SUPPLEMENT TO "RISK PREFERENCES AND THE MACROECONOMIC ANNOUNCEMENT PREMIUM"

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IN THIS SUPPLEMENT, we provide details of the proofs omitted in the main text and the appendices of the paper. In Section S. 1, we prove the equivalence between the Arrow-Debreu setup and the sequential market setup in the two-period model. Section S. 2 contains details of the certainty equivalent functionals of dynamic preferences that Theorems 1 and 2 allow for and the associated A-SDF. Section S. 3 provides details of the continuous-time model in Section 5 of the paper.

S. 1 The two-period model

In this section, we provide a formal derivation of the A-SDF in the two-period model. We also establish the equivalence between Arrow-Debreu markets and sequential markets in the context of our model. We show that both formulations lead to the same set of asset pricing equations. Section S. 1.3 discusses the A-SDF of the recursive utility in the two-period model.

S. 1.1 The Arrow-Debreu market

We use $\left\{ \bar{C}_0, \{\bar{C}_1(s)\}_{s=1}^N \right\}$ to denote aggregate endowment in our two-period model and use $\{C_0(s), \ C_1(s)\}_{s=1}^N$ for the consumption choice of the agent. From an individual agent's perspective, the decision for C_0 is made after the announcement, and therefore is allowed to depend on s. At the aggregat level, $\bar{C}_0$ does not depend on s.

Trading on the Arrow-Debreu market happens in period 0^-. Let $q_0(s)$ be the period 0^- price of an Arrow-Debreu security that delivers one unit of consumption good in period 0^+ and state s, for $s = 1, 2, \cdots, N$. Similarly, let $q_1(s)$ be the Arrow-Debreu price of one unit of consumption good in period one and state s. Because markets are complete, the utility maximization problem of the representative agent can be written as:

$$\max \mathcal{I}\left[u(C_0(s)) + \beta u(C_1(s)) \right]$$

$$subject \ to \ : \ \sum_{s=1}^N \left[q_0(s) C_0(s) + q_1(s) C_1(s) \right] \leq \sum_{s=1}^N \left[q_0(s) \bar{C}_0 + q_1(s) \bar{C}_1(s) \right]$$

In the above setup, because the announcement is made at time 0^+, from the agent's perspective, consumption at time 0^+ is allowed to depend on s, which we write as $C_0(s)$. To save notation, as in the paper, we denote $V_s = u(C_0(s)) + \beta u(C_1(s))$. Optimality implies

$$q_0(s) = \lambda \frac{\partial \mathcal{I}[V]}{\partial V_s} u'(C_0(s)), \ \ q_1(s) = \lambda \frac{\partial \mathcal{I}[V]}{\partial V_s} \beta u'(C_1(s)),$$

where λ is the Lagrangian multiplier of the budget constraint. In equilibrium, market clearing implies that $C_0(s) = \bar{C}_0$ for all s. If we normalize the price of one unit state-non-contingent consumption at time 0^+ to be one, that is, $\sum_{s=1}^{N} q_0(s) = 1$; then, for all s,

$$q_0(s) = \frac{\frac{\partial \mathcal{I}[V]}{\partial V_s}}{\sum_{s=1}^{N} \frac{\partial \mathcal{I}[V]}{\partial V_s}}, \tag{S. 1.1}$$

and $\frac{q_1(s)}{q_0(s)} = \beta \frac{u'(\bar{C}_1(s))}{u'(\bar{C}_0)}$. That is, we can simply use ratios of marginal utilities to compute Arrow-Debreu prices. Clearly, (S. 1.1) implies the expression of the A-SDF in equation (12) of the paper.

S. 1.2   The sequential market

Here, we show that the two-period version of the sequential market setup described in Section 4 leads to the same asset pricing equation, (12). In period 0^-, there is no consumption decision and the agent chooses investment in a vector of announcement returns to maximize:

$$\max_{\{\xi_j\}_{j=1}^{J}} \mathcal{I}\left[V\left(W'\right)\right]$$
$$subject\ to\ :\ W'_s = W - \sum_{j=1}^{J} \xi_j + \sum_{j=1}^{J} \xi_j R_{A,j}(s),\ \ all\ s, \tag{S. 1.2}$$

where $W' = \{W'_s\}_{s=1}^{N}$ is the realizations of wealth in the next period, and $V(W') = \{V_s(W'_s)\}_{s=1}^{N}$ is a vector of value functions. For each s, the value function $V_s(W)$ is defined by the optimal portfolio choice problem on the post-announcement market:

$$V_s(W) = \max_{C_0(s),C_1(s)} u(C_0(s)) + \beta u(C_1(s))$$
$$subject\ to\ :\ C_1(s) = (W - C_0(s)) R_{P,s}. \tag{S. 1.3}$$

Note that $R_{P,s}$ is the return from period 0^+ to period 1 after announcement s. Because the announcement fully reveals the true state of the world, $R_{P,s}$ is a risk-free return.

