ln\frac{V(t,T|X_t)}{V(t^-,T|X_{t^-})}-\ln\frac{b(t,T|X_t)}{b(t^-,T|X_{t^-})}\right\}=\lim_{T\to\infty}\left\{\ln\frac{E[Y_T|X_t]}{E[Y_T|X_{t^-}]}\right\}=0. \tag{18}$$

Proof. See Appendix 5.2. □

As a result, an imperfect correlation between the long-term bond and the long-term equity return, as we documented in the previous section, rejects the independence hypothesis. It is important to note that our independence hypothesis is formulated in terms of stochastic discount factors. It rules out the subtle case where monetary policy does not directly affect the stochastic growth component, $\tilde{Y}_t$. However, it is correlated with other elements of the state variables in X_t that affect the co-movement between S_t and the long-run growth $\tilde{Y}_t$. To understand these more complicated

cases, we need to study the correlation structure between the martingale components of S_t and Y_t, which we turn to in the next sections.

3.3 Expected growth under the long-run risk-neutral probability

As we show in Section 2, empirical evidence strongly rejects the independence hypothesis. In order to account for the responses of long-term assets over short FOMC announcement windows, we need models with stochastic growth, which allow for a nontrivial impact of monetary policy on the long-run levels of output. In general, we can interpret the return on the long-term portfolio as a quantitative measure of the long-run impact of monetary policy. In this section, we provide a proof for Equation (2) in Claim 2, which states that the return on the long-short long-term portfolio can be interpreted as the change in the expected long-run level of output under the long-run risk-neutral probability.

Let $\mathbf{e}_S(X_t)$ be the principal eigenfunction of S_t and $\bar{r}$ be the corresponding eigenvalue; that is, $e^{\bar{r}t}\mathbf{e}_S(X_t)S_t$ is a martingale, and the distribution of X_t is stationary and ergodic under the probability induced by $\hat{S}_t = e^{\bar{r}t}\mathbf{e}_S(X_t)S_t$. The probability induced by $\hat{S}_t$ is called the long-run risk-neutral probability by Hansen and Scheinkman (2009) and Borovička, Hansen, and Scheinkman (2016). The following proposition relates the return on the long-short long-term portfolio to expected long-run growth under the long-run risk-neutral probability.

Proposition 2. *(Expected growth under the long-run risk-neutral measure)*

Let t be an FOMC announcement, and assume that S_t, Y_t, and S_tY_t all admit long-term factorizations.

$$\lim_{T\to\infty}\left\{\ln\frac{V(t,T|X_t)}{V(t^-,T|X_{t^-})}-\ln\frac{b(t,T|X_t)}{b(t^-,T|X_{t^-})}\right\}=\lim_{T\to\infty}\ln\left(\frac{\hat{E}_t^S[Y_T|X_t]}{\hat{E}_{t^-}^S[Y_T|X_{t^-}]}\right), \tag{19}$$

where $\hat{E}^S$ denotes the expectation under the probability induced by $\hat{S}_t$.

Proof. See Appendix 5.2 □

Proposition 2 implies that the response of the long-short long-term portfolio over a short FOMC announcement window can be interpreted as the revision in expected long-run growth under the long-run risk-neutral probability. As shown in Equation (3), our measure of the long-run impact, in general, contains both expected long-run growth under the subjective probability and risk compensation for long-run growth. To provide guidance for structural models of the long-run impact of monetary policy, in the rest of this section, we provide sufficient conditions under which the risk premium term is zero and (19) can be interpreted as an update to subjective beliefs about long-run growth. We also give examples where risk premium may be important when these sufficient conditions fail.

3.4 Subjective belief about long-run growth

In this section, we first provide a necessary and sufficient condition under which the return on the long-short portfolio, $\ln \frac{V(t+\Delta,T)}{V(t,T)} - \ln \frac{b(t+\Delta,T)}{b(t,T)}$, can be interpreted as the update to investors' expectations about long-run growth. We then give several examples of structural models where this sufficient condition is met.

A necessary and sufficient condition Suppose the equilibrium price and quantity dynamics of a stochastic growth model are described by the general setup of (6), (7), and (9). We assume that the real SDF admits an eigen pair $\{\bar{r}, \mathbf{e}_S(X)\}$ and the real output process admits an eigen pair $\{\rho_Y, \mathbf{e}_Y(X)\}$. Given the principal eigenfunctions, we can construct martingales $\hat{S}_t = e^{\bar{r}t}\mathbf{e}_S(X_t) S_t$ and $\hat{Y}_t = e^{-\rho_Y t}\mathbf{e}_Y(X_t) Y_t$. Our necessary and sufficient condition is stated in terms of the quadratic covariation of $\hat{S}_t$ and $\hat{Y}_t$.

Proposition 3. *(Deterministic covariance condition)*

Let $\{-\bar{r}, \mathbf{e}_S(X)\}$ and $\{\rho_Y, \mathbf{e}_Y(X)\}$ be the principal eigenvalue-eigenfunction pairs of S_t and Y_t in their long-term factorizations. Let $\hat{S}_t$ and $\hat{Y}_t$ be martingales constructed from (13). A principal eigenfunction of $S_t Y_t$ can be constructed as $\mathbf{e}_{SY}(X,t) = \mathbf{e}_S(X) \times \mathbf{e}_Y(X) F(t)$ for some periodic function $F(t)$ if and only if

$$\frac{d\left\langle \hat{S}, \hat{Y} \right\rangle_t}{\hat{S}_t \hat{Y}_t} = f(t)\, dt, \tag{20}$$

for some $f(t)$. In this case, $F(t) = \bar{F}(t \mod \tau)$, where $\bar{F}(t) = F_0 e^{\lambda t - \int_0^t f(s)ds}$ for $t \in [0,\tau)$, $F_0 > 0$ is an arbitrary multiplicative constant, and λ is given by:

$$\lambda = \frac{1}{\tau}\int_0^\tau f(s)\, ds + \ln \frac{\int \mathbf{e}_S(y,t)\, \mathbf{e}_Y(y,t)\, zQ(x, d(y,z))}{\mathbf{e}_S(x,t^-)\, \mathbf{e}_Y(x,t^-)}.$$

Proof. See Appendix 5.3. □

Suppose the eigenfunction for $S_t Y_t$ is of the form $\mathbf{e}_{SY}(X,t) = \mathbf{e}_S(X,t) \times \mathbf{e}_Y(X,t) F(t)$, then the return on the long-term portfolio over a short FOMC announcement window is:

$$\lim_{T\to\infty} \frac{V(t,T)/b(t,T)}{V(t^-,T)/b(t^-,T)} = \frac{\mathbf{e}_S(X_t,t)\, \mathbf{e}_Y(X_t,t)\, F(t)/\mathbf{e}_S(X_t,t)}{\mathbf{e}_S(X_{t^-},t)\, \mathbf{e}_Y(X_{t^-},t)\, F(t^-)/\mathbf{e}_S(X_{t^-})} = \frac{\mathbf{e}_Y(X_t)}{\mathbf{e}_Y(X_{t^-})}\frac{F(t)}{F(t^-)}.$$

If $F(t)$ is continuous, then the return on the long-term portfolio over a short FOMC announcement window, $\lim_{T\to\infty} \frac{V(t,T)/b(t,T)}{V(t^-,T)/b(t^-,T)}$, is exactly the update to investors' beliefs about the long-run impact of monetary policy, $\lim_{T\to\infty} \frac{E_t[Y_T]}{E_{t^-}[Y_T]}$, by (12). In general, the jump in $F(t)$ may reflect a periodic announcement premium. After subtracting the announcement premium, we can use the return on the long-term portfolio to recover investor belief about the long-run impact of monetary policy.

In what follows, we will provide several examples of equilibrium models where condition (20) holds or approximately holds. Our first example is a model of the macroeconomic announcement premium taken from Ai and Bansal (2018).

Example 1. (A log-linear example)

Section 5 of Ai and Bansal (2018) presents a continuous-time model with macroeconomic announcements and recursive preferences. The aggregate endowment follows

$$\frac{dC_t}{C_t} = x_t dt + \sigma dB_{C,t}, \tag{21}$$

where x_t is an Ornstein–Uhlenbeck process of the form:

$$dx_t = a\left(\bar{x} - x_t\right) dt + \sigma_x dB_{x,t}, \tag{22}$$

and $B_{C,t}$ and $B_{x,t}$ are independent standard Brownian motions. The true value of x_t is not observable, but investors can learn about x_t through two sources. First, investors can update their beliefs about x_t through their observations of C_t. Second, pre-scheduled FOMC announcements at $t = 0, T, 2T, \cdots$ fully reveals the true value of x_{nT} at the time of the announcements, for $n = 0, 1, 2, \cdots$. Investors update their beliefs about x_t based on the observed realizations of aggregate endowment and the FOMC announcements using Bayes' rule. Because announcements fully reveal x_t at nT, the mean and variance of the posterior distribution of x_t, denoted $(\hat{x}_t, q_t)$, are set to $(x_t, 0)$ at $t = 0, T, 2T, \cdots$. After announcements, because x_t follows (22), $\hat{x}_t$ and x_t diverge, and the law of motion of $(\hat{x}_t, q_t)$ follows a Kalman filter, which we describe in Equations (40) and (41) in Appendix 5.4.