The first order condition for (S. 1.2) with respect to ξ_j implies that for any announcement returns $R_{A,j}$,

$$\sum_{s=1}^{N} \frac{\partial}{\partial V_s} \mathcal{I}\left[V\left(W'\right)\right] \frac{\partial V_s(W'_s)}{\partial W'_s} [R_{A,j}(s) - 1] = 0, \tag{S. 1.4}$$

where W'_s denote the equilibrium wealth of the agent in period 0^+ after announcement s. The envelope condition for (S. 1.3) implies that $\frac{\partial V_s(W'_s)}{\partial W'_s} = u'(C_0(s)) = u'(\bar{C}_0)$, where the second

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equality uses the market clearing condition. As $u' > 0$, equation (S. 1.4) implies

$$\sum_{s=1}^{N} \frac{\frac{\partial}{\partial V_s} \mathcal{I}\left[V\left(W'\right)\right]}{\sum_{s=1}^{N} \frac{\partial}{\partial V_s} \mathcal{I}\left[V\left(W'\right)\right]} R_{A,j}\left(s\right) = 1,$$

as in equation (11) of the paper.

S. 1.3   The example of recursive utility

Here, we provide details of the computation of the A-SDF for the recursive utility in Section 3.2 of the paper. We illustrate that because the announcement in our model leads uncertainty to resolve before the realization of consumption shocks, the computation of utilities and therefore, marginal utilities differ from that in models in which resolution of uncertainty happens at the same time of the realization of the consumption shocks.

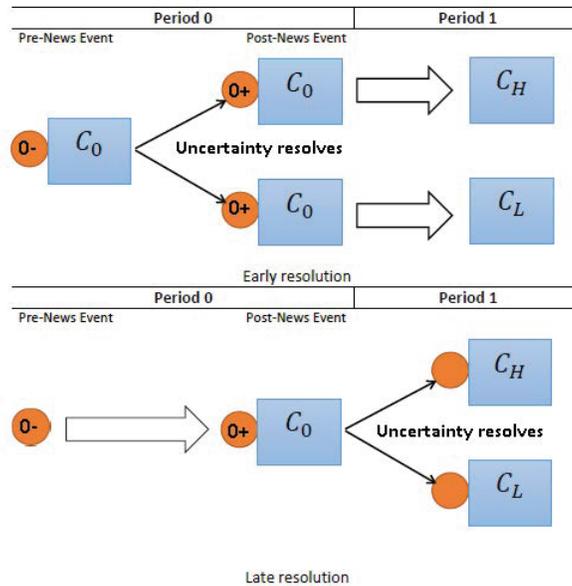Figure S. 1: **Early and Late Resolution of Uncertainty**

Figure S. 1 plots a consumption plan with early resolution of uncertainty (top panel) and a consumption plan with late resolution of uncertainty.

Figure S. 1 illustrates a two-period model with announcement and one without announcement. The top panel is the same as that in Figure 2 in our main text, where the announcement at time 0^+ fully reveals the true state and leads to early resolution of uncertainty. In the bottom panel of Figure S. 1, due to the absence of announcement, the uncertainty is resolved in period 1 when consumption is realized; that is, it is a case of late resolution of uncertainty.[1]

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[1]The comparison between early and late resolution of uncertainty here is the same as that in Figure 2 of Kreps

We denote the utility at 0^- in the case of early resolution as $V^E\left(\{C_0(s),C_1(s)\}_{s=1}^N\right)$. Consistent with our previous notations, in the rest of this section, we will allow C_0 to depend on s when evaluating utilities, and will impose market clearing, $C_0(s) = \bar{C}_0$ for all s when computing stochastic discount factors. In the case of early resolution, because there is no uncertainty in period 0^+, we first aggregate over time to compute the continuation utility as $V_s = \frac{1}{1-\frac{1}{\psi}}C_0^{1-\frac{1}{\psi}}(s) + \beta\frac{1}{1-\frac{1}{\psi}}C_1^{1-\frac{1}{\psi}}(s)$, and then aggregate over uncertain realizations of the announcement to compute its certainty equivalent at 0^- as:

$$V^E = \left\{\sum_{s=1}^N \pi(s)\left[\left\{C_0^{1-\frac{1}{\psi}}(s) + \beta C_1^{1-\frac{1}{\psi}}(s)\right\}^{\frac{1-\gamma}{1-1/\psi}}\right]\right\}^{\frac{1}{1-\gamma}}. \tag{S. 1.5}$$

Clearly, $\forall s$,

$$\frac{\partial V^E}{\partial C_0(s)} = \pi(s)\left(V^E\right)^\gamma V_s^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} C_0^{-\frac{1}{\psi}}(s), \quad \frac{\partial V^E}{\partial C_1(s)} = \beta\pi(s)\left(V^E\right)^\gamma V_s^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} C_1^{-\frac{1}{\psi}}(s).$$

Therefore, the marginal utility of one unit of data-0 state un-contingent consumption can be computed as

$$\sum_{s=1}^N \frac{\partial V^E}{\partial C_0(s)} = \sum_{s=1}^N \pi(s)\left(V^E\right)^\gamma V_s^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}} C_0^{-\frac{1}{\psi}}(s)$$

$$= \left[\sum_{s=1}^N \pi(s) V_s^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}\right]\left(V^E\right)^\gamma \bar{C}_0^{-\frac{1}{\psi}},$$

where the second equality imposes market clearing. Therefore, the price of one unit of consumption good paid in period one, state s, measured in date-0 state un-contingent consumption is:

$$\frac{\frac{\partial V^E}{\partial C_1(s)}}{\sum_{s=1}^N \frac{\partial V^E}{\partial C_0(s)}} = \beta\pi(s)\left(\frac{\bar{C}_1(s)}{\bar{C}_0}\right)^{-\frac{1}{\psi}} \frac{V_s^{\frac{1/\psi-\gamma}{1-1/\psi}}}{\sum_{s=1}^n \pi(s) V_s^{\frac{1/\psi-\gamma}{1-1/\psi}}}. \tag{S. 1.6}$$