The representative investor has a recursive preference with risk aversion γ and a unit IES. Using the results from Ai and Bansal (2018), for $t \in (nT, (n+1)T)$, the SDF takes the form of

$$dS_t = S_t\left[-r\left(\hat{x}_t, t\right) dt - \sigma_S\left(t\right) d\hat{B}_t\right],$$

where $r\left(\hat{x}, t\right) = \beta + \hat{x} - \gamma\sigma^2 - \frac{\gamma-1}{a+\beta} q\left(t\right)$, and $\sigma_S\left(t\right) = \gamma\sigma + \frac{\gamma-1}{a+\beta}\frac{q(t)}{\sigma}$. On the boundary,

$$\frac{S_{nT}}{S_{nT}^-} = e^{-\frac{1}{2}\left(\frac{\gamma-1}{a+\beta}\right)^2 q\left(nT^-\right) - \frac{\gamma-1}{a+\beta}\left(\hat{x}_{nT} - \hat{x}_{nT}^-\right)}.$$

In the above setup, we consider a levered claim on aggregate consumption $Y_t = C_t^b$ for some $b > 0$. In this example, the SDF is discontinuous at announcement times and generates a non-trivial announcement premium. In Appendix 5.4, we show that this example satisfies a more general version of condition (20) that includes jumps in the stochastic discount factor. We also show in the same appendix that the three eigenfunctions take the exponential linear form of:

$$\mathbf{e}_S\left(\hat{x}, t\right) = e^{-\frac{1}{a_x}\hat{x} + g_B(t)};\ \mathbf{e}_Y\left(\hat{x}, t\right) = e^{\frac{b}{a_x}\hat{x} + g_Y(t)};\ \mathbf{e}_{SY}\left(\hat{x}, t\right) = e^{\frac{b-1}{a_x}\hat{x} + g_{SY}(t)}.$$

As a result, the return on the long-term portfolio over an announcement window is given by:

$$\frac{\mathbf{e}_{SY}\left(\hat{x}_t, t\right)/\mathbf{e}_S\left(\hat{x}_t, t\right)}{\mathbf{e}_{SY}\left(\hat{x}_{t^-}, t^-\right)/\mathbf{e}_S\left(\hat{x}_{t^-}, t^-\right)} = e^{\frac{b}{a}\left(x_t - \hat{x}_t^-\right) - \frac{1}{2}\left(\frac{b}{a}\right)^2 q_t} e^{\frac{b}{a}\left(\frac{1}{a} + \frac{\gamma-1}{a+\beta}\right) q_t}, \tag{23}$$

at $t = 0, T, 2T, \cdots$. The first component in (23) is a martingale component and equals the update to expected long-run economic growth:

$$\frac{\mathbf{e}_Y(\hat{x}_t, t)}{\mathbf{e}_Y(\hat{x}_{t^-}, t^-)} = e^{\frac{b}{a}\left(x_t - \hat{x}_t^-\right) - \frac{1}{2}\left(\frac{b}{a}\right)^2 q_t}.$$

The second component, $e^{\frac{b}{a}\left(\frac{1}{a} + \frac{\gamma - 1}{a+\beta}\right) q_t}$ is the announcement premium for the long-term equity. In this example, the update of belief about long-run economic growth can be recovered from the return on the long-term portfolio by subtracting an announcement premium component, which is predictable using pre-announcement information.

Example 2. (Constant nominal rate)

Our second example makes the following assumptions on the joint dynamics of the SDF and the state variable X_t.

ASSUMPTION 1: Constant real interest rate: $r(X_t, t) = r$, where r is constant and $\sigma_S(X_t)^T \sigma_j(X_t) = 0$ for all j.

The above assumption requires that changes in the monetary policy relevant state variable X_t have no impact on either the risk-free interest rate component or the Brownian motion component of the SDF. Our second assumption requires the covariance between the growth rate and the SDF to be a deterministic function of time.

ASSUMPTION 2: $\sigma_S(X_t)^T \sigma_Y(X_t) = -f(t)$ is deterministic.

Under the above assumptions, the eigenfunction of the SDF, $\mathbf{e}_S(X_t, t)$, is a constant. The covariance condition (20) becomes:

$$\frac{d\left\langle \hat{S}, \hat{Y} \right\rangle_t}{\hat{S}_t \hat{Y}_t} = -\left[\sigma_S^T(X_t, t)\, \sigma_Y(X_t, t) + \frac{D_X \mathbf{e}_Y'(X_t, t)}{\mathbf{e}_Y(X_t, t)} \sigma_S^T(X_t, t)\, \Sigma_X(X, t)\right] = -f(t)$$

is satisfied.

In this example, the monetary policy state variable X_t is allowed to affect output dynamics in arbitrary ways. However, it does not have an impact on the stochastic discount factor in the sense that i) the real interest rate is constant and not affected by X_t; ii) the risk premium on output growth, $\sigma_S(X_t)^T \sigma_Y(X_t)$, is deterministic and therefore not affected by X_t. In addition, the above example also requires that the monetary policy state variable X_t is not correlated with the stochastic discount factor in the sense that $\sigma_S(X_t)^T \Sigma(X_t) = 0$.

Essentially, under the above conditions, the return on the long-term portfolio, $\lim_{T \to \infty} \frac{V(t,T)/b(t,T)}{V(t^-,T)/b(t^-,T)}$, fully recovers investors' belief about long-run growth because monetary policy has very limited impact on the long-run risk-neutral probability in a way that expected changes in the long-run risk-neutral probability coincides with expected changes in investor belief. The assumption that monetary policy does not impact the real interest rate is perhaps uninteresting given that it often does through general equilibrium mechanisms in models where it has a non-trivial real impact. The section below provides a class of heterogenous agent models where monetary policy is allowed to

have an impact on the SDF and yet $\lim_{T\to\infty}\frac{V(t,T)/b(t,T)}{V(t^-,T)/b(t^-,T)}$ fully recovers investors' belief about long-run growth.

Heterogenous agents In this section, we show that the conclusion of Proposition 3 is robust to environments with heterogeneous agents under the assumption of expected utility. In representative agent models, individual consumption is the same as aggregate consumption, and the stochastic discount factor processes constructed from all agents in the economy are identical. While heterogeneous agent models may differ in the details of the economic mechanism of heterogeneity, such as limited stock market participation (Basak and Cuoco (1998)), financial intermediation (He and Krishnamurthy (2013)), and dynamic agency such as limited commitment (Ai and Bhandari (2021); Alvarez and Jermann (2000)), the common feature of the above models is that the stochastic discount implied by the marginal investors who actively participate in trading and the stochastic discount factor implied by a hypothetical investor who consumes the aggregate consumption differ. In what follows, we present an irrelevance result: we show that heterogeneity does not affect the applicability of Proposition 3 as long as the consumption share of the marginal investors as a fraction of aggregate consumption is stationary under the assumption of CRRA utility.

We start with an output process Y_t that has a long-term factorization with $\{\rho_Y, \mathbf{e}_Y(X,t)\}$ as the eigenvalue and eigenfunction. We assume that all agents in the economy have CRRA utility of the form $E\left[\int e^{-\beta t}\frac{1}{1-\gamma}C_t^{1-\gamma}dt\right]$, where γ is the risk aversion and β is the discount rate. We assume that the SDF implied by the aggregate consumption, $S_t = e^{-\beta t}C_t^{1-\gamma}$, has a long-term factorization with eigenvalue $-\bar{r}$ and eigenfunction $\mathbf{e}_S(X,t)$. We also assume that the SDF implied by the average investor, S_t, satisfy the conditions in Proposition 3, but allow the consumption share of the marginal investors to depend on the Markov state variable X_t through an arbitrary function $\lambda(X_t,t)$. We state all formal results as follows.

Proposition 4. *(Irrelevance of heterogenous agents)*

Let S_t be the SDF implied by aggregate consumption and assume that $\{S_t, Y_t\}$ jointly satisfy Condition (20) of Proposition 3 so that

$$\mathbf{e}_{SY}(X,t) = \mathbf{e}_S(X) \times \mathbf{e}_Y(X) F(t) \tag{24}$$

for some periodic function $F(t)$. Let $\lambda(X_t,t)$ be the consumption share of the marginal investors. The SDF implied by the marginal investor's marginal rate of substitution also satisfies (24).

The above proposition can be phrased intuitively: heterogeneity does not impact the permanent component of SDF under expected utility. In heterogeneous agent models, the consumption of the marginal investor differs from that of the average or aggregate investor. Heterogeneity can therefore be an important mechanism that impacts the volatility of SDF as shown in the above literature. However, under expected utility, heterogeneity will not impact permanent component of the SDF, and therefore, it will not impact the validity of using the return on the long-term portfolio to measure expected long-run growth.

4 Conclusion

This paper proposes an asset-pricing test of the independence hypothesis and a quantitative measure of the long-run impact of monetary policy. Building on the operator approach of Hansen and Scheinkman (2009), we show that the difference between the log returns on long-term equity and the long-term bond over a short FOMC announcement window differences out the common discount-rate component and identifies the long-run impact of the announcement under the long-run risk-neutral measure. Under the independence hypothesis, these two returns are perfectly correlated. We find that the data strongly reject this hypothesis. We further provide conditions for the measure to coincide with the revision in expected long-run growth under investors' subjective beliefs. Applying the measure, we find a quantitatively large long-run impact of monetary policy: a 25-basis-point expansionary monetary policy shock and a 25-basis-point central bank information shock—identified following Jarociński and Karadi (2020)—are associated, on average, with roughly 100-basis-point and 250-basis-point upward revisions in expected long-run economic growth under the long-run risk-neutral measure, respectively. Under the assumption of rational expectations, we decompose the measure into long-run cash-flow-expectation and long-run risk-premium components and find that each accounts for about half the variance of the measure.

5 Appendix

5.1 Long-term factorization

In this section, we first provide a general result for multiplicative functionals that admit a long-term factorization. We first state two lemmas, one from Hairer and Mattingly (2011) and the other from Friedman (1975), which we will use to prove our main result.

Hairer and Mattingly (2011) consider a discrete-time Markov process. They start with $(\mathbb{X}, \mathcal{B})$, a measurable space with P, a Markov transition kernel defined on it. The convention is that P induces an operator on the space of functions, $(P\varphi)(x) = \int \varphi(y) P(x, dy)$, and an operator on the space of measures by $(\mu P)(A) = \int P(x, A) \mu(dx)$. Hairer and Mattingly (2011) make two assumptions in order to establish their contraction result. The contraction result is stated in their Theorem 1.3; the existence of the invariant measure and its moment bound are in their Theorem 1.2 and Section 3.1.

ASSUMPTION 1 (geometric drift): There exist a function $V : \mathbb{X} \to [0, \infty)$ and constants $\gamma \in (0, 1)$ and $K \geq 0$ such that

$$(PV)(x) \leq \gamma V(x) + K \qquad \text{for all } x \in \mathbb{X}. \tag{25}$$

ASSUMPTION 2 (minorization on a level set): There exist $\alpha \in (0, 1)$, a probability measure ν, and a level $R > \frac{2K}{1-\gamma}$ such that

$$P(x, \cdot) \geq \alpha \nu(\cdot) \qquad \text{for all } x \text{ with } V(x) \leq R. \tag{26}$$

Lemma 1. *(Hairer and Mattingly (2011), Theorem 1.3) For $\zeta > 0$, define the weighted total variation distance*

$$\rho_\zeta(\mu_1, \mu_2) = \int (1 + \zeta V(x)) |\mu_1 - \mu_2|(dx) = \sup_{|\varphi| \leq 1 + \zeta V} \left| \int \varphi d\mu_1 - \int \varphi d\mu_2 \right|. \tag{27}$$

Suppose Assumptions 1 and 2 hold. Then, for any $\alpha_0 \in (0, \alpha)$ and $\gamma_0 \in \left(\gamma + \frac{2K}{R}, 1\right)$, set

$$\zeta = \frac{\alpha_0}{K}, \qquad \bar{\alpha} = \max\left\{1 - (\alpha - \alpha_0), \frac{2 + R\zeta\gamma_0}{2 + R\zeta}\right\} \in (0, 1),$$

the kernel is a strict contraction:

$$\rho_\zeta(\mu_1 P, \mu_2 P) \leq \bar{\alpha} \rho_\zeta(\mu_1, \mu_2) \qquad \text{for all probability measures } \mu_1, \mu_2 \text{ with } \int V d\mu_i < \infty. \tag{28}$$

Consequently, since the space of such measures is complete under ρ_ζ, the following hold: P has a unique invariant probability measure π; $\int V d\pi \leq \frac{K}{1-\gamma}$; and $\rho_\zeta(\mu P^n, \pi) \leq \bar{\alpha}^n \rho_\zeta(\mu, \pi)$, that is, geometric convergence in V-weighted total variation with completely explicit constants.

Assumption 2 above requires a verification of minorization on a level set. In the context of our problem, we use the following result. Part i) of the lemma below follows from the classical parabolic

estimates in Friedman (1975); part ii) follows from part i) by a compactness argument.

Lemma 2. *(Friedman (1975)) Let X_t be a diffusion on $\mathbb{X} \subseteq \mathbb{R}^d$ of the form*

$$dX_t = \mu_X(X_t, t)\, dt + \Sigma_X^T(X_t, t)\, dB_t, \qquad t \in \left[0, \bar{T}\right],$$

where $\mu_X(x,t)$ and $\Sigma_X(x,t)$ are locally Lipschitz on $\mathbb{X} \times \left[0, \bar{T}\right]$, $\Sigma_X^T(x,t)\,\Sigma_X(x,t)$ is positive definite for all (x,t), and X_t neither explodes nor exits $\mathbb{X}$. Then: i) for all $0 \leq s < t \leq \bar{T}$, $P_{s,t}(x, dy) = p_{s,t}(x,y)\, dy$, where the density $(s,t,x,y) \mapsto p_{s,t}(x,y)$ is jointly continuous and strictly positive; and ii) for every compact set $C \subset \mathbb{X}$ and every $t_0 > 0$, setting $\kappa = \min\{p_{s,t}(x,y) : t - s \geq t_0,\ x, y \in C\} > 0$, $\alpha = \kappa \mathrm{Leb}(C)$, and ν the uniform distribution on C, we have

$$P_{s,t}(x, A) \geq \alpha \nu(A) \qquad \text{for all } x \in C, \text{ all } (s,t) \text{ with } t - s \geq t_0, \text{ and all Borel sets } A.$$

We are now ready to state our main proposition, which provides a sufficient condition for the convergence results for the pricing of long-term assets, such as (14) and (16).

Proposition 5. *Let $\tau > 0$ and let X_t be a right-continuous process under probability $\hat{P}$ such that, on each interval $(n\tau, (n+1)\tau)$, $n = 0, 1, 2, \cdots$, X_t is a diffusion process on $\mathbb{X}$ of the form (6); and at each announcement time $n\tau$, for $n = 1, 2, \cdots$, the transition probability of $X_{n\tau}$ given $X_{n\tau^-} = y$ is given by a Markov kernel $Q(y, dx)$ that does not depend on n. Assume the following conditions hold.*

1) The coefficients $\mu_X(x,t)$ and $\Sigma_X(x,t)$ are τ-periodic in the sense that $\mu_X(x,t) = \mu_X(x,s)$ and $\Sigma_X(x,t) = \Sigma_X(x,s)$ if $t = s \mod \tau$ and are locally Lipschitz on $\mathbb{X} \times [0, \infty)$ with Lipschitz constants uniform in t. In addition, $\Sigma_X^T(x,t)\,\Sigma_X(x,t)$ is positive definite for all $x \in \mathbb{X}$ and all t.

2) There exists a $V \in C^2(\mathbb{X})$ such that i) $V \geq 1$ and $V(x) \to \infty$ as $x \to \partial\mathbb{X}$ or $\|x\| \to \infty$; and ii) there exist constants $b, \beta > 0$ such that $\mathcal{L}_t V(x) \leq -\beta V(x) + b$ for all $x \in \mathbb{X}$ and all t.

3) There exist constants $1 \leq \kappa < e^{\beta\tau}$ and $d \geq 0$ such that for all $y \in \mathbb{X}$,

$$\int V(x)\, Q(y, dx) \leq \kappa V(y) + d.$$

Then there is a τ-periodic family of distributions $\{\pi_t\}_{t \in [0,\tau)}$ such that the distribution of X_T converges to $\pi_{T \mod \tau}$ as $T \to \infty$ and there exist constants $C, \delta > 0$ such that for all f with $|f(x,t)| \leq V(x)$ for all t,

$$\left| \hat{E}\left[f(X_T, T) \middle| X_t = x \right] - \int f(y, T)\, \pi_{T \mod \tau}(dy) \right| \leq C V(x)\, e^{-\delta(T-t)}. \tag{29}$$

Moreover, the family satisfies $\pi_s = \pi_0 P_{0,s}$ for $s \in (0, \tau)$, where $P_{0,s}$ denotes the transition kernel of the diffusion (6) over $(0,s)$; and the one-sided limit defined as $\pi_{\tau^-} = \lim_{s \nearrow \tau} \pi_s$ satisfies $\pi_0 = \pi_{\tau^-} Q$. In addition, $\sup_{s \in [0,\tau)} \int V(y)\, \pi_s(dy) \leq \frac{\kappa \frac{b}{\beta} + d}{1 - \kappa e^{-\beta\tau}}$.

Proof. Let $P_{s,t}(x, dy)$, for $n\tau \leq s \leq t \leq (n+1)\tau$ and $n = 0, 1, 2, \cdots$, be the transition prob-

ability of the diffusion process (6) with no announcement in between. By the τ-periodicity of the coefficients, it does not depend on n. This allows us to construct a skeleton Markov chain, $\{X_t : t = n\tau + s, n = 0, 1, 2, \cdots\}$ for all $s \in [0, \tau)$, whose one-period transition kernel is

$$\tilde{Q}_s = P_{s,\tau} Q P_{0,s}, \qquad \tilde{Q}_0 = P_{0,\tau} Q.$$

Because the coefficients are τ-periodic and Q does not depend on n, $\tilde{Q}_s$ is a time-homogeneous Markov kernel.

We first establish that $\tilde{Q}_s$ is a contraction on the space of probability measures on $\mathbb{X}$ by verifying the two assumptions in Hairer and Mattingly (2011). The first assumption is the geometric drift condition (25). Condition 2ii), together with Itô's formula and Grönwall's inequality, implies that, over any time interval that contains no announcement,

$$\hat{E}\left[V\left(X_T\right) \middle| X_t = x\right] \leq e^{-\beta(T-t)} V\left(x\right) + \frac{b}{\beta}\left(1 - e^{-\beta(T-t)}\right). \tag{30}$$

Composing the three segments of $\tilde{Q}_s$ (the diffusion of length $\tau - s$, the jump, and the diffusion of length s) and using condition 3) together with $\kappa \geq 1$, we obtain, for every s,

$$\tilde{Q}_s V \leq \kappa e^{-\beta\tau} V + K, \qquad \textit{where } K := \kappa \frac{b}{\beta}\left(1 - e^{-\beta\tau}\right) + d.$$

This verifies (25) with $\gamma = \kappa e^{-\beta\tau}$, which is strictly below one by condition 3). Neither γ nor K in the above equation depends on s.