In the case of late resolution, 0^- and 0^+ have the same utility level, which we denote as $V^L\left(C_0, \{C_1(s)\}_{s=1}^N\right)$. We first aggregate over uncertain period 1 consumption to compute its certainty equivalent: $\left\{E\left[C_1^{1-\gamma}(s)\right]\right\}^{\frac{1}{1-\gamma}}$, and then aggregate over time to compute V^L as:

$$V^L = \left\{\frac{1}{1-\frac{1}{\psi}}C_0^{1-\frac{1}{\psi}} + \beta\frac{1}{1-\frac{1}{\psi}}\left\{\sum_{s=1}^N \pi(s)\left[C_1^{1-\gamma}(s)\right]\right\}^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}. \tag{S. 1.7}$$

and Porteus [12]. Our top panel corresponds to node $d_0(a)$ and the bottom panel corresponds to node $d_0(b)$ in that Figure.

The Arrow-Debreu price for one unit of consumption in period one measured in period-0 consumption numeraire can be computed as

$$\frac{\frac{\partial V^L}{\partial \bar{C}_1(s)}}{\frac{\partial VL}{\partial \bar{C}_0}} = \pi(s)\beta\left(\frac{\bar{C}_1(s)}{\bar{C}_0}\right)^{-\frac{1}{\psi}}\left\{\frac{\bar{C}_1(s)}{\left[\sum_{s=1}^n \pi(s)\bar{C}_1^{1-\gamma}(s)\right]^{\frac{1}{1-\gamma}}}\right\}^{\frac{1}{\psi}-\gamma}. \tag{S. 1.8}$$

Clearly, the SDF for the early resolution case, (S. 1.6) can be decomposed into the m^* in equation (10) and an SDF that discounts period 1 cash flow into period 0^+ consumption units: $\beta\left(\frac{\bar{C}_1(s)}{\bar{C}_0}\right)^{-\frac{1}{\psi}}$. The SDF in (S. 1.8) takes a familiar form as in many consumption-based asset pricing models where uncertainty is assumed to resolve at the same time of the realization of consumption shocks. In general, the term $\left\{\frac{\bar{C}_1(s)}{\left[\sum_{s=1}^n \pi(s)\bar{C}_1^{1-\gamma}(s)\right]^{\frac{1}{1-\gamma}}}\right\}^{\frac{1}{\psi}-\gamma}$ does not integrate to one unless in the special case of unit IES.

S. 2   Examples of Dynamic Preferences and A-SDF

In this section, we show that most of the non-expected utility proposed in the literature can be represented in the form of (14). We also provide an expression for the implied A-SDF.[2]

- The recursive utility of Kreps and Porteus [12] and Epstein and Zin [4]. The recursive preference can be generally represented as:

$$U_t = u^{-1}\left\{(1-\beta)u(C_t) + \beta u \circ h^{-1}E\left[h(U_{t+1})\right]\right\}. \tag{S. 2.1}$$

For example, the well-known recursive preference with constant IES and constant risk aversion is the special case in which $u(C) = \frac{1}{1-1/\psi}C^{1-1/\psi}$ and $h(U) = \frac{1}{1-\gamma}U^{1-\gamma}$. With a monotonic transformation,

$$V = u(U), \tag{S. 2.2}$$

the recursive relationship for V can be written in the form of (14) with the same u function in equation (S. 2.1) and the certainty equivalent functional:

$$\mathcal{I}(V) = \phi^{-1}\left(\int \phi(V)\,dP\right),$$

where $\phi = h \circ u^{-1}$. the A-SDF can be written as:

$$m^*(V) \propto \phi'(V), \tag{S. 2.3}$$

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[2]Depending on the model, additional conditions may be needed so that the assumptions of Theorem 1 can be verified. We provide the expressions for A-SDF assuming appropriate conditions on the primitive utility functions can be imposed to guarantee its existence.

where we suppress the normalizing constant, which is chosen so that $m^*(V)$ integrates to one.

- The maxmin expected utility of Gilboa and Schmeidler [6]. The dynamic version of this preference is studied in Epstein and Schneider [3] and Chen and Epstein [1]. This preference can be represented as the special case of (14) where the certainty equivalent functional is of the form:

$$\mathcal{I}(V) = \min_{m \in M} \int mV dP,$$

where M is a family of probability densities that is assumed to be convex and closed in the weak* topology. As we show in Section 3.2 of the paper, the A-SDF for this class of preference is the Radon-Nikodym derivative of the minimizing probability measure with respect to P.

- The variational preferences of Maccheroni, Marinacci, and Rustichini [14], the dynamic version of which is studied in Maccheroni, Marinacci, and Rustichini [15], features a certainty equivalent functional of the form:

$$\mathcal{I}(V) = \min_{E[m]=1} \int mV dP + c(m),$$

where $c(m)$ is a convex and weak*$-$lower semi-continuous function. Similar to the maxmin expected utility, the A-SDF for this class of preference is minimizing probability density.

- The multiplier preferences of Hansen and Sargent [7] and Strzalecki [16] is represented by the certainty equivalent functional:

$$\mathcal{I}(V) = \min_{E[m]=1} \int mV dP + \theta R(m),$$

where $R(m)$ denote the relative entropy of the density m with respect to the reference probability measure P, and $\theta > 0$ is a parameter. In this case, the A-SDF is also the minimizing probability that can be written as a function of the continuation utility: $m^*(V) \propto e^{-\frac{1}{\theta}V}$.