Second, to verify the minorization condition (26), choose an $R > \max\left\{\frac{2K}{1-\gamma}, 1\right\}$ (as required by Assumption 2); we need to verify that (26) holds on the sublevel set $\{x : V\left(x\right) \leq R\}$. Because $V \geq 1$ and $V\left(x\right) \to \infty$ as $x \to \partial\mathbb{X}$ or $\|x\| \to \infty$, $\{x : V\left(x\right) \leq R\}$ is compact. It is also non-empty: if $V\left(x\right) > R$ for all x, then $\hat{E}\left[V\left(X_{s+n\tau}\right) \middle| X_s = x\right] > R$ for all n, while iterating the drift bound above implies $\hat{E}\left[V\left(X_{s+n\tau}\right) \middle| X_s = x\right] \leq \gamma^n V\left(x\right) + K/\left(1-\gamma\right) < R$ for n sufficiently large, a contradiction. Under the locally Lipschitz and elliptic condition 1), every kernel $\tilde{Q}_s$ contains a diffusion segment of duration at least $\tau/2$: the segment $P_{s,\tau}$ if $s \leq \tau/2$, and the segment $P_{0,s}$ if $s > \tau/2$. Theorem 2 applies to that segment, and composing with the remaining kernels preserves the minorization;[6] hence (26) holds on $\{x : V\left(x\right) \leq R\}$, and the constants α and ν can be chosen independent of s. As a result, there is a unique $\tilde{Q}_s$-invariant distribution π_s such that the skeleton chain converges to it in the ρ_ζ-norm defined in Theorem 1, that is, for any measure μ on $\mathbb{X}$, $\rho_\zeta\left(\mu\tilde{Q}_s^n, \pi_s\right) \leq \bar{\alpha}^n \rho_\zeta\left(\mu, \pi_s\right)$ with $\bar{\alpha} \in (0,1)$ independent of s.[7]

[6] If a kernel M satisfies $M\left(x, \cdot\right) \geq \alpha\nu\left(\cdot\right)$ for all x in a set C, then for any Markov kernel N, $MN\left(x, \cdot\right) \geq \alpha\left(\nu N\right)\left(\cdot\right)$: a minorization of the leading segment survives composition with later kernels. This settles the case $s \leq \tau/2$ with $M = P_{s,\tau}$ and $N = QP_{0,s}$. For $s > \tau/2$, first note that starting from any x with $V\left(x\right) \leq R$, the leading part $P_{s,\tau}Q$ lands in $C' = \{V \leq R'\}$ with $R' = 2\left[\kappa\left(R + b/\beta\right) + d\right]$ with probability at least $1/2$ by Markov's inequality; applying Theorem 2 to the trailing segment $P_{0,s}$ on C' then yields the minorization with the extra factor $1/2$. In both cases the constants can be chosen uniformly over the phase because the transition densities are jointly continuous in the phase and the duration.

[7] $\bar{\alpha}$ can be chosen independent of s because K, α_0, γ_0, R and α in HM can all be chosen independent of s.

Next, we verify the two identities in the conclusion. For $s \in (0,\tau)$, the measure $\pi_0 P_{0,s}$ is $\tilde{Q}_s$-invariant. To see this, note that the Chapman-Kolmogorov equation implies $P_{0,s}\tilde{Q}_s = P_{0,s}P_{s,\tau}QP_{0,s} = P_{0,\tau}QP_{0,s} = \tilde{Q}_0 P_{0,s}$, and therefore $(\pi_0 P_{0,s})\,\tilde{Q}_s = \left(\pi_0\tilde{Q}_0\right)P_{0,s} = \pi_0 P_{0,s}$. Since $\pi_0 P_{0,s}$ has a finite V-moment by (30), the uniqueness of the $\tilde{Q}_s$-invariant distribution implies $\pi_s = \pi_0 P_{0,s}$. Letting $s \nearrow \tau$ and using the continuity of the diffusion transition kernel in the length of the time interval, the one-sided limit is $\pi_{\tau^-} = \pi_0 P_{0,\tau}$; consequently, $\pi_{\tau^-}Q = \pi_0 P_{0,\tau}Q = \pi_0\tilde{Q}_0 = \pi_0$. Finally, invariance and the drift bound imply $\int V(y)\,\pi_s(dy) = \int \tilde{Q}_s V(y)\,\pi_s(dy) \leq \gamma\int V(y)\,\pi_s(dy) + K$, so that $\int V(y)\,\pi_s(dy) \leq \frac{K}{1-\gamma} \leq \frac{\kappa\frac{b}{\beta}+d}{1-\kappa e^{-\beta\tau}}$ for every s.

Finally, to establish (29), we write $T - t = n\tau + r$ with $r \in [0,\tau)$ and let $\bar{P}_{t,t+r}$ denote the transition kernel of X from t to $t+r$, inclusive of any announcement in between, which implies $\hat{E}\left[f(X_T,T)\middle|X_t = x\right] = \int f d\left[\delta_x\bar{P}_{t,t+r}\tilde{Q}^n_{t+r}\right]$. Since $T \mod \tau = (t+r) \mod \tau$, the distribution $\pi_{T \mod \tau}$ is $\tilde{Q}_{t+r}$-invariant, so that $\pi_{T \mod \tau}\tilde{Q}^n_{t+r} = \pi_{T \mod \tau}$. We now use the contraction property, $\rho_\zeta\left(\delta_x\bar{P}_{t,t+r}\tilde{Q}^n_{t+r}, \pi_{T \mod \tau}\tilde{Q}^n_{t+r}\right) \leq \bar{\alpha}^n\rho_\zeta\left(\delta_x\bar{P}_{t,t+r}, \pi_{T \mod \tau}\right)$, to compute:

$$\begin{aligned}
\zeta\left|\int f(y,T)\,d\left[\left(\delta_x\bar{P}_{t,t+r}\right)\tilde{Q}^n_{t+r}\right](dy) - \int f(y,T)\,\pi_{T \mod \tau}(dy)\right| &\leq \zeta\int V(y)\left|\left[\delta_x\bar{P}_{t,t+r}\tilde{Q}^n_{t+r}\right] - \pi_{T \mod \tau}\right|(dy) \\
&\leq \bar{\alpha}^n\left\{\int(1+\zeta)\,V(y)\left|\delta_x\bar{P}_{t,t+r} - \pi_{T \mod \tau}\right|(dy)\right\} \\
&\leq \bar{\alpha}^n(1+\zeta)\left\{\int V(y)\,\delta_x\bar{P}_{t,t+r}(dy) + \int V(y)\,\pi_{T \mod}\right.
\end{aligned}$$

where the first inequality uses $|f(x,t)| \leq V(x)$, the second uses the contraction property and the fact that $1 \leq V$, and the last inequality is due to $V \geq 0$. The interval $[t, t+r]$ contains at most one announcement, so composing (30) with condition 3) gives

$$\int V(y)\,\delta_x\bar{P}_{t,t+r}(dy) \leq \kappa V(x) + (1+\kappa)\frac{b}{\beta} + d \leq \left[\kappa + (1+\kappa)\frac{b}{\beta} + d\right]V(x),$$

where the last inequality uses $V \geq 1$. In addition, $\int V(y)\,\pi_{T \mod \tau}(dy) \leq \frac{\kappa\frac{b}{\beta}+d}{1-\kappa e^{-\beta\tau}} \leq \frac{\kappa\frac{b}{\beta}+d}{1-\kappa e^{-\beta\tau}}V(x)$. Therefore, (29) holds with $C = \frac{1+\zeta}{\zeta}\bar{\alpha}^{-1}\left[\kappa + (1+\kappa)\frac{b}{\beta} + d + \frac{\kappa\frac{b}{\beta}+d}{1-\kappa e^{-\beta\tau}}\right]$ and $\delta = \tau^{-1}\ln(1/\bar{\alpha})$.[8] □

The above proposition motivates the definition of Lyapunov functions.

Definition 3. (Lyapunov function)

Let $\tau > 0$ and let X_t be a right-continuous process under probability $\hat{P}$ such that on each interval $(n\tau,(n+1)\tau)$, $n = 0,1,2,\cdots$, X_t is a diffusion process on $\mathbb{X}$ of the form (6); and at each announcement time $n\tau$, for $n = 1,2,\cdots$, the transition probability of $X_{n\tau}$ given $X_{n\tau^-} = y$ is given by a Markov kernel $Q(y,dx)$ that does not depend on n. $V \in C^2(\mathbb{X})$ is said to be a Lyapunov function under probability $\hat{P}$ with discounting coefficient (β,κ) and bound (b,d) if there

[8] To translate the discrete time discounting to continuous time, we use $\bar{\alpha}^n \leq \bar{\alpha}^{\frac{T-t}{\tau}-1} = \bar{\alpha}^{-1}e^{\frac{(T-t)}{\tau}\ln\bar{\alpha}} = \bar{\alpha}^{-1}e^{-\delta(T-t)}$.

exist constants $\{\beta, \kappa, b, d\}$ such that conditions 2) and 3) in Proposition 5 hold.

5.2 Proof of Propositions 1 and 2

Proof of 1 First, because S_t admits a long-term factorization, the time-t price of a long-term bond can be computed as (15). Consider the price of zero-coupon equity with maturity T,

$$P_t E\left[\frac{S_T Y_T}{S_t}\psi\left(X_T\right)\middle| X_t\right] = P_t Y_t \left\{E\left[e^{-\bar{r}(T-t)}\frac{\hat{S}_T}{\hat{S}_t}\frac{\mathbf{y}\left(X_T,T\right)}{\mathbf{y}\left(X_t,t\right)}\frac{\mathbf{e}_S\left(X_t,t\right)}{\mathbf{e}_S\left(X_T,T\right)}\psi\left(X_T\right)\middle| X_t\right] E\left[\frac{Y_T^G}{Y_t^G}\middle| X_t\right]\right\}.$$

The term $E\left[e^{-\bar{r}(T-t)}\frac{\hat{S}_T}{\hat{S}_t}\frac{\mathbf{y}(X_T,T)}{\mathbf{y}(X_t,t)}\frac{\mathbf{e}_S(X_t,t)}{\mathbf{e}_S(X_T,T)}\psi\left(X_T\right)\middle| X_t\right]$ can be computed as:

$$E\left[e^{-\bar{r}(T-t)}\frac{\hat{S}_T}{\hat{S}_t}\frac{\mathbf{y}\left(X_T,T\right)}{\mathbf{y}\left(X_t,t\right)}\frac{\mathbf{e}_S\left(X_t,t\right)}{\mathbf{e}_S\left(X_T,T\right)}\psi\left(X_T\right)\middle| X_t\right] = e^{-\bar{r}(T-t)}\frac{\mathbf{e}_S\left(X_t,t\right)}{\mathbf{y}\left(X_t,t\right)}\hat{E}^S\left[\frac{\mathbf{y}\left(X_T,T\right)\psi\left(X_T\right)}{\mathbf{e}_S\left(X_T,T\right)}\middle| X_t\right], \tag{31}$$

where $\hat{E}^S$ denotes expectation under the probability induced by $\hat{S}_t$.

Under our assumption, $\frac{\mathbf{y}(X_T,T)}{\mathbf{e}_S(X_T,T)}$ is dominated by a Lyapunov function, and $\psi\left(X_T\right)$ is bounded. By Proposition 5, $\hat{E}\left[\frac{\mathbf{y}(X_T,T)\psi(X_T)}{\mathbf{e}_S(X_T,T)}\middle| X_t = x'\right] \to \int \frac{\mathbf{y}(y,T)\psi(y)}{\mathbf{e}_S(y,T)}\pi_T\left(dy\right)$, which does not depend on x, and the convergence is uniform on a compact subset of $\mathbb{X}$. The same applies to $\hat{E}\left[\frac{\mathbf{y}(X_T,T)\psi(X_T)}{\mathbf{e}_S(X_T,T)}\middle| X_{t^-} = x\right]$.

Now consider the term

$$\begin{aligned} E\left[\frac{Y_T^G}{Y_t^G}\middle| X_t\right] &= E\left[\frac{\mathbf{y}\left(X_t,t\right)}{\mathbf{y}\left(X_T,T\right)}\frac{Y_T}{Y_t}\middle| X_t\right] \\ &= e^{\rho_Y(T-t)}E\left[\frac{\mathbf{y}\left(X_t,t\right)}{\mathbf{y}\left(X_T,T\right)}\frac{\mathbf{e}_Y\left(X_t,t\right)}{\mathbf{e}_Y\left(X_T,T\right)}\frac{\hat{Y}_T}{\hat{Y}_t}\middle| X_t\right] \\ &= \mathbf{y}\left(X_t,t\right)\mathbf{e}_Y\left(X_t,t\right)\hat{E}^Y\left[\frac{1}{\mathbf{y}\left(X_T,T\right)\mathbf{e}_Y\left(X_T,T\right)}\middle| X_t\right]. \end{aligned} \tag{32}$$

Because $\frac{1}{\mathbf{y}(X_T,T)\mathbf{e}_Y(X_T,T)}$ is dominated by a Lyapunov function under $\hat{E}^Y$, we have $\hat{E}^Y\left[\frac{1}{\mathbf{y}(X_T,T)\mathbf{e}_Y(X_T,T)}\middle| X_t = x'\right] \to \int \frac{\mathbf{y}(y,T)\psi(y)}{\mathbf{e}_S(y,T)}\pi_T\left(dy\right)$, which does not depend on x', and the convergence is uniform on a compact subset of $\mathbb{X}$. The same applies to $E\left[\frac{Y_T^G}{Y_t^G}\middle| X_{t^-} = x\right]$.

Summarizing (31) and (32), we can write the zero-coupon equity price before and after an announcement as:

$$\lim_{T\to\infty} P_t E\left[\frac{S_T Y_T}{S_t}\psi\left(X_T\right)\middle| X_t = x'\right] = P_t Y_t e^{(\rho_Y-\bar{r})(T-t)}\mathbf{e}_S\left(x',t\right)\mathbf{e}_Y\left(x',t\right)\int\frac{\mathbf{y}\left(y,T\right)\psi\left(y\right)}{\mathbf{e}_S\left(y,T\right)}\pi_T\left(dy\right)\times\int\frac{\mathbf{y}\left(y,T\right)\psi\left(y\right)}{\mathbf{e}_S\left(y,T\right)}\pi$$

and

$$\lim_{T\to\infty} P_t E\left[\frac{S_T Y_T}{S_t}\psi\left(X_T\right)\middle| X_{t^-} = x\right] = P_t Y_t e^{(\rho_Y-\bar{r})(T-t)}\mathbf{e}_S\left(x,t^-\right)\mathbf{e}_Y\left(x,t^-\right)\int\frac{\mathbf{y}\left(y,T\right)\psi\left(y\right)}{\mathbf{e}_S\left(y,T\right)}\pi_T\left(dy\right)\times\int\frac{\mathbf{y}\left(y,T\right)\psi\left(y\right)}{\mathbf{e}_S\left(y,T\right)}$$

where due to the continuity assumption on P_t and Y_t, they are the same before and after announce-

ments. As as result,

$$\lim_{T\to\infty}\frac{P_tE\left[\frac{S_TY_T}{S_t}\psi\left(X_T\right)\middle|X_t=x'\right]}{P_tE\left[\frac{S_TY_T}{S_t}\psi\left(X_T\right)\middle|X_{t^-}=x\right]}=\frac{\mathbf{e}_S\left(x',t\right)\mathbf{e}_Y\left(x',t\right)}{\mathbf{e}_S\left(x,t\right)\mathbf{e}_Y\left(x,t\right)}.$$

Proof of 2 First, under the assumption that S_t has a long-term factorization, the announcement return on long-term bonds is given by Equation (14). In addition, because S_tY_t admits a long-term factorization, the price of a zero-coupon equity satisfies (16). As a result,

$$\lim_{T\to\infty}\frac{e^{-\rho_{SY}(T-t)}P_tE\left[\frac{S_TY_T}{S_t}\psi\left(X_T\right)\middle|X_t\right]}{e^{\bar{r}(T-t)}b\left(t,T|\,X_t\right)}=\frac{\mathbf{e}_{SY}\left(X_t,t\right)Y_t\int\frac{\psi(y)}{\mathbf{e}_{SY}(y,T)}\pi_{T\mod\tau}\left(dy\right)}{\mathbf{e}_S\left(X_t,t\right)\int\frac{1}{\mathbf{e}_S(y,T)}\pi_{T\mod\tau}\left(dy\right)}.$$

Because Y_t is continuous, and $\int\frac{1}{\mathbf{e}_S(y,T)}\pi_{T\mod\tau}\left(dy\right)$ and $\int\frac{\psi(y)}{\mathbf{e}_{SY}(y,T)}\pi_{T\mod\tau}\left(dy\right)$ do not depend on X_t, we can write the return on the long-term portfolio over an FOMC announcement window as:

$$\lim_{T\to\infty}\frac{\frac{V(t,T|X_t)}{b(t,T|X_t)}}{\frac{V\left(t^-,T|X_{t^-}\right)}{b\left(t^-,T|X_{t^-}\right)}}=\frac{\frac{\mathbf{e}_{SY}(X_t,t)}{\mathbf{e}_S(X_t,t)}}{\frac{\mathbf{e}_{SY}\left(X_{t^-},t^-\right)}{\mathbf{e}_S\left(X_{t^-},t^-\right)}}.\tag{33}$$