- The second order expected utility of Ergin and Gul [5] can be written as (14) with the following choice of $\mathcal{I}$:

$$\mathcal{I}(V) = \phi^{-1}\left(\int \phi(V) dP\right),$$

where ϕ is a concave function. In this case, the A-SDF can be written as a function of continuation utility:

$$m^*(V) \propto \phi'(V).$$

- The smooth ambiguity preference of Klibanoff, Marinacci, and Mukerji [10] and Klibanoff, Marinacci, and Mukerji [11] can be represented as:

$$\mathcal{I}(V) = \phi^{-1}\left(\int_M \phi\left(\int_\Omega mV dP\right) d\mu(m)\right), \tag{S. 2.4}$$

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where μ is a probability measure on a set of probability densities M. The A-SDF can be written as :

$$m^* (\omega) \propto \int_M \phi' \left(\int_\Omega mV dP \right) m (\omega) d\mu (m).$$ (S. 2.5)

- The certainty equivalent functional $\mathcal{I}$ for the disappointment aversion preference is implicitly defined as $\mathcal{I}[V] = \mu$, where μ is the unique solution to the following equation:

$$\phi(\mu) = \int \phi(V) dP - \theta \int_{\mu \geq V} [\phi(\mu) - \phi(V)] dP,$$

where ϕ is a concave function. The A-SDF can be written as:

$$m^* (V) = \begin{cases} \frac{\phi'(V)}{\phi'(\mu)[1+\theta P(V \leq \mu)]} & if \ V > \mu \\ \frac{(1-\theta)\phi'(V)}{\phi'(\mu)[1+\theta P(V \leq \mu)]} & f \ V \leq \mu \end{cases},$$

whenever $\mathcal{I}[V]$ is differentiable at V.

- Hayashi and Miao [9] develop a class of generalized recursive smooth ambiguity model that takes the following form:[3]

$$\bar{V}_t = u^{-1} \left\{ (1-\beta) u (C_t) + \beta \left[u \circ \nu^{-1} \right] \left(\int_M \left[\nu \circ \phi^{-1} \right] \left(\int m\phi (\bar{V}_{t+1}) dP \right) d\mu (m) \right) \right\},$$
(S. 2.6)

where u, v, and ϕ are all smooth and monotone functions. As in the Klibanoff, Marinacci, and Mukerji [10] model, M is a set of probability densities that represent ambiguous beliefs, and μ is a measure on the set of densities. With a monotonic transformation, $V_t = u(\bar{V}_t)$, the above can be written in the form of (14) with

$$\mathcal{I}(V) = \left[u \circ \nu^{-1} \right] \left(\int_M \left[\nu \circ \phi^{-1} \right] \left(\int m \left[\phi \circ u^{-1} \right] (V) dP \right) d\mu (m) \right).$$

The A-SDF for this class of preferences can be written as:

$$m^* (\omega) \propto \int_M \left[\nu \circ \phi^{-1} \right]' \left(\int m \left[\phi \circ u^{-1} \right] (V) dP \right) m (\omega) \left[\phi \circ u^{-1} \right]' (V (\omega)) d\mu (m).$$

S. 3 Details of the Continuous-time model

In this section, we provide details of the solution of the continuous-time model. Section S. 3.1 provide the solution to the model with periodic announcement in Section 5.1 and 5.2 of the main text of the paper. Section S. 3.2 provides the omitted proofs for the results on time-non-separable

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[3]The model in Hayashi and Miao [9] is more general than (S. 2.6) and may not permit a representation of the form $V_t = u(C_t) + \beta \mathcal{I}[V_{t+1}]$. However, the applied examples of this preference are often special cases of (S. 2.6). See also the related generalized recursive multiple-priors model of Hayashi [8], which can be obtained as a limiting case of (S. 2.6).

preferences discussed in Section 5.3.

S. 3.1   Asset Pricing in the Learning Model

Value function of the representative agent   Because announcements fully reveal the value of x_t at nT, $q_{nT}^+ = 0$. We start from $q_0 = 0$. In the interior of $(0, T)$, the standard optimal filtering implies that the posterior mean and variance of x_t are given by equations (30) and (31). Here q_t has a closed form solution:

$$q(t) = \frac{\sigma_x^2 \left(1 - e^{-2\hat{a}t}\right)}{(\hat{a} - a_x) e^{-2\hat{a}t} + a_x + \hat{a}}, \tag{S. 3.1}$$

where $\hat{a} = \sqrt{a_x^2 + (\sigma_x/\sigma)^2}$. In general, we can write $q_t = q(t \bmod T)$ for all t.[4]

Using the results from Duffie and Epstein [2], the representative consumer's preference is specified by a pair of aggregators $(f, \mathcal{A})$ such that the utility of the representative agent, V_t is the solution to the following stochastic differential equation:

$$dV_t = [-f(C_t, V_t) - \frac{1}{2}\mathcal{A}(V_t)\|\sigma_V(t)\|^2]dt + \sigma_V(t)dB_t,$$

for some square-integrable process $\sigma_V(t)$. We adopt the convenient normalization $\mathcal{A}(V) = 0$ (Duffie and Epstein [2]), and denote $\bar{f}$ the normalized aggregator, and $\bar{V}_t$ the corresponding utility process. Under this normalization,

$$\bar{f}(C, \bar{V}) = \rho \left\{(1 - \gamma) \bar{V} \ln C - \bar{V} \ln \left[(1 - \gamma) \bar{V}\right]\right\}.$$