Next, the expected value of output at time T under the long-run risk-neutral measure, $\hat{E}^S\left[Y_T\right]$, can be computed as:

$$E\left[\frac{\hat{S}_T}{\hat{S}_t}Y_T\middle|X_t\right]=e^{\bar{r}(T-t)}E\left[\frac{\mathbf{e}_S\left(X_T,T\right)}{\mathbf{e}_S\left(X_t,t\right)}\frac{S_T}{S_t}Y_T\middle|X_t\right]=e^{(\rho_{SY}+\bar{r})(T-t)}Y_tE\left[\frac{\mathbf{e}_S\left(X_T,T\right)}{\mathbf{e}_S\left(X_t,t\right)}\frac{\widehat{SY}_T}{\widehat{SY}_t}\frac{\mathbf{e}_{SY}\left(X_t,t\right)}{\mathbf{e}_{SY}\left(X_T,T\right)}\middle|X_t\right],$$

where the second equality uses the multiplicative factorization of S_tY_t. Because $\frac{\mathbf{e}_S(X_T,T)}{\mathbf{e}_{SY}(X_T,T)}$ is dominated by a Lyapunov function,

$$e^{-(\rho_{SY}+\bar{r})(T-t)}\hat{E}^S\left[Y_T|\,X_t=x\right]=\frac{\mathbf{e}_{SY}\left(x,t\right)}{\mathbf{e}_S\left(x,t\right)}Y_t\hat{E}^{SY}\left[\frac{\mathbf{e}_S\left(X_T,T\right)}{\mathbf{e}_{SY}\left(X_T,T\right)}\middle|X_t=x\right]\to\frac{\mathbf{e}_{SY}\left(x,t\right)}{\mathbf{e}_S\left(x,t\right)}Y_t\int\frac{\mathbf{e}_S\left(y,T\right)}{\mathbf{e}_{SY}\left(y,T\right)}\pi_{T\mod\tau}(d\tag{34}$$

and the convergence is uniform in x on a compact subset of $\mathbb{X}$. Because Y_t is continuous at t, we can use (34) to compute the update of beliefs about Y_T over an FOMC announcement as

$$\lim_{T\to\infty}\frac{\hat{E}^S\left[Y_T|\,X_t\right]}{\hat{E}^S\left[Y_T|\,X_t^-\right]}=\frac{\frac{\mathbf{e}_{SY}(X_t,t)}{\mathbf{e}_S(X_t,t)}}{\frac{\mathbf{e}_{SY}\left(X_{t^-},t^-\right)}{\mathbf{e}_S\left(X_{t^-},t^-\right)}},\tag{35}$$

for all realizations of X_t within a compact subset of $\mathbb{X}$. We establish Equation (19) by combining (33) and (35).

5.3 Proof of Proposition 3

We first derive an expression for $d\left\langle \hat{S},\hat{Y}\right\rangle_t$. Given the eigenfunctions $\mathbf{e}_S(X,t)$ and $\mathbf{e}_Y(X,t)$, $\hat{S}_t = e^{\bar{r}t}\mathbf{e}_S(X_t,t)\,S_t$ and $\hat{Y}_t = e^{-\rho_Y t}\mathbf{e}_Y(X_t,t)\,Y_t$ and we can write

$$\frac{d\left\langle \hat{S},\hat{Y}\right\rangle_t}{\hat{S}_t\hat{Y}_t} = \left[\frac{D_X\mathbf{e}_S(X_t,t)}{\mathbf{e}_S(X_t,t)}\Sigma_X^T(X,t) - \sigma_S^T(X_t,t)\right]\left[\sigma_Y(X_t,t) + \frac{D_X\mathbf{e}_Y'(X_t,t)}{\mathbf{e}_Y(X_t,t)}\Sigma_X(X,t)\right]dt = f(t)\,dt \tag{36}$$

by the assumption of Proposition 3.

Because $\{-\bar{r},\mathbf{e}_S(X,t)\}$ and $\{\rho_Y,\mathbf{e}_Y(X,t)\}$ are the eigen pairs of the eigenvalue problems for S_t and Y_t. In the interior, $\mathbf{e}_S(X,t)$ and $\mathbf{e}_Y(X,t)$ must satisfy:

$$[-\bar{r}+r(X,t)] = \frac{D_X\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)}\left[\mu_X(X,t) - \Sigma_X'(X,t)\,\sigma_S(X,t)\right] + \frac{1}{2}tr\left[\frac{D_{XX}^2\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)}\Sigma_X'(X,t)\,\Sigma_X(X,t)\right] + \frac{D_t\mathbf{e}_S(X}{\mathbf{e}_S(X,} \tag{37}$$

Similarly,

$$[\rho_G - \mu_Y(X,t)] = \frac{D_X\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)}\left[\mu_X(X,t) + \Sigma_X'(X,t)\,\sigma_Y(X,t)\right] + \frac{1}{2}tr\left[\frac{D_{XX}^2\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)}\Sigma_X'(X,t)\,\Sigma_X(X,t)\right] + \frac{D_t\mathbf{e}_Y}{\mathbf{e}_Y(} \tag{38}$$

In addition, the martingale condition implies that for all Δ,

$$\mathbf{e}_S(X_t,t) = E\left[e^{\bar{r}\Delta}\mathbf{e}_S(X_{t+\Delta},t+\Delta)\,\frac{S_{t+\Delta}}{S_t}\middle|\,X_t\right],$$

which implies the boundary condition $\mathbf{e}_S(x,t^-) = E\left[\mathbf{e}_S(X_t,t)\,\frac{S_t}{S_{t^-}}\middle|\,X_{t^-}=x\right]$ and $\mathbf{e}_Y(x,t^-) = E\left[\mathbf{e}_Y(X_t,t)\middle|\,X_{t^-}=x\right]$ for all $x\in\mathbb{X}$ and for all $t=0,\tau,2\tau,\cdots$.

To derive conditions under which $\{\rho,\mathbf{e}_S(X)\,\mathbf{e}_Y(X)\,F(t)\}$ solves the eigenvalue problem for S_tY_t for some ρ and $F(t)$, we note that this guessed functional form implies that $\frac{D_X[\mathbf{e}_S(X)\mathbf{e}_Y(X)F(t)]}{\mathbf{e}_S(X)\mathbf{e}_Y(X)F(t)} = \frac{D_X\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)} + \frac{D_X\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)}$, and $\frac{D_{XX}^2[\mathbf{e}_S(X)\mathbf{e}_Y(X)F(t)]}{\mathbf{e}_S(X)\mathbf{e}_Y(X)F(t)} = \frac{D_X^2\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)} + \frac{D_X^2\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)} + \frac{2D_X\mathbf{e}_S(X,t)D_X^T\mathbf{e}_Y(X,t)}{\mathbf{e}_S(X,t)\mathbf{e}_Y(X,t)}$. The corresponding PDE for

$$\begin{aligned}\left[\rho + r(X,t) - \mu_Y(X,t) + \sigma_Y'(X,t)\,\sigma_S(X,t)\right] = &\left[\frac{D_X\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)} + \frac{D_X\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)}\right]\left[\mu_X(X,t) + \Sigma_X'(X,t)\,(\sigma_Y(X,t) -\right.\\ &+\frac{1}{2}tr\left\{\left[\frac{D_X^2\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)} + \frac{D_X^2\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)} + \frac{2D_X\mathbf{e}_S(X,t)\,D_X^T\mathbf{e}_Y(X,t)}{\mathbf{e}_S(X,t)\,\mathbf{e}_Y(X,t)}\right]\right.\end{aligned} \tag{39}$$

Using Equations () and () to eliminate terms, we can write the above equation as:

$$\begin{aligned}\rho + \bar{r} - \rho_Y + \sigma_Y'(X,t)\,\sigma_S(X,t) = &\frac{D_X\mathbf{e}_S(X,t)}{\mathbf{e}_S(X,t)}\Sigma_X'(X,t)\,\sigma_Y(X,t) - \frac{D_X\mathbf{e}_Y(X,t)}{\mathbf{e}_Y(X,t)}\Sigma_X'(X,t)\,\sigma_S(X,t)\\ &+\frac{F'(t)}{F(t)} + tr\left\{\frac{D_X\mathbf{e}_S(X,t)\,D_X^T\mathbf{e}_Y(X,t)}{\mathbf{e}_S(X,t)\,\mathbf{e}_Y(X,t)}\Sigma_X'(X,t)\,\Sigma_X(X,t)\right\},\end{aligned}$$

which can be written as

$$\rho + \bar{r} - \rho_Y = \frac{F'(t)}{F(t)} + f(t).$$

by using (36).

Therefore, $\mathbf{e}_S(X,t)\,\mathbf{e}_Y(X,t)\,F(t)$ is a solution to the PDE in (39) if and only if $F'(t) = [\lambda - f(t)]\,F(t)$ for some λ, or $F(t) = F(0)\,e^{\lambda t - \int_0^t f(s)ds}$. The boundary condition as announcement times implies that λ needs to satisfy:

$$\mathbf{e}_S\left(x,t^-\right)\mathbf{e}_Y\left(x,t^-\right)e^{\lambda\tau-\int_0^\tau f(s)ds} = E\left[\mathbf{e}_S(X,t)\,\mathbf{e}_Y(X,t)\,\frac{S_t}{S_{t^-}}\middle| X_{t^-}=x\right].$$

5.4 Details of Examples 1 and 2

Example 1: A log-linear example As shown in Ai and Bansal (2018), the posterior distribution of x_t is a Gaussian distribution with mean $\hat{x}$ and variance q. For $t \in (nT, (n+1)T)$,

$$d\hat{x}_t = a(\bar{x} - \hat{x}_t)\,dt + \frac{q_t}{\sigma}d\hat{B}_t, \tag{40}$$

where the Brownian motion $\hat{B}_t$ is defined as $d\hat{B}_t = \frac{1}{\sigma}\left[\frac{dC_t}{C_t} - \hat{x}_t dt\right]$. The posterior variance q_t satisfies a Riccati equation in the form

$$dq_t = \left[\sigma_x^2 - 2aq_t - \frac{1}{\sigma^2}q_t^2\right]dt. \tag{41}$$

On the boundary, $\hat{x}_T = \frac{1}{q_{nT}}\left[\frac{1}{\sigma_S^2}s_{nT} + \frac{1}{q^-_{nT}}\hat{x}^-_{nT}\right]$. In addition, q_t cycles deterministically across announcements and satisfies $q_t = q_{\tau(t)}$ with $\frac{1}{q_{nT}} = \frac{1}{\sigma_S^2} + \frac{1}{q^-_{nT}}$.

5.5 Heterogeneous agents

$\{-\bar{r}, \mathbf{e}_S(X,t)\}$ and $\{\rho_Y, \mathbf{e}_Y(X,t)\}$ are the eigen pairs of the eigenvalue problems for S_t and Y_t, where S_t is the SDF derived from the marginal utility of a hypothetical investor that consumes aggregate consumption. That is,

$$\mathbf{e}_S(X,t) = E\left[e^{\bar{r}\Delta}\frac{S_{t+\Delta}}{S_t}\mathbf{e}_S(X_{t+\Delta}, t+\Delta)\right], \tag{42}$$

and

$$\mathbf{e}_Y(X_t,t) = E\left[e^{-\rho_Y\Delta}\frac{Y_{t+\Delta}}{Y_t}\mathbf{e}_Y(X_{t+\Delta}, t+\Delta)\right], \tag{43}$$

for all $\Delta > 0$. Because (S_t, Y_t) satisfy the conditions of Proposition 3, $\mathbf{e}_S(X,t)\,\mathbf{e}_Y(X,t)$ is the eigenfunction of S_tY_t in the sense that

$$\mathbf{e}_S(X,t)\,\mathbf{e}_Y(X,t) = E\left[e^{\rho\Delta}\frac{S_{t+\Delta}Y_{t+\Delta}}{S_tY_t}\mathbf{e}_S(X_{t+\Delta}, t+\Delta)\,\mathbf{e}_Y(X_{t+\Delta}, t+\Delta)\right], \tag{44}$$

for some ρ for all $\Delta > 0$.

Because the marginal investor's consumption share is $\lambda(X_t,t)$, the marginal utility of the marginal investor is $e^{-\beta t}\left[\lambda(X_t,t)C_t\right]^{-\gamma}=\lambda(X_t,t)^{-\gamma}S_t$, which is the true stochastic discount factor of the economy. As a result, multiplying both sides of Equation (), we get:

$$\lambda(X_t,t)^{\gamma}\mathbf{e}_S(X,t)=E\left[e^{\bar{r}\Delta}\frac{S_{t+\Delta}}{S_t}\left(\frac{\lambda(X_{t+\Delta},t+\Delta)}{\lambda(X_t,t)}\right)^{-\gamma}\lambda(X_{t+\Delta},t+\Delta)^{\gamma}\mathbf{e}_S(X_{t+\Delta},t+\Delta)\right].$$

The term $\frac{S_{t+\Delta}}{S_t}\left(\frac{\lambda(X_{t+\Delta},t+\Delta)}{\lambda(X_t,t)}\right)^{-\gamma}$is the true SDF implied by the marginal investors. Therefore, $\lambda(X_t,t)^{\gamma}\mathbf{e}_S(X,t)$ is the eigenfunction of the true SDF. Similarly, Equation (44) implies

$$\lambda(X_t,t)^{\gamma}\mathbf{e}_S(X,t)\mathbf{e}_Y(X,t)=E\left[e^{\bar{r}\Delta}\frac{S_{t+\Delta}}{S_t}\left(\frac{\lambda(X_{t+\Delta},t+\Delta)}{\lambda(X_t,t)}\right)^{-\gamma}\lambda(X_{t+\Delta},t+\Delta)^{\gamma}\mathbf{e}_S(X_{t+\Delta},t+\Delta)\mathbf{e}_Y(X_{t+\Delta},t+\right.$$

that is, $\lambda(X_t,t)^{\gamma}\mathbf{e}_S(X,t)\mathbf{e}_Y(X,t)$ is the eigenfunction of $\tilde{S}_tY_t$. This implies that

$$\lim_{T\to\infty}\frac{\frac{V(t,T|X_t)}{b(t,T|X_t)}}{\frac{V\left(t^-,T|X_{t^-}\right)}{b\left(t^-,T|X_{t^-}\right)}}=\frac{\frac{\mathbf{e}_{SY}(X_t,t)}{\mathbf{e}_S(X_t,t)}}{\frac{\mathbf{e}_{SY}\left(X_{t^-},t^-\right)}{\mathbf{e}_S\left(X_{t^-},t^-\right)}}=\frac{\frac{\lambda(X_t,t)^{\gamma}\mathbf{e}_S(X,t)\mathbf{e}_Y(X,t)}{\lambda(X_t,t)^{\gamma}\mathbf{e}_S(X_t,t)}}{\frac{\lambda\left(X_{t^-},t^-\right)^{\gamma}\mathbf{e}_S\left(X_{t^-},t^-\right)\mathbf{e}_Y\left(X_{t^-},t^-\right)}{\lambda\left(X_{t^-},t^-\right)^{\gamma}\mathbf{e}_S\left(X_{t^-},t^-\right)}}=\frac{\mathbf{e}_Y(X,t)}{\mathbf{e}_Y\left(X_{t^-},t^-\right)}$$

is the update of belief about long-run growth.

6 Data appendix

Monetary policy shocks We obtain monetary policy shocks and Fed information shocks from Jarociński and Karadi (2020). By exploiting the negative and positive comovement between short rates and S&P500 prices, they decompose the first principal component of the surprises in interest rate derivatives with maturities from one month to one year into pure monetary policy shocks and Fed information shocks. The set of derivatives includes MP1, FF4, ED2, ED3 and ED4. Their decomposition allows both types of shocks to be present in each announcement. In addition, we obtain both the unadjusted and orthogonalized monetary policy shocks from Bauer and Swanson (2023). The unadjusted shock is the first principal component of the surprises in ED1, ED2, ED3 and ED4 and the orthogonalized shock is its residual from a regression on macroeconomic and financial news.

Long term equity To classify the stocks as long-term equity or not, we obtain stock-related variables from monthly CRSP. Specifically, we obtain the price per share (prc), the number of common shares outstanding (shrout), the holding period returns including dividends (ret) and excluding dividends (retx). A firm is defined as not paying dividends in month t if ret = retx. We rebalance our long term equity portfolio at the end of each year. Each year, we assign a firm to the non-dividend-paying portfolio if it had paid no dividends as of the end of the previous year. Our sample is restricted to U.S. common stocks listed on the NYSE, AMEX and NASDAQ. We exclude

utilities (SIC codes 4900-4999) and financial firms (SIC codes 6000-6999) .

10-K annual reports We download the cleaned 10-K annual reports in text files from Bill McDonald's "Software Repository for Accounting and Finance" website, which is based on the work in Loughran and McDonald (2016). Using these text files, we parse out the Item 5 ("Market for Registrant's Common Equity, Related Stockholder Matters and Issuer Purchases of Equity Securities") in each report. We prompt GPT-4o-mini to read the extracted Item 5 text and return two binary indicators for whether the firm states that it does not anticipate paying dividends in the foreseeable future and whether it intends to retain earnings to finance growth.

Dividend futures We obtain daily prices of dividend futures on the S&P 500 index from Bansal, Miller, Song, and Yaron (2021) for January 2007 to August 2014, and from Bloomberg from November 2015 onwards. The dividend futures expire on the third Friday of December. As is common in the literature, we linearly interpolate futures prices between two adjacent maturities to obtain prices at constant annual maturities up to seven years for each trading day. The first date with valid data for the nearest five years in our sample is December 2009.

Other data We obtain the daily nominal yield data from Gürkaynak, Sack, and Wright (2007) and the daily S&P 500 index from CRSP. We obtain daily series for VIX and for the yields on Moody's Seasoned Baa and Aaa Corporate Bonds from FRED.

7 Additional Tables and Figures

Table 7.1: High-frequency yield changes on Nominal and Real long term bonds

	$y^{\infty}_{B,real}$
	(1)
$y^{\infty}_{B,nomi}$	0.98^{***}
	(0.11)
Const.	-0.0007
	(0.001)
Adj. R^2	0.88
N	151

Notes: The table report OLS estimates of $y^{\infty}_{B,real} = \beta_0 + \beta_1 y^{\infty}_{B,nomi} + u$, where $y^{\infty}_{B,real}$ is the changes in the on-the-run 30Y TIPS yield and $y^{\infty}_{B,nomi}$ is the changes in the on-the-run 30Y nominal Treasury yield, measured over the 30-minute window around FOMC announcements. Yield changes are from the US Monetary Policy Event-Study Database of the Federal Reserve Bank of San Francisco. The sample is 151 scheduled announcements from Feb 2005 to Dec 2023. Heteroskedasticity-robust standard errors are reported in brackets.$^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$.

Table 7.2: High Frequency revisions in long-run growth in response to MPS and CBI (10-K portfolio)

	Panel (A): $\ln R_E^\infty - \ln R_B^\infty$			Panel (B): $\ln R_E^\infty$		
	(1)	(2)	(3)	(4)	(5)	(6)
MPS	-3.76^{***}			-7.89^{***}		
	(1.38)			(0.41)		
CBI	11.29^{***}			9.24^{***}		
	(2.23)			(0.64)		
BS		-0.55			-4.36^{***}	
		(1.08)			(0.68)	
$BS^\perp$			-1.42			-4.93^{***}
			(1.19)			(0.79)
Const.	2.25	-1.23	-1.76	0.64	0.23	-0.21
	(3.69)	(4.16)	(4.19)	(1.45)	(3.10)	(3.12)
N	231	231	225	231	231	225
Adj. R^2	0.36	0.00	0.01	0.84	0.20	0.21

Notes: This table provide a robustness check for the table (2.5), using the alternative long-term equity portfolio constructed from the 10-K reports. The table reports OLS estimates from regressions of (A) the revisions in long-run growth expectations under long-run risk neutral measure $\ln R_E^\infty - \ln R_B^\infty$ and (B) the log returns on the long-term equity portfolio ($\ln R_E^\infty$) on the monetary policy surprises. Heteroskedasticity-robust standard errors are reported in brackets.$^* p < 0.1,^{**} p < 0.05,^{***} p < 0.01$.