Due to homogeneity, the value function is of the form[5]

$$\bar{V}(\hat{x}_t, t, C_t) = \frac{1}{1 - \gamma} H(\hat{x}_t, t) C_t^{1-\gamma}, \tag{S. 3.2}$$

where $H(\hat{x}, t)$ satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:

$$-\frac{\rho}{1 - \gamma} \ln H(\hat{x}_t, t) H(\hat{x}_t, t) + \left(\hat{x}_t - \frac{1}{2}\gamma\sigma^2\right) H(\hat{x}_t, t) + \frac{1}{1 - \gamma} H_t(\hat{x}_t, t)$$

$$+ \left[\frac{1}{1 - \gamma} a_x (\bar{x} - \hat{x}_t) + q_t\right] H_x(\hat{x}_t, t) + \frac{1}{2}\frac{1}{1 - \gamma} H_{xx}(\hat{x}_t, t) \frac{q_t^2}{\sigma^2} = 0, \tag{S. 3.3}$$

with the boundary condition that for all $n = 1, 2 \cdots$

$$H(\hat{x}_{nT}^-, nT) = E\left[H(\hat{x}_{nT}^+, nT)\big| \hat{x}_{nT}^-, q_{nT}^-\right]. \tag{S. 3.4}$$

The solution to the partial differential equation (PDE) (S. 3.3) together with the boundary

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[4]We use the notation $t \bmod T$ for the remainder of t divided by T.

[5]As $\Delta \to 0$, the discrete time approximation, (32) converge to the following monotonic transformation of $\bar{V}$: $V_t = \frac{1}{1-\gamma} \ln\left[(1 - \gamma) \bar{V}_t\right]$.

condition (S. 3.4) is separable and given by:

$$H\left(\hat{x},t\right)=e^{\frac{1-\gamma}{a_x+\rho}\hat{x}+h(t)},$$

where $h\left(t\right)$ satisfy the following ODE:

$$-\rho h\left(t\right)+h'\left(t\right)+f\left(t\right)=0, \tag{S. 3.5}$$

where $f\left(t\right)$ is defined as:

$$f\left(t\right)=\frac{\left(1-\gamma\right)^2}{a_x+\rho}q\left(t\right)+\frac{1}{2}\frac{\left(1-\gamma\right)^2}{\left(a_x+\rho\right)^2}\frac{1}{\sigma^2}q^2\left(t\right)-\frac{1}{2}\gamma\left(1-\gamma\right)\sigma^2+a_x\bar{x}\frac{1-\gamma}{a_x+\rho}.$$

The general solution to (S. 3.5) is of the following form on $\left(0,T\right)$:

$$h\left(t\right)=h\left(0\right)e^{\rho t}-e^{\rho t}\int_0^t e^{-\rho s}f\left(s\right)ds.$$

We focus on the steady state in which $h\left(t\right)=h\left(t\bmod T\right)$ and use the convention $h\left(0\right)=h\left(0^+\right)$ and $h\left(T\right)=h\left(T^-\right)$. Under these notations, the boundary condition (S. 3.4) implies $h\left(T\right)=h\left(0\right)+\frac{1}{2}\left(\frac{1-\gamma}{a_x+\rho}\right)^2 q\left(T^-\right)$.

Asset prices For $n=1,2,\cdots$, in the interior of $\left(nT,\left(n+1\right)T\right)$, the law of motion of the state price density, π_t satisfies the stochastic differential equation of the form:

$$d\pi_t=\pi_t\left[-r\left(\hat{x}_t,t\right)dt-\sigma_\pi\left(t\right)d\tilde{B}_{C,t}\right],$$

where

$$r\left(\hat{x},t\right)=\rho+\hat{x}-\gamma\sigma^2+\frac{1-\gamma}{a_x+\rho}q\left(t\right)$$

is the risk-free interest rate, and

$$\sigma_\pi\left(t\right)=\gamma\sigma+\frac{\gamma-1}{a_x+\rho}\frac{q\left(t\right)}{\sigma}$$

is the market price of the Brownian motion risk.

We denote $p\left(\hat{x}_t,t\right)$ as the price-to-dividend ratio. For $t\in\left(nT,\left(n+1\right)T\right)$, the price of the claim to the dividend process can then be calculated as:

$$p\left(\hat{x}_t,t\right)D_t=E_t\left[\int_t^{\left(n+1\right)T}\frac{\pi_s}{\pi_t}D_s ds+\frac{\pi_{\left(n+1\right)T}}{\pi_t}p\left(\hat{x}_{\left(n+1\right)T}^-,\left(n+1\right)T^-\right)D_{\left(n+1\right)T}\right].$$

The above present value relationship implies that

$$\pi_t D_t + \lim_{\Delta \to 0} \frac{1}{\Delta} \left\{ E_t \left[\pi_{t+\Delta} p \left(\hat{x}_{t+\Delta}, t+\Delta \right) D_{t+\Delta} \right] - \pi_t p \left(\hat{x}_t, t \right) D_t \right\} = 0. \tag{S. 3.6}$$