References

Acosta, Miguel, Andrea Ajello, Michael D Bauer, Francesca Loria, and Silvia Miranda-Agrippino, 2025, Financial market effects of FOMC communication: Evidence from a new event-study database, *Working Paper*.

Ai, Hengjie, and Ravi Bansal, 2018, Risk Preferences and the Macroeconomic Announcement Premium, *Econometrica* 86, 1383–1430.

Ai, Hengjie, and Anmol Bhandari, 2021, Asset pricing with endogenously uninsurable tail risk, *Econometrica* 89, 1471–1505.

Ai, Hengjie, and Leitao Fu, 2026, Using Asset Prices to Measure Long-run Growth Expectations, *Working paper, University of Wisconsin-Madison*.

Alvarez, Fernando, and Urban Jermann, 2005, Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth, *Econometrica* 73, 1977–2016.

Alvarez, Fernando, and Urban J Jermann, 2000, Efficiency, equilibrium, and asset pricing with risk of default, *Econometrica* 68, 775–797.

Backus, David, Mikhail Chernov, and Stanley Zin, 2014, Sources of Entropy in Representative Agent Models, *The Journal of Finance* 69, 51–99.

Bakshi, Gurdip, and Fousseni Chabi-Yo, 2012, Variance bounds on the permanent and transitory components of stochastic discount factors, *Journal of Financial Economics* 105, 191–208.

Bakshi, Gurdip, Fousseni Chabi-Yo, and Xiaohui Gao, 2018, A recovery that we can trust? Deducing and testing the restrictions of the recovery theorem, *The Review of Financial Studies* 31, 532–555.

Bansal, Ravi, and Bruce N. Lehmann, 1997, Growth-Optimal Portfolio Restrictions on Asset Pricing Models, *Macroeconomic Dynamics* 1, 333–354.

Bansal, Ravi, Shane Miller, Dongho Song, and Amir Yaron, 2021, The term structure of equity risk premia, *Journal of Financial Economics* 142, 1209–1228.

Basak, Suleyman, and Domenico Cuoco, 1998, An equilibrium model with restricted stock market participation, *The Review of Financial Studies* 11, 309–341.

Bauer, Michael, and Mikhail Chernov, 2024, Interest rate skewness and biased beliefs, *The Journal of Finance* 79, 173–217.

Bauer, Michael D, Carolin E Pflueger, and Adi Sunderam, 2024, Perceptions about monetary policy, *The Quarterly Journal of Economics* 139, 2227–2278.

Bauer, Michael D, and Eric T Swanson, 2023, A reassessment of monetary policy surprises and high-frequency identification, *NBER Macroeconomics Annual* 37, 87–155.

Bernanke, Ben S., and Kenneth N. Kuttner, 2005, What Explains the Stock Market's Reaction to Federal Reserve Policy?, *The Journal of Finance* LX, 1221–1257.

Bianchi, Francesco, Howard Kung, and Gonzalo Morales, 2019, Growth, slowdowns, and recoveries, *Journal of Monetary Economics* 101, 47–63.

Blanchard, Olivier, 2018, Should We Reject the Natural Rate Hypothesis?, *Journal of Economic Perspectives* 32, 97–120.

Borovička, Jaroslav, Lars Peter Hansen, and José A Scheinkman, 2016, Misspecified recovery, *The Journal of Finance* 71, 2493–2544.

Christensen, Timothy M, 2017, Nonparametric stochastic discount factor decomposition, *Econometrica* 85, 1501–1536.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans, 1999, Monetary Policy Shocks: What Have We Learned and to What End?, in *Handbook of Macroeconomics* (Elsevier,).

Cieslak, Anna, and Paymon Khorrami, 2025, Risk Premia, Subjective Beliefs, and Forward Guidance, *Working Paper*.

Cochrane, John H, and Monika Piazzesi, 2002, The fed and interest rates - a high-frequency identification, *American economic review* 92, 90–95.

Fornaro, Luca, and Martin Wolf, 2023, The scars of supply shocks: Implications for monetary policy, *Journal of Monetary Economics* 140, S18–S36.

Friedman, Avner, 1975, (*Stochastic Differential Equations and Applications*).

Friedman, Milton, 1968, The Role of Monetary Policy, *American Economic Review* 58, 1–17.

Galí, Jordi, 2015, *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. (Princeton University Press).

Garga, Vaishali, and Sanjay R Singh, 2021, Output Hysteresis and Optimal Monetary Policy, *Journal of Monetary Economics* 117, 871–886.

Gertler, Mark, and Peter Karadi, 2015, Monetary policy surprises, credit costs, and economic activity, *American Economic Journal: Macroeconomics* 7, 44–76.

Golez, Benjamin, and Ben Matthies, 2025, Fed information effects: Evidence from the equity term structure, *Journal of Financial Economics* 165, 103988.

Gürkaynak, Refet S., Brian Sack, and Eric T. Swanson, 2005, Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements, *International Journal of Central Banking* 1.

Gürkaynak, Refet S, Brian Sack, and Jonathan H Wright, 2007, The US Treasury yield curve: 1961 to the present, *Journal of monetary Economics* 54, 2291–2304.

Hairer, Martin, and Jonathan C. Mattingly, 2011, Yet another look at Harris' ergodic theorem for Markov chains, in *Seminar on Stochastic Analysis, Random Fields and Applications VI* (Springer,).

Hansen, Lars Peter, and José A Scheinkman, 2009, Long-term risk: An operator approach, *Econometrica* 77, 177–234.

He, Zhiguo, and Arvind Krishnamurthy, 2013, Intermediary asset pricing, *American Economic Review* 103, 732–770.

Jarociński, Marek, and Peter Karadi, 2020, Deconstructing monetary policy surprises - the role of information shocks, *American Economic Journal: Macroeconomics* 12, 1–43.

Jensen, Christian Skov, David Lando, and Lasse Heje Pedersen, 2019, Generalized recovery, *Journal of Financial Economics* 133, 154–174.

Jordà, Òscar, Sanjay R Singh, and Alan M Taylor, 2024, The Long-Run Effects of Monetary Policy, *The Review of Economics and Statistics*.

Kekre, Rohan, and Moritz Lenel, 2022, Monetary policy, redistribution, and risk premia, *Econometrica* 90, 2249–2282.

Kekre, Rohan, Moritz Lenel, and Federico Mainardi, 2025, Monetary policy, segmentation, and the term structure, *Working paper*.

Kuttner, Kenneth N, 2001, Monetary policy surprises and interest rates: Evidence from the Fed funds futures market, *Journal of monetary economics* 47, 523–544.

Loughran, Tim, and Bill McDonald, 2016, Textual analysis in accounting and finance: A survey, *Journal of accounting research* 54, 1187–1230.

Lucas, Robert E, 1972, Expectations and the Neutrality of Money, *Journal of Economic Theory* 4, 103–124.

Lucca, David O, and Emanuel Moench, 2015, The Pre-FOMC Announcement Drift, *The Journal of finance* 70, 329–371.

Lustig, Hanno, Andreas Stathopoulos, and Adrien Verdelhan, 2019, The term structure of currency carry trade risk premia, *American Economic Review* 109, 4142–4177.

Mehra, Rajnish, and Edward C. Prescott, 1985, The Equity Premium: A Puzzle, *Journal of Monetary Economics* 15, 145–161.

Moran, Patrick, and Albert Queralto, 2018, Innovation, productivity, and monetary policy, *Journal of Monetary Economics* 93, 24–41.

Nagel, Stefan, and Zhengyang Xu, 2025, Movements in yields, not the equity premium: Bernanke-kuttner redux, *Working paper*.

Nakamura, Emi, and Jon Steinsson, 2018, High Frequency Identification of Monetary Non-Neutrality: The Information Effect, *Quarterly Journal of Economics* 133, 1283–1330.

Pflueger, Carolin, and Gianluca Rinaldi, 2022, Why does the Fed move markets so much? A model of monetary policy and time-varying risk aversion, *Journal of Financial Economics* 146, 71–89.

Qin, Likuan, and Vadim Linetsky, 2017, Long-term risk: A martingale approach, *Econometrica* 85, 299–312.

Ramey, Valerie A, 2016, Macroeconomic shocks and their propagation, *Handbook of macroeconomics* 2, 71–162.

Romer, Christina D, and David H Romer, 2004, A New Measure of Monetary Shocks: Derivation and Implications, *American Economic Review* 94, 1055–1084.

Ross, Steve, 2015, The recovery theorem, *The Journal of Finance* 70, 615–648.

Savor, Pavel, and Mungo Wilson, 2013, How Much Do Investors Care About Macroeconomic Risk? Evidence from Scheduled Economic Announcement, *Journal of Financial and Quantitative Analysis* 48, 343–375.

Stadler, George W, 1990, Business cycle models with endogenous technology, *The American Economic Review* pp. 763–778.

Stock, James H, and Mark W Watson, 2018, Identification and estimation of dynamic causal effects in macroeconomics using external instruments, *The Economic Journal* 128, 917–948.

Van Binsbergen, Jules H, and Ralph SJ Koijen, 2017, The term structure of returns: Facts and theory, *Journal of Financial Economics* 124, 1–21.

Woodford, Michael, 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*. (Princeton University Press).