Equation (S. 3.6) can be used to show that the price-to-dividend ratio function must satisfy the following PDE:

$$1 - p \left(\hat{x}, t \right) \varpi \left(\hat{x}, t \right) + p_t \left(\hat{x}, t \right) - p_x \left(\hat{x}, t \right) \nu \left(\hat{x}, t \right) + \frac{1}{2} p_{xx} \left(\hat{x}, t \right) \frac{q^2 \left(t \right)}{\sigma^2} = 0, \tag{S. 3.7}$$

where the functions $\varpi \left(\hat{x}, t \right)$ and $\nu \left(\hat{x}, t \right)$ are defined by:

$$\varpi \left(\hat{x}, t \right) = \rho - \mu + \phi \bar{x} + \left(1 - \phi \right) \hat{x} + \left(\phi - 1 \right) \left[\gamma \sigma^2 + \frac{\gamma - 1}{a_x + \rho} q \left(t \right) \right]$$

$$\nu \left(\hat{x}, t \right) = a_x \left(\hat{x} - \bar{x} \right) + \left(\gamma - \phi \right) q \left(t \right) + \frac{\gamma - 1}{a_x + \rho} \left(\frac{q \left(t \right)}{\sigma} \right)^2.$$

Also, equation (S. 3.6) can be used to derive the following boundary condition for $p \left(\hat{x}, t \right)$:

$$p \left(\hat{x}_T^-, T^- \right) = \frac{E \left[e^{\frac{1 - \gamma}{a_x + \rho} \hat{x}_T^+} p \left(\hat{x}_T^+, T^+ \right) \Big| \hat{x}_T^-, q_T^- \right]}{e^{\frac{1 - \gamma}{a_x + \rho} \hat{x}_T^- + \frac{1}{2} \left(\frac{1 - \gamma}{a_x + \rho} \right)^2 \left[q_T^- - q_T^+ \right]}}. \tag{S. 3.8}$$

Again, we focus on the steady-state and denote $p \left(\hat{x}, 0 \right) = p \left(\hat{x}, nT^+ \right)$, and $p \left(\hat{x}, T \right) = p \left(\hat{x}, nT^- \right)$. Under this condition PDE (S. 3.7) together with the boundary condition can be used to determined the price-to-dividend ratio function.

We define $\mu_{R,t}$ to the instantaneous risk premium, that is,

$$\mu_{R,t} dt = \frac{1}{p \left(\hat{x}_t, t \right) D_t} \left\{ D_t dt + E_t \ d \left[p \left(\hat{x}_t, t \right) D_t \right] \right\}. \tag{S. 3.9}$$

In the interior of $\left(nT, \left(n+1 \right) T \right)$, the instantaneous risk premium, $\mu_{R,t} - r \left(\hat{x}, t \right)$ can be computed as

$$\left[\mu_{R,t} - r \left(\hat{x}, t \right) \right] dt = -Cov_t \left[\frac{d \left[p \left(\hat{x}_t, t \right) D_t \right]}{p \left(\hat{x}_t, t \right) D_t}, \frac{d \pi_t}{\pi_t} \right].$$

We have:

$$\mu_{R,t} - r \left(\hat{x}, t \right) = \left[\gamma \sigma + \frac{\gamma - 1}{a_x + \rho} \frac{q \left(t \right)}{\sigma} \right] \left[\phi \sigma + \frac{p_x \left(\hat{x}, t \right)}{p \left(\hat{x}, t \right)} \frac{q \left(t \right)}{\sigma} \right]. \tag{S. 3.10}$$

To gain a better understanding on how the risk premium and the announcement premium depend on the parameters, let $\varrho \left(\hat{x}, t \right) = \ln p \left(\hat{x}, t \right)$, then equation (S. 3.7) can be written as:

$$e^{-\varrho \left(\hat{x}, t \right)} - \varpi \left(\hat{x}, t \right) + \varrho_t \left(\hat{x}, t \right) - \varrho_x \left(\hat{x}, t \right) \nu \left(\hat{x}, t \right) + \frac{1}{2} \left[\varrho_{xx} \left(\hat{x}, t \right) + \varrho_x^2 \left(\hat{x}, t \right) \right] \frac{q^2 \left(t \right)}{\sigma^2} = 0. \tag{S. 3.11}$$

Note that $\hat{x}_t$ is itself an Ornstein-Uhlenbeck process with steady state $\bar{x}$. Using a log-linear approximation around $\hat{x} = \bar{x}$, we can replace the term $e^{-\varrho \left(\hat{x}, t \right)}$ with $e^{-\varrho \left(\hat{x}, t \right)} \approx e^{-\bar{\varrho}} - e^{-\bar{\varrho}} \left[\varrho \left(\hat{x}, t \right) - \bar{\varrho} \right]$,

where we denote $\bar{\varrho} \equiv \varrho(\bar{x}, t)$, and write

$$e^{-\bar{\varrho}}\left[1 + \bar{\varrho} - \varrho(\hat{x}, t)\right] - \varpi(\hat{x}, t) + \varrho_t(\hat{x}, t) - \varrho_x(\hat{x}, t)\,\nu(\hat{x}, t) + \frac{1}{2}\left[\varrho_{xx}(\hat{x}, t) + \varrho_x^2(\hat{x}, t)\right]\frac{q^2(t)}{\sigma^2} = 0.$$
(S. 3.12)

We conjecture that $\varrho(\hat{x}, t) = A\hat{x} + B(t)$, and equation (S. 3.12) can be used to solve for A and $B(t)$ by the method of undetermined coefficients to get $A = \frac{\phi - 1}{a_x + e^{-\bar{\varrho}}}$.

Using the log-linearization result to evaluate equation (S. 3.10) at $\hat{x} = \bar{x}$, we obtain (35). In addition, using $p\left(\hat{x}_T^+, T^+\right) \approx e^{A\hat{x}_T^+ + B(T^+)}$, we can compute the expectation in (S. 3.8) explicitly and obtain (36).

Numerical Solutions To solve the PDE (S. 3.7) with the boundary condition (S. 3.8), we consider the following auxiliary problem:

$$p(x_t, t) = E\left[\int_t^T e^{-\int_t^s \varpi(x_u, u)\,du}\,ds + e^{-\int_t^T \varpi(x_u, u)\,du}\,p(x_T, T)\right],$$
(S. 3.13)

where the state variable x_t follows the law of motion;

$$dx_t = -\nu(\hat{x}, t)\,dt + \frac{q(t)}{\sigma}\,dB_t.$$
(S. 3.14)

Note that the solution to (S. 3.13) and (S. 3.8) satisfies the same PDE. Given an initial guess of the pre-announcement price-to-dividend ratio, $p^-(x_\tau, \tau)$, we can solve (S. 3.13) by the Markov chain approximation method (Kushner and Dupuis [13]):

(i) We first start with an initial guess of a pre-announcement price-to-dividend ratio function, $p(x_T, T)$.

(ii) We construct a locally consistent Markov chain approximation of the diffusion process (S. 3.14) as follows. We choose a small dx, let $Q = |\nu(\hat{x}, t)|\,dx + \left(\frac{q(t)}{\sigma}\right)^2$, and define the time increment $\Delta = \frac{dx^2}{Q}$ be a function of dx. Define the following Markov chain on the space of x:

$$\Pr(x + dx\,|x) = \frac{1}{Q}\left[-\nu(\hat{x}, t)^+\,dx + \frac{1}{2}\left(\frac{q(t)}{\sigma}\right)^2\right],$$

$$\Pr(x - dx\,|x) = \frac{1}{Q}\left[-\nu(\hat{x}, t)^-\,dx + \frac{1}{2}\left(\frac{q(t)}{\sigma}\right)^2\right].$$

One can verify that as $dx \to 0$, the above Markov chain converges to the diffusion process (S. 3.14) (In the language of Kushner and Dupuis [13], this is a Markov chain that is locally consistent with the diffusion process (S. 3.14)).

(iii) With the initial guess of $p(x_T, T)$, for $t = T - \Delta$, $T - 2\Delta$, etc, we use the Markov chain

approximation to compute the discounted problem in (S. 3.13) recursively:

$$p(x_t, t) = \Delta + e^{-\varpi(x,t)\Delta} E\left[p(x_{t+\Delta}, t+\Delta)\right],$$

until we obtain $p(x, 0)$.

(iv) Compute an updated pre-announcement price-to-dividend ratio function, $p(x_T, T)$ using (S. 3.8):

$$p\left(\hat{x}_T^-, T^-\right) = \frac{E\left[e^{\frac{1-\gamma}{a_x+\rho}\hat{x}_T^+}p\left(\hat{x}_T^+, 0\right)\middle| \hat{x}_T^-, q_T^-\right]}{e^{\frac{1-\gamma}{a_x+\rho}\hat{x}_T^- + \frac{1}{2}\left(\frac{1-\gamma}{a_x+\rho}\right)^2\left[q_T^- - q_T^+\right]}}.$$

Go back to step 1 and iterate until the function $p(x_T, T)$ converges.

Our numerical example is based on the following choice of parameter values:

Choice of parameter values The numerical example we presented in the paper uses parameter values in the standard long-run risk model:

ρ	γ	ψ	$\bar{x}$	a_x	σ_x	σ	h_0	ϕ	σ_S^2
0.02	10	2	1.5%	0.10	0.025%	3%	5	3	0

All parameters are annual. We assume that announcements are made at the monthly frequency, that is, $T = \frac{1}{12}$.

Pre-announcement drift The density of communication in the top panel of Figure 4 is generated from a Beta distribution with parameter $\alpha = 2$, $\delta = 3$ on $[-6, 0]$ hours before announcement. The density of the Beta distribution is

$$f\left(y\middle| \alpha, \delta\right) = B\left[\sigma, \delta\right]^{-1} y^{\alpha-1}\left(1-y\right)^{\delta-1}, \quad for\ y \in (0, 1),$$

where $B\left[\sigma, \delta\right]$ is the Beta function. In our example, the density of the occurrence of a communication h hours before announcement is $f\left(1 - \frac{h}{6}\middle| \alpha, \delta\right)$.

During a small interval dt, the expected return of the dividend claim is $\mu_{R,t}dt$ if the announcement does not occur. The expected return is $\frac{E[p(\hat{x}_T^+, T^+)|\hat{x}_T^-, q_T^-]}{p(\hat{x}_T^-, T^-)}$ if the announcement return occurs during dt. Given that the probability of an announcement during hour $(k-h, k)$ is $\int_{k-h}^k f\left(1 - \frac{t}{6}\middle| \alpha, \delta\right) dt$, the expected return of the dividend claim during hour $(k-h, k)$ can be written as

$$E\left[\int_{k-h}^k \left\{f\left(1 - \frac{t}{6}\middle| \alpha, \delta\right) \frac{E\left[p\left(\hat{x}_{T+\frac{t}{2880}}^+, \left(T+\frac{t}{2880}\right)^+\right)\middle| \hat{x}_{T+\frac{t}{2880}}^-, q_{T+\frac{t}{2880}}^-\right]}{p\left(\hat{x}_{T+\frac{t}{2880}}^-, \left(T+\frac{t}{2880}\right)^-\right)} + \mu_{R,T+\frac{t}{2880}}\right\} dt\right].$$

$$(S.\ 3.15)$$

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The above calculation assumes that there are 360 days per year and 8 hours per day. Because t is measured in hours, it needs to be divided by $360 \times 8 = 2880$ to translate into annual unit. Numerically, because the pre-announcement drift happens within hours before T, replacing $T + \frac{t}{2880}$ with T does not make any material difference in the evaluation of (S. 3.15). In addition, the term $\int_{k-h}^{k} \mu_{R,T+\frac{t}{2880}} dt$ is negligible. We can therefore approximate the average return during hour $(k-h, k)$ as

$$E \left[\int_{k-h}^{k} f \left(1 - \frac{t}{6} \middle| \alpha, \delta \right) dt \right] \times E \left[\frac{E \left[p \left(\hat{x}_T^+, T^+ \right) \middle| \hat{x}_T^-, q_T^- \right]}{p \left(\hat{x}_T^-, T^- \right)} \right].$$

S. 3.2   Time-non-separable Utilities

To guarantee that the model is well defined, we make the following assumptions on the weighting function $\{\xi(t,s)\}_{s=0}^{t}$.

$$\int_{0}^{t} \xi(t,s) \, ds \leq 1, \text{ for all } t > 0. \tag{S. 3.16}$$

$$\int_{0}^{\infty} \xi(t+s, t) \, ds < \infty, \quad \text{for all } t > 0. \tag{S. 3.17}$$

$$\left(1 - \int_{0}^{t} \xi(t,s) \, ds \right) H_0 + \int_{0}^{t} \xi(t,s) C_s ds < C_t, \quad \text{for all } t > 0. \tag{S. 3.18}$$

The first assumption requires that $\{\xi(t,s)\}_{s=0}^{t}$ is an appropriate weighting function, that is, total weights is less than one. The second assumption implies that the contribution of C_t to future habit stock is finite, and the last assumption ensures $C_t - H_t > 0$ so that the utility function is well defined.

External habit   Under the assumption of complete markets, the state-price density can be constructed from the marginal utility of the representative agent. In the external habit model,

$$\pi_t = e^{-\beta t} u' \left(C_t + b H_t \right).$$

Internal habit   In this case, the calculation of the state price density must take into account of the impact of C_t on future habit stock. Therefore, the state price density is given by (39). Because announcement fully reveals x_t, we need to show that

$$E \left[\int_{0}^{\infty} e^{-\beta s} \xi(t+s, t) u' \left(C_{t+s} + b H_{t+s} \right) ds \middle| x_t = x \right] \tag{S. 3.19}$$

is a decreasing function of x. Without loss of generality, we assume $t = 0$ in the following lemma.

Lemma S.1. *Fixing the path of Brownian motions $\{B_{C,s}, B_{x,s}\}_{s=0}^{\infty}$,*

$$\frac{\partial}{\partial x_0} \left[C_t + b H_t \right] > 0 \quad for \quad all \quad t > 0. \tag{S. 3.20}$$

Proof. Using the law of motion of C_t, we have

$$\ln C_t = \ln C_0 - \frac{1}{2}\sigma^2 t + \int_0^t \sigma dB_{C,s} + \int_0^t x_s ds.$$

Since x_t is an Ornstein-Uhlenbeck process, we can solve $\int_0^t x_s ds$ explicitly:

$$\int_0^t x_s ds = (x_0 - \bar{x})\frac{1}{a_x}\left[1 - e^{-a_x t}\right] + \bar{x}t + \frac{1}{a_x}\int_0^t \left[1 - e^{a_x(s-t)}\right]\sigma_x dB_{x,s}.$$

Therefore, for given realizations of the Brownian motion paths,

$$\frac{\partial}{\partial x_0}C_t = C_t\frac{1}{a_x}\left[1 - e^{-a_x t}\right],$$

and

$$
\begin{aligned}
\frac{\partial}{\partial x_0}H_t &= \int_0^t \xi(t,s)\frac{\partial C_s}{\partial x_0}ds \\
&= \int_0^t \xi(t,s)C_s\frac{1}{a_x}\left[1 - e^{-a_x s}\right]ds \\
&< \int_0^t \xi(t,s)C_s ds\frac{1}{a_x}\left[1 - e^{-a_x t}\right] \\
&< C_t\frac{1}{a_x}\left[1 - e^{-a_x t}\right],
\end{aligned}
$$

where the first inequality is true because $s < t$, and the second is due to the fact that $\int_0^t \xi(t,s)C_s ds \le H_t < C_t$. The inequality (S. 3.20) follows because $b \in (-1,0)$. $\qquad\square$

Consider two initial conditions, $x_0 = x$ and $x_0 = x'$. The above lemma implies that $x > x'$ implies that $C_{t+s} + bH_{t+s}$ first order stochastic dominate $C'_{t+s} + bH'_{t+s}$. Because $u'(\cdot)$ is a strictly decreasing function, we conclude that (S. 3.19) must be a decreasing function of x.

Consumption substitutability   Because (S. 3.19) is decreasing function of x, with $b > 0$, the state price density in (39) must be a decreasing function of x_t as well. As a result, the announcement premium must be positive for any payoff that is increasing in x_t.

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